"Second-order magic" radio-frequency dressing for magnetically trapped ^{87}Rb atoms

Georgy A. Kazakov, Thorsten Schumm

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ATOMINSTITUT

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Outline

Introduction

- Instability of atomic clock: Allan deviation
- Magic magnetic trapping: “1^{st} order magic” DC field
- Magic trapping in Ioffe-Pritchard trap

Second order magic conditions

- Modification of adiabatic potential with RF dressing
- A few words about calculation
- Second-order magic conditions

Role of the field uncertainty

- DC and RF field magnitudes
- RF field polarization
- Influence on the position-dependent shift

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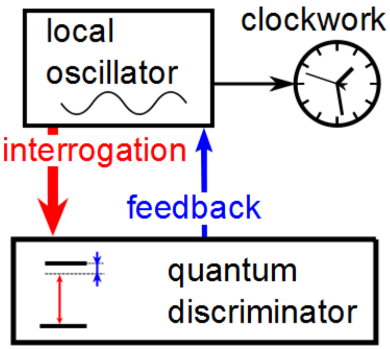
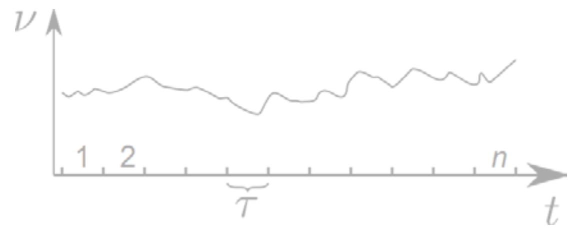


Allan deviation: measure of instability



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Shot noise limit of the short-term stability:

$$\sigma_{y,QPN}(\tau) = \frac{1}{2\pi\nu_a T_R I}\sqrt{\frac{T_c}{N\tau}}$$

Total cycle time

clock transition frequency

Ramsey time

fringe contrast

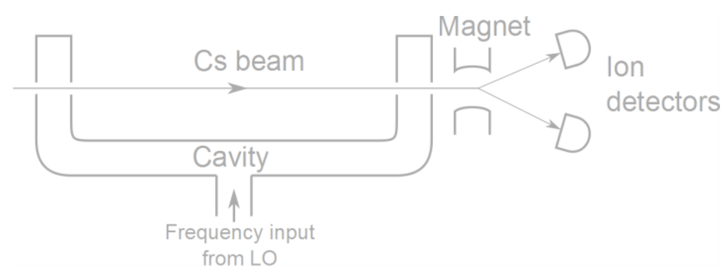
number of atoms

Microwave clocks with free atoms:

Cs beam standard (PTB CS2):

$T_R = 8$ ms, $\sigma_y(1s) = 3.6\times10^{-12}$

A. Bauch, Meas. Sci. Technol. ***14****, 1159 (2003)*



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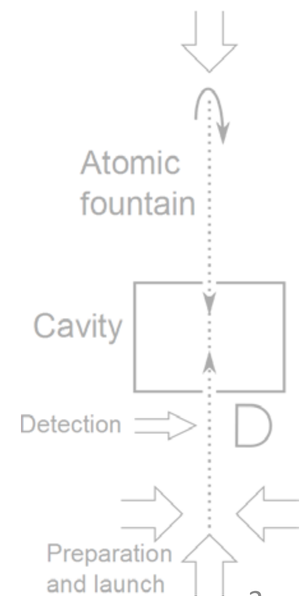

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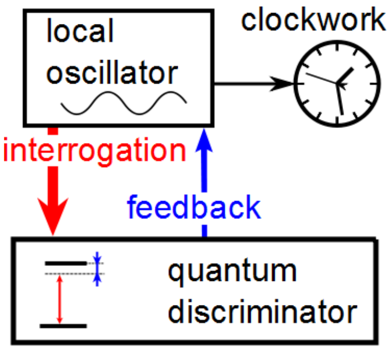

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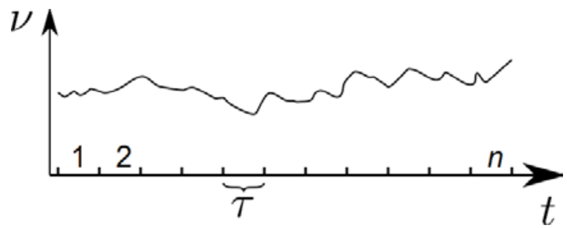

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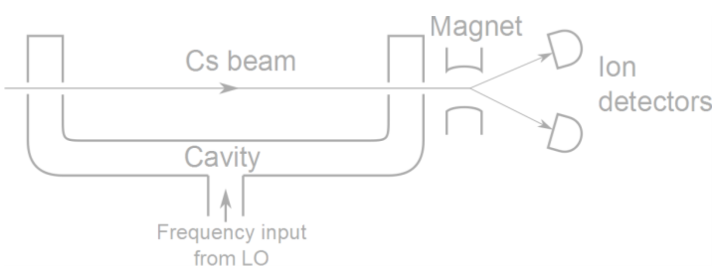

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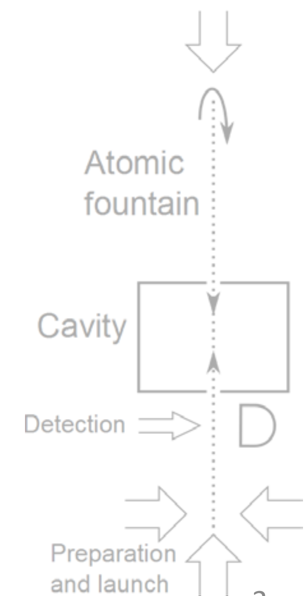

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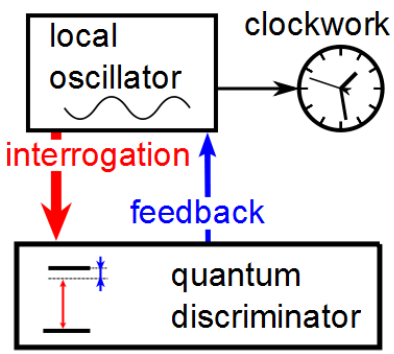

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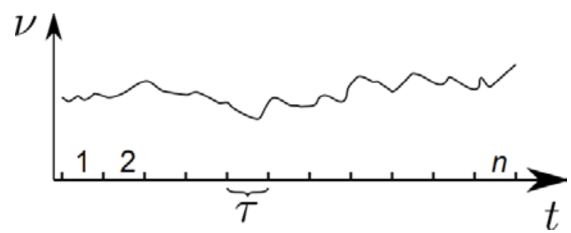

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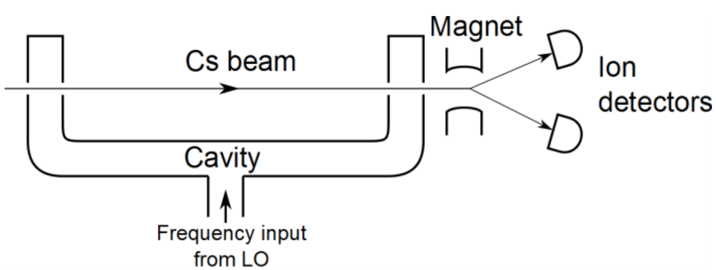

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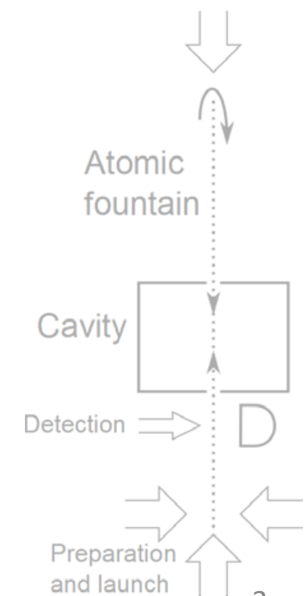

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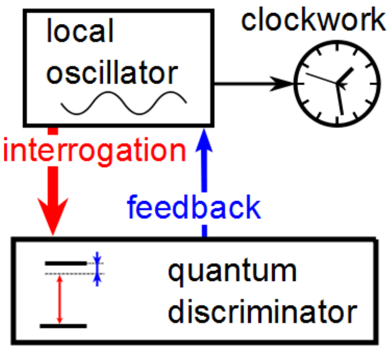

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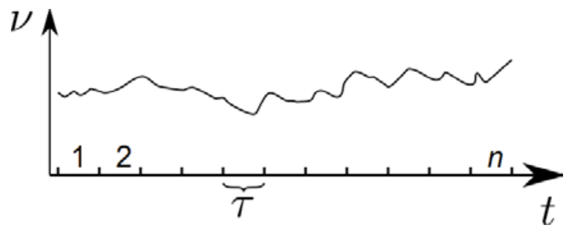

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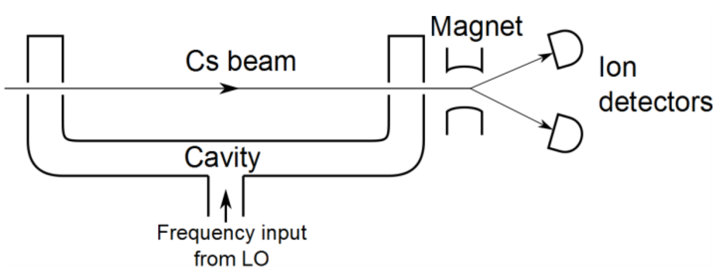

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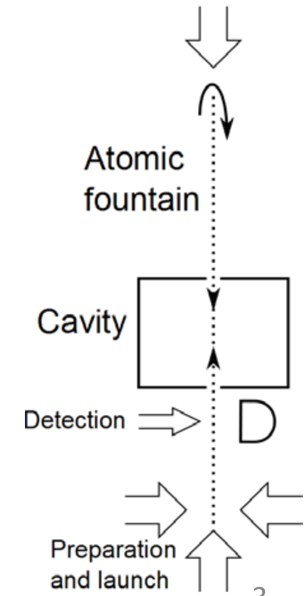

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Magic magnetic trapping: "1st order magic" DC field

Trapped atoms instead of ballistic

Using of trapped atoms allows to:

1. Increase T_R
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Trapping is a modification of the energy levels in some external field.
For the clock states such modification should be equal (*magic trap*).
It is **impossible** to compensate the difference in **all orders** of the potential energy, but possible in the 1st order („*1st order magic*" *trap*),
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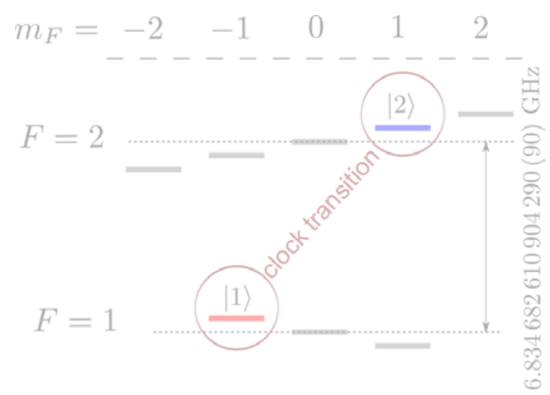

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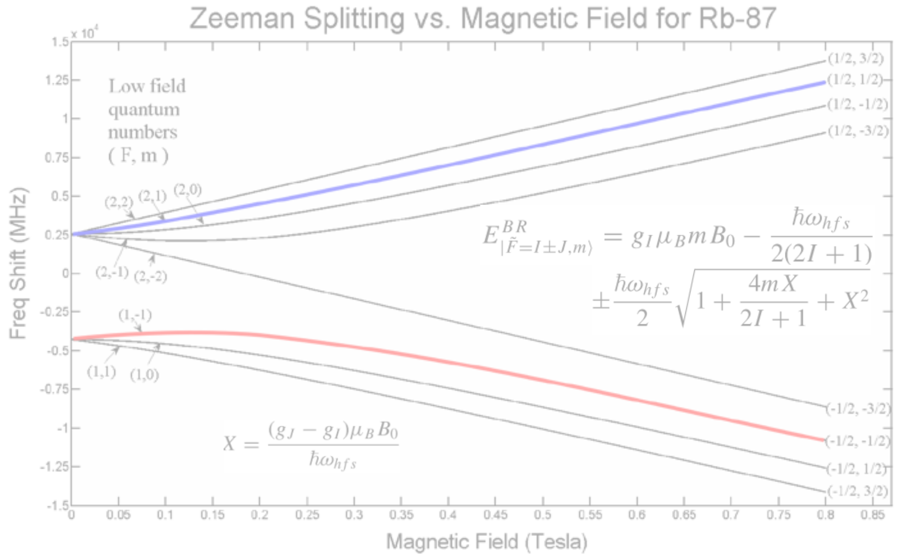

"1st order magic" DC field

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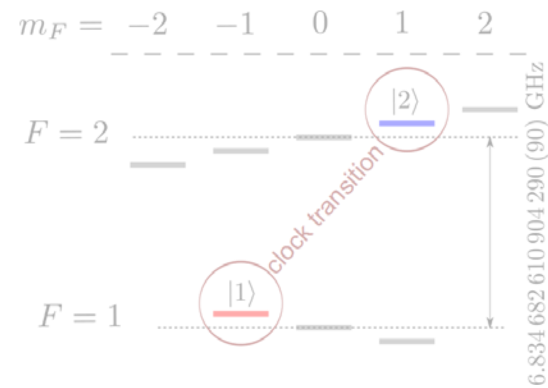

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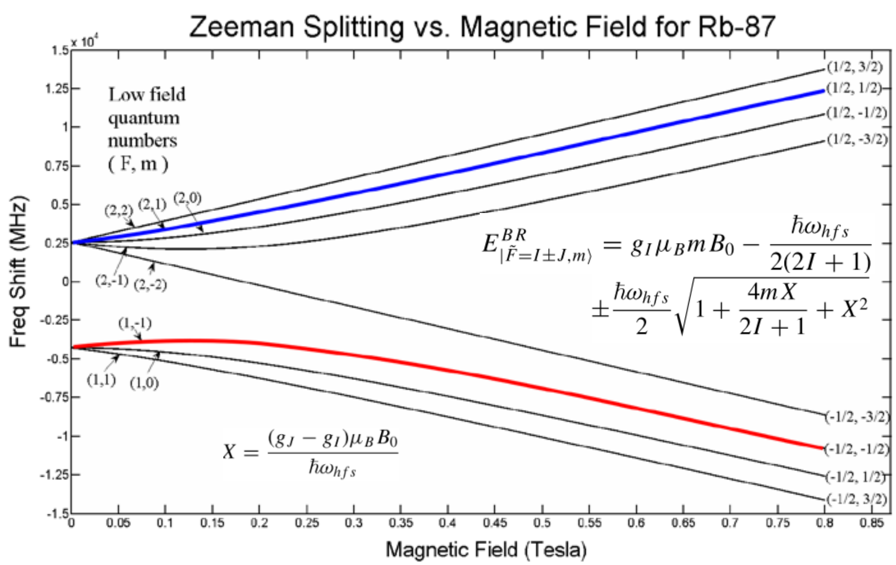

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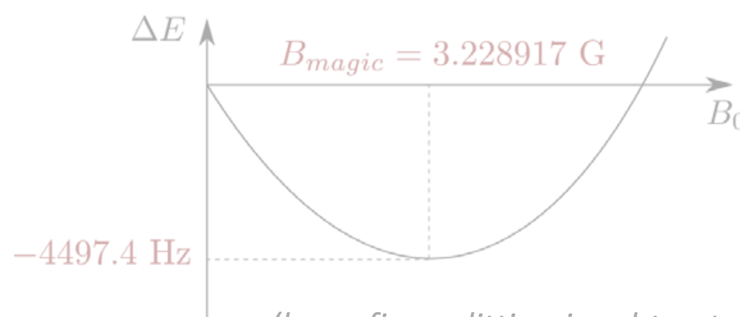

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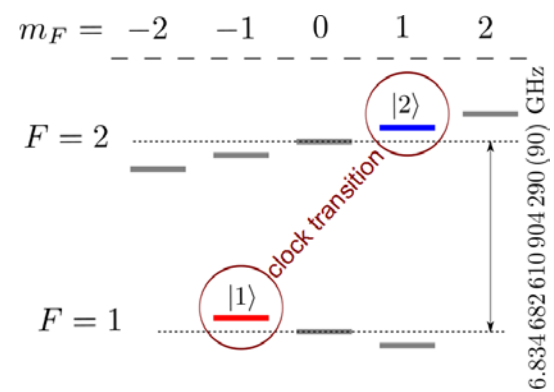

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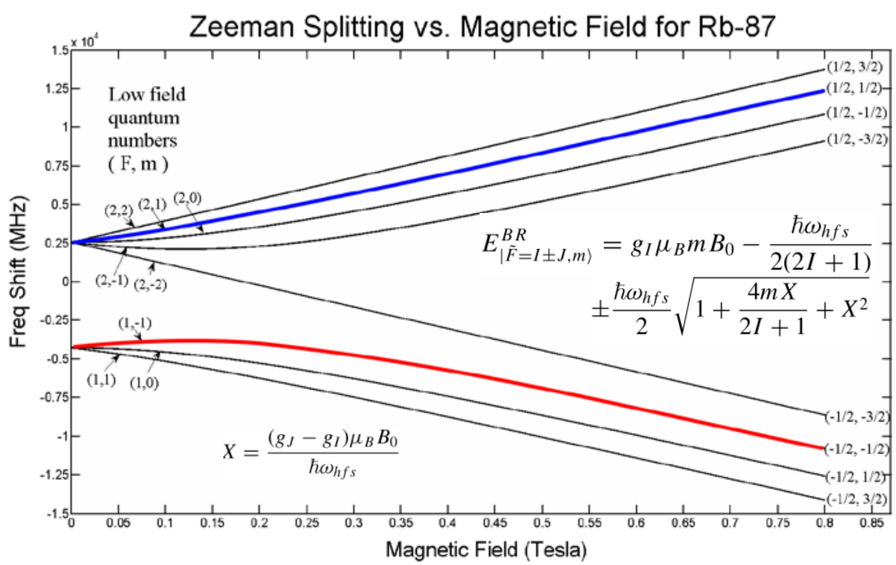

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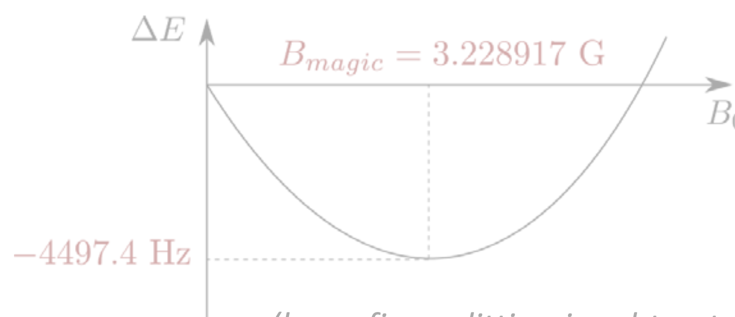

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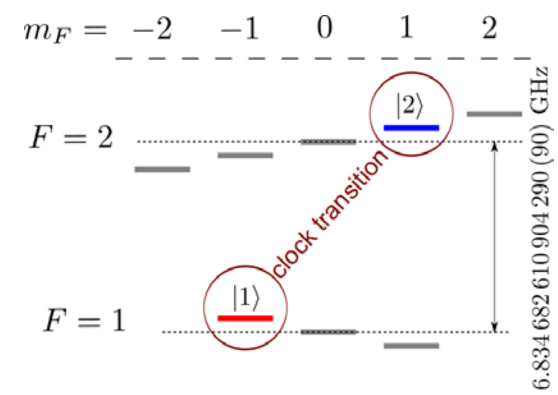

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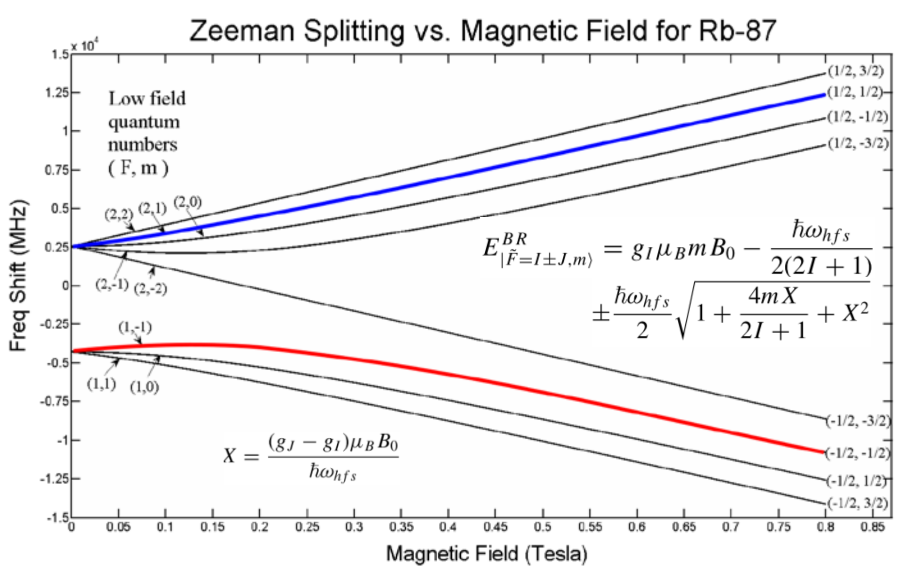

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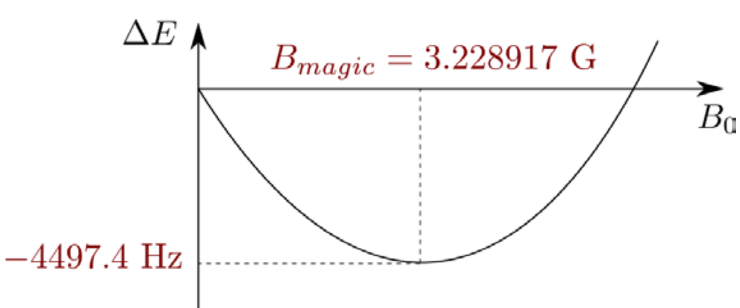

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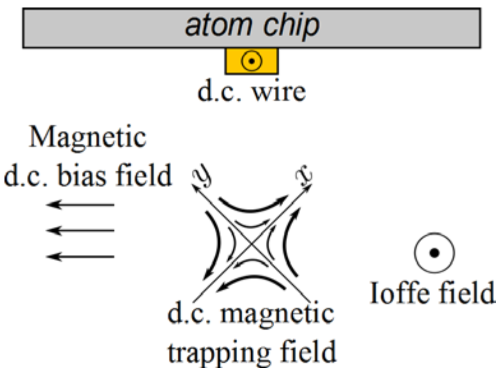
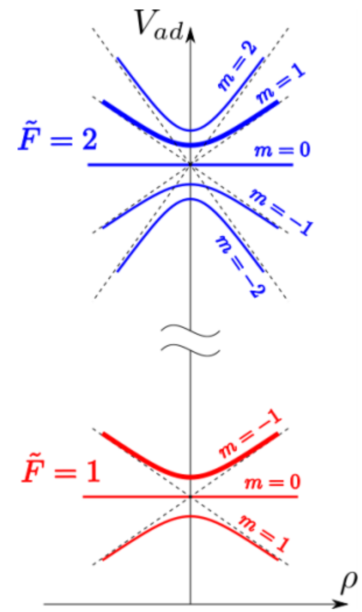


$$\vec{B}_0 = \vec{e}_z B_I + G(\vec{e}_x x - \vec{e}_y y)$$

*P. Rosenbusch, Appl. Phys. B **95**, 227 (2009)*

Energy levels:

Spatial dependence of the adiabatic potential for different Zeeman states



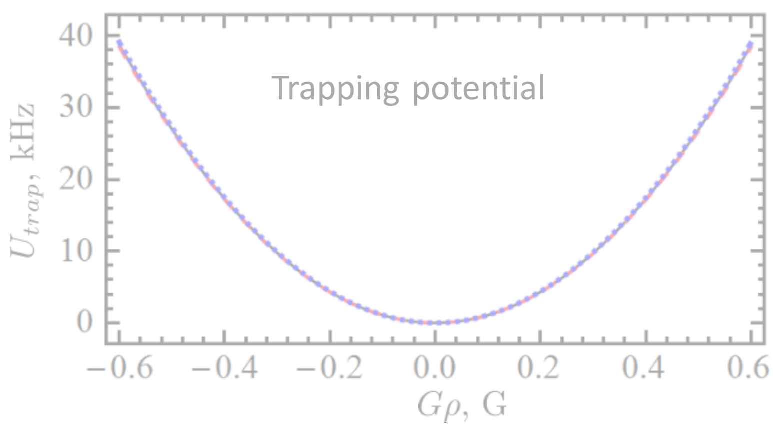
For „1st order magic“ conditions:

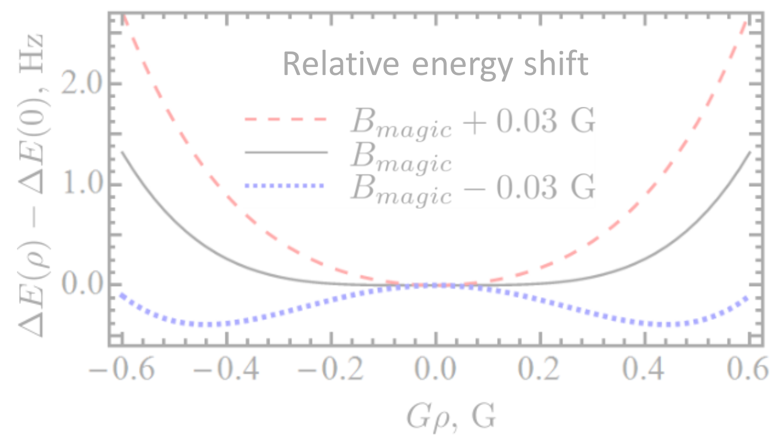
$$\Delta E(\chi) = A_0 + A_2\chi^2 + A_3\chi^3 + \cdots$$

$$A_0 \approx -4497.4 \text{ Hz}, \qquad \chi = G^2\rho^2$$

$$A_2 = \frac{1}{2}\frac{1}{4B_{\text{magic}}^2}\frac{\partial^2 \Delta E}{\partial B^2}\bigg|_{B=B_{\text{magic}}} \approx 10.34 \text{ Hz/G}^4$$

Trapping potential and relative energy shift as a functions of position





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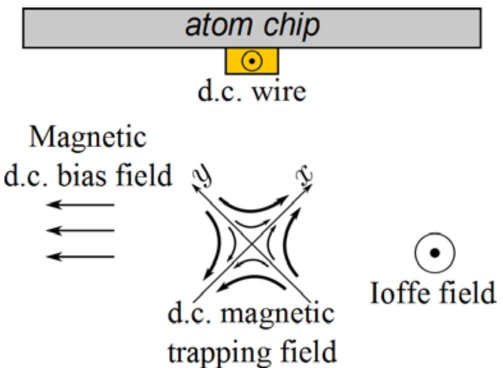

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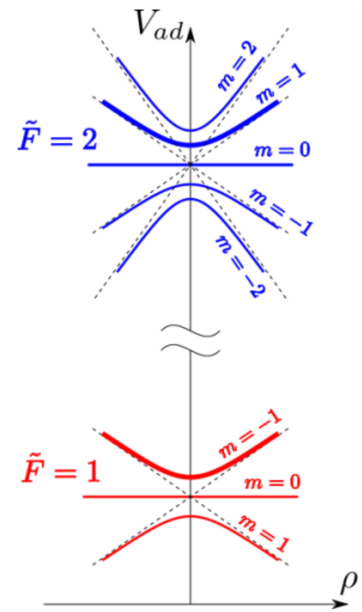

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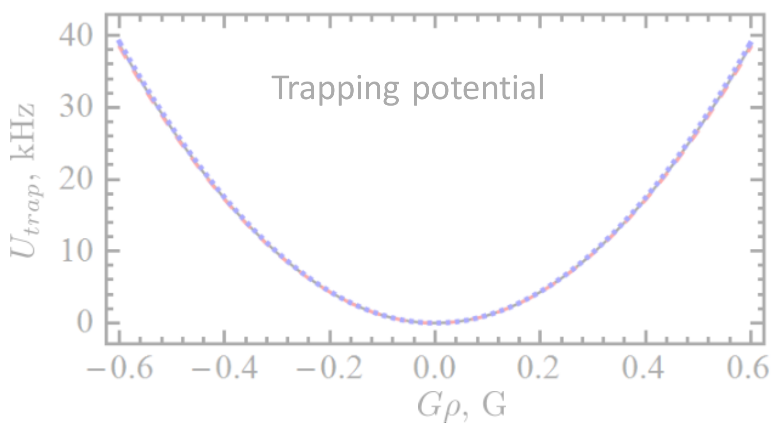

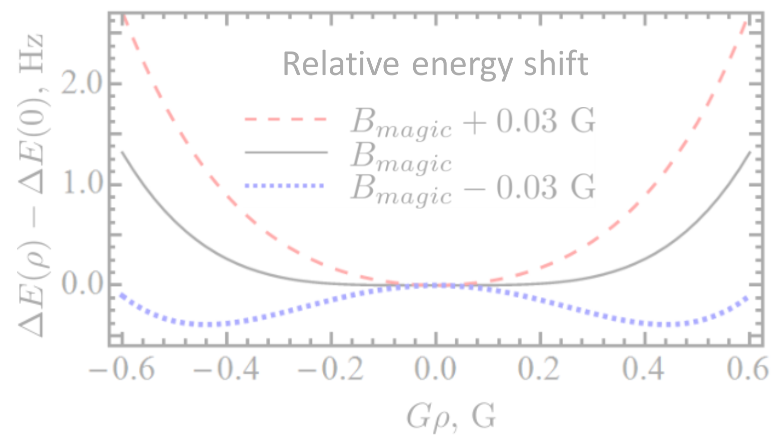

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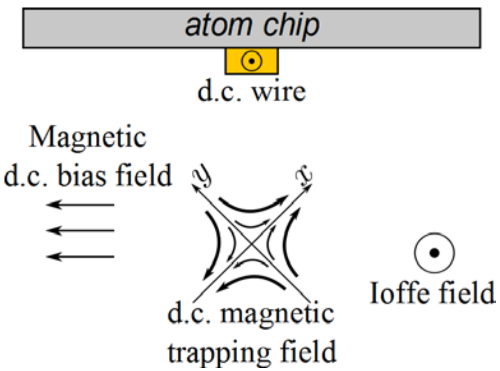

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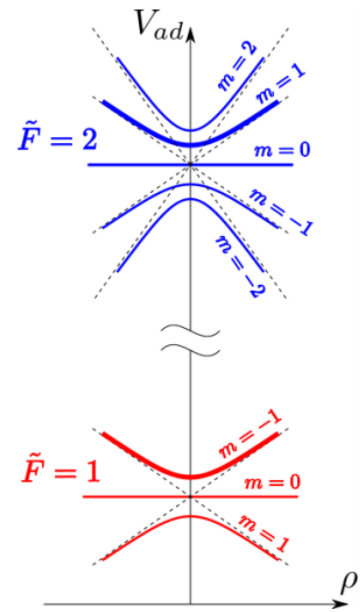

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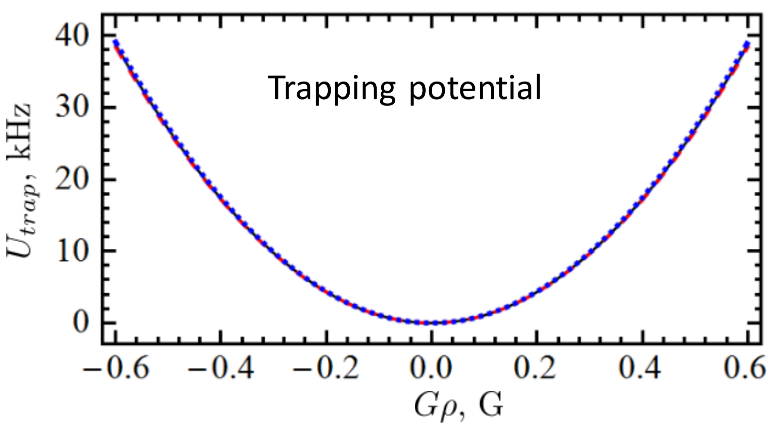

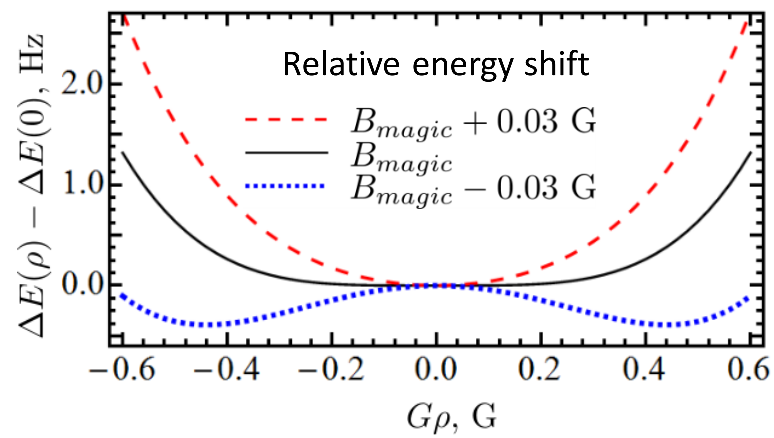

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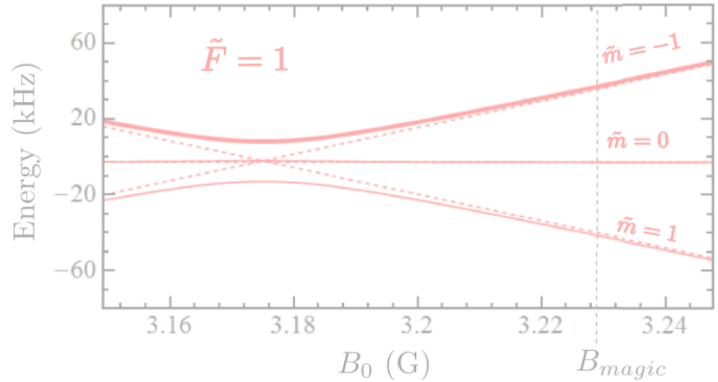

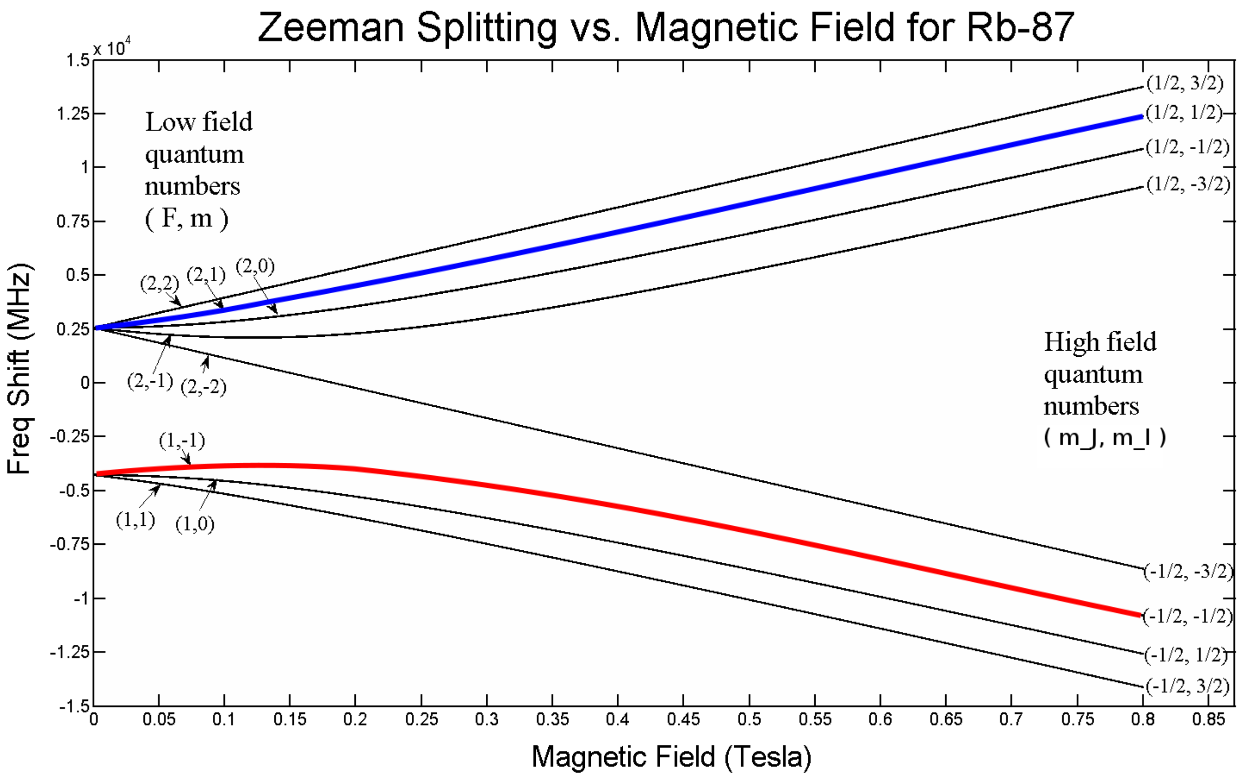

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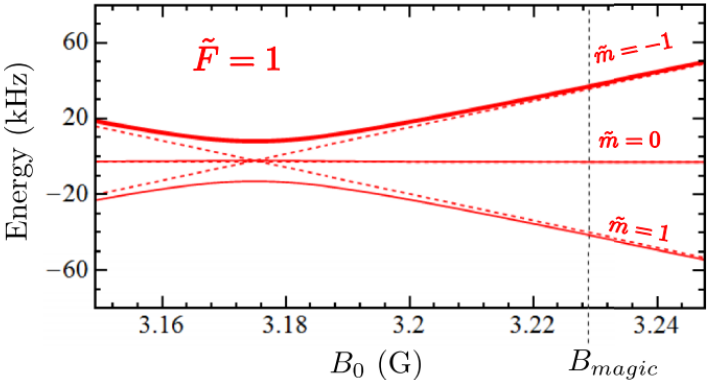

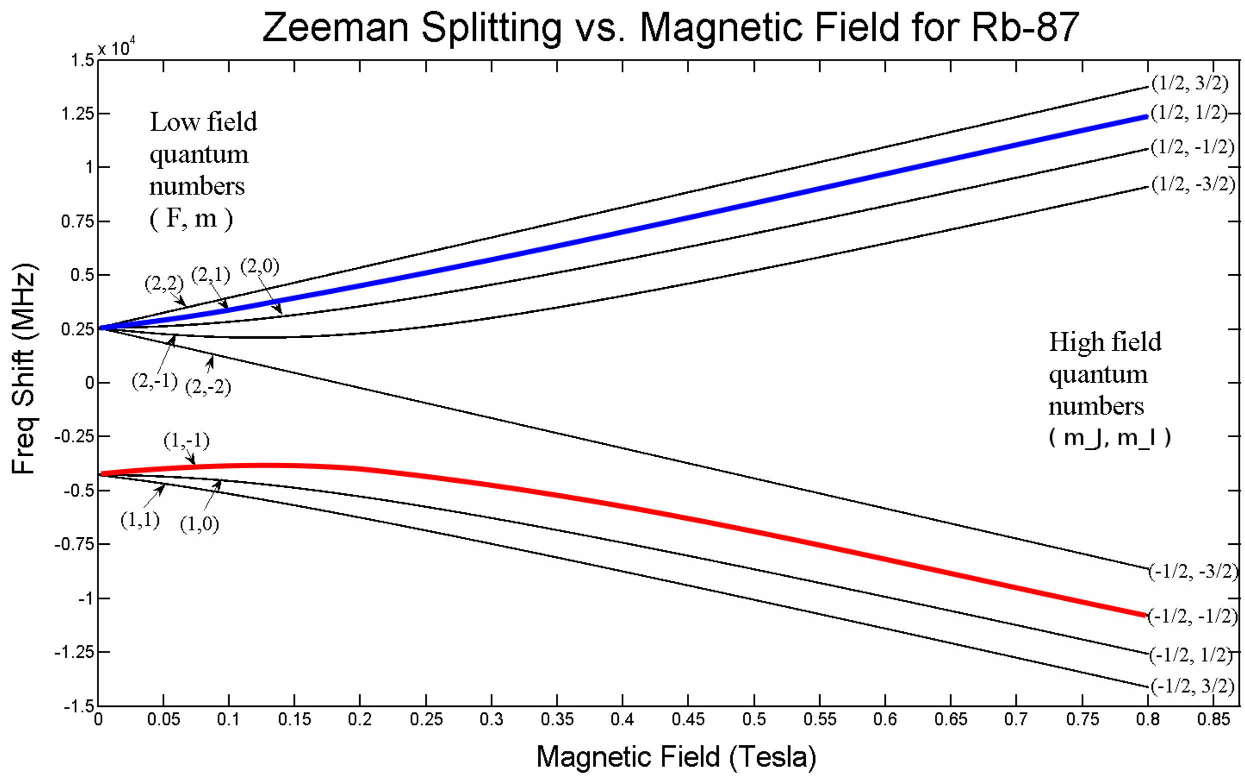

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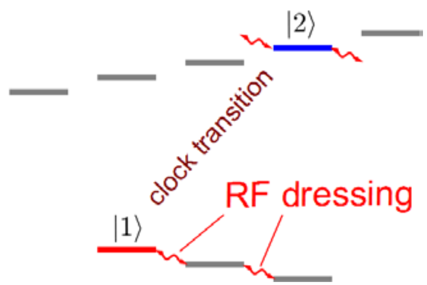
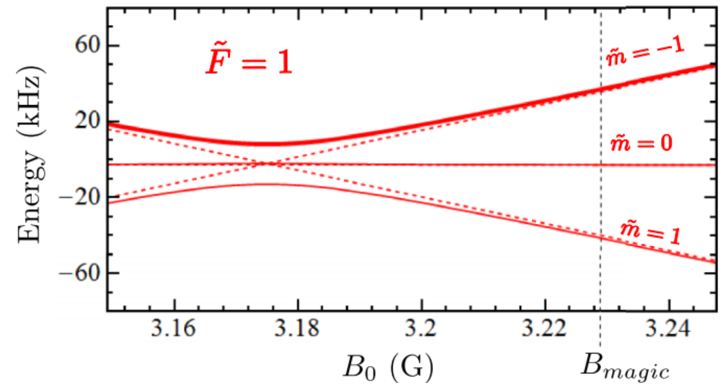
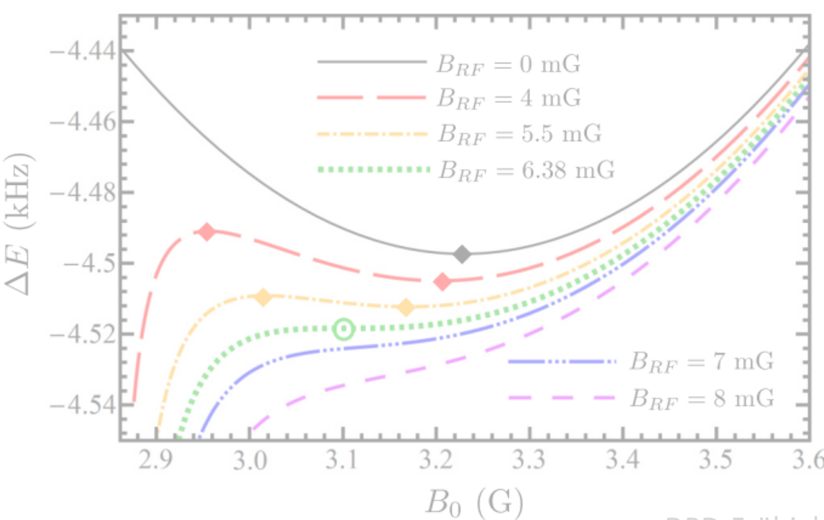
⇒ **use circularly polarized RF field for dressing**



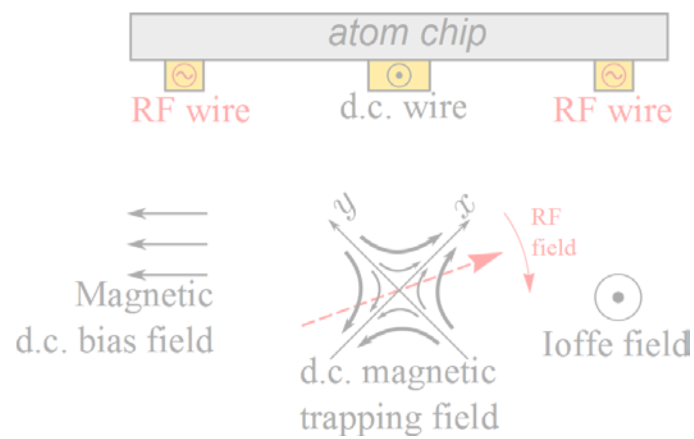


Modification of ΔE(B_0) with RF dressing

$\vec{B}_0 \perp \vec{B}_{rf}, \quad \omega = 2\pi \cdot 2\ \mathrm{MHz}$, *left-hand polarization*



Schematic of a chip-based Ioffe-Pritchard trap with rf dressing



$$\vec{B}_0 = \vec{e}_z B_I + G(\vec{e}_x x - \vec{e}_y y)$$

$$\vec{B}_{\mathrm{rf}} = \frac{B_{\mathrm{rf}}}{2}[(\vec{e}_x \cos\delta - i\vec{e}_y \sin\delta)e^{i\omega t} + \mathrm{c.c.}]$$

Second order magic conditions

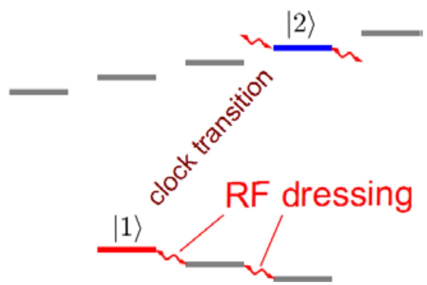
Modification of adiabatic potential with RF dressing

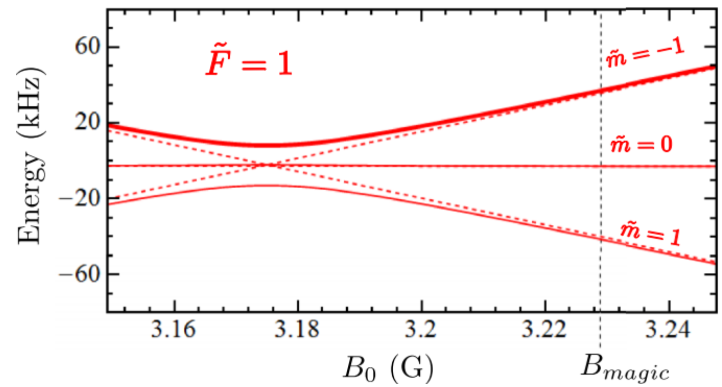
Idea:

Adiabatic potential of the state |1⟩ is **concave** (as a function of B) but that of the state |2⟩ is **convex**

$\Rightarrow$ **use red-detuned RF dressing to modify the trapping potential** in order to **make** the adiabatic potential of the **state |1⟩ also convex**

$\Rightarrow$ **use circularly polarized RF field for dressing**





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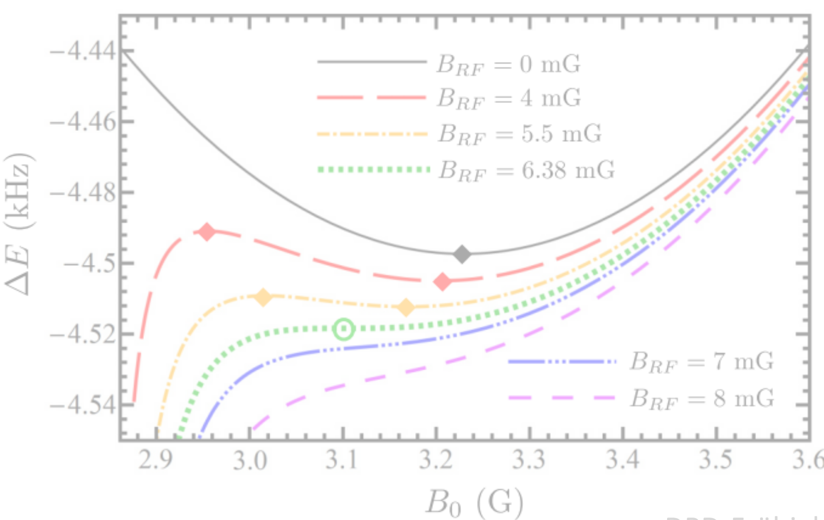

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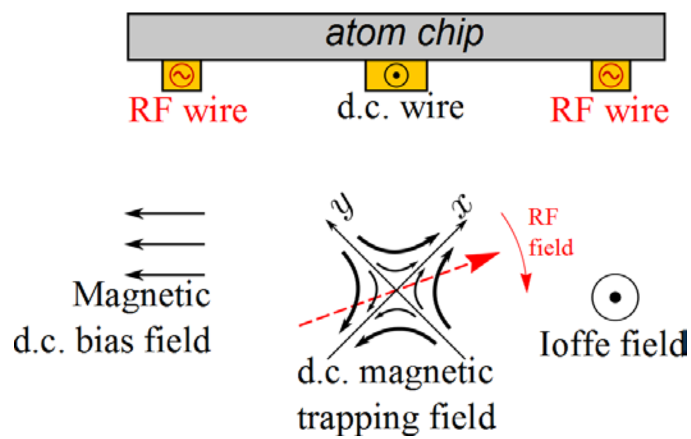

$$\vec{B}_0 = \vec{e}_z B_I + G(\vec{e}_x x - \vec{e}_y y)$$

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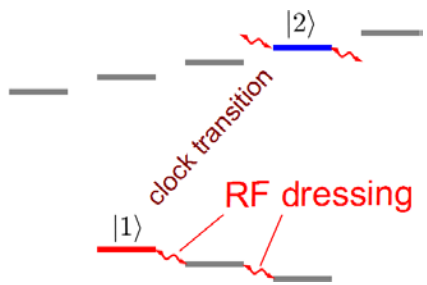

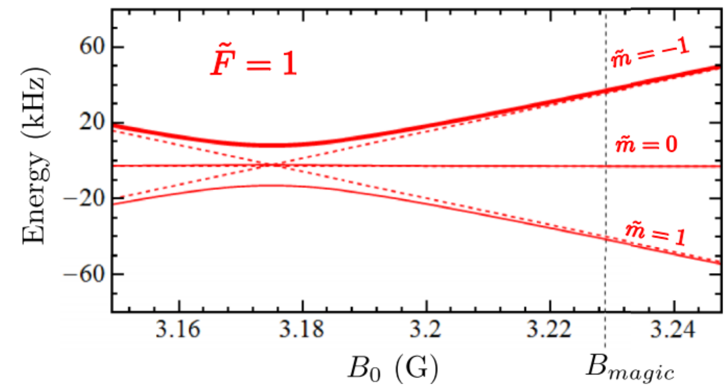

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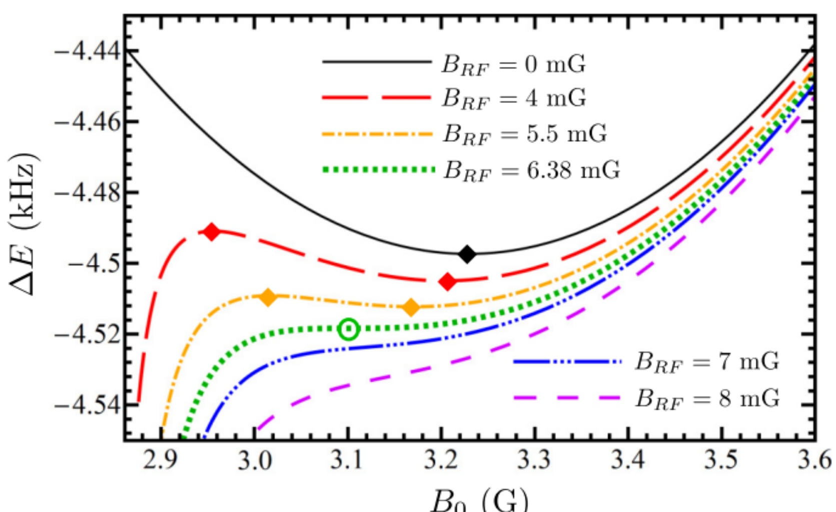
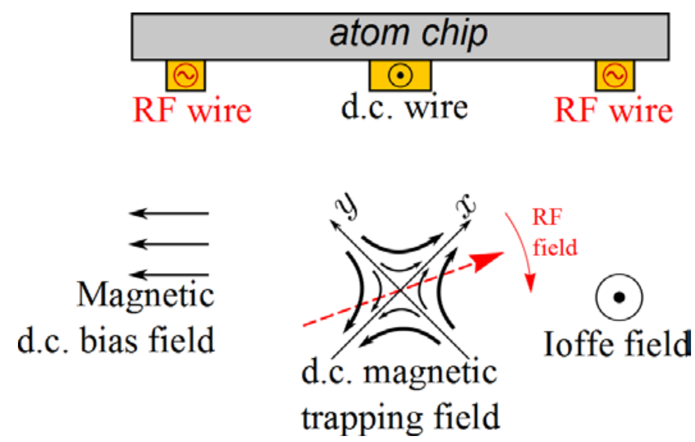
$\vec{B}_0 \perp \vec{B}_{rf}$, $\omega = 2\pi \cdot 2$ MHz, *left-hand polarization*



Schematic of a chip-based Ioffe-Pritchard trap with rf dressing



$$\vec{B}_0 = \vec{e}_z B_I + G(\vec{e}_x x - \vec{e}_y y)$$

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Second order magic conditions

A few words about calculation

Hamiltonian

It is possible either to apply Floquet theory directly, or to transform the Hamiltonian into the rotating frame using the following ***weak-field Floquet approximation (WFFA):***

$$\langle \tilde{F},m|\hat{\vec{I}}\cdot\vec{B}_{\mathrm{rf}}|\tilde{F},m'\rangle \approx \langle F,m|\hat{\vec{I}}\cdot\vec{B}_{\mathrm{rf}}|F,m'\rangle$$

$$\langle \tilde{F},m|\hat{\vec{J}}\cdot\vec{B}_{\mathrm{rf}}|\tilde{F},m'\rangle \approx \langle F,m|\hat{\vec{J}}\cdot\vec{B}_{\mathrm{rf}}|F,m'\rangle$$

It is valid, if $\omega \ll \omega_{hfs}$ and $B_{\mathrm{rf}} \ll B_0 \ll \hbar\omega_{hfs}/\mu_B$

Then we apply the unitary transformation

$$\hat{H} = \hat{U}_R^{+}\hat{H}^i\hat{U}_R - i\hbar\hat{U}_R^{+}\left(\frac{\partial \hat{U}_R}{\partial t}\right) = \hat{H}_1 + \hat{H}_2$$

$$\hat{U}_R = \exp[i(\hat{P}_{\tilde{1}} - \hat{P}_{\tilde{2}})\hat{F}_{z'}\omega t]$$

and express the components of the RF field in the local frame where the z‘ is along the dc field

$$\vec{B}_{\mathrm{rf}} = \frac{e^{i\omega t}}{2}[\vec{e}_{x'}B_{x'} - i\vec{e}_{y'}B_{y'} + \vec{e}_{z'}B_{z'}] + \mathrm{c.c.}$$

$$B_{x'} = B_{\mathrm{rf}}(\cos\alpha\ \cos\theta\ \cos\delta - i\ \sin\alpha\cos\theta\sin\delta)$$

$$B_{y'} = B_{\mathrm{rf}}(\cos\alpha\sin\delta - i\ \sin\alpha\cos\delta)\,,$$

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$$\boxed{\hat{H}^i = \frac{\hbar\omega_{hfs}}{2}\hat{\vec{J}}\cdot\hat{\vec{I}} + \mu_B(g_J\hat{\vec{J}} + g_I\hat{\vec{I}})\cdot[\vec{B}_0 + \vec{B}_{\mathrm{rf}}(t)]}$$

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In the absence of dressing: Breit-Rabi

$$E^{BR}_{|\tilde{F}=I\pm J,m\rangle} = g_I\mu_B m B_0 - \frac{\hbar\omega_{hfs}}{2(2I+1)} \pm \frac{\hbar\omega_{hfs}}{2}\sqrt{1 + \frac{4mX}{2I+1} + X^2}$$

$$X = \frac{(g_J - g_I)\mu_B B_0}{\hbar\omega_{hfs}}$$

It gives in WFFA (in RWA only $H^{(0)}$ remains)

$$\hat{H}_{\tilde{F}} = \sum_{n=-2}^{2}\hat{H}^{(n)}_{\tilde{F}}\exp(in\omega t)$$

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$$+\frac{\mu_B g_F}{4}[\hat{F}_{\pm}(B_{x'} \mp B_{y'}) + \hat{F}_{\mp}(B^*_{x'} \mp B^*_{y'})]$$

$$\hat{H}^{(1)}_{\tilde{F}} = \frac{\mu_B g_F}{2}B_{z'}\hat{F}_{z'}, \quad \hat{H}^{(2)}_{\tilde{F}} = \frac{\mu_B g_F}{4}\hat{F}_{\mp}(B_{x'} \mp B_{y'})$$

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Floquet theory: J. H. Shirley, Phys. Rev. ***138****, B979 (1965)*

Second order magic conditions
A few words about calculation

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$$\hat{H}_{\tilde{F}}^{(1)} = \frac{\mu_B g_F}{2}B_{z'}\hat{F}_{z'}, \quad \hat{H}_{\tilde{F}}^{(2)} = \frac{\mu_B g_F}{4}\hat{F}_\mp(B_{x'} \mp B_{y'})$$
$$\hat{H}_{\tilde{F}}^{(-1)} = \hat{H}_{\tilde{F}}^{(1)+}, \quad \hat{H}_{\tilde{F}}^{(-2)} = \hat{H}_{\tilde{F}}^{(2)+}$$

Floquet theory: J. H. Shirley, Phys. Rev. ***138****, B979 (1965)*

Second order magic conditions
A few words about calculation

ATOMINSTITUT — TU WIEN

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$$\vec B_0 = \vec e_z B_I + G(\vec e_x x - \vec e_y y)$$

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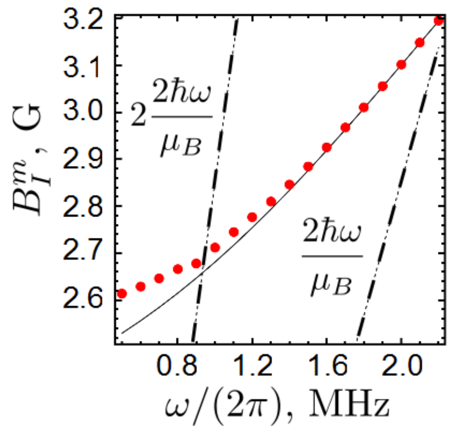
&\quad + \frac{\mu_B g_F}{4}[\hat F_\pm(B_{x'} \mp B_{y'}) + \hat F_\mp(B^*_{x'} \mp B^*_{y'})]\\
\hat H_{\tilde F}^{(1)} &= \frac{\mu_B g_F}{2}B_{z'}\hat F_{z'},\quad \hat H_{\tilde F}^{(2)} = \frac{\mu_B g_F}{4}\hat F_\mp(B_{x'} \mp B_{y'})\\
\hat H_{\tilde F}^{(-1)} &= \hat H_{\tilde F}^{(1)+},\quad \hat H_{\tilde F}^{(-2)} = \hat H_{\tilde F}^{(2)+}
\end{aligned}}$$

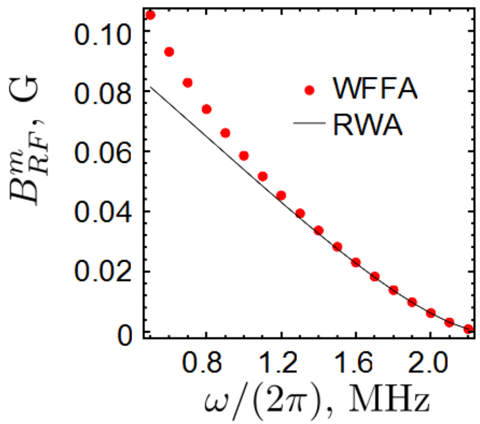
Floquet theory: J. H. Shirley, Phys. Rev. ***138****, B979 (1965)*

Second order magic conditions

Second-order magic conditions

Second-order magic values of dc Ioffe and rf field





Quantitative characterization

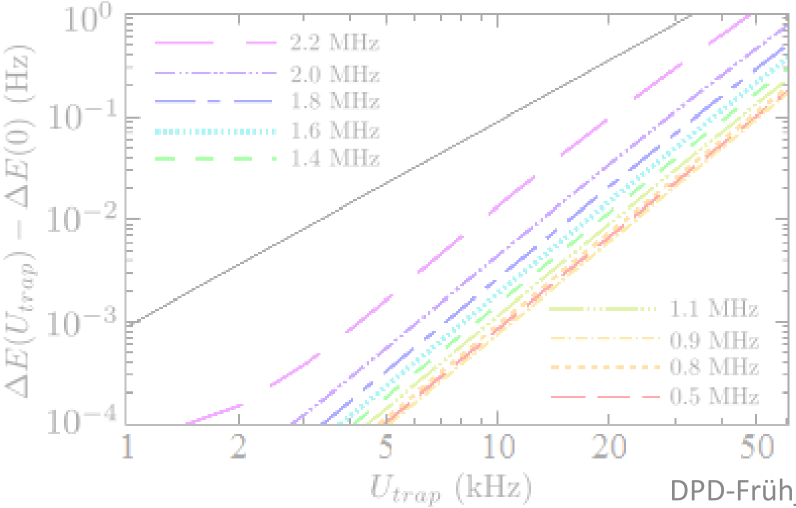
The potential energy is determied mainly by dc field B_0:

$$B_0 = \sqrt{B_I^2 + \chi} \simeq B_I + \chi/(2B_I)$$
$$\chi = G^2\rho^2$$

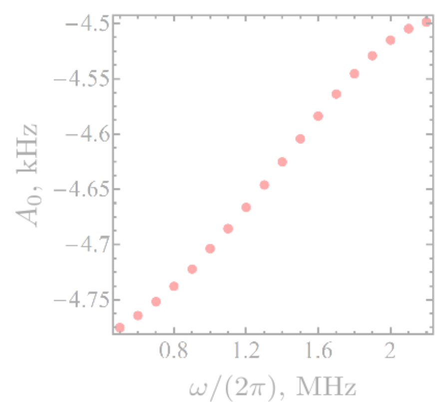
The relative energy shift is

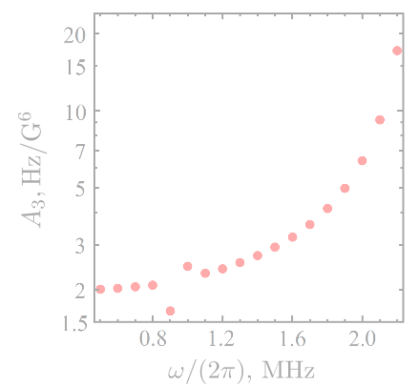
$$\Delta E(\chi) = A_0 + \cancel{A_1}\chi + \cancel{A_2}\chi^2 + A_3\chi^3 + \ldots$$

Relative energy shift for 1st- and 2nd order magic conditions



Coefficients A_0 and A_3





Second order magic conditions

Second-order magic conditions

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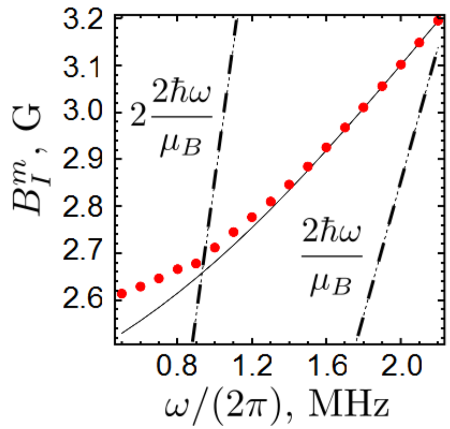

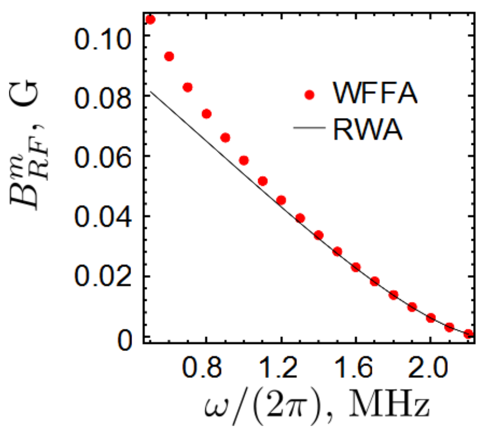

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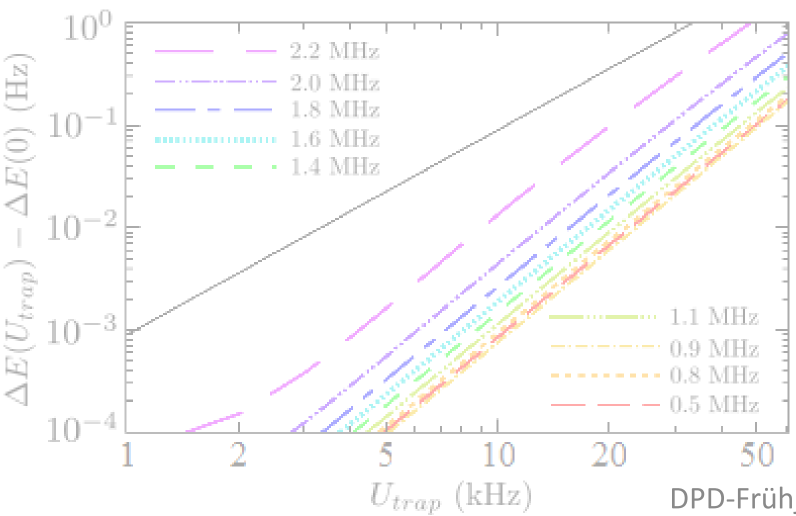

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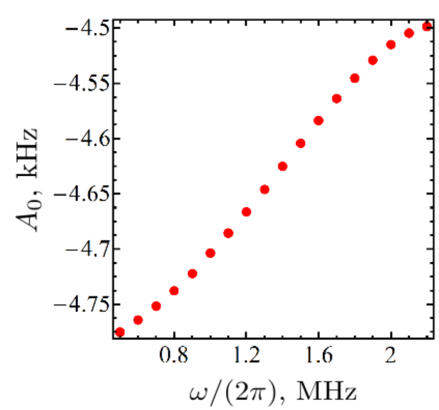

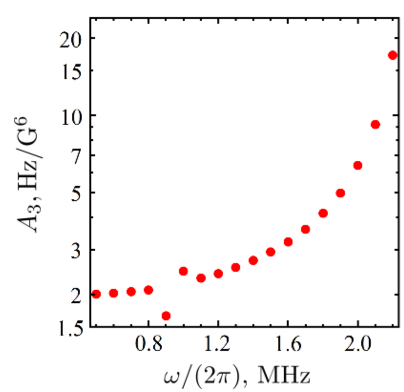

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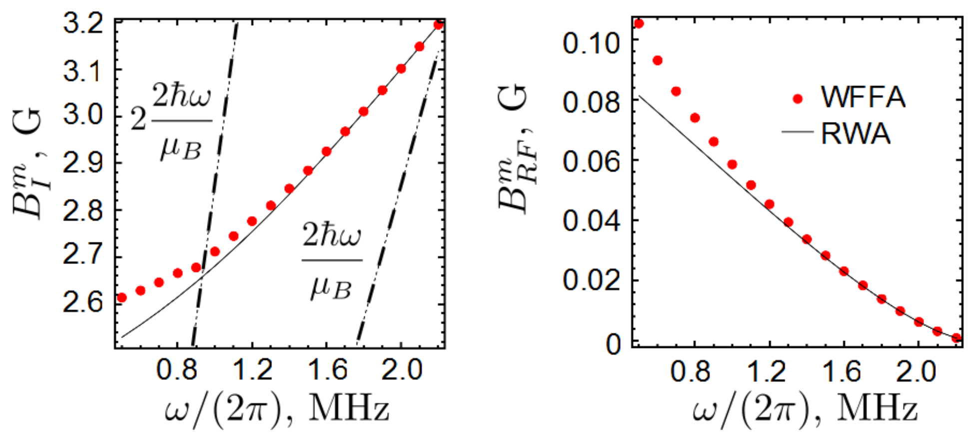

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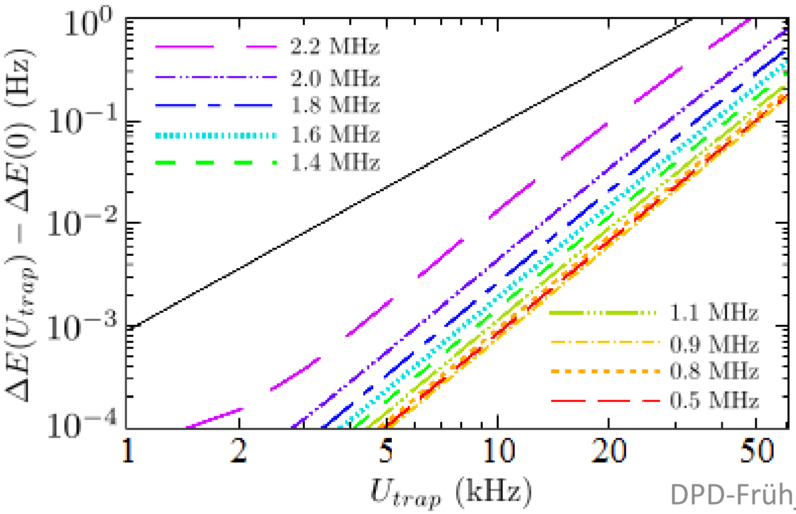

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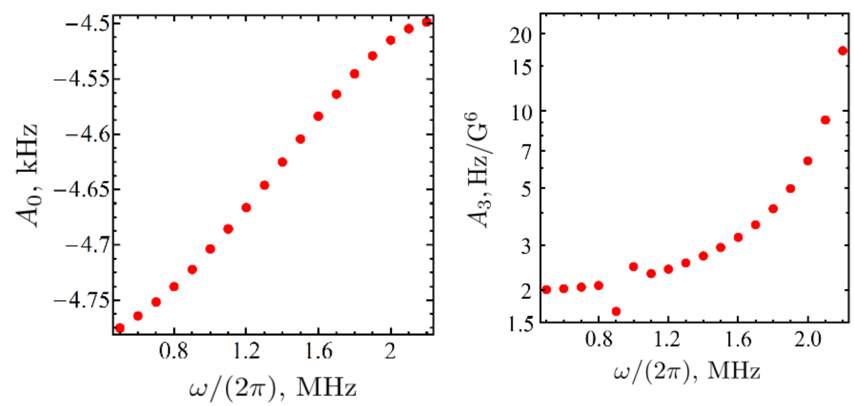

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- DC and RF field magnitudes
- RF field polarization
- Influence on the position-dependent shift

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Role of the field uncertainty

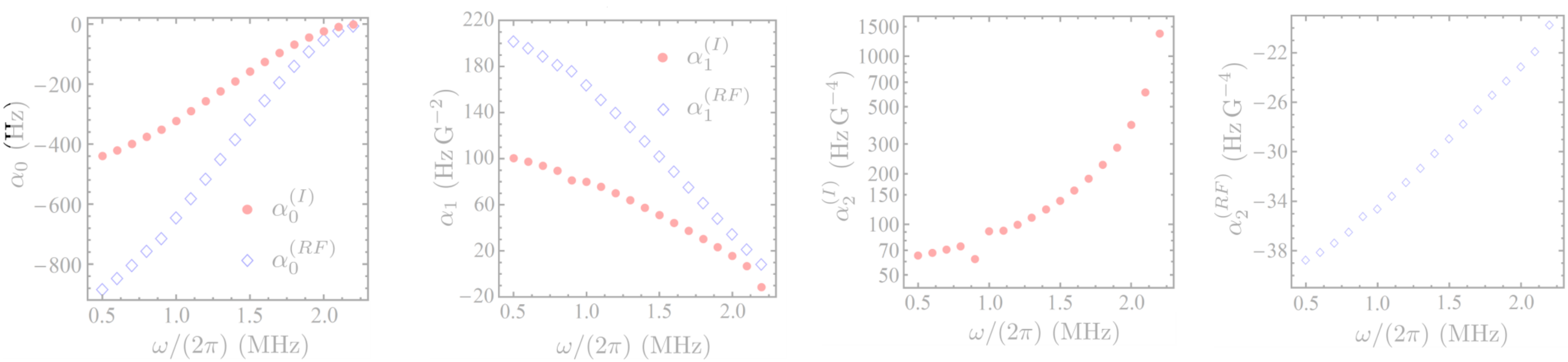
Robustness to variation of field magnitudes

The fields can be controlled up to some extent only. The second-order magic conditions should be robust!
What happens if the magnitudes of Ioffe and rf fields are slightly differ from their magic values?

$$\Delta E(B_I, B_{rf}, \chi) = \sum_n A_n(B_I, B_{rf})\chi^n \qquad \chi = G^2\rho^2$$

$$A_n(B_I, B_{rf}) = A_n + \alpha_n^{(I)}\frac{B_I - B_I^m}{B_I} + \alpha_n^{(rf)}\frac{B_{rf} - B_{rf}^m}{B_{rf}} + \ldots$$

Coefficients α :



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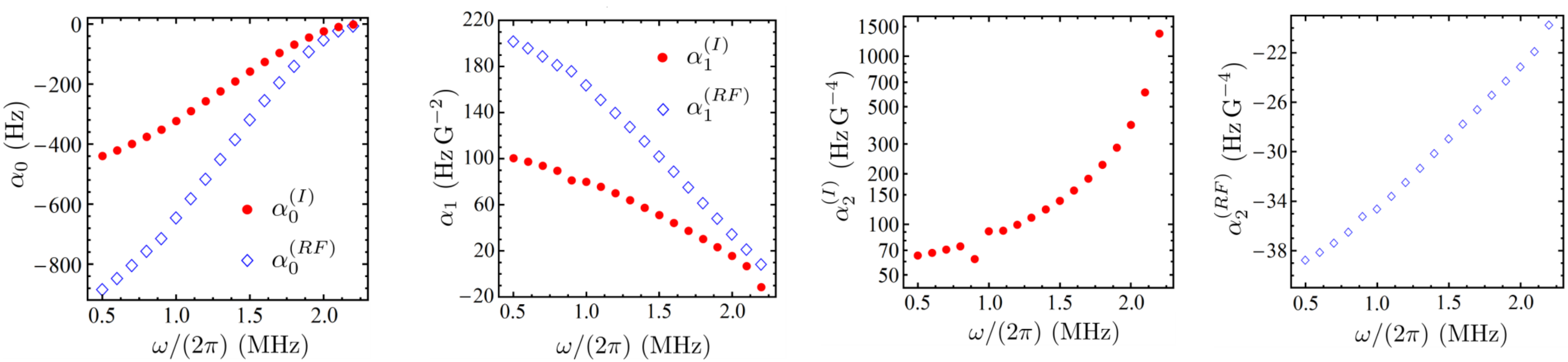

Role of the field uncertainty

Robustness to variation of RF field polarization

The polarization of rf field can be controlled up to some extent only.
Non-perfect circular polarization gives the angular dependence of the relative energy shift

$$\Delta E(\epsilon, \alpha) = \sum_n A_n(\epsilon, \alpha)\chi^n$$

$$A_n(\epsilon, \alpha) = A_n + \beta_n \cos(2\alpha)\epsilon + \gamma_n \epsilon^2 + \dots$$

Here the fields:

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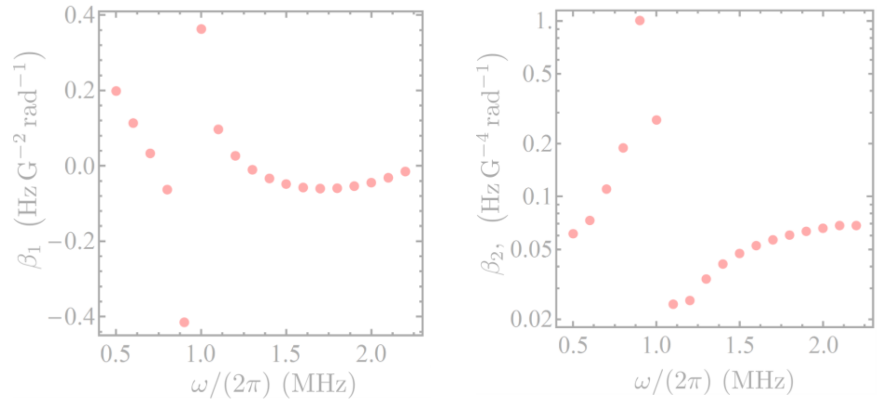
$$\vec{B}_{\mathrm{rf}} = \frac{B_{\mathrm{rf}}}{2}[(\vec{e}_x \cos\delta - i\vec{e}_y \sin\delta)e^{i\omega t} + \mathrm{c.c.}]$$

$$x = \rho\cos\alpha$$

$$y = -\rho\sin\alpha$$

$$\delta = -\pi/4 + \epsilon$$

Coefficients β:



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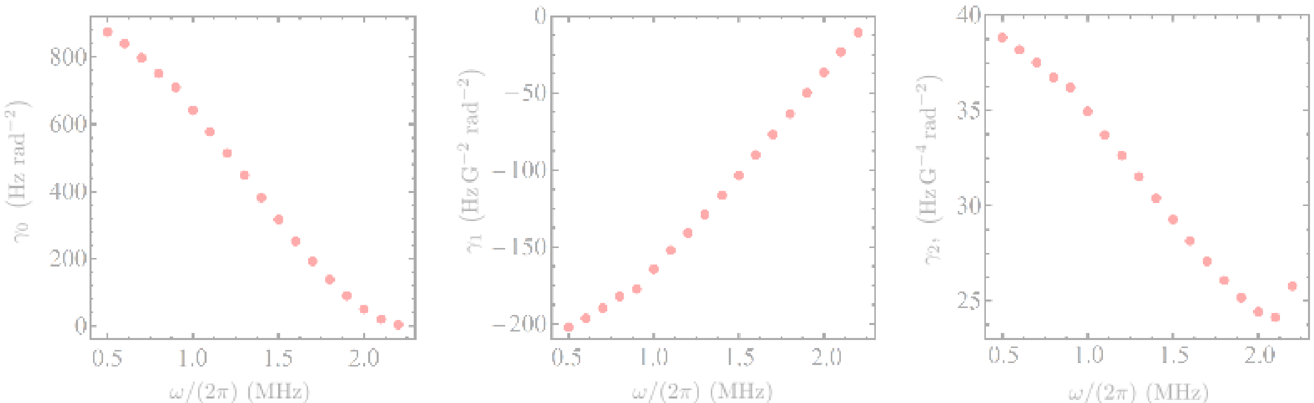

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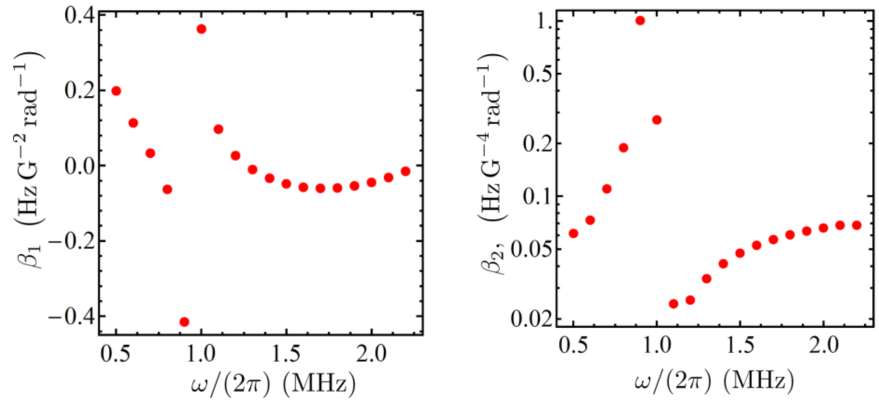

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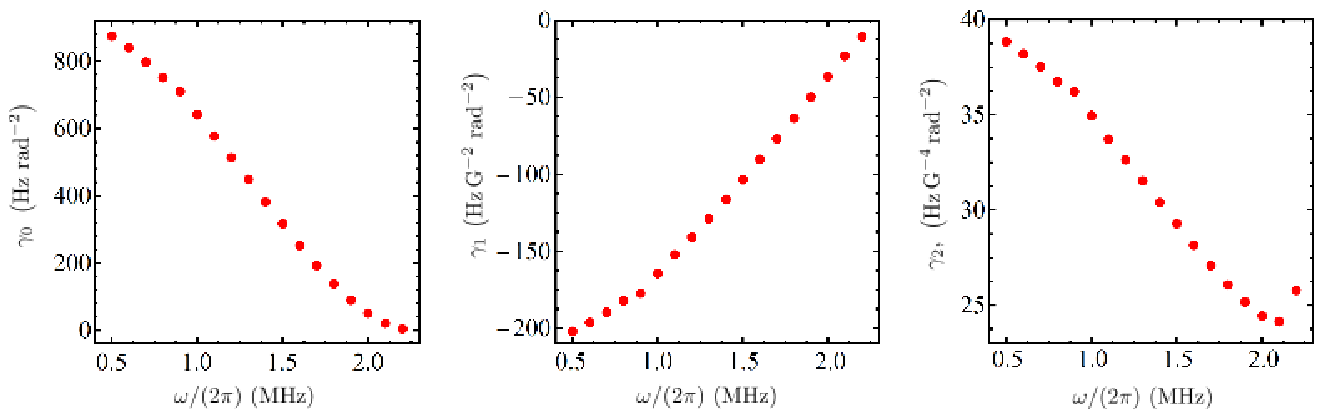


Role of the field uncertainty

Influence on the position-dependent shift

Here we consider how the field uncertainties affect the position-dependent mean-square variation of the relative energy shift:

$$\delta E = \sqrt{\left(\frac{\partial \Delta E}{\partial B_I}\delta B_I\right)^2 + \left(\frac{\partial \Delta E}{\partial B_{RF}}\delta B_I\right)^2 + \left(\frac{\partial \Delta E}{\partial \epsilon}\epsilon\right)^2}$$

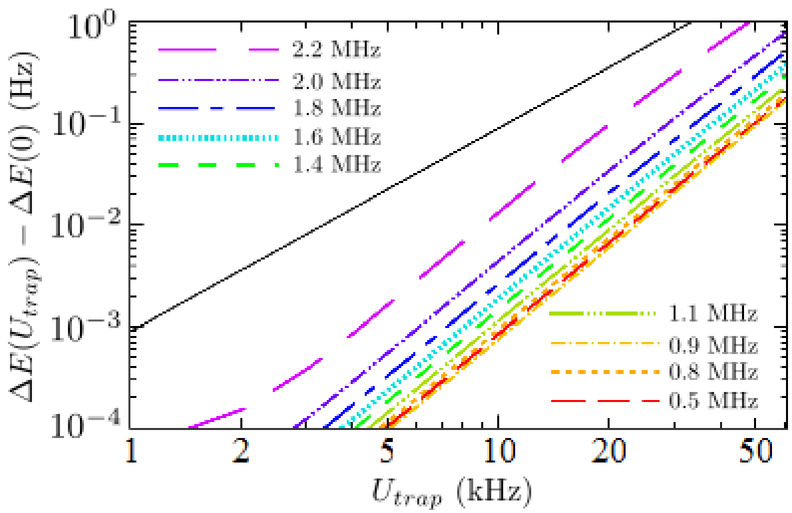
Parameters:

$$\frac{\delta B_I}{B_I} = 2.5 \times 10^{-4} \qquad \frac{\delta B_{rf}}{B_{rf}} = 5 \times 10^{-4} \qquad \epsilon = 0.2^\circ$$

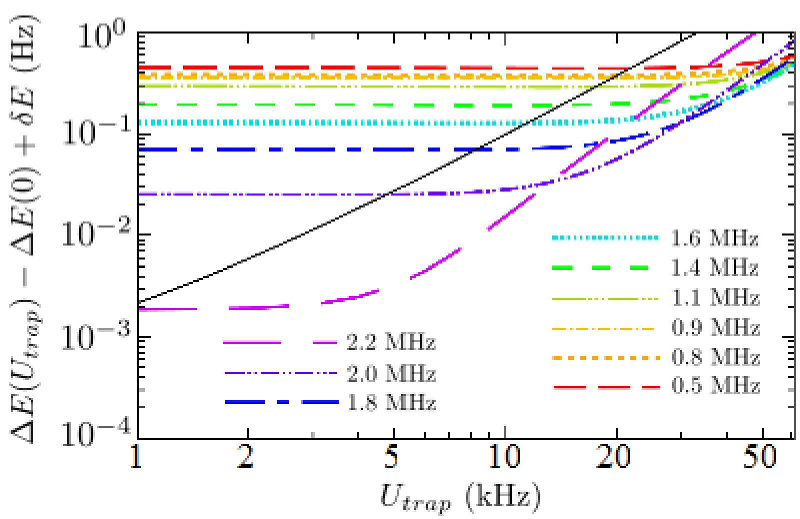
*T.Berrada et al, Nature Communications **4**, 2077 (2013)*

Relative energy shift

without deviations



with magnitude and polarization deviations



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*See our paper PRA **91,** 023404 (2015) for more details!*

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Thank you for your attention!