Methods for Cellular Automata and Evolution Systems in Modelling and Simulation

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1. INTRODUCTION

Cellular automata are in many occasions perceived and treated as *natural systems* consisting of a grid of cells with locally characterised dynamic behaviour. A partially different perception regards cellular automata as a method for modelling and simulation. Of course in both cases the basic ideas and the structure are mostly identical. In the latter case the conception of cellular automata is however used to depict a natural system as an abstract conceptual model and to describe the simplified system in a mathematical fashion. Also the term *cellular automaton* itself is controversial in this case.

A more abstract approach to cellular automata can be formalised in a functional analytic way as *(locally characterised) evolution systems* or as strongly continuous semigroups. This is the basis for connecting cellular automata and evolution systems with parabolic partial differential equations or abstract evolution equations (Goldstein (1985); Engel and Nagel (2000); ...).

2. EXAMPLE: AGE-STRUCTURED SIR MODEL

An important demographic parameter in connection with epidemiology is age. An age-structured population was for example investigated by Iannelli and Martcheva (2003) and modelled by the following partial differential equation where S, I and R as well as $P := S + I + R$ are functions $T \times [0, a_{\max}] \to \mathbb{R}_+$ where $a_{\max}$ is the maximum age and μ is the *force of mortality* (or age-dependent natural death-rate).

$$\begin{aligned} \partial_t S - \partial_a S &= -\lambda(t,a)S - \mu(a)S \\ \partial_t I - \partial_a I &= \lambda(t,a)S - \beta(a)I - \mu(a)I \\ \partial_t R - \partial_a R &= \beta(a)I - \mu(a)R \end{aligned} \tag{1}$$

The parameter

$$\lambda(t,a) := \int_0^\infty \kappa(a,s)\frac{I}{P}(t,s)\,ds \tag{2}$$

depends on the contact behaviour (κ) between different age-groups (a and s) of the population.

However for real contact data between individuals with specific age the corresponding evolution system is not locally characterised, because the typical contact behaviour of a population $\kappa(\cdot\,,\cdot)$ exhibits a shape as illustrated in Fig. 1.

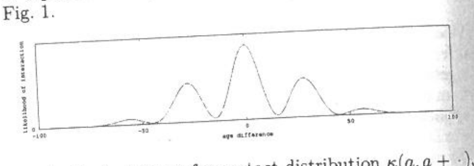

Fig. 1. Typical form of a contact distribution $\kappa(a, a + \cdot)$, describing the likelihood of interaction of a person aged a to other persons with age in $[a - 100, a + 100]$. This figure only shows the general form of a contact distribution. For example ages below 0 are not possible such that this function is not necessarily symmetric. The peaks at -30 respectively $+30$ indicate the interaction of children with their parents and vice versa (i.e. generations).

3. CONCLUSION AND OUTLOOK

Analytic investigations often have to deal with nonlinearities (e.g. Iannelli and Martcheva (2003)). We try to compare experimental data from a cellular automaton implementation with analytic results for our application example especially in context of local characterisation. Also a stochastic cellular automaton model can be used to simulate nonlinear systems. General methods for relating cellular automata and parabolic partial differential equations should be investigated.

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