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Stabilization of the radial equilibrium of a tethered satellite with respect to out-of-plane initial perturbations

Alois Steindl[1,*]

[1] Institute for Mechanics and Mechatronics
Vienna University of Technology
Wiedner Hauptstr. 8–10
A1040 Wien

We investigate the stabilization of the radial equilibrium of a tethered satellite by tension control, if both in-plane and out-of-plane deviations from the vertical position are taken into account. While in-plane perturbations can be eliminated in finite time, the length rate change of the tether acts as parametric control on the out-of-plane deviations. Therefore the control becomes less effective, if the perturbation decreases.

In order to improve the convergence we investigate the optimal control problem assuming no costs for the applied tension force but restricting the control force to a finite interval. For tether cconfigurations close to the vertical target position this approach yields a solution with singular control, which leads to a faster decay of the deviations.

1 Introduction

We consider a satellite, which is connected by a massless tether to a space station rotating around the earth along a Keplerian circle with constant angular speed. By only excerting a tension force on the tether we try to steer the satellite to a steady local vertical position. By assuming, that the tether's motion is restricted to the orbital plane, we could already show, that the desired target position can be reached in finite time ([4]). In this article we take also out-of-plane perturbations into account.

1.1 Mechanical model

Assuming that the satellite's mass is negligible compared to the main station, the satellite's scaled equations of motion read ([1]):

$$[\ddot\psi + \{(\dot\vartheta-1)^2 + 3\cos^2\vartheta\}/2\sin 2\psi]\ell + 2\dot\ell\dot\psi = 0, \tag{1}$$

$$[(\ddot\vartheta + 3/2\sin 2\vartheta)\cos\psi - 2(\dot\vartheta-1)\dot\psi\sin\psi]\ell + 2\dot\ell(\dot\vartheta-1)\cos\psi = 0, \tag{2}$$

$$\ddot\ell + [1 - \{(\dot\vartheta-1)^2 + 3\cos^2\vartheta\}\cos^2\psi - \dot\psi^2]\ell = -u. \tag{3}$$

* Corresponding author: e-mail Alois.Steindl@tuwien.ac.at, phone +43 (1) 58801 325208, fax ++43 (1) 58801 9325208

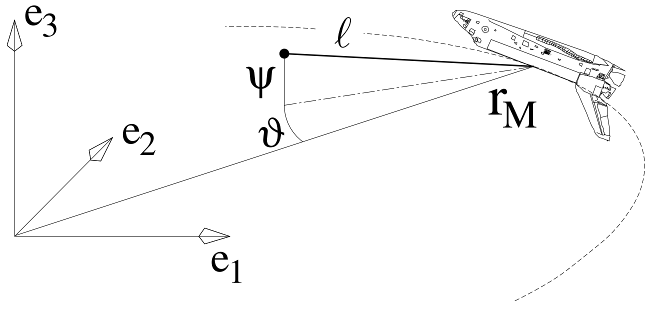

Fig. 1: Simplified tether model admitting in-plane (ϑ) and out-of-plane (ψ) deviations from the local vertical position.

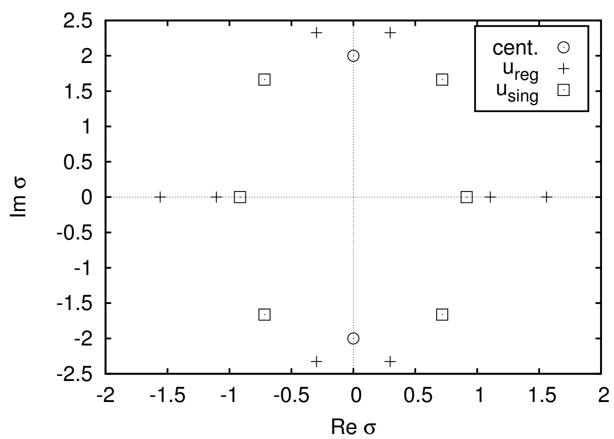

Fig. 2: Spektrum of the linearized system for regular and singular control. For the out-of-plane subsystem a non-semi-simple pair of pure imaginary eigenvalues $\pm 2i$ occurs.

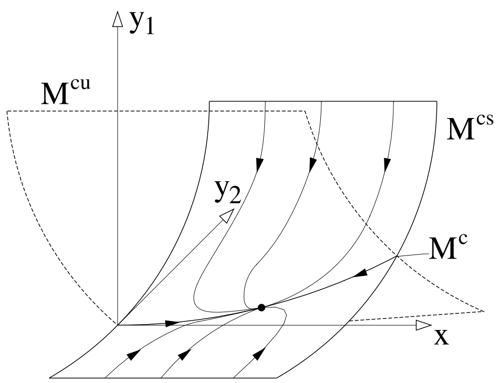

Fig. 3: Invariant manifolds for Hamiltonian equations

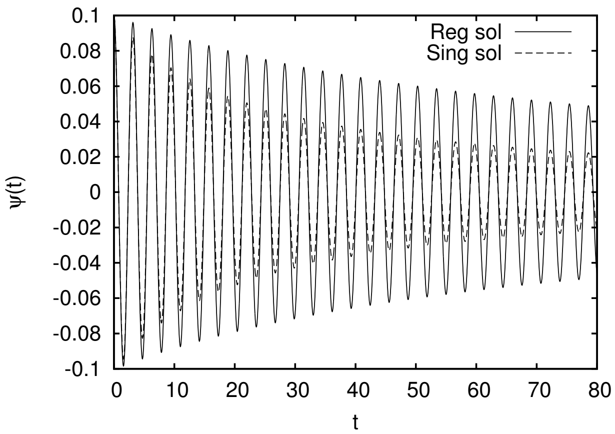

Fig. 4: Decay of the out-of-plane oscillations of the regularly and singularly controlled problem.

Here ψ and ϑ are the angles of the out-of-plane and in-plane positions, respectively. The scaled length of the tether is denoted by ℓ, the control variable u denotes the tension force on the tether. Derivatives w.r.t. the scaled time t are denoted by $\dot{(\cdot)}$. It should be noted, that the scaled time t is chosen such that the orbital period is 2π.

2 Optimal Control solution

In order to steer the subsatellite from an arbitrary initial position to the locally vertical relative equilibrium with $\ell = 1$, we define the cost

$$C = \int_0^\infty \left(\psi^2 + \dot\psi^2/4 + \vartheta^2 + \dot\vartheta^2/4 + (\ell-1)^2 + \dot\ell^2 + (u-3)^2\right)/2dt, \tag{4}$$

where $u = 3$ denotes the tension force in the equilibrium configuration, and apply Pontryagin's Maximum principle to obtain a cost minimizing trajectory. In [3] it is shown, that due to the parametric influence of the length rate change on the out-of-plane oscillations the linearized Hamiltonian equations have a non-semisimple pair of purely imaginary eigenvalues, leading to a Hamiltonian Hopf szenario in the out-of-plane sub-system ([2]). Since the remaining eigenvalues are hyperbolic (see Fig. 2), the global solution structure can be described as depicted in Fig. 3: The state and co-state variables for the in-plane oscillations and the tether length converge along the Center-Stable manifold M^{cs} to the 4-dimensional Center Manifold M^c. Along the Center Manifold the system converges algebraically slowly to the stationary solution.

2.1 Singular Control

In order to accelerate the decay, the term $(u-3)^2/2$ for the control action was removed from the cost function (4). Since now the control variable u appears linearly in the Hamiltonian, we expect singular control during the final phase of the solution, because a steady bang-bang control would lead to periodic oscillations of the in-plane solution. In this model the singular control can be obtained simply by regarding the length rate change $\dot\ell$ as new control variable and investigating the reduced system. In order to obtain the tension force in the singular control regime, u has to be calculated from (3). It turns out, that the linearized system has the same structure as for the regular control: The double pair of imaginary eigenvalues persists and the remaining eigenvalues are still hyperbolic, (Fig. 2), but the solution converges faster to the steady state. Of course, the convergence is still only algebraically slow and in reality one would need some steering device, that excerts a transversal force on the satellite.

References

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