Selfsensing Unbalance Rejection and Reduction of the Gyroscopic Effect for an Active Magnetic Bearing System

Markus Hutterer, Matthias Hofer and Manfred Schrödl
Vienna University of Technology
Institute of Energy Systems and Electrical Drives, Austria, 1040 Vienna, Gusshausstraße 25-27
Email:markus.hutterer@tuwien.ac.at

***Abstract*—The unbalance of a magnetically levitated rotor causes synchronous oscillations in the current and position signals. This oscillations can lead to saturation of the magnetic actuator. To deal with that problem a selfsensing unbalance controller is developed to reduce the unbalance oscillations in the current signals. The selfsensing unbalance controller consists of two parts. The first part is the unbalance observer which has the task to detect the angular velocity and the angle of the synchronous unbalance information. The second part is the unbalance controller which uses the information of the unbalance observer to reject the unbalance oscillations in the current signal. For the unbalance controller a two modulation step Notch filter is used. The unbalance controller is not the only device which needs the angular velocity. With the angular velocity information also a parameter variant control path can be developed which has the task to reduce the gyroscopic effect. The last part of this paper deals with experimental results of the proposed control system which is implemented on an industrial magnetic bearing system.**

***Keywords*—AMB System, Gyroscopic effect, Unbalance control, Selfsensing.**

I. Introduction

Vibrations caused by mass unbalance are a common problem in magnetic bearing applications. Unbalance occurs if the principal axis of inertia of the rotor is not coincident with its axis of geometry [4]. In most cases it is almost impossible to balance the rotor, because the unbalance distribution is changed during operation. In the case of ball bearings, reaction forces occurs due to the unbalance [8]. This reaction forces where transmitted to the machine housing, which leads to unwanted vibrations. Compared to this conventional bearing types with active magnetic bearings it is able to provide an unbalance compensation. This additional component of the control structure allows the rotor to spin around its inertial axis. This unbalance compensation can have the following tasks:

- Rejection of synchronous bearing forces: The synchronous bearing current is approximately a quadratic function of the rotational speed. Therefore the amplifier will saturate for high speeds and the system will get unstable. The aim of this compensation is to reject the synchronous bearing current.
- Rejection of the unbalance vibration: The aim is to reject the vibration due to the reaction forces of the unbalance and the housing. To get a suitable rejection the system needs high damping forces which can also caused a saturation of the amplifiers.
- Rejection of the displacement orbits when the rotational speed crosses the rigid body modes

The focus of this paper is the rejection of the synchronous bearing forces and the reduction of the gyroscopic effect using the unbalance information to estimate the angular velocity. The simplest method is the insertion of a Notch filter in the feedback path. The drawback of this method is that an open loop designed filter can introduce instability for the closed loop system [9]. This reason of instability is eliminated by an observer based design [10]. The drawback of this design method is that a very accurate plant model is needed and the computing time is very large compared to the other methods. A converse approach is the adaptive feedforward method [11], which has the advantage that they cannot introduce instability, if the adaption process itself is stable. Most often, complex nonlinear adaption processes were used and convergence could not be proved in all cases. In this paper a multi variable Notch filter which is designed for the closed loop system like it was demonstrated in [7] is used. The unbalance information to calculate the coefficients is estimated using a special unbalance observer. This observer is able to estimate the angle and the rotational speed of the unbalance of the rotor. The reduction of the synchronous current is not the only application where this unbalance information can be used. Also the gyroscopic effect is able to be reduced significantly with this selfsensing unbalance method. In [3], [5] and [6] the gyroscopic effect is reduced by using a special parameter variant structure.

II. Unbalance Control using a two modulation step approach

For the rejection of the synchronous bearing forces the two modulation step approach of [7] is used. The structure of this method is shown in Fig. 1. The closed loop system with $\mathbf{C}(s)$ and $\mathbf{G}(s)$ is assumed to be stable. The sensor signal $\mathbf{y}(t)$ contains a sinusoidal of the frequency Ω which correspondents with the unbalance of the signal. The idea of this compensation is to generate a compensation signal $\mathbf{c}(t)$, which has the same phase, frequency and amplitude like the sensor signal $\mathbf{y}(t)$ and subtract it. To generate the compensation signal the sensor signal is multiplied by $\sin\Omega t$ and $\cos\Omega t$ to shift the frequency Ω down to zero. Then this signal is integrated and is shifted back to the frequency Ω by multiply it with $\sin\Omega t$ and $\cos\Omega t$. With

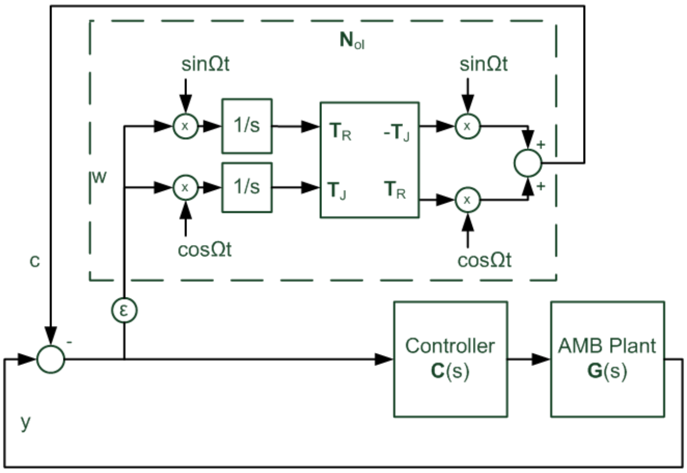

Fig. 1. Two modulation step Notch filter

the integration action the DC value of the signal is calculated which corresponds with the amplitude of the unbalance signal. The convergence speed can be changed with the value ϵ. It is possible to replace the multiplications with the trigonometric functions by a transformation in a rotating frame. But this transformation cannot handle oval rotor orbits.

The compensation signal $\mathbf{c}(t)$ is:

$$\begin{aligned}\mathbf{c}(t) &= [\sin(\Omega t)\mathbf{I}\ \cos(\Omega t)\mathbf{I}] \begin{bmatrix}\mathbf{T}_r & -\mathbf{T}_j \\ \mathbf{T}_j & \mathbf{T}_r\end{bmatrix} \\ & \cdot \int \begin{bmatrix}\sin(\Omega t) \\ \cos(\Omega t)\end{bmatrix} dt \end{aligned} \tag{1}$$

The bold symbols denotes multivariable matrices and $\mathbf{I}$ is a Idenity matrix. Transform equation (1) in the s-domain leads to the input output equivalent:

$$\mathbf{N}_{ol} = \frac{1}{s^2+\Omega^2}\left(s\mathbf{T}_R - \Omega\mathbf{T}_J\right) \tag{2}$$

After closing the feedback loop of the two modulation Notch filter the transfer function has the following form:

$$\mathbf{N}_{cl} = \frac{\mathbf{e}}{\mathbf{y}} = \left(s^2+\Omega^2\right)\left(s^2\mathbf{I} + s\mathbf{A}_1 + \mathbf{A}_0\right)^{-1} \tag{3}$$

with

$$\mathbf{A}_1 = \epsilon\mathbf{T}_R,\ \mathbf{A}_0 = \Omega^2\mathbf{I} - \epsilon\Omega\mathbf{T}_J \tag{4}$$

From equation (3) the notch characteristic can be seen where ϵ defines the bandwidth of the system. The above description shows that the two modulation notch filter has the same input output description like the common LTI Notch filter. Nevertheless the two modulation steps has some advantages compared to the common LTI implementation.

- The two modulation Notch filter can be used as an ideal feedforward compensation if ϵ is kept to zero. This operation is impossible for the LTI implementation.
- With the integrator outputs the amplitude of the unbalance of the rotor can be calculated

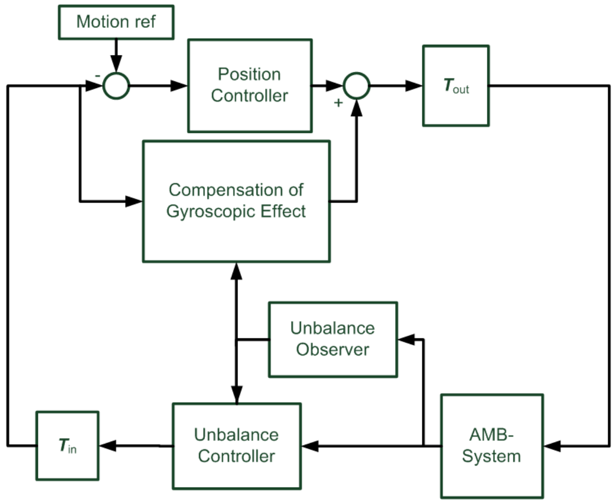

Fig. 2. Control structure

- In the narrow band case the two modulation notch filter shows no numerical errors compared to the classical LTI implementation

Stability analysis of this two modulation notch filter was done in [7]. In this paper a decentralized Notch filter design is used.

III. Reduction of the Gyroscopic Effect

The gyroscopic effect decreases the performance of the regarded system in two ways.

- The fast decentralized Notch filter can only be used for system with low cross couplings. If the gyroscopic effect is too high the easy decentralized implementation has to be replaced by a more complex method which needs much more computing time.
- The gyroscopic effect splits up the rigid body modes. For such a parameter variant system either the performance decreases or a complex controller is needed.

To overcome this problem a parameter variant feedback structure is developed. The system description of the magnetic bearing system with a rigid rotor in the linearized form is [4]:

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{B}\mathbf{K}_S\mathbf{B}^T\mathbf{x} &= \mathbf{B}\mathbf{K}_i\mathbf{i} \\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned} \tag{5}$$

with the mass matrix $\mathbf{M}$, the gyroscopic matrix $\mathbf{G}(\Omega)$, the matrix of the negative stiffness $\mathbf{K}_S$, the matrix of the force to current factors $\mathbf{K}_i$, the input matrix $\mathbf{B}$, the output matrix $\mathbf{C}$, the center of gravity (COG) coordinates $\mathbf{x}$, the sensor coordinates $\mathbf{y}$ and the current vector $\mathbf{i}$. Equation (5) shows that the only parameter variant term is the matrix of the gyroscopic effect $\mathbf{G}(\Omega)$. To use the linear time invariant (LTI) control theory $\mathbf{G}(\Omega)$ has to be cancelled or even rejected. Fig. 2 shows the overall control structure schematically. To affect the tilting and translation rigid body modes independent from each other an input $\mathbf{T}_{in}$ and output transformation $\mathbf{T}_{out}$ is used to transform the system in the COG coordinates. If

$$\mathbf{T}_{in} = \text{inv}\left(\mathbf{C}\right) \tag{6}$$

and

$$\mathbf{T}_{out} = \text{inv}\left(\mathbf{BK}_i\right) \tag{7}$$

is used, the system equation is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{BK}_S\mathbf{B}^T\mathbf{x} = \mathbf{i} \tag{8}$$

The invertibility of the matrix $\mathbf{C}$ and $\mathbf{BK}_i$ is given in the most cases, because the matrices for such a system are a quadratic ones and the determinant is only zero for singular points. After the transformation the tilting and translation rigid body modes are nearly decoupled. The only coupling term which is left of both modes is $\mathbf{BK}_S\mathbf{B}^T$. But this term is normally quite low and can be neglected.

In the COG coordinate system the gyroscopic effect can be reduced with a parameter variant controller of the following form:

$$\mathbf{i}_{komp} = C_r\mathbf{G}(\Omega)\dot{\mathbf{x}} \tag{9}$$

After this compensation the system is nearly parameter invariant and the LTI control theory and the decentralized Notch filter can be used. In [3] is suggested that a complete elimination of the gyroscopic effect is not very robust against dead times, and therefore a factor C_r is introduced. The system equation with the parameter variant feedback term equation (9) is:

$$\mathbf{M}\ddot{\mathbf{x}} + \left(1 - C_r\right)\mathbf{G}(\Omega)\dot{\mathbf{x}} + \mathbf{BK}_S\mathbf{B}^T\mathbf{x} = \mathbf{i} \tag{10}$$

Because nearly the whole parameter variant term is compensated, the whole feedback term could be treated as a linear time invariant system for stability analysis.

Equation (9) needs the velocities of the COG system. Because usually position sensors are used for the feedback path to stabilize the magnetic bearing system, the velocities are not measuring variables. In the last years also a few sensorless control strategies were developed, like the INFORM method which is described in [1] and [2]. To get the velocities the position signals can be differentiate. But differentiation usually increases the measuring noise significantly. To overcome this problem a Kalman observer is developed, which estimates the states of the AMB system. For the reduction of the gyroscopic effect only the tilting velocities of the AMB system are needed. This fact can be used to improve the Kalman observer. If the Kalman observer is developed in the COG coordinate system the observer can split up into an observer for the tilting movement and one for the translation movement. If this knowledge is used the computing time of the Kalman observer is reduced by a factor of four. The angular velocity information for the reduction of the gyroscopic effect is estimated with a unbalance observer. The initial guess of the Kalman observer is chosen to be zero. How such a Kalman observer can be developed is shown in [6].

IV. Unbalance observer

To reduce the gyroscopic effect and the unbalance of the rotor the angular velocity has to be known. For the system which is presented in this paper the angular velocity and the angle of the synchronous unbalance is estimated using a special observer. The unbalance equation for a forward rotating system has the following form:

$$\begin{aligned} x &= Acos(\varphi) \\ y &= Asin(\varphi) \end{aligned} \tag{11}$$

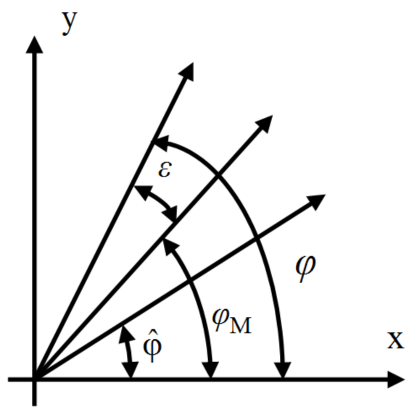

Fig. 3. Substitute variables

To estimate the unbalance information the following linear observer model is used

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varphi \\ \Omega \end{bmatrix} + \begin{bmatrix} k_{1\varphi} & k_{2\varphi} \\ k_{1\Omega} & k_{2\Omega} \end{bmatrix}\begin{bmatrix} x - \hat{x} \\ y - \hat{y} \end{bmatrix} \tag{12}$$

where φ is the angle of the maximum of the elongation caused by the unbalance and Ω is the angular velocity. When the measuring equations (11) are inserted into the observer model equation (12) the failure dynamic

$$\begin{aligned} \begin{bmatrix} \dot{e}_\varphi \\ \dot{e}_\Omega \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e_\varphi \\ e_\Omega \end{bmatrix} + \begin{bmatrix} k_{1\varphi} & k_{2\varphi} \\ k_{1\Omega} & k_{2\Omega} \end{bmatrix} \\ &\cdot \begin{bmatrix} A\left(cos(\varphi) - cos(\hat{\varphi})\right) \\ A\left(sin(\varphi) - sin(\hat{\varphi})\right) \end{bmatrix} \end{aligned} \tag{13}$$

is nonlinear. For a nonlinear failure dynamic, stability cannot be proven in all cases. To solve this problem the feedback variables k_φ and k_Ω can be chosen as a function of the states and the input [12]. With this nonlinear feedback variables it is possible to get a linear failure dynamic. The first step to get a linear failure dynamic is to formulates the feedback term

$$x - \hat{x} = A\left(cos(\varphi) - cos(\hat{\varphi})\right) \tag{14}$$

in a different way. To do this substitute variables are introduced according to Fig. 3.

$$\begin{aligned} \varphi_M &= \hat{\varphi} + \epsilon \\ \epsilon &= \frac{\varphi - \hat{\varphi}}{2} \end{aligned} \tag{15}$$

With this variables the failure can formulate as:

$$\begin{aligned} x - \hat{x} &= A\left(cos(\varphi_M + \epsilon) - cos(\varphi_M - \epsilon)\right) \\ &= -2Asin(\varphi_M)sin(\epsilon) \end{aligned} \tag{16}$$

When it is assumed that ϵ is very small, then $\varphi_M \approx \varphi$ and the taylor expansion can be used for $sin(\epsilon)$ and the failure can be formulate as:

$$x - \hat{x} = -Asin(\varphi)(\varphi - \hat{\varphi}) \tag{17}$$

For the y direction the derivation is the same and the result is:

$$y - \hat{y} = Acos(\varphi)(\varphi - \hat{\varphi}) \tag{18}$$

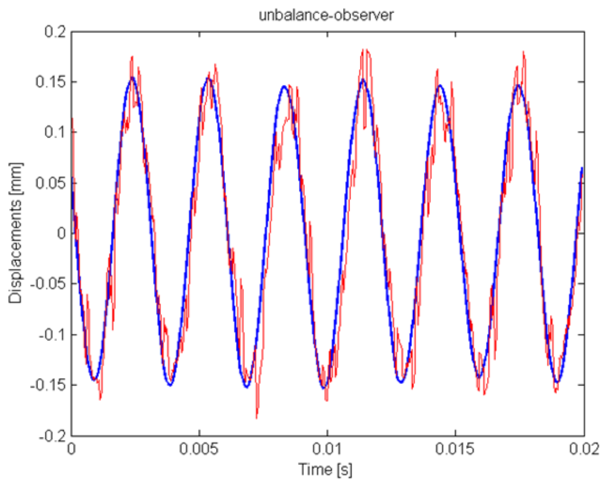

Fig. 4. Simulated performance of the unbalance observer

To get a linear fault dynamic it is possible to compensate the nonlinearities. An idea for compensation could be to dived by the sine or cosine. But this approach causes numerical problems at the zero crossing of the sine and cosine. Therefore an other method of compensation is:

$$\begin{aligned} k_{1\varphi} &= \frac{\tilde{k_\varphi}}{A} sin(\varphi) \qquad & k_{2\varphi} &= \frac{\tilde{k_\varphi}}{A} cos(\varphi) \\ k_{1\Omega} &= \frac{\tilde{k_\Omega}}{A} sin(\varphi) \qquad & k_{2\Omega} &= \frac{\tilde{k_\Omega}}{A} cos(\varphi) \end{aligned} \tag{19}$$

With this feedback variables the failure dynamic after some algebraic steps is

$$\begin{aligned} \begin{bmatrix} \dot{e}_\varphi \\ \dot{e}_\Omega \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_\varphi \\ e_\Omega \end{bmatrix} + \begin{bmatrix} \tilde{k}_\varphi & \tilde{k}_\varphi \\ \tilde{k}_\Omega & \tilde{k}_\Omega \end{bmatrix} \\ &\cdot \begin{bmatrix} e_\varphi \\ e_\varphi \end{bmatrix}. \end{aligned} \tag{20}$$

This failure dynamic is now a linear time invariant system. For such a system stability can be proved with the calculation of the eigenvalues. Compared to other nonlinear observers this observer do not need a high computing time. With the presented method it is possible to estimates the angle and angular velocity of the synchronous unbalance from the sensor signal. This unbalance observer in combination with a unbalance controller do not need informations from external devices (like the motor controller) to get the angular velocity. Thus this combination is a selfsensing unbalance rejection method.

V. Simulation Results

For the simulations a rigid body model is used and analyses are performed on MATLAB/SIMULINK. The controller and observer was implemented with a Matlab-function block, where the digitization is considered. To simulate the measuring noise a white noise with an amplitude of 10μm is applied on the sensor signals. Fig. 4 shows the estimated sine wave of the unbalance observer in blue and the sine wave of the output in red. It can be seen that the observer provides also a good solution when the output has a quite high measuring noise.

The next simulation should proof the functionality of the compensation of the gyroscopic effect. If the gyroscopic effect

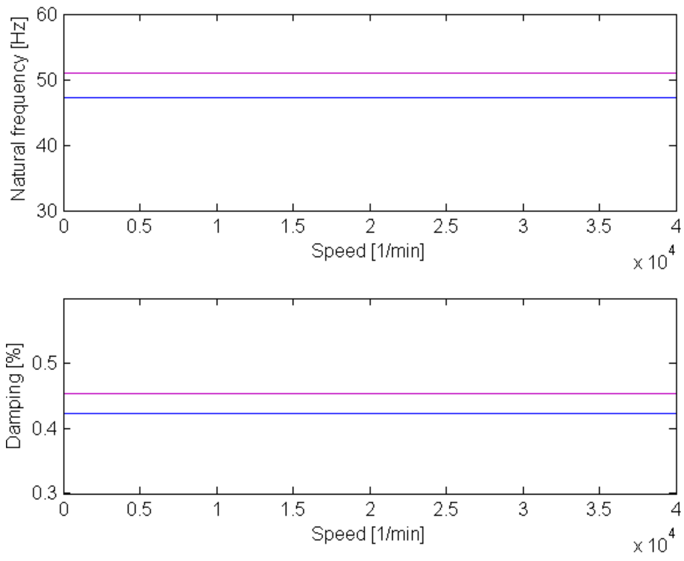

Fig. 5. Simulated Campbell Diagram

is compensated completely, the natural frequencies should not be dependent on the angular velocity. To show this context the Campbell diagram is used according to Fig. 5. The natural frequencies are speed independent. This is a big advantage compared to the original structure, because speed dependent poles decreases the performance of the system and could lead to instability. The compensated system is linear and time invariant. This means that an optimal LTI controller is optimal for all angular velocities. Because the damping ratio is positive in the whole speed range stability is proven.

VI. Experimental Results

Fig. 6 shows the performance of the unbalance observer. Where Ch1 is the estimated angle CH2 is the position signal of the unbalance and Ch3 is the sine wave which is calculated from the observer plus a 90 phase shift. As it can be seen the observer estimates a nearly linear angle. With this angle and the estimated amplitude the sine is now able to get reproduced. The estimated sine correlates well with the measured one.

The next experiment is the testing of the performance of the two modulation Notch filter. Fig. 7 shows the performance of the Notch Filter when it is switched on at $10000rpm$. Where CH1 and CH2 are position signals of the position sensors and Ch3 and Ch4 are current signals which are calculated from the position controller. It can be seen that the unbalance excites a low frequency oscillation. When the unbalance controller is switched on the unbalance part from the current signal is significantly reduced and no low frequency oscillation occurs at the position signals. With this experiment the functionality of the unbalance controller is proven.

To verify, if the compensation of the gyroscopic effect works in the real system, in the next experiment the dynamic behaviour at standstill and operating speed, were compared. For comparison the compliance transfer functions of the tilting and translation movement were used. Due to the symmetry of the dynamic behaviour in the x and y direction, only one tilting and one translation transfer function is necessary. Fig. 8

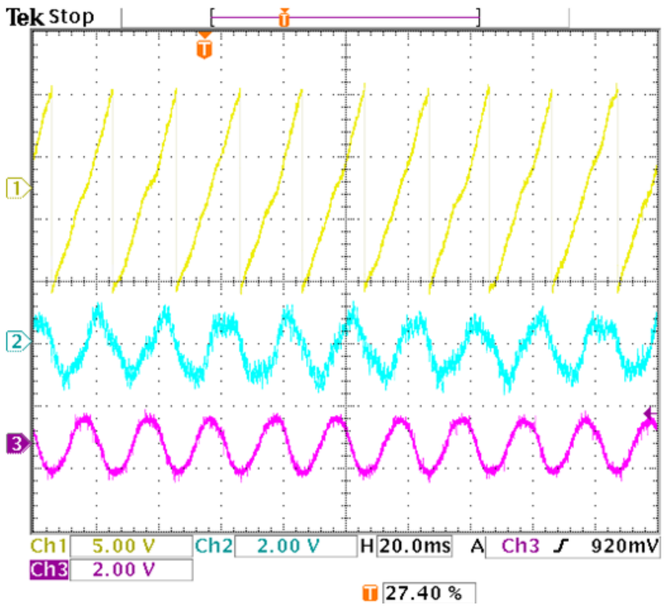

Fig. 6. Measured performance of the unbalance observer

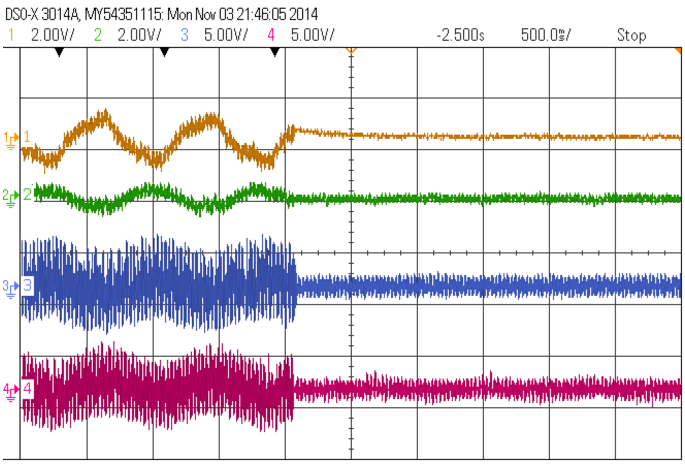

Fig. 7. Performance of the unbalance controller

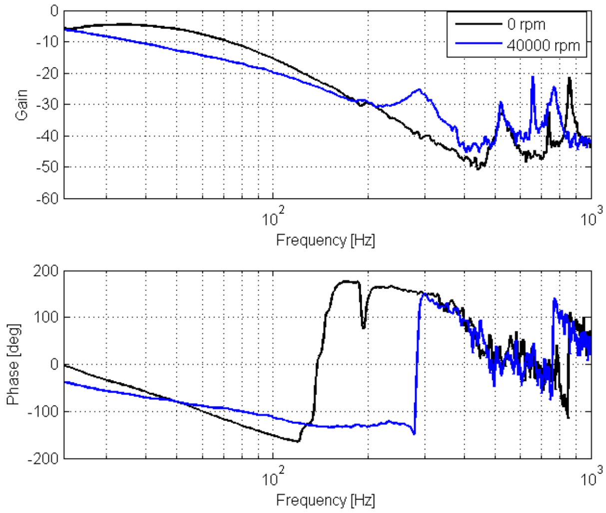

Fig. 8. Measured compliance function of the tilting movement

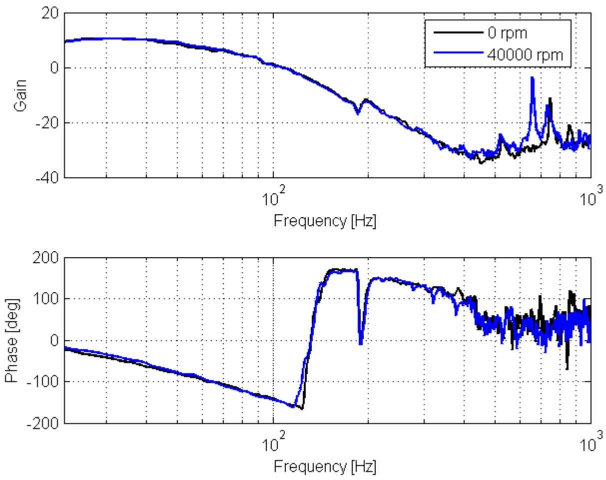

Fig. 9. Measured compliance function of the translation movement

shows a comparison of the tilting compliance functions. To get a higher robustness C_r was chosen with 0.6. The gain of both transfer function differs slightly. The reason is a not modelled effect, caused by the AMB application. The phase shows that the rigid body modes of the tilting movements are nearly the same for both operating speeds and the natural frequencies are at about 60Hz. The natural frequency of the first bending mode at standstill is at about 860Hz and splits up for operating speed into one bending mode with a backward whirl at about 780Hz and one with a forward whirl which cannot be seen in this transfer function. In this paper only the gyroscopic effect of the rigid body modes is compensated, because all the other effects do not show stability problems. In contrast to this the rigid body modes without this compensation will have stability problems, due to the split up caused by the gyroscopic effect.

Fig. 9 shows the comparison of the translation compliance functions. Both transfer functions are nearly equal. This fact proves the functionality of the transformation in the COG coordinate system. From the phase plot can be seen that the natural frequency of the translation rigid body modes are at about 60Hz. The phase plot do not show a phase step due to the first bending mode. The reason is that the first bending mode is not well observable for translation movements with this rotor. In summary, can be stated that the designed decoupled controller fullfills the requirements for a stable and robust system.

VII. Conclusion

This paper presents a selfsensing unbalance rejection for a high gyroscopic rotor. This selfsensing unbalance rejection consists of a unbalance observer which have the task to estimate the angle and the angular velocity of the unbalance from the unbalance information of the position signals and an unbalance controller which has the task to reject the unbalance part of the current signals. For the unbalance controller a two modulation step Notch filter was used. A decentralized structure of the unbalance controller could only be used, if the cross couplings like the gyroscopic effect is low. To use such an decentralized structure also for high gyroscopic rotors a parameter variant rejection of the gyroscopic effect was designed. This rejection of the gyroscopic effect needs also the information of the angular velocity Ω and of the velocities of the system equation $\dot{\mathbf{x}}$. Ω can be used from the unbalance observer and for the estimation of $\dot{\mathbf{x}}$ a Kalman observer was designed. The unbalance observer is designed in a nonlinear way, that the resulting failure dynamic is linear. The performance of the complete control system was proved by simulations and experiments. In summary can be stated, that the presented selfsensing control structure has a good performance and offers a redundancy to the angular velocity information of the motor controller. This means, if the motor controller has an error it is possible to run the system only with the unbalance observer.

ACKNOWLEDGMENT

This work is conducted within the project P21631-N22 of the Vienna University of Technology with the Austrian Science Fund (FWF). The authors thank the Austrian Science Fund for supporting this project.

REFERENCES

[1] M. Hofer, *Design and Sensorless Position Control of a Permanent Magnet Biased Radial Active Magnetic Bearing*, PhD thesis, TU Vienna, 2013.

[2] M. Hofer, E. Schmidt and M. Schrödl, *Design of a Three Phase Permanent Magnet Biased Radial Active Magnetic Bearing Regarding a Position Sensorless Control*, IEEE, 2009.

[3] M. Ahrens and L. Kucera, *Cross feedback control of a magnetic bearing system controller design considering gyroscopic effect*, Proceedings of the Third International Symposium on Magnetic Bearings, Federal Institute of Technology Zrich,, pp. 177191.

[4] G. Schweitzer, *Magnetic Bearings Theory, Design, and Applications to Rotating Machinery*, Springer Berlin, 2009.

[5] M. Hutterer, M. Hofer, T. Nenning and M. Schrödl, *LQG Control of an Active Magnetic Bearing with a Special Method to consider the Gyroscopic Effect*, 14th International Symposium on Magnetics Bearings, Linz, 2014.

[6] M. Hutterer, M. Hofer and M. Schrödl, *Decoupled Control of an Active Magnetic Bearing System for a High Gyroscopic Rotor*, International Conference on Mechatronics, Nagoya, 2015.

[7] R. Herzog, P. Bühler,, C. Gähler and R. Larsonneur, *Unbalance Compensation Using Generalized Notch Filters in the Multivariable Feedback of Magnetic Bearings*, IEEE Transactions on Control Systems Technology, September 1996.

[8] B. Shaifi, S. Beale, P. LaRocca and E. Cusson, *Magnetic bearing control systems and adaptive forced balancing*, IEEE Contr. Syst. Mag., vol. 14, no.2, Apr. 1994.

[9] C.R. Knospe, *Stability and performance of notch filter controllers for unbalance response*, Int. Symp. Magn. Suspension Technol.,NASA Langley Research Center, Hampton, VA, NASA Conf, Pub.3152, 1991.

[10] N.K. Rutland, P.S. Keogh and C.R. Burrows *Comparison of controller design for attenuation of vibration in a rotor-bearing system under synchronous and transient condition*, 4th International Symposium on Magnetics Bearings, Zürich, 1994.

[11] N. Taguchi, T. Ishimatsu, S.J. Woo and C. Ghler *Unbalance compensation of magnetic bearings*, IECON, Beregna, 1994.

[12] J. Adamy, *Nichtlineare Systeme und Regelungen*, Springer Berlin, 2014, ISBN 978-3-642-45012-9.