

## STATIONARY SOLITARY WAVES IN TURBULENT OPEN-CHANNEL FLOW

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Steady two-dimensional turbulent free-surface flow in a channel is considered, with the Froude number  $Fr$  of the oncoming flow being slightly above the critical value, i.e.  $(2/3)(Fr-1) = \varepsilon \rightarrow 0$ . The Reynolds number is assumed to be very large, and the slope of the channel bottom  $\alpha$  is of the order of  $\varepsilon^2$ , as required for an analysis that is free of turbulence modelling. It is known that for a plane bottom with constant roughness, stationary solitary waves in turbulent open-channel flow cannot exist. Thus, various perturbations of the conditions at the channel bottom are considered.

In JFM 726:137-259, 2013, the roughness of the channel bottom was assumed to vary with the space coordinate to allow small variations in the bottom friction coefficient. The result of the asymptotic analysis was an extended Korteweg–deVries (KdV) equation for the non-dimensional, first-order surface elevation  $H$ . The steady-state version of this equation has solutions that describe stationary solitary waves in steady flow. The existence of the predicted wave was confirmed experimentally (J. Hydr. Eng. 142/1:04015035, 2016).

In the present work, the previous analysis is modified to study very small variations in the shape of the bottom (ramps, bumps) of the order of  $\alpha\varepsilon^{1/2}$  that also give rise to stationary solitary waves. The bottom roughness is taken constant. We show that there exists an analogy between the bottom shape function  $\psi$  and the function that describes the variation in the bottom roughness in the previous analysis.

Numerical solutions of the extended KdV equation and uniformly valid analytical solutions for small values of the dissipation parameter  $\beta = (1/3)\alpha\varepsilon^{-3/2}$  are presented. In the latter case, the position of the wave crest is determined from integral conservation conditions. Solitary waves with strong decay, i.e. solitary waves of the classical type, are obtained for ramps of any shape that have a particular non-dimensional height far downstream. As an example, a ramp with constant slope is considered, see Fig. 1. In case of a bump, the solutions describe stationary solitary waves with a downstream ‘tail’ of negative surface elevation. Fig. 2 shows solutions of a stationary solitary wave above a symmetric bump with triangular cross section.

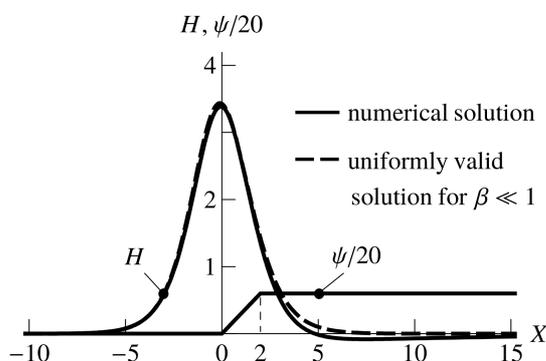


Fig.1 Bottom shape  $\psi$  of a ramp and surface elevation  $H$ , respectively, dissipation parameter  $\beta=0.1$

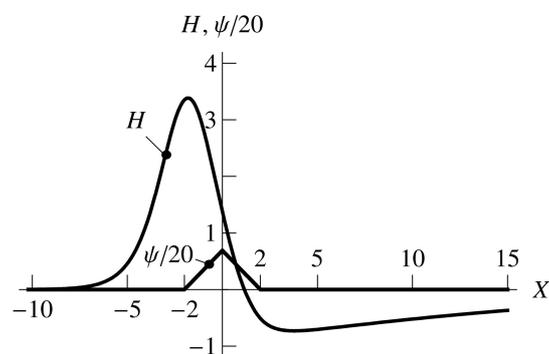


Fig.2 Bottom shape  $\psi$  of a bump and surface elevation  $H$ , respectively, for a wave with a tail;  $\beta=0.1$