Convection-Diffusion Equations used for pollution studies

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**Abstract.** In the following the behaviour of the convection-diffusion equation in one dimension is analysed. Different methods are used to implement this equation which can be applied for the simulation of groundwater pollution. There are different fields of studies dealing with the analysis of different kind of diffusion equations. In chemistry as well as in biology the reaction diffusion equation plays an important role. Another form of this equation can also be used to recreate pattern formations of fish’s skin or cat’s fur. There are also many physical applications, e.g. the heat equation. Diffusion is also used to foresee the behaviour of buyers of stocks in the financial market.  
For the analysis of the behaviour the one dimensional form of the convection diffusion equation is used. There are certain initial and boundary conditions considered. The initial condition describes an instantaneous release of pollution at the source. The boundary conditions are trivial. Due to these conditions an analytical solution can be given.  
Regarding numerical approaches two certain methods are used. On the one hand finite differences are used to approximate the behaviour of diffusion. This method is very easy to apply and the results are quite useable. On the other hand the finite element method is implemented. This method is based on formulating variations of the differential equation. An advantage of this method is the suitability to any geometry. Additionally the accuracy of the results is higher than for the finite difference method.  
The third approach deals with a Gaussian based method to analysis this question. This method can be extracted using the analytical solution of the equation. So the approaches range from analytical over numerical to stochastically methods.   
  
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