Modeling Visibility in 3D Space: a Qualitative Frame of Reference

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Paolo Fogliaroni*

Dept. for Geodesy and Geoinformation,

Vienna University of Technology, Gusshausstr. 27-29, A-1040 Vienna

Eliseo Clementini†

Dept. of Industrial and Information Engineering and Economics,

University of L'Aquila, via G. Gronchi 18, I-67100 L'Aquila, Italy

Abstract

This paper introduces and formalizes a frame of reference for projective relations in 3D space that can be used to model human visual perception. While in 2D space visibility information can be derived from the concept of collinearity (thus, as ternary relations), in 3D space it can be derived from coplanarity, which calls for quaternary relations. Yet, we can retain ternary relations by anchoring our frame to an ubiquitous reference element: a general sense of vertical direction that, on Earth, can be the expression of gravity force or, in other cases, of the asymmetries of an autonomous agent, either human or robotic, that is, its vertical axis. Based on these observations, the presented frame of reference can be used to model projective and visibility information as ternary relations. Granularity and complexity of the models can be adjusted: we present two differently detailed realizations and discuss possible applications in Geographic Information Systems.

*paolo@geoinfo.tuwien.ac.at

†eliseo.clementini@univaq.it

I. INTRODUCTION

In the last two decades, qualitative spatial representation and reasoning (QSR) has distinguished itself as a multi–disciplinary, full-fledged research area aiming at understanding mechanisms underlying human spatial cognition [cf. 7, for an overview]. The mainstream of this research has focused on the definition and analysis of so-called qualitative spatial calculi: formal theories capable of resembling human spatial representation and reasoning abilities.

Literature offers many examples of qualitative calculi modeling different aspects of space like topology [16], direction [6, 10, 12], and distance [14]. The majority of these theories have been designed to model entities embedded in either 1D or 2D space, like points, line segments, and regions. However, we live in a 3D (4D, considering temporal dimension) universe and, while some aspects can be satisfactorily approximated into a lower–dimensional space, there exist certain spatial properties that are inextricably related to all three spatial dimensions. One example is visibility.

Visibility is defined[1] as "the ability to see or be seen" and is the expression of the probably most important sensory experience through which we cognize space: visual perception. Visibility is a fundamental property in many research and applied fields, spanning from spatial cognition to architectural design, urban planning, and computer graphics.

Visibility is, intrinsically, a three-dimensional concept and to satisfactorily capture its semantics we cannot reduce it to lower-dimensional spaces, unless it is intended as a projection of 3D space on the plane, like in the approach of [19] where visibility is defined on a 2D projective framework[2]. This approach, indeed, only works for overly-simplified environments where certain properties of physical objects can be ignored but that poorly fit real world variety. For example, in a "block-world" (a world consisting only of cuboidal objects), visibility can be satisfactorily modeled in 2.5D space (i.e., a 2D reference system enriched with height information). Simply by allowing our objects to have holes or concavities, a 2.5D modeling does not suffice any longer. Thus, capturing semantics of visibility in real-world scenarios calls for full 3D modeling.

[1] http://www.merriam-webster.com/dictionary/visibility

[2] If living in an imaginary 2D space, visibility would follow quite different rules as in the perilous life experience of Flatland characters [1].

This paper introduces a frame of reference for 3D space that allows for empowering, refining, and simplifying previously–presented qualitative models for projective and visibility relations (see Section II for details). The presented frame of reference lies upon two reference solids and is anchored to a direction vector. The two reference objects represent an observer agent (human or robotic) and an obstacle possibly occluding his sight field, respectively. The direction vector can be the expression of gravity force: "an important asymmetric factor in the world" [11]. If the observer assumes a different vertical axis than that of gravity force—e.g. if the observer is flying or is out of the gravity field—the direction vector can be associated to the feet/head axis of human body or an equivalent axis of a robot.

The remainder of the paper is structured as follows. In Section II we review related work on qualitative visibility and projective relations in 2D and 3D. Section III describes generating elements of the frame of reference and introduces two qualitative models for projective and visibility relations, respectively. In Section IV we discuss how the model can be applied to objects with holes and concavities. Properties and possible applications of the proposed models are discussed in Section V. Section VI concludes the paper and describes future work.

II. RELATED WORK

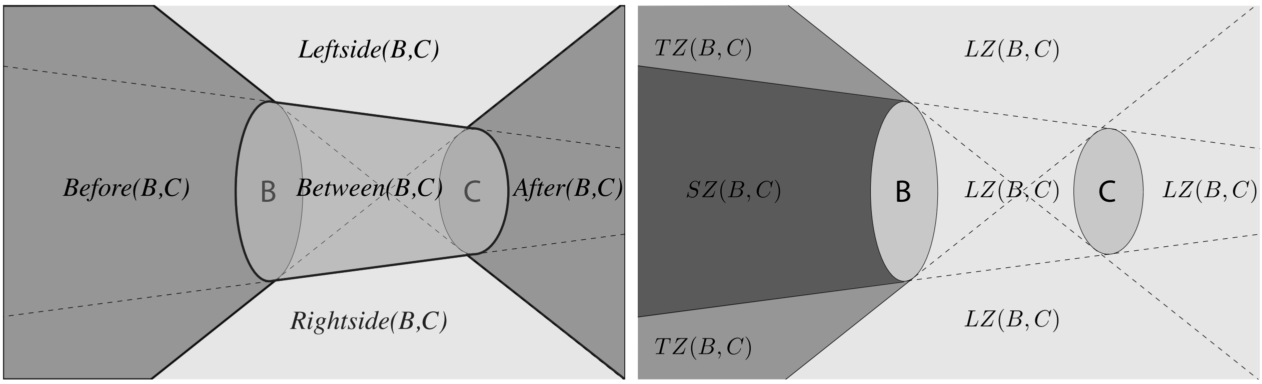
Projective relations in 2D space have been modeled by Billen and Clementini [3] as ternary relations of the form $R_{proj}(A, B, C)$ where A is the located (or primary) object with respect to two reference objects B and C. If the convex hulls of B and C are disjoint, it is possible to trace four lines tangent to both referents. These lines are used to partition the plane into five *acceptance areas*[3], as shown in Figure 1(a). To each acceptance area is associated a so–called *single–tile relation* that holds if A falls completely in that area. The case that A spans multiple acceptance areas is modeled through so–called *multi–tile relations.* Usually, multi–tile relations are named after the and–concatenation of the single tile relations associated to the acceptance areas spanned by the primary object. The number of multi–tile relations equals the number of admissible combinations of single–tile relations. Whether a combination is admissible or not depends on the topology of the objects under considera-

[3] The acceptance area of an *n-ary* relation, can be defined as a parametric subset of the relation's domain, having as a parameter a tuple of $n-1$ domain objects.

tion. For example, considering connected objects, the relation *before* $\wedge$ *after*(A, B, C) is not admissible. Indeed, it is impossible for A to span the areas *Before*(B, C) and *After*(B, C) without intersecting any other area—e.g., *Between*(B, C). The resulting projective model consists of both single– and multi–tile relations whose reasoning properties are discussed in [6].

An extension of the model to cope with solids in 3D space is presented in [4]. This work presents two frames of reference grounded on concepts of projective geometry, such as collinearity and coplanarity. The first frame of reference allows for defining four, ternary, single–tile relations: *before*, *between*, *after*, and *aside*. The other frame of reference allows for defining the relative position of a primary object with respect to a configuration of three reference objects and defines the following, quaternary, single–tile relations: *coplanar* (further refined into *internal* and *external*) and *non_coplanar* (further refined into *above* and *below*).

Bartie et al. [2] integrate the ternary 3D model of Billen and Clementini with other qualitative calculi and use it to represent and reason about the environmental visual perception of an agent modeled as a point. The application of the resulting model to produce cognitively–sound route instructions based on landmarks—as suggested by Raubal and Winter [18]—is discussed.



(a) 5-Intersection model [3].

(b) Visibility model [19].

FIG. 1: Frames of reference for 5-Intersection (a) and visibility model (b).

A qualitative model for visibility relations in 2D space is introduced in [19]. The provided relations express the property of an observed (primary) object of being seen from an observer when a third object acts as an obstacle (reference objects). Inspired by the frame of reference for projective relations presented by Billen and Clementini [3], the authors design the frame of reference depicted in Figure 1(b). In this frame, the observer object (C) is imagined to be a source of light and the acceptance areas are named accordingly: light zone (LZ), shadow zone (SZ) and twilight zone (TZ). The single–tile, ternary, relations associated to these zones are called: *visible* (V), *occluded* (O), and *partially visible* (PV), respectively.

Fogliaroni et al. [9] refine the visibility model by also considering the direction where a partially visible object is perceived with respect to the obstacle. Drawing upon this finer–grained model, they define a qualitative coordinate system based on visibility and show its application for robotic localisation and navigation. De Felice et al. [8] exploit the model for environmental learning.

A different strategy to modeling visibility is adopted by Galton [13]: The viewpoint of the observer is fixed in space and the arrangements of obstacle and observed object are described by binary relations. In this way Galton defines a lines of sight calculus providing 14 qualitative relations and discusses its reasoning properties.

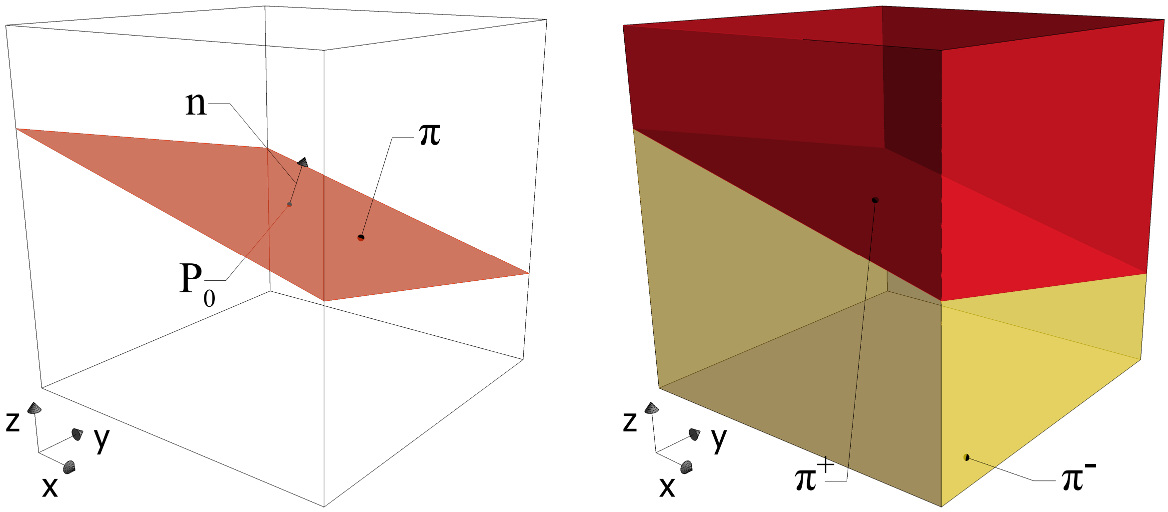
Randell et al. [17] extend Galton's work by allowing the treatment of concave objects. Köhler [15] introduces an occlusion calculus that draws inspiration from both region connection calculus [16] and Galton's lines of sight calculus, and allows for describing configurations of two convex solids in 3D as perceived from a fixed viewpoint.

Finally, [20] try to extend the visibility model of Tarquini et al. [19] to 3D by applying the same principles used in the 2D variant: tracing tangential planes among two polyhedra to split the space into acceptance volumes. They discover that the approach becomes cumbersome as the number of tangent planes is not fixed (contrarily to linear tangents in 2D) and depends on the shape of the objects as well as on their relative position and orientation.

III. A FRAME OF REFERENCE FOR PROJECTIVE RELATIONS IN 3D SPACE

In this section we present a frame of reference generating from two reference solids (B and C) and a direction vector ($\mathbf{d}$) that allows for partitioning 3D space in different manners. In the next two sections we show how two such partitions can be used to model relative directions and visibility, respectively.

We assume, without loss of generality, that the direction vector $\mathbf{d}$ is parallel to the z axis



(a) Point-vector definition of a plane.

(b) half–spaces.

FIG. 2: A point P and a vector $\mathbf{v}$, uniquely define the plane π passing through P and orthogonal to $\mathbf{v}$. A plane in $\mathbb{E}^3$ (a) splits the space into two half–spaces (b).

of a canonical Cartesian coordinate system. Moreover, we assume that the convex hulls of the two reference objects (B and C) are disjoint.

Before proceeding, let us shortly recall some basic concepts of analytic geometry of space. A plane π in 3D Euclidean space $\mathbb{E}^3$ is a flat surface that is uniquely identified by one of its points P_0 and an orthogonal non-zero vector $\mathbf{n}$—cf. Figure 2(a). Thus, a plane is the locus of points $P = (x, y, z)$ such that $(P - P_0) \cdot \mathbf{n} = 0$. Assuming that $P_0 = (x_0, y_0, z_0)$ and that the vector $\mathbf{n} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is expressed in terms of its components along an orthonormal basis, we have that:

$$\pi : ax + by + cz + d = 0 \tag{1}$$

with $d = -ax_0 - by_0 - cz_0$. Any plane π splits $\mathbb{E}^3$ into two parts called *half–spaces and identified by the following inequalities:*

$$\pi^+ : ax + by + cz + d > 0 \tag{2}$$

$$\pi^- : ax + by + cz + d < 0 \tag{3}$$

where the half–space π^+ is the one "aimed" by the vector $\mathbf{n}$—cf. Figure 2(b).

For simplicity, assume that B and C are spheres. Call Π the infinite set of planes tangential to both. Assume each such plane π is given in a point–vector representation, with the point being the intersection point with C and the vector going from this point to the centre of C. That is, C falls completely in the half–space π^+—except its intersection point with π. Tangential planes can be split into two disjoint sets: (*i*) the set of *external tangent planes* Π_{ext} consists of all the planes π such that the other reference object B also falls into π^+; (*ii*) the set of *internal tangent planes* Π_{ext} consists of all the planes π such that B falls into π^+.

We now have all the constructive elements to define the frame of reference shown in Figure 3(f) consisting of two pairs of cones[4] and three planes.

The first pair of cones Δ_{ext}, shown in Figure 3(a), are called *external cones.* The cone Δ^+_{ext} (resp. Δ^-_{ext}) is the subspace obtained by intersecting the half–spaces π^+_{ext} (resp. π^-_{ext}) identified by all the external tangent planes π_{ext}:

$$\Delta^+_{ext} = \bigcap_{\pi \in \Pi_{ext}} \pi^+ ; \Delta^-_{ext} = \bigcap_{\pi \in \Pi_{ext}} \pi^- \tag{4}$$

Note that, when B and C have same radius, the cone Δ^+_{ext} degenerates into a cylinder and Δ^-_{ext} does not exist—i.e. is the empty subspace.

Figure 3(b) depicts the second pair of cones Δ_{int}, called *internal cones.* The cone Δ^+_{int} (resp. Δ^-_{int}) is the subspace resulting from the intersection of the π^+_{int} (resp. π^-_{int}) half–spaces defined by all the internal tangent planes π_{int}:

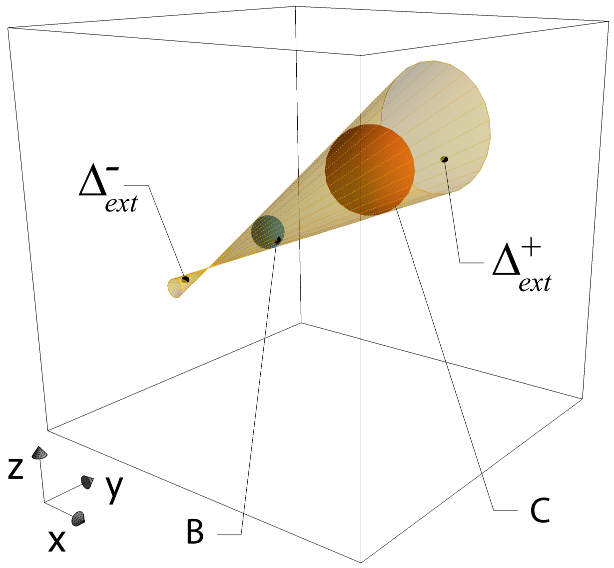
$$\Delta^+_{int} = \bigcap_{\pi \in \Pi_{int}} \pi^+ ; \Delta^-_{int} = \bigcap_{\pi \in \Pi_{int}} \pi^- \tag{5}$$

Figure 3(c) shows how the two pairs of cones are located with respect to each other.

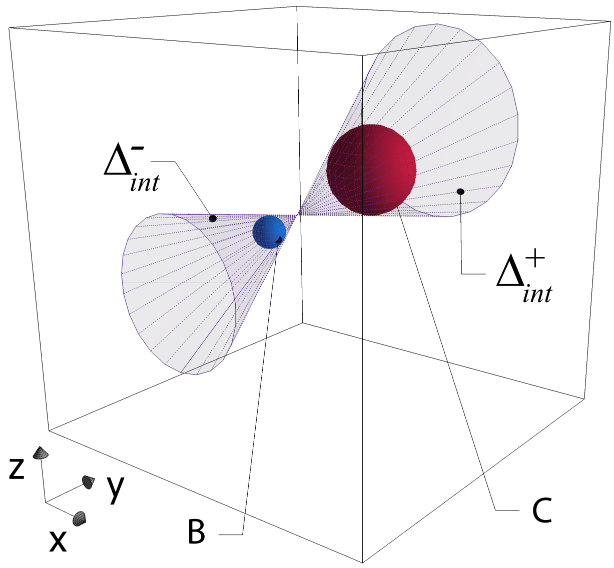
The plane *Above* (resp. *Below*), cf. Figure 3(d), is the external tangent plane whose intersection point with B has max (resp. min) value along the direction vector $\mathbf{d}$—i.e., z coordinate, assuming $\mathbf{d}$ aligned to z axis. Note that *Above* (resp. *Below*) splits $\mathbb{E}^3$ into two half–spaces: *Above*$^+$ (resp. *Below*$^+$) is the half–space containing B and C; *Above*$^-$ (resp. *Below*$^-$) is the other half–space.

Finally, the plane *Central*, cf. Figure 3(e), is parallel to the direction vector $\mathbf{d}$ and passes through the centroids of B and C. We call *Central*$^+$ (resp. *Central*$^-$) the half–space on the right (resp. left) of *Central*, as going from B to C.

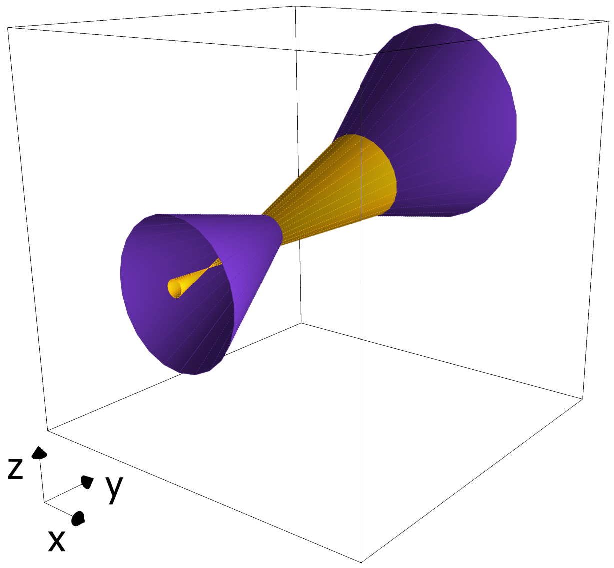
[4] Pyramids, if considering polyhedral objects.



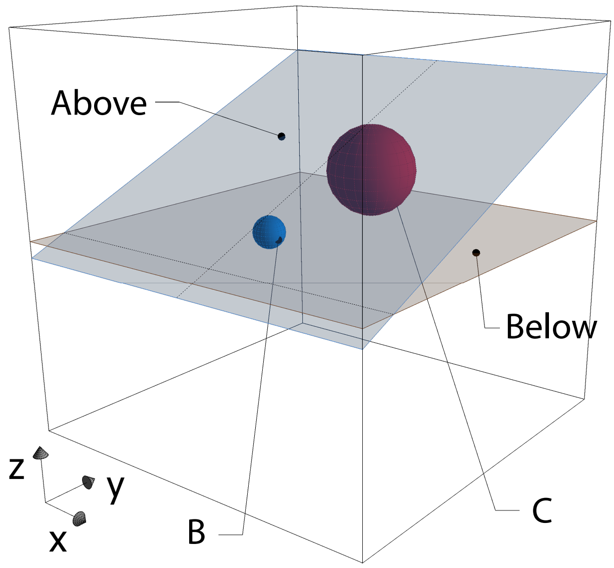
(a) External cones.



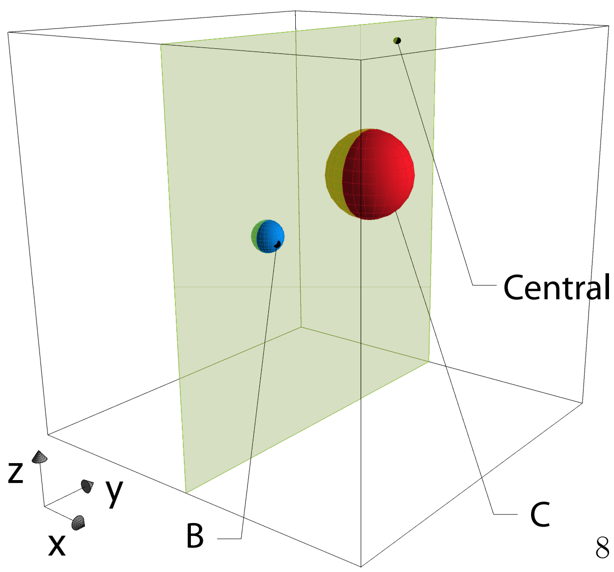
(b) Internal cones.



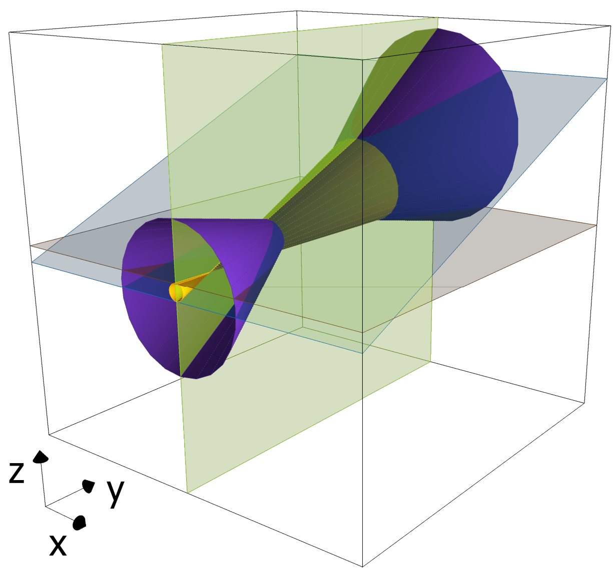
(c) External and internal cones.



(d) Plane *Above* and *Below*.



(e) Plane *Central*



(f) Frame of reference.

FIG. 3: Constructing elements of the frame of reference.

TABLE I: Definition of the acceptance volumes depicted in Figure 4(a).

Volume	Definition
$Between(B, C)$	$ConvexHull(B \cup C)$
$Before(B, C)$	$\Delta^{-}_{int} \setminus ConvexHull(B \cup C)$
$After(B, C)$	$\Delta^{+}_{int} \setminus ConvexHull(B \cup C)$
$AboveLeft(B, C)$	$(Central^{-} \cap Above^{-} \cap Below^{+}) \setminus (\Delta_{ext} \cup \Delta_{int})$
$Left(B, C)$	$(Central^{-} \cap Above^{+} \cap Below^{+}) \setminus (\Delta_{ext} \cup \Delta_{int})$
$BelowLeft(B, C)$	$(Central^{-} \cap Above^{+} \cap Below^{-}) \setminus (\Delta_{ext} \cup \Delta_{int})$
$AboveRight(B, C)$	$(Central^{+} \cap Above^{-} \cap Below^{+}) \setminus (\Delta_{ext} \cup \Delta_{int})$
$Right(B, C)$	$(Central^{+} \cap Above^{+} \cap Below^{+}) \setminus (\Delta_{ext} \cup \Delta_{int})$
$BelowRight(B, C)$	$(Central^{+} \cap Above^{+} \cap Below^{-}) \setminus (\Delta_{ext} \cup \Delta_{int})$

A. Relative Direction Relations

This section shows how the frame of reference introduced before can be used to design a ternary qualitative model for relative directions in 3D space. The model provides relations of the form $R_{dir}(A, B, C)$ that express the location of a primary object A with respect to two reference objects B and C, heading from B to C.

Beyond the constructive components of the frame of reference discussed in section above, we resort to the Convex Hull operation to partition $\mathbb{E}^3$ into nine volumes parametric with respect to B and C, as shown in Figure 4(a). The definition of the volumes is given in Table I.

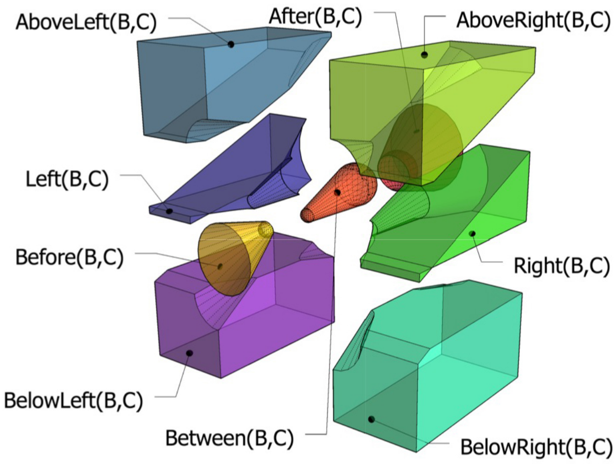
These volumes are referred to as *acceptance volumes*[5] and to each of them is associated a single–tile direction relation named accordingly. So, for example, if the primary object A falls completely in the volume *Before*(B, C) the relation *Before*(A, B, C) holds. The cases when A spans multiple acceptance volumes are covered by multi–tile relations. That is, relations named after the and-concatenation of the relations associated to the spanned volumes. For example, if A overlaps the volumes *Before*(B, C) and *LeftSide*(B, C), the multi–tile relation *Before*$\wedge$*LeftSide*(A, B, C) holds. As explained in Section II, which combination of single–tile

[5] Acceptance volumes are the generalization to 3D space of the notion of acceptance areas, discussed in Section II

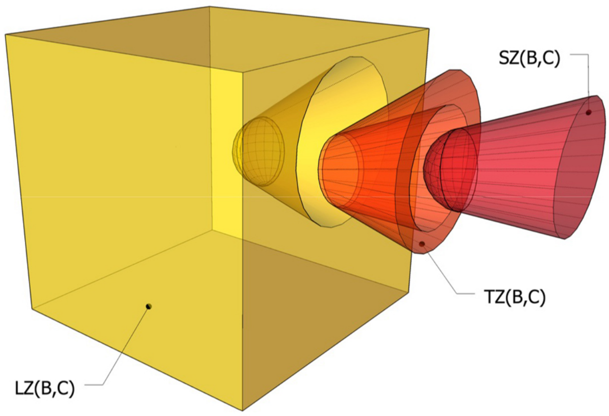
relations is allowable depends on the topology of the objects considered.

The qualitative relation $R_{dir}(A,B,C)$ can be expressed by a 3x3 matrix of void or non-void intersections:

$$\begin{pmatrix} A \cap AboveLeft(B,C) & A \cap After(B,C) & A \cap AboveRight(B,C) \\ A \cap Left(B,C) & A \cap Between(B,C) & A \cap Right(B,C) \\ A \cap BelowLeft(B,C) & A \cap Before(B,C) & A \cap BelowRight(B,C) \end{pmatrix}$$



(a) Acceptance volumes for relative directions (exploded).



(b) Acceptance volumes for visibility (exploded).

FIG. 4: Two possible interpretations of the frame of reference of Figure 3(f).

B. Visibility Relations

This section presents a different interpretation of the frame of reference presented before that provides a ternary qualitative model for visibility in 3D space. The model provides spatial predicates of form $R_{vis}(A,B,C)$ capturing the semantics of the visibility relation holding between an observer (B) and an observed object (A) when a third, opaque, object (C) acts as an obstacle.

Given two reference objects B and C it is possible to split $\mathbb{E}^3$ as depicted in Figure 4(b). We follow the naming convention adopted by Tarquini et al. [19] that, imagining the observer to be a source of light, distinguish between three main volumes whose definitions are given in Table II:

Light Zone (LZ) According to the source–of–light metaphor, this is the well–lighted subspace, or, in other terms, is the set of points visible from any point of the observer. That is, it is possible to trace a straight line segment between any point of the observer B and any point of LZ(B, C) such that no point of the obstacle C belongs to such a segment. If an object A falls into this zone we say it is *visible* from C and denote it by $Visible(A, B, C)$.

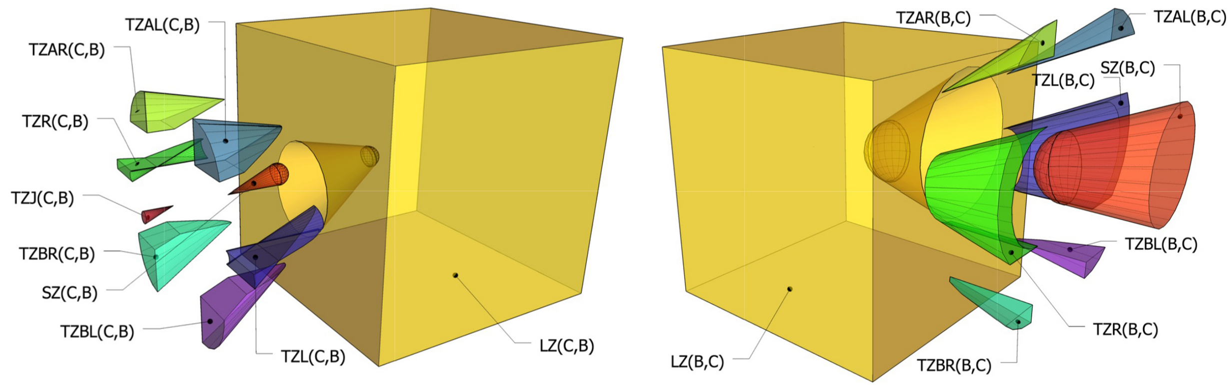
Shadow Zone (SZ) Conceptually, this can be imagined as the shadow cone casted by the obstacle C when the observer B is a light source. SZ(B, C) is the subspace whose points are occluded from every point of the observer B by the obstacle C. That is, it is impossible to trace a straight line segment between any point of B and any point of SZ(B, C) that does not intersect C. If an object A falls into this zone we say it is *occluded* from B by C and denote it by $Occluded(A, B, C)$.

Twilight Zone (TZ) This is a transition zone between well–lighted and obscured parts of the space. It is the set of points that are occluded from some points of the observer B but visible from others. If an object A falls into this zone we say it is *partially visible* from B and denote it by $PartiallyVisible(A, B, C)$.

Besides this coarse partition, a finer–grained one can be obtained from the same constructive elements that splits Twilight Zone into smaller acceptance volumes. This is similar to the refinement in granularity operated on the 2D model in [9] and allows for distinguishing where the *partially visible* object is perceived with respect to the obstacle. We use the plane *Central*—cf. Figure 3(e)—to distinguish between left and right side and the planes *Above*

TABLE II: Definition of the acceptance volumes depicted in Figure 4(b).

Name	Volume	Definition
Light Zone	$LZ(B, C)$	$\mathbb{E}^3 \setminus (\Delta^+_{int} \setminus (ConvexHull(B \cup C) \setminus C))$
Shadow Zone	$SZ(B, C)$	$(\Delta^+_{ext} \setminus \Delta^-_{int}) \setminus (ConvexHull(B \cup C) \setminus C)$
Twilight Zone	$TZ(B, C)$	$\Delta^+_{int} \setminus \Delta^+_{ext}$



(a) The case of a finite Shadow Zone (SZ) also yields the Twilight Zone Joint (TZJ).

(b) An infinite Shadow Zone (SZ) and the partition of the Twilight Zone according to directional primitives.

FIG. 5: Acceptance volumes for visibility relations (exploded).

and *Below*—cf. Figure 3(d)—to differentiate among objects perceived above and below the obstacle, respectively. The resulting partition is depicted in Figure 5(b); names and definitions for the refined Twilight Zones are provided in Table III. The qualitative relations associated to these zones are named *PartiallyVisible*$*$, where $*$ is a placeholder for *AboveRight*, *AboveLeft*, and so on. The semantics of these partially visible relations is straightforward: let us assume, for example, that an object A falls into the volume TZL(B, C)—cf. Figure 5(b). Then, B sees A partially and perceives it on the left side of C: *PartiallyVisibleLeft*(A, B, C). Such a finer distinction turns very helpful for certain applications, as will be discussed in Section V.

It is worth noting that the Shadow Zone can be finite or infinite according to the relative size and position of the reference objects. This is depicted in Figure 5(a), where the roles of observer and obstacle are swapped. In this case two main things are worth to be noted. (i) Shadow Zone does not extend to infinity. (ii) It is possible to distinguish a new zone called Twilight Zone Joint (TZJ): if an object A falls into this zone, it is partially visible and perceived "all around" (i.e., simultaneously AL, AR, L, R, BL, and BR) the obstacle B. We denote this by $PartiallyVisibleJoint(A, B, C)$.

Similarly to the case of relative directions, visibility relations $R_{vis}(A, B, C)$ can be single– and multi–tile and can be represented by a matrix of void or non–void intersections. For

TABLE III: Definition of the refined Twilight Zones depicted in Figure 5 in terms of the coarse visibility relations in Table II and the of the direction relations in Table I.

Name	Volume	Definition
Twilight Zone Left	$TZL(B,C)$	$TZ(B,C) \cap Left(B,C)$
Twilight Zone Right	$TZR(B,C)$	$TZ(B,C) \cap Right(B,C)$
Twilight Zone Above Left	$TZAL(B,C)$	$TZ(B,C) \cap AboveLeft(B,C)$
Twilight Zone Above Right	$TZAR(B,C)$	$TZ(B,C) \cap AboveRight(B,C)$
Twilight Zone Below Left	$TZBL(B,C)$	$TZ(B,C) \cap BelowLeft(B,C)$
Twilight Zone Below Right	$TZBR(B,C)$	$TZ(B,C) \cap BelowRight(B,C)$
Twilight Zone Joint	$TZJ(B,C)$	Δ^{-}_{ext}

the coarse model we need a 1x3 matrix of the kind:

$$\begin{pmatrix} A \cap LZ(B,C) & A \cap TZ(B,C) & A \cap SZ(B,C) \end{pmatrix}$$

For the finer–grained model we need a 3x3 matrix of the kind:

$$\begin{pmatrix} A \cap TZAL(B,C) & A \cap LZ(B,C) & A \cap TZAR(B,C) \\ A \cap TZL(B,C) & A \cap TZ(B,C) & A \cap TZR(B,C) \\ A \cap TZBL(B,C) & A \cap TZJ(B,C) & A \cap TZBR(B,C) \end{pmatrix}$$

IV. TREATING HOLES AND CONCAVITIES

So far we assumed that the objects to be modeled are convex and hole–free. For the sake of an easier treatment, we also assumed that they are spheres and that the convex hulls of the reference objects are disjoint. However, in real cases (e.g. in a 3D Geographic Information System) objects are usually modeled as polyhedra and may have holes and concavities. While the treatment of polyhedra, rather than spheres, is no special case, treatment of holes and concavities must be discussed.

First, consider the case that only the primary object has holes or concavities. In this case the frame of reference—thus the acceptance volumes of the two models—is not affected and the only difference concerns the multi–tile relations that can be deemed allowable. For example, if A is convex and without holes, the relation $Left \wedge AboveLeft \wedge AboveRight \wedge Right(A, B, C)$ can never hold. Indeed, in order to intersect the acceptance volumes corresponding to this multi–tile relation, A should necessarily span either $Before(B, C)$, or $Between(B, C)$, or $After(B, C)$. Yet, this is no longer the case if A is concave—c.f. Figure 4(a) for a visual aid.

Now, consider the case that reference objects are not convex and hole–free. We first address the case of concave objects and, to tackle it, we suggest to (i) decompose concave objects into a number of convex parts such that $B = B_1 \cup \ldots \cup B_m$ and $C = C_1 \cup \ldots \cup C_n$; (ii) compute the acceptance volumes for each pair (B_i, C_j); (iii) aggregate the obtained volumes by applying different aggregation rules according to the semantics of the relation each volume is associated with. Using this approach allows for reducing the problem of treating concavities to that of treating multi–objects (i.e. objects consisting of several parts). Convex decomposition has been widely treated in the field of computational geometry and optimal methods have been found [5]. Computation of acceptance volumes between pairs of parts is no different than applying the model on convex objects. Thus, what remains to be defined are the aggregation rules.

Due to space limitations we only discuss coarse visibility. Moreover, for simplicity, we resort to a 2D example. Since the frame of reference presented in this paper is an extension to 3D of the projective frames of reference in Figure 2, we stay assured that the same concepts apply to the corresponding 3D relations.

Say the observer B and the obstacle C are concave, call $B_1, \ldots, B_m$ and $C_1, \ldots, C_n$ their convex parts, and denote by I and J the index sets $\{1, \ldots, m\}$ and $\{1, \ldots, n\}$.

First, fix an arbitrary part C_j of the obstacle and aggregate on B. The Light Zone (resp. Shadow Zone) is the subspace whose points are *simultaneously* visible (resp. occluded) from every point of the observer. Since the observer consists of several parts one could rather say: to every point of every part. That is:

$$LZ(B, C_j) = \bigcap_{i \in I} LZ(B_i, C_j) \; ; \; SZ(B, C_j) = \bigcap_{i \in I} SZ(B_i, C_j) \tag{6}$$

Twilight Zone can be obtained by subtracting the union of the other zones from $\mathbb{E}^2$.

Now, fix an arbitrary part B_i of the observer and aggregate on C. Since the obstacle consists of multiple parts—that is equivalent to have multiple obstacles—the visibility of any point in space is affected by all the parts at the same time. In order to reflect this property

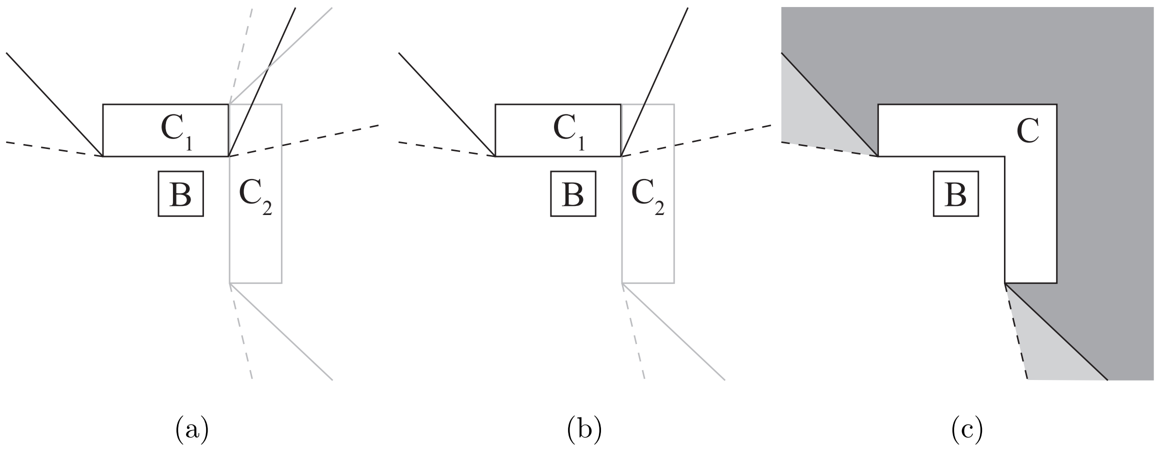


FIG. 6: The three steps of the aggregation procedure for concave objects.

on the acceptance zones, one shall apply the following procedure: (i) trace acceptance zones for every pair (B_i, C_j); (ii) remove all the tangents to B_i and C_j that intersect another C_k between B_i and C_j; (iii) if a tangent to B_i and C_j intersects another C_k beyond C_j, cut the tangent at the intersection with C_k and start considering C_j and C_k as a unique part $C_l = C_j \cup C_k$ (because, from the perspective of B, C_j and C_k are perceived as a continuum). For a visual exemplification refer to Figure 6. Enumerate the parts resulting from the application of the procedure above by the index set $K = 1, \ldots, p$. Then we have:

$$LZ(B_i, C) = \bigcap_{k \in K} LZ(B_i, C_k) \ ; \ SZ(B_i, C) = \bigcup_{k \in K} SZ(B_i, C_k) \tag{7}$$

Again, the Twilight Zone can be obtained as set difference.

Finally let us tackle the treatment of objects with holes. The semantics of direction relations is not affected by the fact that one or both reference objects have holes: e.g., the fact that an object A lies on the left of (B, C) is not affected by the fact that B or C have holes. Conversely, when dealing with visibility relations, the case when the obstacle B has holes must be treated specially. Again, we suggest to look at the obstacle as consisting of different parts: the real object and its holes. Acceptance volumes must be computed for all parts (which, if concave, must be split into convex subparts), but the role of Shadow and Light Zones must be swapped in the case of holes. Finally, we apply the aggregation rules defined for multi–objects.

V. APPLICATION TO WAYFINDING

The models presented in this paper can be applied in a variety of fields, spanning from cognitive robotics to architectural design, urban planning, and computer graphics. In this section we discuss how to generate cognitively–sound route instructions based on landmark visibility, as suggested by [18].

The idea draws inspiration from previous work on qualitative localisation and navigation where Fogliaroni et al. [9] show that, by repeatedly applying the 2D visibility frame of reference on every pair of objects in the environment, it is possible to split the navigable space into regions. Each such region corresponds to the intersection of a number of visibility acceptance areas, thus, specific visibility relations hold between each point of a region and objects in the environment. In particular, Fogliaroni et al. show that, if the observer is modeled as a point b, internal and external tangents collapse into each other, *PartiallyVisible* relations are not defined, and the following properties hold when reasoning on change of perspective:

$$\begin{aligned} Visible(A,b,C) &\iff Visible(b,A,C) \\ Occluded(A,b,C) &\iff Occluded(b,A,C) \\ Visible \wedge Occluded(A,b,C) &\iff PartiallyVisible^{*}(b,A,C) \end{aligned} \tag{8}$$

where $*$ stands for one of the fine–grained partial visibility relations.

Moreover, if $Visible \wedge Occluded(A,b,C)$, b only sees a part of A and, in 2D, this part is perceived either on the *Left* or on the *Right* side of C. Fogliaroni et al. show that there is a neat relation between these relative directions and the relation $PartiallyVisible^{*}(b,A,C)$ obtained when changing perspective. Say A_v is the part of A visible from b then we have the following properties:

$$\begin{aligned} Right(A_v,b,C) &\iff PartiallyVisibleLeft(b,A,C) \\ Left(A_v,b,C) &\iff PartiallyVisibleRight(b,A,C) \end{aligned} \tag{9}$$

Similarly, when operating a change of perspective in 3D, we have that *Right* relations map onto $PartiallyVisibleLeft$ ones, $Left$ onto $PartiallyVisibleRight$, *Above* onto $PartiallyVisibleBelow$, and *Below* onto $PartiallyVisibleAbove$.

Thanks to these properties, one can say if, how, and in which direction with respect to a certain obstacle, an object is perceived from an observer standing in a visibility region.

Fogliaroni et al. also highlight the fact that, since the number of acceptance zones is finite, so is the number of boundaries among them and that for each boundary two qualitative navigation behaviours are defined that lead from one zone to the neighbour and back. Such behaviours can be mapped onto natural language instructions like “walk around C keeping it on your right until you start perceiving A on C’s left”, or “walk towards C, until A disappears from your sight”.

So, given a 3D urban dataset D consisting of buildings—a subset of which is denoted as landmarks—and of navigable spaces (e.g., roads), one can compute visibility spaces of each landmark l by applying the frame of reference of Section III to every pair (o, l), where o is a building, and intersect them with the walkable spaces.

Using a standard routing algorithm for pedestrian routes one can (i) compute the shortest path from A to B, (ii) intersect it with the visibility spaces of landmarks, and (iii) generate natural language instructions by considering which boundaries between visibility volumes are crossed as going from A to B.

VI. CONCLUSIONS

We presented a qualitative 3D frame of reference that is both, an extension of the 2D projective models presented in [3, 19] and a refinement of the 3D models discussed in [4].

[4] presented two qualitative models for projective relations, one ternary and one quaternary. Thanks to the anchorage to a direction vector, the relative direction model presented in Section III A allows for maintaining a lower arity (ternary) while augmenting expressiveness (differentiating relations along the vertical direction).

In Section III B we showed how the newly introduced frame of reference allows for extending the visibility model presented in [19] and the finer–grained variant discussed in [9].

We also discussed how the two models we introduce can be applied when considering objects with holes and concavities and, finally, we speculated on a possible application on landmark–based route instructions.

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