The Jahn-Teller distortion in praseodymium dioxide (PrO$_2$) has been studied using density functional theory with the PBE+$U$ functional for the exchange-correlation energy. Various possible distortions were suggested based on recent neutron diffraction experiments [C. H. Gardner et al., Phys. Rev. B 70, 024415 (2004)]. We could identify the most likely distorted structure with a magnitude of the distortion in very good agreement with the experiment. This structure is the one with the lowest total energy and the corresponding calculated electronic and magnetic properties agree well with the experiment. We investigated the dependency of various properties on the Coulomb parameter $U_{\text{eff}}=U-J$ and suggest $U_{\text{eff}}=6$ eV as the most appropriate value, for which about 1.65 4f electrons (one localized plus 0.65 delocalized) per Pr atom are found. We also show that magnetism (antiferromagnetism or ferromagnetism) has only a minor influence on the energy and magnitude of the distortion, which is consistent with the fact that the Néel temperature (13.5 K) is much smaller than the temperature of 120 K below which the distortion is observed.

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I. INTRODUCTION

The compounds CeO$_2$, PrO$_2$, and TbO$_2$ are the only lanthanide dioxides which have been successfully synthesized. CeO$_2$ occurs naturally, while a positive oxygen pressure is necessary to form PrO$_2$ and TbO$_2$. Most of the other lanthanide dioxides which have been successfully synthesized have been studied and reported using density functional theory (DFT) calculations of the geometrical and electronic structures of CeO$_2$ and Ce$_2$O$_3$ have recently appeared. From these studies, it became apparent that it is necessary to go beyond the local density approximation (LDA) or generalized gradient approximation (GGA) in order to accurately reproduce the experimental results of these two Ce oxides, in particular, for Ce$_2$O$_3$, for which the formal valence is Ce$^{3+}$ (i.e., one 4f electron per Ce atom).

More generally, it is well known that LDA and GGA functionals usually lead to qualitatively wrong results when they are applied to solids containing localized electrons (e.g., transition-metal and lanthanide oxides). The main reason for this shortcoming is the self-interaction error (the electrostatic interaction of an electron with itself) that is contained in these approximate functionals and their associated potentials (functional derivative). The LDA(GGA)+$U$ method deduced that PrO$_2$ is a highly correlated and partly covalent oxide, which can be described by a mixing of Pr$^{4+}$ and Pr$^{3+}$ configurations (a 4f occupancy of 1.5–1.6 electrons is calculated). From reflectivity and resonant photoemission, Kimura et al. observed the coexistence of the Pr$^{4+}$ and Pr$^{3+}$ states in PrO$_2$ from x-ray absorption spectroscopy, it was suggested in Refs. 39 and 40 that the ground state consists of one localized 4f electron (Pr$^{4+}$ configuration) plus an additional delocalized state having some 4f character hybridized with O 2p states. More recently, it was deduced from neutron diffraction experiments that for 90% of the Pr atoms, the configuration is Pr$^{4+}$ (Ref. 29) and that below the Néel temperature of $T_N=13.5$ K, PrO$_2$ has a complicated noncollinear magnetic structure which consists of two components, with a magnetic moment on Pr of 0.65±0.02 and 0.35±0.04 $\mu_B$, respectively. Previous experimental studies reported a total magnetic moment of 0.6±0.03, 0.572±0.012, and 0.68±0.07 $\mu_B$, respectively. Previous experimental studies reported a total magnetic moment of 0.6±0.03, 0.572±0.012, and 0.68±0.07 $\mu_B$, respectively. Previous experimental studies reported a total magnetic moment of 0.6±0.03, 0.572±0.012, and 0.68±0.07 $\mu_B$, respectively. Previous experimental studies reported a total magnetic moment of 0.6±0.03, 0.572±0.012, and 0.68±0.07 $\mu_B$, respectively. Previous experimental studies reported a total magnetic moment of 0.6±0.03, 0.572±0.012, and 0.68±0.07 $\mu_B$, respectively.
whereas a conducting behavior was reported in Ref. 42.

The number of DFT calculations on PrO$_2$ available in literature is rather small. Koelling et al.\textsuperscript{43} reported the first calculation on PrO$_2$. They used LDA for the exchange-correlation energy functional and neglected correlation. In Refs. 44 and 45, LDA calculations on PrO$_2$ were performed using the pseudopotential and tight-binding linear muffin-tin orbital methods, respectively. PrO$_2$ is among the numerous lanthanide oxides considered by Svane and co-workers\textsuperscript{6,26,27} who used the SIC-LDA functional. Finally, we also mention the crystal field calculations for PrO$_2$ reported by Diviš and co-workers\textsuperscript{46–48}. The DFT calculations presented in these cited papers were done assuming the perfect fluorite structure, i.e., none of them considered the Jahn-Teller distortion.

The paper is organized as follows. In Sec. II, the theoretical method is briefly outlined and the computational details are given. In Sec. III, we present the considered distorted structures, and in Sec. IV the results are discussed. Finally, in Sec. V, the conclusion is given.

II. THEORETICAL METHOD AND COMPUTATIONAL DETAILS

The calculations were done with the WIEN2K code\textsuperscript{50–52} which is based on the full-potential (linearized) augmented plane-wave and local orbitals (FP-LAPW+lo) method\textsuperscript{53,54} to solve the Kohn-Sham equations\textsuperscript{5} of DFT. The Brillouin zone integrations were performed with $k$ meshes which were large enough to obtain well converged results. For instance, for calculations in a tetragonal unit cell (see Fig. 1) containing 8 f.u., a $10 \times 10 \times 5$ special point grid was used. $R_{\text{MT}}^{\text{min}}K_{\text{max}}=7$ (the product of the smallest of the atomic sphere radii $R_{\text{MT}}$ and the plane-wave cutoff parameter $K_{\text{max}}$) was used for the expansion of the basis set since test calculations with $R_{\text{MT}}^{\text{min}}K_{\text{max}}=8$ led only to minor changes in the results.

The sphere radii $R_{\text{MT}}$ of Pr and O atoms were chosen as 2.5 and 1.7 a.u., respectively. The spin-orbit (SO) coupling was included in the calculations based on the second variational approach\textsuperscript{55–57} using a basis set of states up to 60 eV above the Fermi energy. The inclusion of SO coupling was necessary in order to obtain a magnetic moment on the Pr atom (containing spin and orbital contributions) in good agreement with the experimental values (see Sec. IV). In addition to the symmetry lowering due to the distortion of the oxygen sublattice and antiferromagnetic ordering, a further symmetry lowering was sometimes (depending on the distorted structure) necessary in order to reach the lowest-energy state. For such systems, the lowering of symmetry can be necessary, and in Ref. 17 this aspect has been discussed in detail for lanthanide mononitrates. We should also mention that we neglected the noncollinearity of the magnetic structure of PrO$_2$,\textsuperscript{60,31} which can be justified by the fact that the temperature $T_D=120$ K, below which the distortion is observed, is much larger than the Néel temperature $T_N=13.5$ K.\textsuperscript{30,31} Nevertheless, we considered different collinear magnetic phases in order to study the influence of the magnetic structure on other properties (electronic structure and geometry).

Concerning the exchange-correlation energy, we chose the PBE+ $U$ functional

$$E_{xc}^{\text{PBE+U}} = E_{xc}^{\text{PBE}} + E_{ee} - E_{dc},$$

(1)

where $E_{xc}^{\text{PBE}}$ is the nonempirical GGA functional developed by Perdew et al.,\textsuperscript{28}

$$E_{ee} = \frac{1}{2} \sum_{\sigma,\sigma'} m_{1m_1} n_{\sigma m_{1m_1}} \langle \langle m_{1m_1} | \sigma \rangle | m_{1m_1} \rangle - \langle \langle m_{1m_1} | \sigma \rangle | m_{2m_2} \rangle \delta_{\sigma\sigma'}$$

(2)

is an electron-electron (ee) interaction energy of Hartree-Fock-type\textsuperscript{58} for the electrons of a selected atom and angular momentum $\ell$, and

$$E_{dc} = U \frac{n_e(n_e-1)}{2} - \frac{1}{2} \sum_{\sigma} n_{\sigma}^e(n_{\sigma}^e-1)$$

(3)

is the atomic limit version of the double-counting (dc) term\textsuperscript{59}. In Eq. (2), $n_{\sigma m_{1m_1}}^e$ ($m_1=-\ell, \ldots, \ell$ and $\sigma$ is the spin index) is the occupation matrix and $u_{\sigma m_{1m_1}}^e$ is a screened Coulomb operator. In Eq. (3), $n_1 = n_j^e + n_j^e$, where $n_1^e$ is the total number of spin-$\sigma$ electrons of angular momentum $\ell$, and $U$ and $J$ [related to the integrals in Eq. (2) (Ref. 58)] are the screened Coulomb and exchange parameters, respectively, which are treated as empirical parameters or can be estimated by a constrained calculation.\textsuperscript{60,61} Contrary to $E_{xc}^{\text{PBE}}$, $E_{ee}$ is free of self-interaction errors (important for localized $d$ and $f$ electrons) and thus leads to a better description of the electrons of angular momentum $\ell$. $E_{dc}$ is subtracted to remove the interactions among these electrons contained in $E_{xc}^{\text{PBE}}$. Note that the calculations presented in this work were...
TABLE I. The space groups and Wyckoff positions of the Pr and O atoms of the four considered PrO$_2$ distorted structures (see Figs. 1 and 2) without considering any magnetic order.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Space group</th>
<th>Pr Wyckoff positions</th>
<th>O Wyckoff positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>JT1</td>
<td>Ibam (No. 72)</td>
<td>4$a[0,0, \frac{1}{2}]$, 4$d[\frac{1}{2},0,0]$</td>
<td>16$c[\frac{1}{2}, \frac{1}{2}+d, \frac{1}{2}]$</td>
</tr>
<tr>
<td>JT2</td>
<td>I4$_1$/acd (No. 142)</td>
<td>8$b[0,\frac{1}{2},\frac{1}{2}]$</td>
<td>16$e[\frac{1}{2}\pm d,0,\frac{1}{2}]$</td>
</tr>
<tr>
<td>JT3</td>
<td>I4$_1$/acd (No. 142)</td>
<td>8$a[0,\frac{1}{2},\frac{1}{2}]$</td>
<td>16$e[\frac{1}{2}\pm d,0,\frac{1}{2}]$</td>
</tr>
<tr>
<td>JT4</td>
<td>I4/mcm (No. 140)</td>
<td>4$a[0,0,\frac{1}{2}]$, 4$d[0,\frac{1}{2},0]$</td>
<td>16$e[\frac{1}{2}+d,\frac{1}{2}-d, \frac{1}{2}]$</td>
</tr>
</tbody>
</table>

done with $U$ replaced by $U_{\text{eff}}=U-J$ and $J=0$ in Eqs. (2) and (3), which leads to the simpler expression

$$E_{\text{exc}}-E_{\text{loc}}=\frac{U_{\text{eff}}}{2}\sum_{\sigma}\sum_{m_1}\sum_{m_2}\left(n_{m_1m_1}^\sigma-n_{m_1m_2}^\sigma n_{m_2m_1}^\sigma\right). \quad (4)$$

For details about the implementation of Eqs. (2) and (3) (and their associated potentials) within the FP–(L)APW+lo method, see Ref. 62. All calculations were done at the experimental lattice constant of 5.386 Å, which was determined by x-ray diffraction experiments at about 10 K.30

III. DISTORTED STRUCTURES

As mentioned in the Introduction, Gardiner et al.30 reported a Jahn-Teller distortion below $T_p=120$ K. They performed neutron diffraction experiments on single crystals of PrO$_2$. Their data did not allow a unique structure determination but left several proposals for possible distorted structures of the oxygen sublattice (the praseodymium sublattice is undisturbed). In their models, the unit cell is doubled along one crystal axis and the distortion consists of displacing the oxygen atoms perpendicular to the direction along which the unit cell is doubled. The displacement from the ideal (fluorite structure) position is the same for all oxygen atoms, and thus only one parameter ($d$) is necessary to characterize it.

Two distorted structures are discussed in detail, whose space groups (without taking into account the lowering of symmetry due to an antiferromagnetic phase) and Wyckoff positions are indicated in Table I. The first one is JT1 [Fig. 1(a)], which leads to two inequivalent Pr atoms, whose first-neighboring atoms (oxygen cage) are shown in Figs. 2(a) and 2(b). The second distorted structure, called JT2, is shown in Fig. 1(b) and leads to Pr atoms which are all equivalent [the oxygen cage is Fig. 2(c)]. The JT2 structure corresponds to the structure shown in Fig. 8 of Ref. 30, but we should mention that in Ref. 30, it was indicated in the text that for JT2 structure the Pr atoms are at positions $8a[0,1/4,3/8]$ which are, however, not in accordance with their Fig. 8. Assuming the positions $8a[0,1/4,3/8]$ for the Pr atoms leads to structure JT3, which is shown in Fig. 1(c), and the corresponding oxygen cage in Fig. 2(d). Since we do not know which positions of the Pr atoms [$8b[0,1/4,1/8]$ or $8a[0,1/4,3/8]$] were actually favored by the authors,30 we considered both JT2 and JT3 distorted structures for our calculations.

JT1 and JT2 (or JT3) are not the only structures which are consistent with the measured reflections. Gardiner et al.30 also mentioned a structure that consists of a superposition of two JT1 distortions but with oxygen displacements in mutually perpendicular directions. For such a structure, the overall displacement of an oxygen atom is $d=\sqrt{2}\times 0.0511=0.0723$ Å. Therefore, we also considered two of such distorted structures. The first, called JT4 [Fig. 1(d) and Table I], leads to two inequivalent Pr sites, whose oxygen cages are shown in Figs. 2(e) and 2(f). The second structure we considered (not shown) led to no distortion; therefore, we will not consider it further in the present paper. Note that the difference between cages 2(a) and 2(f) is that for cage 2(a), there are two different O–O bond lengths among the O–O bonds parallel to the sheet plane, whereas for cage 2(f), all these O–O bonds have the same length. From Fig. 2, we can see that for cages 2(b) and 2(d), the four short Pr–O bonds form a tetrahedron, while for cages 2(a), 2(c), and 2(f), the four short Pr–O bonds are situated in a plane [only approximately for 2(c)]. In cage 2(e), all Pr–O bonds have the same length.
and magnetic phase) by starting such calculations with different electron densities. For the PBE+U+SO calculations, we considered several starting configurations of the 4f electrons. These include the solution(s) obtained by PBE, the crystal field states $t_{2g}$, $e_{g}$, and $a_{2g}$, and the pure $m$ states satisfying all or some of Hund’s rules. We also found slightly different results when we started from PBE calculations, with and without SO coupling, and then used the corresponding occupation matrix and electron density to start the PBE +U+SO calculations.

Among the different PBE+U+SO results, the most stable one was obtained by starting from a PBE+SO solution. Figure 3 shows the corresponding spin magnetization density $\rho_{\uparrow}-\rho_{\downarrow}$ (in the AF1 magnetic phase), whose shape is dominated by a single fully spin-polarized localized Pr 4f electron. Therefore, the shape of the spin magnetization density reflects the shape of the electron density of this localized 4f electron. We can see that each atomic spin magnetization density has eight lobes and, in fact, by choosing an appropriate orientation of a local coordinate system to express the Pr 4f occupation matrix, the 4f electron density roughly corresponds to an electron in an orbital whose angular part is a linear combination of the spherical harmonics $Y_{1,-2}$ and $Y_{3,2}$. Such a local coordinate system would be, for instance, one whose $z$ axis points in the [111] cubic direction. Among the eight lobes, four (those situated in a plane perpendicular to the direction along which the unit cell is doubled) point in a direction such that the oxygen atoms are maximally avoided, whereas the other four (those situated in a plane parallel to the direction along which the unit cell is doubled) point in a direction slightly deviating from a nearest-neighboring oxygen atom. This feature of the electronic structure will be important later on to explain which one of the considered distorted structures [(a)–(d) of Fig. 1] leads to the lowest total energy and largest distortion. All the results discussed below were obtained with this Pr 4f electronic configuration.

In Fig. 4, we show the variation of the fundamental band gap $\Delta$, magnetic moments ($\mu_{\text{tot}}, \mu_{S}$, and $\mu_{4f}$), distortion $d$ of the oxygen sublattice, and number of 4f electrons per Pr atom (within the atomic sphere $R_{\text{MT}}^{\text{c}}=2.5$ a.u.) with respect to the Coulomb parameter $U_{\text{eff}}$ which was varied between 0 (corresponding to a PBE calculation) and 8 eV. These calculations were done for the JT2 distorted structure (which leads to the lowest total energy, see below) and the AF1 magnetic phase. From Fig. 4(a), we can see that the band gap $\Delta$ increases rapidly from 0.09 eV (for $U_{\text{eff}}=0$ eV) to 1.65 eV (for $U_{\text{eff}}=4$ eV), while the increase between $U_{\text{eff}}=4$ and 8 eV is only 0.16 eV. From a simple estimate using our calculated fundamental band gap and the measured activation energy of 0.262 eV for the electrical conductivity, a value for $U_{\text{eff}}$ not larger than 0.5 eV seems to be more appropriate. The best agreement is obtained with $U_{\text{eff}}=0.2$ eV (the second data point) which yields $\Delta=0.26$ eV. Nevertheless, the mechanism responsible for the reported conductivity is unknown and different conduction mechanisms (ionic and electronic) were suggested. We would not expect such a small value for $U_{\text{eff}}$ for the present system. For instance, for CeO$_2$ and Ce$_2$O$_3$, the best agreement with experiment is obtained for $U_{\text{eff}}$ lying in the range 2–5 eV. Furthermore, the measured onset of the optical conductivity of PrO$_{2.8}$ is $\sim 2$ eV.\n
FIG. 3. (Color online) Isosurface of the spin magnetization density $(\rho_{\uparrow}-\rho_{\downarrow})$ of PrO$_2$ in the AF1 magnetic phase. The red and blue colors correspond to positive and negative values ($\pm 0.07$ electrons/Å$^3$), respectively. The small red balls are O atoms.

Most calculations were done with the antiferromagnetic phase AF1 which consists of ferromagnetic Pr planes stacked antiferromagnetically with respect to the axis along which the unit cell is doubled (see Fig. 3). Additional calculations were also done for two other magnetic phases for the distorted structure with the lowest energy (which is JT2, see Sec. IV). The first one is AF2, which consists of antiferromagnetic planes stacked antiferromagnetically [see Fig. 10(a) of Ref. 30], and the second one is the ferromagnetic phase (F).

IV. RESULTS AND DISCUSSION

For atoms with a partially filled 4f or 5f shell, it is a difficult task to find the corresponding electronic configuration which leads to the lowest total energy. The situation can become particularly tricky when an orbital-dependent potential (as, for example, PBE+U) is used since such a potential usually leads to numerous local minima in the electronic configuration space. In the case of PrO$_2$, already at the PBE level (with an orbital-independent potential), it was possible to stabilize a few different solutions (for a given geometry
which is better reproduced with a large value of $U_{\text{eff}}$ (see below). Figure 4(b) shows that the total magnetic moment reaches its maximum value (0.52 $\mu_B$) at approximately $U_{\text{eff}} = 2$ eV, whereas for $U_{\text{eff}} = 0$ eV (PBE only), $\mu_{\text{tot}}$ would be clearly lower than the experimental values [0.5–0.75 $\mu_B$ (Refs. 30, 33, and 41)]. Nevertheless, we recall that an atomic magnetic moment is a quantity for which a unique definition does not exist and that its value depends on the atomic radius $R_{\text{MT}}$. We can also observe that the magnitudes of the spin $\mu_S$ and orbital $\mu_L$ magnetic moments behave differently with respect to the variation of $U_{\text{eff}}$ and that the variation of $\mu_{\text{tot}}$ is mainly due to the $\mu_L$ component. The curve [Fig. 4(c)] for the oxygen distortion $d$ shows, at $U_{\text{eff}} = 1$ eV, a maximum of 0.086 Å, which is slightly above the experimental range 0.065–0.078 Å. The best agreement...
FIG. 6. (Color online) PBE+U+SO spin-up DOS of PrO$_2$ in the \textit{AF1} magnetic phase and JT2 structure for $U_{\text{eff}}=6$ eV. The Fermi energy is set at zero. No broadening was applied.

would be obtained for $U_{\text{eff}}=0.2$ and 4 eV, both yielding $d=0.072$ Å, while for $U_{\text{eff}}=0$ and 8 eV, $d=0.045$ Å, which is clearly too small compared to experiment. Figure 4(d) shows that the number of 4$f$ electrons per Pr atom $n_{4f}$ decreases linearly with $U_{\text{eff}}$ from 1.89 electrons (at $U_{\text{eff}}=0$ eV) to 1.56 electrons (at $U_{\text{eff}}=8$ eV). Assuming the 4$f$ occupancy of 1.5–1.6 electrons calculated in Refs. 35–37 makes a large value of $U_{\text{eff}}$ more appropriate.

In order to have more information about a proper value of $U_{\text{eff}}$ for PrO$_2$, we used the method of Anisimov and Gunnarsson$^{60}$ (constrained calculation) to calculate $U_{\text{eff}}$ in an \textit{ab initio} way [see Ref. 61 for details about the method when used with the FP- (L)APW+lo basis set]. Using this method, we obtained a value of about $U_{\text{eff}}=7.5$ eV, but we want to stress that the calculations had to be done assuming a Pr$^{3+}$ state (i.e., two 4$f$ electrons per Pr atom) since it is not possible to apply this method for less than two 4$f$ electrons. Therefore, we can expect the value of $U_{\text{eff}}$ to be slightly smaller since in PrO$_2$ each Pr atom contains about 1.5 electrons. Nevertheless, the result of this \textit{ab initio} calculation of $U_{\text{eff}}$ indicates that the use of a large $U_{\text{eff}}$ is more appropriate for PrO$_2$.

In Fig. 5, the density of states (DOS) of the \textit{AF1} magnetic phase of PrO$_2$ in the JT2 distorted structure is plotted, and three values of $U_{\text{eff}}$ (0.2, 2, and 6 eV) were chosen in order to show the evolution of the DOS with respect to the increase of $U_{\text{eff}}$. For $U_{\text{eff}}=0.2$ eV [Fig. 5(a)], we can see that the highest occupied states (sharp peak) are of Pr 4$f$ character and are situated about 1 eV above the top of O 2$p$ bands. The fundamental band gap (0.26 eV) separates this sharp occupied Pr 4$f$ peak from the rest of the Pr 4$f$ bands. Then, at about 5 eV above the Fermi energy, states predominantly of Pr 5$d$ character begin to appear. The main effect of increasing $U_{\text{eff}}$ is to separate the occupied from the unoccupied Pr 4$f$ states. For $U_{\text{eff}}=2$ eV [Fig. 5(b)], a large part of the occupied Pr 4$f$ states is now situated at the highest O 2$p$ states, which make the top of the valence band having a mixed Pr 4$f$/O 2$p$ character, where hybridization is evident from the relatively broad 4$f$ partial DOS. Applying $U_{\text{eff}}=6$ eV [Fig. 5(c)] shifts the occupied Pr 4$f$ states down to about 4 eV below the Fermi energy and concentrates them into a sharp peak. The latter corresponds to a single localized 4$f$ electron, while between $\sim-3$ and 0 eV, the integration of the Pr 4$f$/DOS yields about 0.65 4$f$ electrons (spin up plus spin down) in bands of mainly O 2$p$ character. This is shown in more detail in Fig. 6. One can clearly see the peak around $-4$ eV corresponding to the localized Pr 4$f$ electron. Note that the peak from the O 2$p$ DOS at $-4$ eV comes from separate bands without 4$f$ contributions and thus these 4$f$ states are localized and not hybridized with the O 2$p$ states. On the other hand, the small Pr 4$f$/DOS in the range $-3$–0 eV corresponds to hybridized 4$f$ states. A further indication of this Pr 4$f$/O 2$p$ hybridization can be observed from some similarities in the positions of the peaks at $-0.8$ and $-0.3$ eV between the Pr 4$f$ and O 2$p$ curves. Note that the spin-up and spin-down Pr 4$f$ DOSs above $-3$ eV (i.e., Pr1 4$f$ and Pr2 4$f$) are very similar.

Figure 7 shows the electron density (spin up plus spin down) of the bands (a) around $-4$ eV and (b) due to the states within the interval $-3$–0 eV. The electron density of Fig. 7(a) is very similar to the spin magnetization density (see Fig. 3) due to the full spin polarization of the localized
4f electron (i.e., the electron density of the other spin is almost negligible). The orientation of the lobes of this localized 4f electron is closely related to the Jahn-Teller distortion (discussed below). In Fig. 7(b), a large (spherical) electron density around the O atoms is evident but a small Pr 4f density is also visible. The lobes of this 4f electron density point toward the O atoms and indicate a small hybridization (bonding) between Pr 4f and O 2p states in this energy range. Note the different orientations of the lobes coming from the localized [Fig. 7(a)] or hybridized [Fig. 7(b)] 4f electrons. The former tend to avoid the O atoms, whereas the latter point toward them.

This picture for the electronic structure of PrO$_2$ (one localized 4f electron plus $\sim$0.65 delocalized 4f electron) is consistent with the conclusions drawn from experimental studies. Derived from experimental data, van der Kolk and Dorenbos proposed a model to predict the insulating or metallic behavior and chemical stability of lanthanide materials. From their model, it was predicted that for PrO$_2$, the 4f empty states are situated at about 2 eV above the top of the valence band and 4 eV below the 5d conduction band, a feature which is quite well reproduced by our calculated DOS.

Figure 8 shows the real part of the optical conductivity which was obtained with $U_{\text{eff}}$=6 eV. The onset of the conductivity at $\sim$2 eV and the position of the most important peaks are in fair agreement with the measured spectrum of PrO$_2$. As indicated in Fig. 8, the transitions below 6 eV are mainly from O 2p bands (which also contain some Pr 4f and Pr 5d characters) into Pr 4f bands. Between 6 and 16 eV, the transitions are from the O 2p bands into bands of mixed Pr 5d/O 2p character, while above 16 eV, the transitions involve the Pr 5p/O 2x occupied and Pr 5d/O 2p unoccupied bands.

The variation of the PBE+$U$+SO total energy with respect to the oxygen distortion $d$ is shown in Fig. 9 for $U_{\text{eff}}$=6 eV. We can see that three curves have a minimum situated at 0.059 Å (see also Table II) which is only 0.006 Å smaller than the lower bound of the range 0.065–0.078 Å of the experimental values. These three curves correspond to different magnetic phases (AF1, AF2, and F) but all for the JT2 distorted structure. The same minimum value for the distortion $d$ indicates that the magnetic ordering has a negligible influence on the Jahn-Teller effect. This is in accordance with the experimental fact that the Néel temperature $T_N$=13.5 K is much lower than the distortion temperature of $T_D$=120 K. The JT2-AF1 and JT2-F curves are nearly indistinguishable (about 0.01 mRy/f.u. of difference at the minimum), while the JT2-AF2 curve is about 0.1 mRy/f.u. higher in energy. Concerning the other distorted structures, the minimum of structures JT1 and JT4 are situated at $d$=0.032 and 0.038 Å, respectively, which are clearly smaller than the lower bound of the experimental values, while structure JT3 does not distort at all.

The reason why the JT2 structure leads to the largest distortion and lowest total energy is quite simple. As described previously, the Pr 4f electron density has eight dominating lobes [see Fig. 3 or 7(a)], four of them (situated within the same plane) pointing approximately in the direction of first-neighboring oxygen atoms. For the JT2 structure, these four lobes point in the directions of the oxygen atoms which correspond to a long Pr–O bond [see Fig. 2(c)], which is the energetically most favorable situation. For the case of the distorted structures JT1 and JT4, this situation happens only for 50% of the Pr atoms, namely, the Pr atoms which are in oxygen cages (a) and (f) of Fig. 2, respectively, while the other 50% Pr atoms are in cages which do not lead to such a
TABLE II. Properties of PrO$_2$ for several magnetic phases obtained with PBE+$U+SO$ ($U_{eff}=6$ eV) for the JT2 structure. $d$ (in Å) is the magnitude of the distortion and $E$ (in mRy/f.u.) is the total energy at $d$ ($E=0$ was set at the minimum of the lowest-energy curve). $\mu_S$, $\mu_L$, and $\mu_{tot}$ (in $\mu_B$) are the spin, orbital, and total magnetic moments of Pr atom, and $\Delta$ (in eV) is the fundamental band gap.

<table>
<thead>
<tr>
<th></th>
<th>$d$</th>
<th>$E$</th>
<th>$\mu_S$</th>
<th>$\mu_L$</th>
<th>$\mu_{tot}$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF1</td>
<td>0.059</td>
<td>0</td>
<td>1.11</td>
<td>-0.63</td>
<td>0.48</td>
<td>1.7</td>
</tr>
<tr>
<td>AF2</td>
<td>0.059</td>
<td>0.11</td>
<td>1.11</td>
<td>-0.62</td>
<td>0.49</td>
<td>1.8</td>
</tr>
<tr>
<td>F</td>
<td>0.059</td>
<td>0.01</td>
<td>1.11</td>
<td>-0.62</td>
<td>0.48</td>
<td>1.5</td>
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<td></td>
<td></td>
<td></td>
<td>0.5–0.75$^b$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Reference 30.
$^b$References 30, 33, and 41.

favorable situation [(b) and (e) of Fig. 2, respectively]. In the case of structure JT3, such a favorable situation never happens since the corresponding oxygen cage is (d) of Fig. 2, where the four short Pr–O bonds form a tetrahedron.

As already noted above, we can see from Table II that the orbital contribution $\mu_L$ to the magnetic moment is important with $-0.6 \, \mu_B$ and the opposite sign of the spin contribution $\mu_S$. By taking both into account, we find a total magnetic moment $\mu_{tot}$ in good agreement with the experimental range $0.5–0.75 \, \mu_B$. Similarity as for the distortion, the magnetic moments and fundamental band gap do not vary much among the different magnetic phases.

V. CONCLUSION

Bulk PrO$_2$ was studied using the PBE+$U$ functional for the exchange-correlation energy including SO coupling. The influence of the Coulomb parameter $U_{eff}=U-J$ (which was varied from 0 to 8 eV) on the electronic, magnetic, and structural properties was investigated. The results showed that good agreement with experiment can be obtained for the magnetic moment and magnitude of the distortion of the oxygen sublattice for values of $U_{eff}$ situated between 0.2 and 6 eV. $U_{eff}=6$ eV seems to be a good choice since it leads to qualitative agreement with the experiment for the electronic structure and is relatively close to the $ab$ initio value of $U_{eff}$ which was obtained from a constrained calculation. Nevertheless, $U_{eff}=6$ eV leads to a magnitude for the distortion from the cubic fluorite structure which is slightly smaller than the reported experimental value, while a perfect agreement can be obtained with, e.g., $U_{eff}=0.2$ or 4 eV. With $U_{eff}=6$ eV, about 1.65 4f electrons (one localized plus 0.65 delocalized) per Pr atom are found, which means that the Pr ions are intermediate between trivalent Pr$^{3+}$ and tetravalent Pr$^{4+}$. We also studied the effects of the long-range magnetic ordering on the calculated properties but found them not to be important. These results were obtained for the distorted structure JT2, which has the lowest total energy among the four models of distortion we considered. Therefore, our calculations strongly support the JT2 structure over the other distortion models considered. Only for this case we obtain results that are consistent with the available experimental data.

Note added in proof. Recently we became aware of two papers by Webster et al.$^{63}$ (x-ray diffraction experiment) and Jensen$^{64}$ (theoretical calculations using a model Hamiltonian) on PrO$_2$. In both Refs. 63 and 64 it is concluded that the distorted structure JT2 is the correct one, which is in accordance with our results.

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