

# Towards active optical standard based on a bad-cavity superradiant laser: challenges and approaches

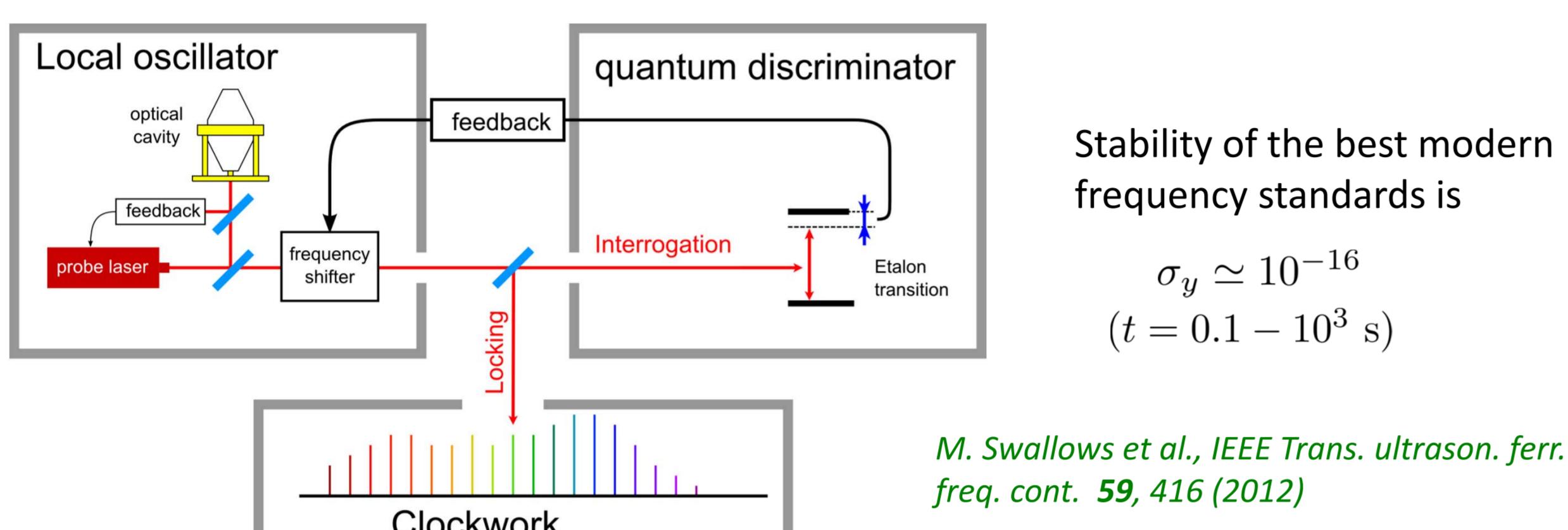
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We consider various approaches to the creation of a high-stability active optical frequency standard, where the atomic ensemble itself produces a highly stable and accurate frequency signal. The short-time frequency stability of such standards may overcome the stability of lasers stabilized to macroscopic cavities which are used as local oscillators in the modern optical frequency standard systems. The main idea is to create a ``superradiant'' laser operating deep in the bad cavity regime, where the decay rate of the cavity field significantly exceeds the decoherence rate of the lasing transition. Two main approaches towards the realization of an active optical frequency standard have been proposed already: the optical lattice laser, and the atomic beam laser. We consider these and some alternative approaches, and discuss the parameters of atomic ensembles necessary to attain the metrology relevant level of the short-time frequency stability.

## Passive optical frequency standards

### Scheme of conventional passive optical frequency standard



### Instability of LO is a source of stability limit for passive standard:

Dick effect:  $\sigma_{y,\text{lim}}(\tau) = \frac{1}{\tau} \sum_{m=1}^{\infty} \left[ \left( \frac{g_m^c}{g_o} \right)^2 + \left( \frac{g_m^s}{g_o} \right)^2 \right] S_y^f \left( \frac{m}{T_o} \right)$

Error signal  $\epsilon = \int_{t_k}^{t_{k+1}} g(t-t_k) \delta\omega dt$   $\left( \frac{g^s}{g^c} \right) = \frac{1}{T_c} \int_0^{T_c} g(\theta) \cos(2\pi f\theta) d\theta$

*C. Audoin et al., IEEE Trans. ultrason. Ferroel. freq. control 45, 877 (1998)*

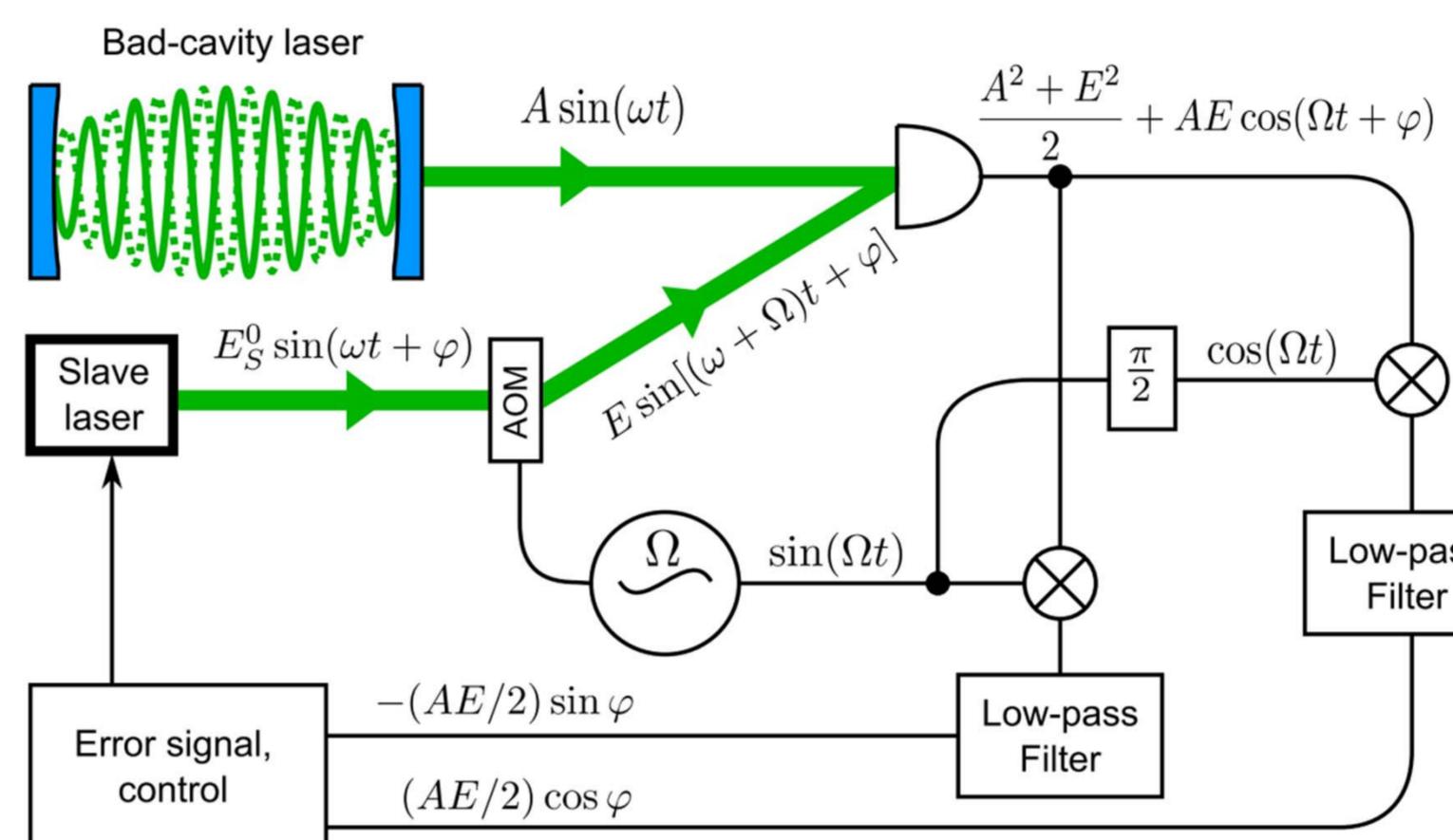
Limitation of the interrogation time:  $\sigma_y(T) = \frac{C}{\text{SNR}} \frac{\Delta\nu}{\nu_0} \sqrt{\frac{T}{T}}$

## Requirement for metrology-relevant active optical standard

Further improvement of the short-term stability of local oscillator may be attained not only by means of refinement of the macroscopic reference cavity (what seems to be a challenging task), but also by locking the interrogation laser to another, more stable reference. Active optical frequency standard is a prominent candidate this role. Here we define the requirements to be fulfilled by such standard.

*J. Chen, arXiv:physics/0512096; D. Meiser et. al., PRL 102, 163601 (2009); D. Yu, J. Chen, PRA 78, 013846 (2008)*

### Locking to the AOFS



### In the shot-noise limit:

Phase noise spectral power density:  $S_\varphi^{1s} = \frac{\hbar\omega}{P}$

Allan deviation (master/slave):  $\sigma_y(\tau) = \frac{1}{\tau} \sqrt{\frac{3\hbar f_h}{P\omega}}$

Taking  $f_h \sim 1000$  Hz,  $\omega = 2\pi \times 429$  THz ( $^{87}\text{Sr}$ ), we get that to obtain  $\sigma_y = 10^{-17}$  at  $\tau = 1$ s, one need

$P \sim 1$  pW

Master laser itself must be narrow:

$\Delta\omega = 10^{-17}\omega \simeq 2\pi \times 4.3$  mHz

## Active optical frequency standards: approaches and discussion

### Concept and qualitative description of behavior

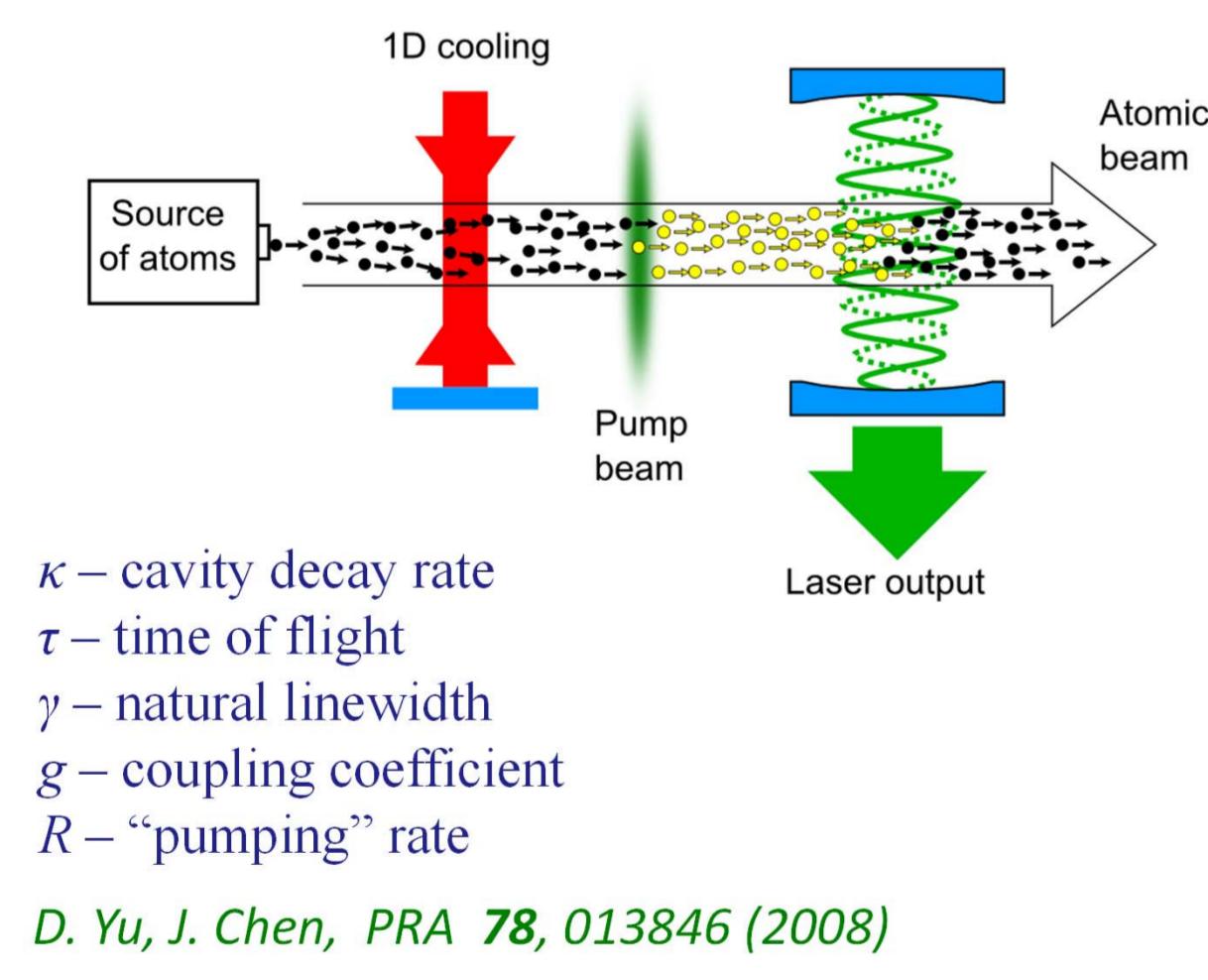
The main idea is to create a ``superradiant'' laser operating deep in the bad cavity regime, where the decay rate  $\kappa$  of the cavity field significantly exceeds the decoherence rate  $\gamma$  of the lasing transition. Specific expressions for the fundamental linewidth  $\Delta\omega$  of the steady-state operating laser and for the frequency  $\omega$  of the emitted radiations depends on the lasing scheme. As a qualitative illustration we present here the results for a laser with a random injection of excited atoms into the cavity

Linewidth:  $\Delta\omega = \frac{g^2 N_{a0}}{I_0 \gamma_{ab}} \left( \frac{\gamma_{ab}}{\kappa/2 + \gamma_{ab}} \right)^2$

Cavity pulling:  $\omega = \frac{2\omega_c \gamma_{ab} + \omega_a \kappa}{2\gamma_{ab} + \kappa}$ .

*Kolobov et.al., PRA 47, 1431 (1993); Kuppens et.al., PRL 72, 3815 (1994)*

### Atomic beam laser



Cavity pulling:  $\frac{\delta}{\Delta'} = \frac{\kappa\tau}{2\epsilon\tau \sin(\epsilon\tau/2)} \left( 1 - \frac{\sin(\epsilon\tau)}{\epsilon\tau} \right)$

In the bad cavity regime  $\delta \gg \Delta'$

Linewidth:  $\Delta\nu = \frac{g^2}{\kappa}$

Threshold:  $R > R_{th} = \frac{\pi^3 v^2}{3\lambda^2 \gamma F C_{CG}^2}$   $F = \frac{\pi c}{L \kappa}$

Regime of generation:  $\sin \frac{\epsilon\tau}{2} = \pm \frac{\epsilon\tau}{2} \sqrt{\frac{\kappa}{R\tau^2 g^2}}$

Single solution:  $1 < \sqrt{\frac{R\tau^2 g^2}{\kappa}} < 4.587$

### Main advantage:

Preparation and lasing are separated in space, not in time!

### Main issues:

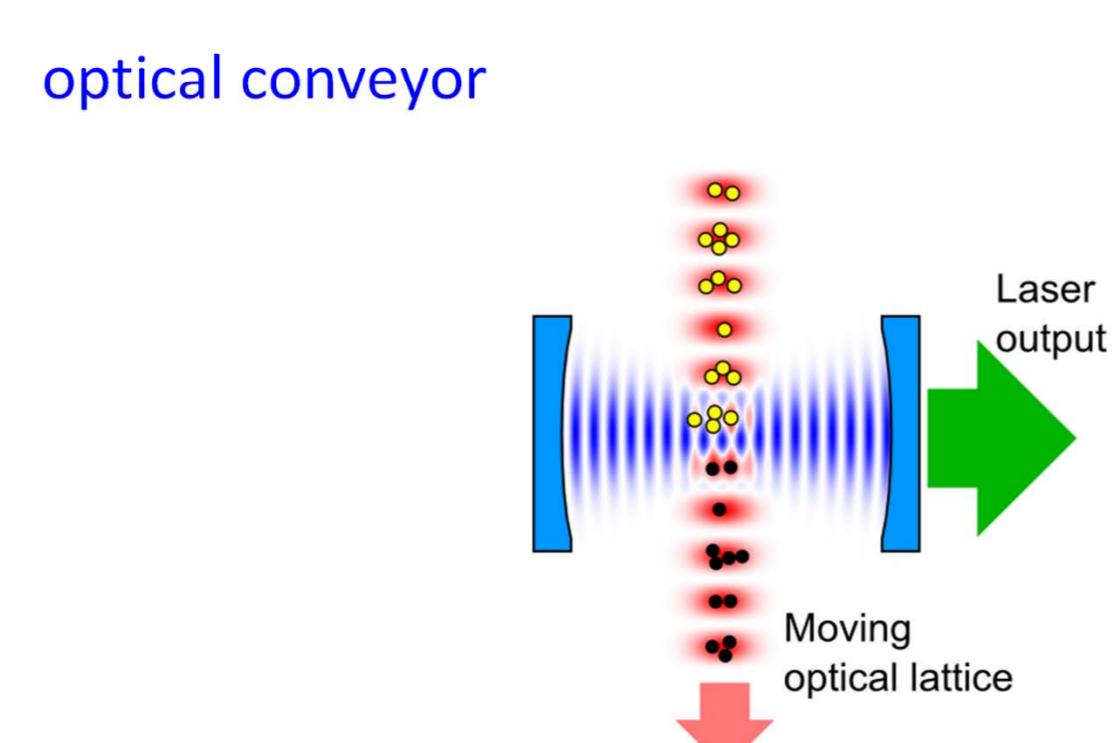
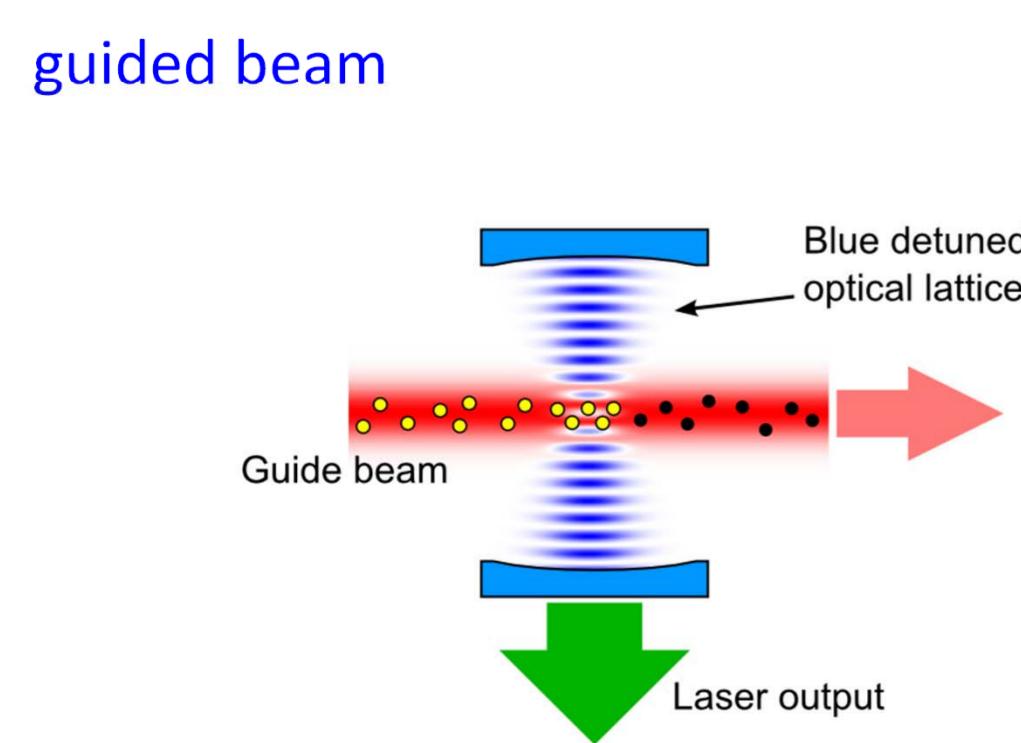
1. for free beam, threshold flux is too large

$F = 10^{-5}$   
 $v = 10^2 \text{ cm/s}$   $\Rightarrow R_{th} = 5.42 \times 10^9 \text{ s}^{-1} !!!$   
 for  $^{1s}_0 - ^3p_0$  transition in  $^{87}\text{Sr}$

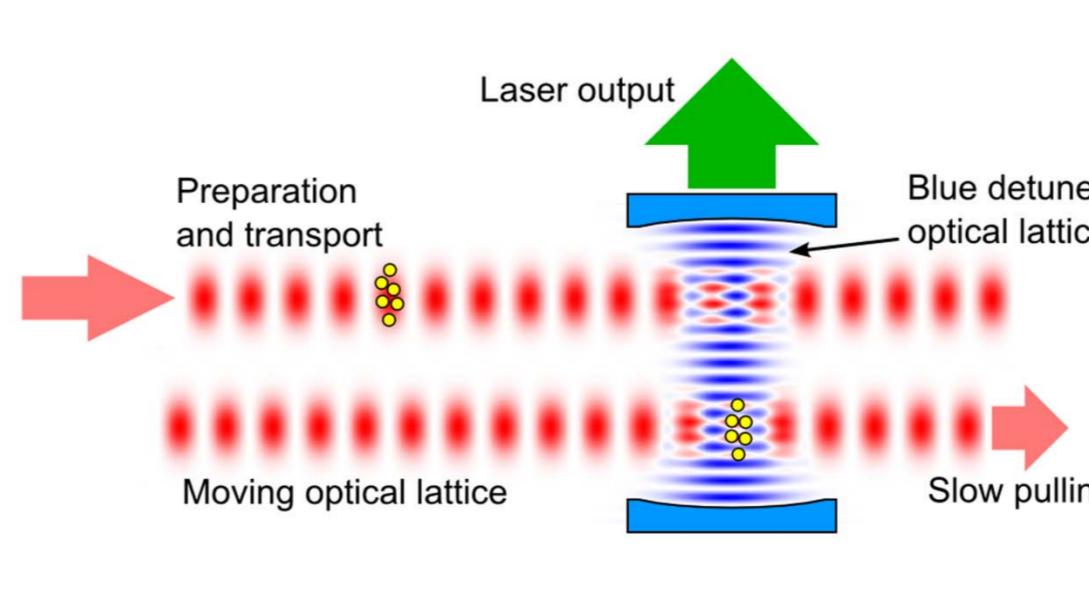
2. 1<sup>st</sup> order Doppler shift:

$\Delta v_\perp = 1 \text{ cm/s} \Rightarrow \frac{\Delta\nu_\perp}{\nu} = 3.3 \cdot 10^{-11}$

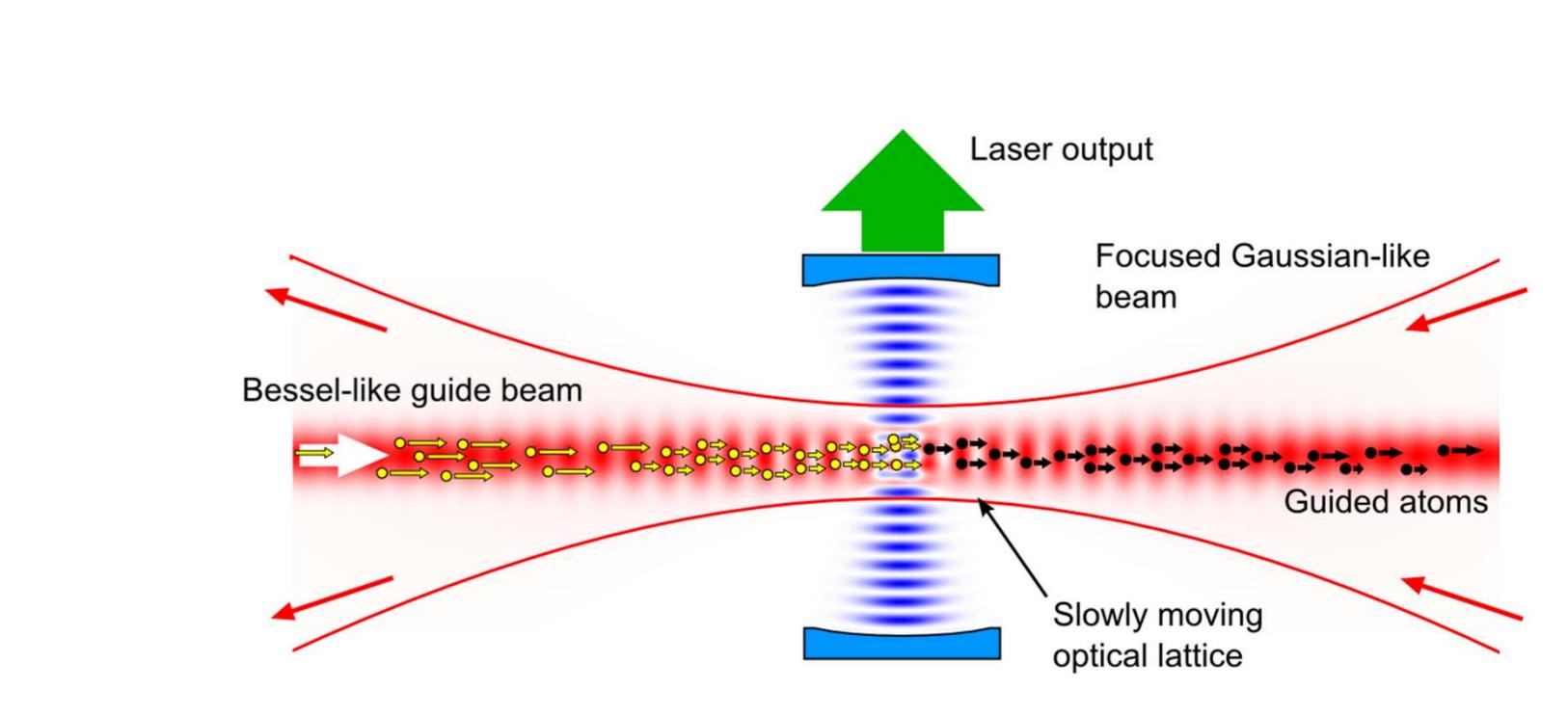
### Mixed approaches



### schemes with irregularly moved atoms



G. Kazakov, T. Schumm: PRA 87, 013821 (2013)



arXiv:1503.03998 [physics.atom-ph]

### Optical lattice laser

Atoms are kept in the optical lattice. This suppress the first order Doppler shift and increase the atom-field interaction time

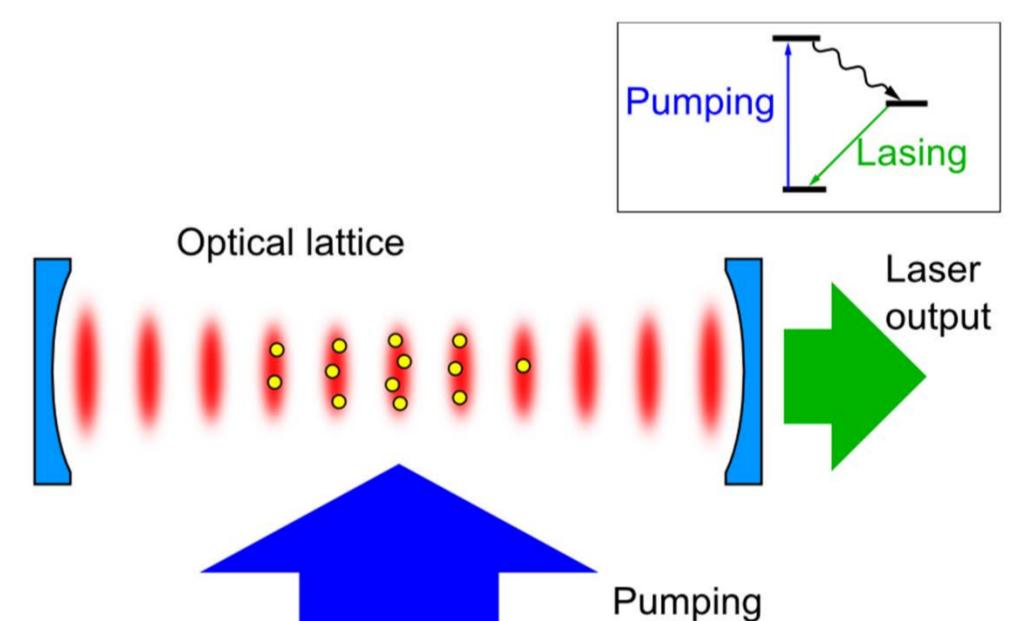
Candidate transitions:  $^{1s}_0 - ^3P_0$  in alkali-earth-like

Proposal: *J. Chen, arXiv:physics/0512096*  
*D. Meiser et. al., PRL 102, 163601 (2009);*

Theory: *PRA 81, 033387 (2010); PRA 81, 063827 (2010); PRA 87, 062101 (2013); PRL 113, 154101 (2014); PRA 89, 013806 (2014); Optics Express 22, 13269 (2014)*

Proof-of-principle experiments:

*Nature Letters 484, 78 (2012); PRL 109, 253602 (2012); App. Phys. Lett. 101, 261107 (2012); PRA 88, 013826 (2013); PRA 90, 053845 (2014)*



Atoms are kept in the optical lattice. This suppress the first order Doppler shift and increase the atom-field interaction time

Estimations for  $^{1s}_0$  ( $m = 9/2$ ) -  ${}^3p_0$  ( $m = 9/2$ ) transition in  $^{87}\text{Sr}$ :

$\omega_a \simeq 2\pi \times 429$  THz

$\gamma = 2\pi \times 7.6$  mHz

Dipole transition matrix element

$$d_{eg} = \frac{3}{\sqrt{11}} \sqrt{\frac{3\hbar c^3}{4\omega_a^3}} \simeq 8 \times 10^{-5} e a_0$$

Coupling coefficient:

$$g = d_{eg} \sqrt{8\pi\omega_a / (h\nu_{eff})} \simeq 52 \text{ s}^{-1}$$

Cooperativity parameter:

$$C = g^2 / (\kappa\gamma)$$

$$P_{\max} = \hbar\omega_a N^2 C \gamma / 8$$

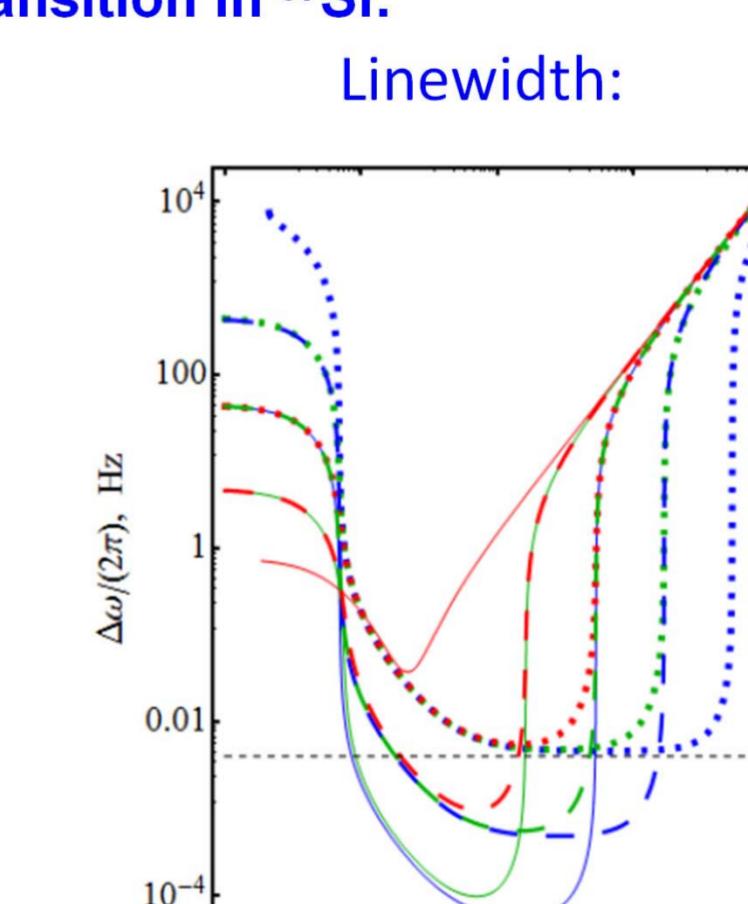
$$\Delta\nu_{min} = C\gamma$$

$$\gamma < w < NC\gamma$$

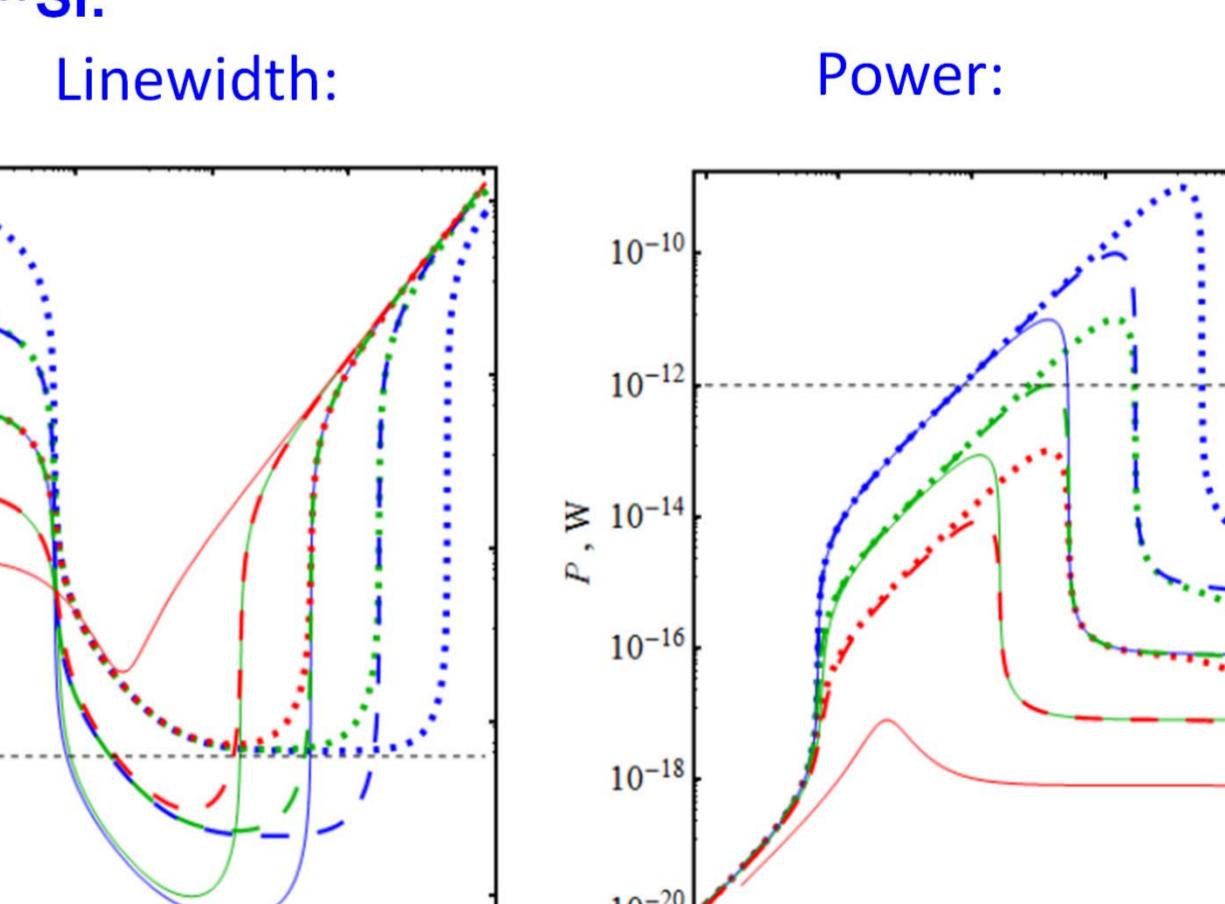
Main advantage:

Atoms are confined in the Lambd-Dicke regime along the cavity axis.

Linewidth:



Power:



Main issues:

1. Limited lifetime of atoms in the optical lattice
2. Large number of atoms is required
3. Heating of atoms and decoherence of etalon transition by repumping.