Spatial Effects in Stochastic Microscopic Models - Case Study and Analysis

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1. INTRODUCTION

In this paper we will not discuss different motivations and applications of microscopic models, but are presenting a technique to investigate the theoretical background of microscopic models. In context of our work we use the term *microscopic* model for dynamic (hereby used to indicate temporal dependent) models, consisting of a high number of similar sub-models, which will furthermore be denoted as *actors*. By this term we want to cover big classes of models like e.g. agent-based models, cellular automata or microsimulation models. As a microscopic model does not necessarily contain a spatial structure we will focus on those kind of models though the presented technique can also be applied on models without spatial relationships.

We furthermore lay special emphasis on the analysis of so called aggregated numbers, typically some kind of sums or statistics. We are analysing the behaviour of those quantities in case of a very large number, respectively in the limit case, an infinite number of individual actors. We especially focus on the influence of spatial relationships between the actors on the aggregated number. Furthermore we apply the results of the theoretical research on three different microscopic models, each of them chosen to particularly point onto an important observation.

2. METHODS

We use the diffusion approximation approach derived N.G. Van Kampen (see Kampen, N. G. van (1982)) in order to predict the temporal behaviour of aggregated numbers of the model by differential equations. Diffusion approximation respectively at least its results are, by knowledge of the author, still core of all theorems developed to perform aggregated analysis of microscopic models - so called mean-field theorems (some examples: Boudec et al. (2007). Benoit et al. (2006)).

Applying the method we derive the mean-field theorem (1) to approximate the expectancy value $\vec{\phi}(t)$ of the aggregated microscopic model by the solution of a differential equation and show some convergence results.

$$\frac{d\vec{\phi}}{dt} = \sum_{i\neq j}(\vec{e_i}-\vec{e_j})\,\omega_{\vec{\phi},\vec{\phi}+\frac{1}{N}(\vec{e_i}-\vec{e_j})}, \quad \vec{\phi}(0) = \vec{\phi}_0. \qquad (1)$$

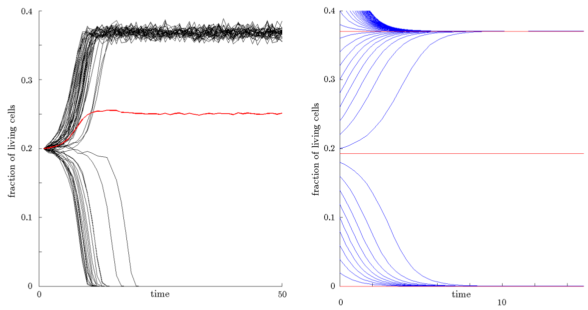

Fig. 1. Left: Sample simulation runs of a modified "Game of Life" model for specific initial condition including mean value (red). Right: Steady state analysis of (1).

3. TEST CASES

Furthermore three simple test cases are used to show how the presented technique can be applied to especially calculate means of counting variables for very simplified toy-models. The three cases are chosen properly to draw attention onto three different issues. A simple agent-based model based on a SIR strategy will be used for a direct verification of the mean-equation (1) an how it is applied. A cellular automaton (CA) based on John H. Conways Game of Life furthermore shows that the technique can fail occasionally caused by clustering effects and spatial relationships. A modification of the CA finally helps to understand the value of the theoretical analysis as a steady-state analysis of equation (1) needs to be performed (see Figure 1).

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