Index Reduction and Regularisation Methods for Multibody Systems

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1. INTRODUCTION

The use of object–oriented simulation tools for modelling of physical or mechanical systems leads to systems of differential–algebraic equations (DAEs) with a high differential index. The differential index indicates the minimal number of differentiations of the system which are necessary to extract a system of ordinary differential equations from the differentiated system. Especially in mechanics the use of a global coordinate system to describe the different occurring states leads to a DAE that usually has differential index three. In general the numerical solution of DAEs with high index by conventional solution methods for ordinary differential equations is very complex. Therefore methods for solving this problem are necessary, which leads to the so–called index reduction. In the following seven methods, where most of them are discussed in detail in Hairer, and Wanner (2002), are considered.

2. DIFFERENTIATION AND SUBSTITUTION OF THE CONSTRAINTS

In this approach the way of reducing the index is to differentiate the constraints $g(x) = 0$ and substitute the constraints by their derivatives, until the system has differential index one. The problem with this method is that due to the differentiation there is a loss of information and so the necessary initial values for the back-integration are unknown and so the numerical "drift–off" occurs.

3. BAUMGARTE–METHOD

The Baumgarte–Method can only be used for DAEs with differential index three. The initial point of this method is the index–1–formulation of the DAE with index three. The constraint equations $\ddot{g}(x) = 0$ are substituted by a linear combination of g, $\dot{g}$ and $\ddot{g}$ of the form where the parameters α and β occur and have to be chosen so that the differential equation is asymptotically stable. The problem of this approach is the exact choice of the parameters α and β.

4. PANTELIDES–ALGORITHM

This algorithm solves the DAE using the so–called "Dummy Derivatives", i.e. if there is a constraint equation an integrator which is connected with the constraint is eliminated by the replacement of a derivative by a dummy variable. On the one hand the algorithm may create a lot of variables and equations, on the other hand the differential index has not to be known for using this method.

5. STABILISATION BY PROJECTION

If the numerical solution does not fulfill the constraints after an integration step, the numerical solution is projected onto the solution manifold, which is given by the constraints $g(x) = 0$ and some of their derivatives with respect to t. A certain procedure leads to a system of differential equations $\dot{y} = f(t, y)$ on the manifold. There are two methods using projection, the orthogonal projection method and the symmetric projection method.

6. METHODS BASED ON LOCAL STATE SPACE TRANSFORMATION

The DAE is not solved on the whole state space, but on a manifold. The obtained system of differential equations on the solution manifold (see section 5) which is solved by the introduction of local coordinate transformations. The difficulty of this method is to find suitable coordinates.

7. GEAR–GUPTA–LEIMKUHLER FORMULATION

The Gear–Gupta–Leimkuhler formulation aims to include the description of the solution manifold by the constraint equations into the equation system. This leads to an overdetermined system, so a correction term is introduced. This DAE has differential index two and can be solved for example with BDF methods or implicit Runge–Kutta methods.

8. CONCLUSION AND OUTLOOK

In this paper, all in all seven methods for the regularisation of DAEs with differential index three were presented. Every method can be applied to solve the DAEs resulting from the equations of motion of mechanical systems. In further studies these methods will be tested by means of several case studies and compared regarding the distance of the numerical solution to the solution manifold, their numerical accuracy and applicability for different tasks. Additionally, methods suitable for DAEs of arbitrary index will be tried out on other than mechanical systems.

REFERENCES

Hairer, E. and Wanner, G. (2002). *Solving ordinary differential equations II*. Springer, Germany.