Entanglement Entropy and AdS/CFT

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The main messages of this talk

• Entanglement entropy is a **measure for entanglement** in quantum systems.

(Other measures have been suggested, but they are not so well suited for doing calculations.)

Entanglement entropy is difficult to compute in quantum field theories.

(Analytic results are available for 1+1 dim. CFTs.)

The AdS/CFT correspondence brings a huge simplification:
Entanglement entropy = area of extremal "surfaces"

(This makes entanglement entropy (numerically) tractable in higher dimensional quantum field theories.)

Outline

1. Introduction

- Entanglement Entropy: Definition, QM, QFT
- Holographic Principle & AdS/CFT
- Holographic Entanglement Entropy: Extremal Surfaces

2. Applications

- Holographic Thermalization
- Example: Holographic Quantum Revivals

3. Summary

Definition of entanglement entropy

Consider a quantum system in a **pure state** $|\psi\rangle$.

The **density matrix** of this state is given by $\rho = |\psi\rangle\langle\psi|$.

Divide the system into **two parts** A,B. The total Hilbert space is factorized:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

The reduced density matrix of A is obtained by the trace over $\mathcal{H}_{\mathcal{B}}$

$$\rho_A = \mathrm{Tr}_B \rho$$

Entanglement entropy is defined as the **von Neumann entropy** of ρ_A :

$$S_A = -\mathrm{Tr}_A \rho_A \mathrm{log} \rho_A$$



Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's. Observer Alice has only access to one spin and Bob to the other spin.



Entanglement entropy is a measure for how much a given quantum state is entangled.

Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method.**

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated!**



Message: Computing entanglement entropy in interacting QFTs is complicated and analytically only possible in 1+1 dim. CFTs.

The **holographic principle** provides a **simpler method** that works also in **higher dimensions**.

Ising spin chain in transverse magnetic field



Holographic principle & AdS/CFT correspondence



We can use general relativity to solve a strongly coupled gauge theory!

Holographic entanglement entropy

Entanglement entropy in the 4 dim. CFT can be computed from extremal surfaces in the 5 dim. gravity theory.

(Extremal surfaces are saddle points of the area functional in the 5 dim. geometry.)



Surfaces are infinitely "long", entanglement entropy in QFTs diverges - need a cut off.

Some illustrative examples



Holographic Thermalization

• The **central question**:

How evolves an excited quantum system to thermal equilibrium?

- Holographic entanglement entropy serves as an observable to study thermalization.
- The holographic dual of **thermalization in CFT** is the **formation** of a **black hole** in the **gravity theory**.
- Has to consider **time dependent geometries** in on the gravity side → one has to do **numerical relativity**.
- To compute the time evolution of entanglement entropy one has to compute extremal surfaces in these numerical spacetimes.

Holographic quantum revivals

Quantum revival: collapse and **reappearance** of a coherent quantum state.

Experimentally realized in **Bose Einstein** condensate trapped in an optical lattice.

The Holographic dual is a oscillating matter shell in the gravity theory.

Entanglement entropy is a bridge between string theory (general relativity) and cond-mat.







[Greiner-Mandel-Hänsch-Bloch 02]

Summary

Entanglement entropy is a **measure for quantum entanglement**.

In **condensed matter** systems entanglement entropy can be used to identify new **quantum phases**. (quantum order parameter)

Except in **1+1 dim. CFTs** computing entanglement entropy directly in **QFT's** is **problematic**.

The **AdS/CFT** correspondence provides a **powerful alternative** which works also in **higher dimensions**.

Entanglement entropy = area of extremal "surfaces"

Thermalization in CFT = **black hole formation** in gravity theory.

Entanglement entropy can be a useful **bridge between general relativity** (string theory) and **condensed matter physics** (and **experiment?**)