Optimization of assigned Power and Bandwidth in Macro-Femto Cellular System using Geometric Programming

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Abstract—Femtocells are used to increase indoor coverage and network capacity of traditional macro BS deployments. Interference is key performance limiting factor in such heterogeneous mobile networks. In order to minimize the interference between macro- and femto base stations, we use a constrained optimization technique to optimally allocate bandwidth and power to macrocell and femtocell UEs. Matlab’s CVX library is used to solve the optimization problem. Depending on the UEs’ position from their serving base station (macro or femto) and their demand for higher data rates than minimum requirement data rate, the optimal power and bandwidth is assigned to each base station.

I. INTRODUCTION

Heterogeneous Cellular Networks (HetNet), are a combination of macro base stations (eNodeBs) and small cells such as femtocells and picocells. These topologies are becoming important due to their advantages in terms of low cost operation and traffic offloading capabilities. Femto base stations (HeNBs) cover a small area and are typically used in closed environments such as buildings homes, offices, shopping malls, train stations etc. Such deployments impose new challenges on the system design. New interference management techniques applied in HetNets improve inter-cell interference that is quite present in these networks. Orthogonal Frequency Division Multiple Access (OFDMA) is used as a modulation scheme in downlink for HetNets in LTE network [1], [2]. Different multi-carrier inter-cell interference techniques have been reported recently: interference management via carrier partitioning [3], [4], enhanced carrier reuse with power control [4], and carrier aggregation based inter-cell interference coordination [5]. Further, dynamic carrier management schemes have been proposed for LTE-Advanced systems [6]. The maximization of minimum data rate for all UEs in a homogeneous cellular network under variable bandwidth and power allocation is a non-convex problem [7]. However in [8], such problem is shown to be transformable into a convex optimization problem, and solved in a homogeneous network for Partial Frequency Reuse (PFR) in [9]. To the best of our knowledge, currently there are no studies that consider the maximization of minimum rate under variable bandwidth and power for HetNets in Geometric Programming (GP) form.

This paper is organized as follows. Section II shows the system model. Section III presents a problem of maximizing the minimum data rate in heterogeneous mobile networks with eNodeBs and HeNBs. Soft Frequency Reuse (SFR) pattern is used between macro- and femto nodes. To find the optimal power and bandwidth assignment for macro- and femto UEs, the presented maximization problem is transformed from a Generalized Geometric Programming (GGP) into a GP [8], [10]. Using MATLAB simulator, and CVX as a package for specifying and solving the disciplined convex programs we solve the transformed optimization problem [11], [12]. Section IV shows the simulation results considering one macro UE and one femto UE per cell. Conclusions are drawn in Section V.

II. SYSTEM MODEL

In our system model we consider seven macro sites each equipped with three sector antennas, yielding three cells per site. In each cell HeNBs are placed in an unplanned random manner. It is considered that $N^m$ UEs per cell served by each eNodeB, and $N^f$ UEs are served by each HeNB. Our system model is illustrated in Fig.1. The Fig.2 shows frequency reuse pattern as applied in our model. We consider a Soft Frequency Reuse 1 in both macro- and femto cells. Macro UEs are served by a portion of bandwidth $B^m$, and femto UEs are served by a portion of bandwidth $B^f$. Our system use optimization techniques for both power and bandwidth assignment.

The macro UEs located in sector $S_{01}$, which presents sector 1 of base station $BS_0$, receive interference from their own sector antennas (sectors $S_{02}$ and $S_{03}$) and from all the sectors of the other base stations, $BS_k$, $k = 1, \ldots, 6$. For macro UEs, we assume that HeNB’s transmission power is low and neglect their interference contribution. A femto UE located in the first sector $S_{01}$ of the first base station ($BS_0$) receives interference from all sectors of each eNodeB.

In our system, the data rate achieved by the $n^{th}$ macro UE in sector $S_{01}$ is given by [9]:

$$R^m_n = B^m_n \log_2 \left( 1 + \frac{G^m_{0n}P^m_0}{N_0 B^m_n + \sum_{k=1}^{6} G^m_{kn}P^m_k} \right)$$

where $B^m_n$ is the bandwidth utilized for the $n^{th}$ macro UE in sector $S_{01}$ and $N_0$ is the noise spectral density. Index $m$ stands for macro UE, index $n$ stands for sectors of each eNodeB, and...
index 0 stands for the first BS (where the UE is located). The large scale path-loss attenuation in dB is expressed as [8]:

\[ G = -[128.1 + 10\alpha \log_{10}(r) + A + L_p + X_\sigma + F] \tag{2} \]

where \( \alpha \) is path-loss exponent, \( r \) is the distance between mobile station and base station in \( km \), \( A \) is the sum of UE and eNodeB antenna gain in dB, \( L_p \) is the penetration loss, and \( F \) is the fast fading channel coefficient. The large scale PL attenuation of the directed channels \( G_{0n}^m \), is defined by Equation (2). Similarly we use the Equation (2) for defining the large scale PL attenuation of interference channels \( G_{kn}^m \), except that the fast fading \( F \) is not taken into account. The transmit power assigned to macro UEs is marked as \( P_0^m \), and the interference power transmitted from interfering eNodeBs is marked as \( P_k^m \), where the index \( k \) shows the number of interfering BS \( k = 1, 2, \ldots, 6 \).

The UEs located in femtocells will also receive interference from eNodeBs. The rate as achieved by the \( q^{th} \) femto UE in sector \( S_{01} \) is given by:

\[ R_q^f = B_q^f \log_2 \left( 1 + \frac{G_{0q}^f P_0^f}{N_0 B_q^f + \sum_{k=1}^{6} g_{qk}^m p_k^m} \right) \tag{3} \]

where \( G_{0q}^f \) is the large scale path loss of directed channels as defined in Equation (4). In this case the penetration loss of the desired signal is assumed 0 dB [5]. The large scale PL attenuation is given by:

\[ G_{0q}^f = -[128.1 + 10\alpha \log_{10}(r) + A^f + X_\sigma + F^f] \tag{4} \]

where \( A^f \) is the sum of UE and HeNB antenna gain. We neglect the interference between HeNBs due to the low transmission power of these nodes and also because these stations are well separated in distance. The large scale PL attenuation for macro interference channels is referred from Equation (2) neglecting the fast fading \( F^f \).

III. OPTIMIZATION OF POWER ALLOCATION AND BANDWIDTH ASSIGNMENT

The use of multiple antennas at the transmitter and receiver can improve the Signal-to-Interference-plus-Noise Ratio (SINR). The increase of SINR allows for a corresponding increase in the achievable data rates. But in bandwidth-limited operations, when the bandwidth-limited range is reached, the achievable data rates start to saturate unless the bandwidth is also allowed to increase. In order to understand this saturation in the achievable data rates it is to consider the normalization of achievable data rates based on normalized channel capacity in [13].

Then, for data rate achieved by a macro UE, Equation (1) reformulates as

\[ R_q^m = \beta_q^m B_{max} \log_2 \left( 1 + \frac{P_0^m}{n_q^m \beta_q^m + \sum_{k=1}^{6} g_{km}^m p_k^m} \right) \tag{5} \]

Similarly the data rate achieved by a femto UE is defined as

\[ R_q^f = \beta_q^f B_{max} \log_2 \left( 1 + \frac{P_0^f}{n_q^f \beta_q^f + \sum_{k=1}^{6} g_{qk}^m p_k^m} \right) \tag{6} \]

The maximization of minimum rate, employing bandwidth and power allocation formulations:

\[ \text{maximize} \quad \min \left( \beta_1^m t^m \log(2), \ldots, \beta_6^m t^m \log(2) \right), \]

subject to

\[ t^m \leq \log(1 + \frac{P_0^m}{n_1^m \beta_1^m + \sum_{k=1}^{6} g_{1k}^m p_k^m}) \tag{7a} \]
\[ t^m \leq \left( \log(1 + \frac{p^m_0}{n^m_{\beta_0} + \sum_{k=1}^6 g^m_{kn} p^m_k}) \right) \]

\[ t^f \leq \left( \log(1 + \frac{p^f_0}{n^f_{\beta_1} + \sum_{k=1}^6 g^f_{kn} p^f_k}) \right) \]

\[ \sum_{n=1}^N \beta^m_n + \sum_{q=1}^Q \beta^f_q \leq 1 \]

\[ K_f p^m + K_m p^f \leq P_{\text{max}} \]

\[ p^m \geq P_{\text{min}}, \quad p^f \geq P_{\text{min}} \]

The optimization problem formulated in (7) is a GGP. The constraints in (7a)-(7b) show the normalized macro UE rates \((t^m)\) subject to a minimum required macro UE rate. Equivalently the constraints in (7c)-(7d) show the normalized femto UE rates \((t^f)\) subject to a minimum required femto UE rate. The constraints in (7e) and (7f) are bandwidth and power constraints, respectively. In Equation (7f) two coefficients are used in order to express the portion of power generated from each node (macro/femto). The term \(K_m\) refers to the weight of the eNodeB and \(K_f\) presents the weight of the HeNB.

To solve this optimization problem, first we transform it from a GGP to a GP and convert it to a convex optimization problem. The transformation of the max-min-rate problem into a GP optimization problem is done using the proposition in [8]. It uses a variable \(u\) which acts as lower bound in the objective \((7)\). Then we end up with the minimization of its inverse value. Similarly the constraints are converted to a GP applying the exponential on both sides of each constraint. The optimization problem now formulates as

\[
\text{maximize} \quad \min \left\{ \frac{1}{u} \right\} \\
\beta^m_n, \beta^f_q, u \in R^+, p \geq 0
\]

subject to

\[ \beta^m_1 t^m \log(2) \geq u \]

\[ \beta^m_n t^m \log(2) \geq u \]

\[ \beta^f_1 t^f \log(2) \geq u \]

\[ \beta^f_q t^f \log(2) \geq u \]

\[ n^m_1 \beta^m_1 + \sum_{k=1}^6 g^m_{k1} p^m_k \geq (e^{t^m} - 1) \]

\[ n^f_1 \beta^f_1 + \sum_{k=1}^6 g^f_{k1} p^f_k \geq (e^{t^f} - 1) \]

\[ n^m_n \beta^m_n + \sum_{k=1}^6 g^m_{kn} p^m_k \geq (e^{t^m} - 1) \]

\[ n^f_n \beta^f_n + \sum_{k=1}^6 g^f_{kn} p^f_k \geq (e^{t^f} - 1) \]

\[ \sum_{n=1}^N \beta^m_n + \sum_{q=1}^Q \beta^f_q \leq 1 \]

\[ K_f p^m + K_m p^f \leq P_{\text{max}} \]

\[ p^m \geq P_{\text{min}}, \quad p^f \geq P_{\text{min}} \]

Refering to [10], the optimization problem given by Equation \((8)\) is in GP form. The constraints \((8a)-(8d)\) are monomials and posynomials. In the constraints \((8e)-(8h)\), the expressions on the left hand side are posynomials, referring to the rule that a posynomial divided by a monomial is still a posynomial. The constraints \((8i)-(8j)\) are posynomials. The optimization problem that maximizes the minimum rate is written in a general case for \(N\) macro UEs, and \(Q\) femto UEs. This general case can be adapted into simulations for a desired portion of UEs, depending on the simulation objectives. In our case, only two UEs are considered, one UE served by eNodeB and one UE served by HeNB. This is done to simplify comparisons between macro- and femto UEs.

IV. Simulation results

Our system model is modelled in MATLAB using polar coordinates, and assuming as reference point \(BS_0\). The eNodeB and HeNB sites have fixed position. For simulations two UEs are considered in the network. One macro UE located in polar coordinates \((200m, 160^\circ)\) and one femto UE located in \((400m, 160^\circ)\). The system model is referred from Section II. For simplicity, only one HeNB is considered in sector \(S_{01}\) at the polar coordinates \((410m, 160^\circ)\). The simulation parameters are listed in Table I.

The eNodeB antennas are 120° sectoral antennas with horizontal pattern per sector as:
TABLE I
SIMULATION PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cellular layout</td>
<td>Hexagonal grid, 7 cell sites, 3 sectors per site</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2.0 GHz</td>
</tr>
<tr>
<td>Maximum base station bandwidth $B_{\text{max}}$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>eNodeB maximum power $P_{\text{max}}$</td>
<td>40 W</td>
</tr>
<tr>
<td>HeNodeB minimum power $P_{\text{min}}$</td>
<td>0.0001 W</td>
</tr>
<tr>
<td>Macro power weight coefficient $K_m$</td>
<td>400</td>
</tr>
<tr>
<td>Femto power weight coefficient $K_f$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Noise spectral density $N_0$</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>Pathloss exponent $\alpha$</td>
<td>3.75</td>
</tr>
<tr>
<td>Penetration loss in macro case $L_P$</td>
<td>20dBi</td>
</tr>
<tr>
<td>Shadowing $X_s$</td>
<td>$N(0, 8) dB$</td>
</tr>
<tr>
<td>Fast Fading $F$</td>
<td>$CN(0, 1) dB$</td>
</tr>
<tr>
<td>Inter base station distance</td>
<td>700m</td>
</tr>
<tr>
<td>Maximum cell range</td>
<td>$(2/3)R_m$</td>
</tr>
</tbody>
</table>

$$A(\theta) = \min \left[ 12 \left( \frac{\theta}{\theta_{\text{3dB}}} \right)^2, 20 \right] ,$$  \(9\)

where $\theta \in [-180^\circ, 180^\circ]$ and $\theta_{3dB} = 70^\circ$. The antenna gain is 14 dBi. The HeNBs employ omni-directional antennas with 0 dBi gain.

The optimization problem given by Equation (8), can be simplified to the case when only two UEs are served by the network.

$$\begin{align*}
\text{maximize} & \quad \min \left\{ \frac{1}{u} \right\} \\
\text{subject to} & \quad \beta_{1m}^{\ell} \log(2) \geq u \\
& \quad \beta_{1f}^{\ell} \log(2) \geq u \\
& \quad \frac{\beta_{1m}}{p_0^m} + \sum_{k=1}^{6} g_{1k} p_k^m \geq (e^{m} - 1) \quad (10a) \\
& \quad \frac{\beta_{1f}}{p_0^f} + \sum_{k=1}^{6} g_{1k} p_k^f \geq (e^{f} - 1) \quad (10b) \\
& \quad \beta_{1m}^{\ell} + \beta_{1f}^{\ell} \leq 1 \quad (10c) \\
& \quad K_m p_m^{\ell} + K_f p_f^{\ell} \leq P_{\text{max}} \quad (10d) \\
& \quad p_m^{\ell} \geq P_{\text{min}}, \quad p_f^{\ell} \geq P_{\text{min}} \quad (10e)
\end{align*}$$

CVX is used to solve the optimization problem, and in following we present the simulation results.

Fig. 3 and Fig.4 show the power assignment depending on minimum data rate requirement for macro- and femto UEs respectively.

When the minimum data rate requirement for the macro UE increases, the power is also increased. According to Equation (1), the transmission power directly relates to the data rate. From the plot in Fig.3 we observed a logarithmic curve that expresses the relation between the power assigned to the macro UE and the normalized minimum data rate requirement per UE. In this case modulation is not taken into account, so the relation between normalized data rate and assigned power follows (5), while being subject to conditions from the optimization problem (10). Fig.3 shows that when normalized macro UE rate exceeds 12 bit, the assigned power reaches up to 25 W. Otherwise, for low normalized macro UE rates, the assigned power is much lower, since interference from the Network is low.

Fig.4 shows assigned power for the femto UE. It is observed that the assigned power is in range of milliwatts, due to the power constraints from (10f). According to this Equation (which in fact represents the weighted power coefficients inequality), the sum of assigned power in each cell can not exceed a certain value, in our case 40 W. Results show the same logarithmic curve of assigned power over normalized femto UE rates, following (6). When the minimum data rate requirement for femto UE increases, the power assigned is
From Figures 3 and 4 we show that the power assignment in the macro UE case only depends on the normalized macro UE rate and is independent from the femto UE rate. The same behaviour is observed for femto UE. From our optimization problem (8) there is a relation between assigned power to both macro and femto cases. This relation (expressed with (10f)) prevents interference between eNodeBs and HeNBs within the same cell and keeps transmit power in optimal terms.

We have compared the normalized bandwidth assigned to both macro- and femto UEs. Fig.5 shows the normalized bandwidth as assigned to the macro UE, dependent on the normalized data rates of both macro- and femto UE. It should be noted that when the requirement for high data rate is increased for macro UE, the assigned bandwidth is increased too. But, as Soft Frequency Reuse 1 is used, the plot shows the tendency for bandwidth assignment when both macro and femto UEs are served by the network. When the requirement for high data rate increases for the femto UE, the assigned bandwidth to the macro UE is decreased. At the same data rate requirement, the same bandwidth portion is assigned to both UEs, 50% of the bandwidth is assigned to macro UE, and 50% to the femto UE.

Fig.6 shows the normalized bandwidth assigned to the femto UE dependent on the normalized data rates of both macro- and femto UE. The bandwidth assigned to the femto UE increases when the requirement of minimum data rate for the femto UE increases. Due to the increase of minimum data rate of the macro UE, the bandwidth assigned to the femto UE decreases. This tendency of bandwidth allocation proves that under equal conditions (same number of UEs served by each eNodeB and HeNB), is assigned an equal portion of bandwidth to macro- and femto UEs. In the general case, the number of UEs served by an eNodeB is larger than the number of UEs served by a HeNB. Based on our optimization problem in (8), there is a trade off between assigned power and bandwidth allocation. When the number of UEs connected to HeNBs increases, also the portion of bandwidth assigned to the femto UEs is increased, instead the assigned power is decreased in order to maintain interference lower. When the number of UEs connected to HeNBs decreases, also the bandwidth portion assigned to femto UEs is decreased, but the assigned power is increased.

Using (1), we have simulated the data rates achieved by the macro UE depending on the minimum data rate requirement. In the same way using (3), we have simulated the data rates achieved by the femto UE depending on minimum data rate requirement.

Fig.7 shows the data rates achieved by macro UE, and Fig.8 shows the data rates achieved by femto UE. Due to the fact that the maximum bandwidth of 20 MHz in LTE is assigned to each base station, and that only one UE is connected to each base station in our system, the data rates achieved by each UE is pretty high. The data rate achieved by the macro UE is lower than data rate achieved by the femto UE, because the
macro UE is further away from its serving base station. The femto UE is only 10 meters apart from its serving station, thus assuming that it is in the same building as the HeNodeB. Thus the path loss is very small compared to the case of macro UE which is 200 meters away from the eNodeB.

V. CONCLUSIONS

Using MATLAB and CVX, we have solved a constrained optimization problem, that optimally allocates bandwidth and transmit power in heterogeneous cellular networks. Such optimization problem is formulated as constrained non-convex optimization. Since non-convex optimization problems are hard to solve, we transformed it into a geometric programming form, and then solved it by using disciplined convex programming. The power assigned to macro- and femto UEs depends on the minimum data rate requirements for macro- and femto UE respectively. For the macro UE, the assigned power is of ranges 0-25 W, and for the femto UE the assigned power is of ranges 0-2.7 mW. Taking into account the portion of UEs served by eNodeBs and HeNBs, we see from our simulation results that the bandwidth and transmit power gets increased as the minimum data rate requirement is increased. For a larger number of UEs connected to both macro- and femto base station types, the optimal solution to assign power and bandwidth represents a trade-off between assigned power and bandwidth.

REFERENCES