

# COMBINED SEISMIC ACTIVATION OF A SDOF-BUILDING WITH A PASSIVE TLCD ATTACHED

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## SUMMARY

One of the effective means of reducing the vibration amplitude of tall buildings, due to horizontal seismic activation, is the installation of tuned liquid column dampers (TLCDs). TLCDs are innovative vibration absorbing devices in the low frequency range, which have been studied extensively in the last decade. However, most research work concentrates on the suppression of horizontal motions of structures, but the vertical component of the seismic activation influences more or less the damping characteristics of an attached TLCD, apparently including the possibility of occurrence of dangerous parametric resonance, likewise to the pendulum type TMD. A detailed study of an SDOF-building with a TLCD attached, under combined horizontal and vertical excitation, is presented in the form of computer modeling and verified with an experimental model setup. The experimental results turns out to be in good agreement with the theoretical predictions of the computer model and both, theoretical and experimental investigations indicate that the vertical component of seismic activation should be considered carefully since in case of parametric resonance the TLCD dynamics would become unstable. However, it will be shown that sufficient damping of the fluid motion is crucial for stability and the optimally required damping coefficient usually turns out to keep the attached TLCD on the safe side. The over-linear turbulent damping presented in the TLCD stabilizes further and contributes one more feature of superiority over conventional TMDs.

## INTRODUCTION

In civil engineering there is a trend to construct relatively light and flexible structures, which are consequently more sensitive to dynamic loads such as wind gusts and earthquakes. Hence, undesired large vibration amplitudes appear which endanger the building and bring uncomfort to their inhabitants. It has been shown that installing secondary tuned structures is one of the effective means of reducing these vibrations, see e.g. *Soong* [1]. Successful and practical, control of vibrations has been achieved by the innovative TLCDs, which have been studied extensively during the last decade, see e.g. *Hochrainer* [2]. TLCDs rely on the motion of a liquid mass in a tube-like container to counteract the external motion while a built-in orifice plate induces turbulent damping forces that dissipate kinetic energy. The most important

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advantages over other types of damping devices are: easy tuning of frequency and damping, little additional mass since water is stored in buildings for fire protection, easy accommodation in structures even in retrofit, the construction is very simple and cheap and maintenance costs are nearly zero. The comparable or even better performance to the conventional tuned mass dampers (TMDs) is evident. Therefore, it is a preferable device for low frequency vibration control of high-rise buildings and long span bridges. However, most scientific work focuses on the suppression of horizontal motion of structures due to earthquake activation and neglected the vertical component. Hence, the objective of this study is to develop a more general formulation of a SDOF building with a TLCD attached and to investigate the influence of the vertical seismic activation on the damping characteristic of TLCDs. It will be shown, that due to vertical excitation the TLCD dynamics becomes nonlinear and quite sensitive to parametric resonance. Parametric resonance causes an unstable dynamic system and leads to an undesired increase of the TLCD vibration amplitude. Thus, the damping behavior of the TLCD might get lost. However, it will be shown that damping strongly influences the occurrence of parametric resonance and due to sufficiently high damping of the TLCD parametric resonance is not observed. Hence, the optimal tuning of the TLCD, (tuned by frequency and equivalent linear damping term), usually turns out a sufficient high value of fluid damping for the considered range of vertical excitation amplitudes and thus, keeps the attached TLCD on the safe side. It has to be mentioned that the danger of parametric resonance also exists for the conventional pendulum type TMD damper whose point of suspension moves vertically. It will be shown that the nonlinear turbulent damping of the TLCD stabilizes the firstly unstable motion and thus, represents a further great advantage of TLCDs.

In order to get further insight into the phenomenon of parametric resonance, a detailed study of a SDOFbuilding with a TLCD attached, considered under combined and assigned horizontal and vertical excitation, is presented in the form of computer modeling and experimentally verified with a model setup. The small scale testing facility has been constructed in the laboratory of the Institute of Rational Mechanics at the Vienna University of Technology. Series of free and electro-dynamically forced vibration tests with time harmonic signals are performed. The experimental results are compared with those derived by computational simulations and indicate an excellent agreement. Furthermore, numerical simulations are performed when considering the combined seismic activation through the realistic European Friuli earthquake, which appeared in the year 1976 in northern Italy. Both, theoretical and experimental investigations indicate that the vertical component of seismic activations should always be considered since in case of parametric resonance a sufficiently high value of fluid damping is crucial in order to prevent undesired worsening-effects on the damping characteristics of the attached passive TLCD. It is further recommended to use sealed tubes and to include the over-linear air-spring effect to further counteract parametric resonance.

## MECHANICAL MODEL

### **Substructure - separated TLCD**

The equation of motion for the main structure, a SDOF shear frame, coupled with a TLCD is derived by the substructure synthesis method, which splits the problem into two parts. Hence, in a first step the TLCD is separated from the main structure and considered under combined and assigned total horizontal  $w_t$  and total vertical  $v_t$  floor excitation, illustrated in Fig. 1. The TLCDs parameters are the horizontal length of the liquid column *B*, the length of the liquid column in the inclined pipe section at rest *H*, i.e. its total length is B+2H, the inclined and horizontal cross-sectional areas  $A_H$  und  $A_B$ , respectively, and the opening angle of the inclined pipe section  $\pi/6 < \beta < \pi/2$ . Further let  $\rho$  denote the liquid density, e.g. water  $\rho = 1000 kg/m^3$ . The relative and incompressible flow of the liquid inside the container is described by the liquid surface displacement u. In the considered mechanical model of the TLCD an open piping system is assumed, thus the pressure difference  $\Delta p = p_2 - p_1$  becomes approximately zero.



Fig. 1. TLCD with symmetrical shape under combined and assigned horizontal and vertical excitation

Applying the modified Bernoulli equation along the relative non-stationary streamline in the moving frame, see *Ziegler* [3, p. 497], yields the nonlinear, parametric excited equation of motion of the TLCD,

$$\ddot{u} + \delta_L \left| \dot{u} \right| \dot{u} + \omega_A^2 \left[ 1 + \frac{\ddot{v}_t}{g} \right] u = -\kappa \ddot{w}_t , \quad \kappa = \frac{B + 2H \cos \beta}{L_{eff}}, \quad L_{eff} = 2H + \frac{A_H}{A_B} B , \tag{1}$$

where  $\kappa$  and  $L_{eff}$  denote a geometry dependent coupling factor and the effective length of the liquid column. The circular eigen frequency of the undamped TLCD is given by

$$\omega_A = \sqrt{\frac{2g\sin\beta}{L_{eff}}},\tag{2}$$

where g is the gravity constant. Furthermore,  $\delta_L$  denotes the head loss coefficient due to turbulent losses along the relative streamline and additional losses due to the built in orifice. The values of the head loss coefficient, in case of stationary flow, for relevant pipe elements and cross-sections are tabulated and given e.g. in *Idelchick* [4]. In Eq. (1), the vertical excitation  $v_t$  causes the conventional equation of motion for the plane TLCD with constant stiffness parameter, see e.g. [2], to become time variant. Due to this time-variant term the motion of the lightly and linear damped TLCD might become unstable, resulting in infinite vibration amplitudes. A detailed study of this phenomenon, called parametric resonance, is presented in the section Vertical Excitation. In view of an optimal design of the attached TLCD, see section Optimal Tuning of TLCDs, the nonlinear equation of motion, Eq. (1), has to be transformed into an equivalent linear one. Basically, the method of harmonic balance, see [3], is used to transform the nonlinear turbulent damping term  $\delta_L$  into its equivalent linear viscous damping term  $\zeta_A$  for an assigned maximum vibration amplitude. After applying this method, Eq. (2) can be rewritten as

$$\ddot{u} + 2\zeta_A \,\omega_A \,\dot{u} + \omega_A^2 \left[ 1 + \frac{\ddot{v}_t}{g} \right] u = -\kappa \ddot{w}_t \,, \quad \zeta_A = 4U_0 \,\delta_L \,/ \,3\pi \,, \tag{3}$$

where  $U_0$  denotes the relative vibration displacement amplitude of the liquid surface. Thereby, the value of  $U_0$  is determined out of numerical simulations of the linear system and commonly chosen as  $U_0 = U_{\text{max}}$ . Subsequently, optimal tuning of TLCDs renders the required eigen frequency  $\omega_A$  and equivalent linear viscous damping coefficient  $\zeta_A$ . Having established the equation of motion for the TLCD, in a second step, the resultant interaction forces have to be determined. Assuming that the dead weight of the TLCD  $m_T = m - m_f$  is added to the floor mass, only the interaction forces between the massless, rigid, liquid filled piping system and the supporting floor are considered. Conservation of momentum applied to a virtual, massless container with fluid mass  $m_f$  in an instant configuration, renders the resultant components of the external force in x' and z' direction, (acting on the fluid body),

$$F_{x'} = m_f \left( \ddot{w}_t + \bar{\kappa} \ddot{u} \right), \quad \bar{\kappa} = \frac{B + 2H\cos\beta}{L_1}, \quad m_f = \rho \left( 2HA_H + BA_B \right) = \rho A_H L_1, \tag{5}$$

$$F_{z'} = m_f \left( \ddot{v}_t + \overline{\kappa}_1 \frac{1}{H} \left( u \ddot{u} + \dot{u}^2 \right) \right), \quad \overline{\kappa}_1 = \frac{2H \sin\beta}{L_1}, \quad L_1 = 2H + \frac{A_B}{A_H} B \quad , \tag{6}$$

where  $\overline{\kappa}$  and  $\overline{\kappa}_1$  define additional geometry dependent factors and  $L_1$  denotes a cross sectional dependent factor which becomes equal  $L_{eff}$  in case of constant cross sectional areas  $A_H = A_B$ . Furthermore, conservation of angular momentum with respect to the accelerated reference point A, see Fig. 1, yields the dynamic part of undesired moment, in addition to the static moment  $m_f g \overline{\kappa} u$ , acting about the y axis,

$$M_A = m_f \left[ -\overline{\kappa}_1 \frac{B}{2} \ddot{u} + \overline{\kappa}_1 \frac{1}{2H} \left( H^2 + u^2 \right) \ddot{w}_t - \overline{\kappa} u \, \ddot{v}_t \right]. \tag{7}$$

However, it is common practice to neglect the influence of this undesired static and dynamic moments which partly also exist for conventional TMDs. In view of implementing TLCDs to long span bridges, the undesired moment from gravity due to large vibration amplitudes of the liquid surface, must be considered.

#### Substructure - separated SDOF shear frame with assigned interaction forces

In this section the equation of motion of the separated main structure, a SDOF shear frame, with assigned interaction forces from the TLCD dynamics is derived. For this sake, the free body diagram of the main structure is considered, shown in Fig. 2.



Fig. 2. Free body diagram of main structure with acting interaction forces from the TLCD dynamics

In Fig. 2, the main structure is assumed to be a SDOF shear frame with its deformation, due to horizontal ground excitation  $\ddot{w}_g$ , given by the displacement w. Further, the main structure moves vertically due to vertical ground excitation  $\ddot{v}_g$ . This latter movement of the main structure is approximately considered as a rigid body motion, (no time-variant  $P-\Delta$  effect in the CC-elastic columns). The main structure has parameters defined by the moving floor mass M, which includes the dead weight  $m_T = m - m_f$  of the TLCD and modal masses of the columns, the total stiffness of the massless columns k and linear damping coefficient  $\zeta_S$ . The total stiffness k of the CC-columns may be effectively corrected according to the compressive axial force from the dead weight of the supporting floor, see *Clough-Penzien* [5]. Applying the basic law of conservation of momentum to the free body diagram of the main structure, Fig. 2, and neglecting the undesired moment  $M_A$  and the eccentricity of the interaction force  $F_{x'}$ , which also exists for conventional TMDs, yield the linear equation of motion

$$\ddot{w} + 2\zeta_S \,\Omega_S \,\dot{w} + \Omega_S^2 \,w = -\ddot{w}_g - \frac{1}{M} F_{x'} \,, \qquad \zeta_S = \frac{r}{2M \,\Omega_S} \,, \tag{8}$$

where  $\Omega_S$  and  $\zeta_S$  denote the circular eigen frequency of the main structure given by  $\Omega_S = \sqrt{k/M}$  and the linear viscous damping coefficient, which is small,  $\zeta_S < 0.20$ , and represents the material damping in Eq. (8). Inserting the coupling force  $F_{X'}$ , Eq. (5), into Eq. (8) renders together with Eq. (1) the coupled equations of motion for the two DOF system, main structure/TLCD,

$$\ddot{w} + 2\zeta_S \,\Omega_S \,\dot{w} + \Omega_S^2 \,w = -\ddot{w}_g - \mu \,\ddot{w}_t - \mu \,\overline{\kappa} \,\ddot{u} ,$$

$$\ddot{u} + \delta_L \left| \dot{u} \right| \dot{u} + \omega_A^2 \left[ 1 + \frac{\ddot{v}_t}{g} \right] u = -\kappa \,\ddot{w}_t , \qquad w_t = w_g + w, \quad v_t \approx v_g ,$$
(9)

where  $\mu$  denotes the TLCD fluid mass to main structure mass ratio determined by

$$\mu = \frac{m_f}{M} \ll 1. \tag{10}$$

The equations of motion are coupled due to the geometry dependent factors  $\kappa$  and  $\overline{\kappa}$ , defined in Eqs. (5) and (6). In case of constant cross sectional area of the piping system  $A_H = A_B = A$ , the geometry dependent factors  $\kappa$  and  $\overline{\kappa}$  become equal. In order to provide highest possible energy transfer from the main structure to TLCD and thus, dissipation of kinetic energy due to turbulent damping of the fluid flow inside the container, the coupling factors should be maximized. Of course, it must be mentioned that increasing the coupling factors leads to a high interaction force  $F_{x'}$  and hence, to an increasing of the undesired moment which has been neglected in the formulation of moment of momentum.

#### VERTICAL EXCITATION

Comparing the nonlinear, parametric excited equation of motion of separated TLCD, Eq. (1), with the conventional equation of motion without any vertical excitation, see e.g. [1], indicates an additional time-variant stiffness parameter due to vertical forcing. This time-variant stiffness leads to parametric excited oscillations of the liquid surface displacement u and, under special conditions, to the undesired instability phenomenon of parametric resonance. The important task of this section is to find a criterion in form of a required linear damping value  $\zeta_A$  in order to prevent the occurrence of parametric resonance.

Subsequently, assuming absence of any horizontal excitation  $\ddot{w}_t$  and further neglecting the turbulent and/or any viscous damping term, Eq. (1) simplifies to the time-variant undamped oscillator equation,

$$\ddot{u} + \omega_A^2 \left[ 1 + \frac{\ddot{v}_t}{g} \right] u = 0.$$
<sup>(11)</sup>

Assigning time-harmonic total vertical excitation  $v_t = v_{t0} \cos v_z t$ , Eq. (11) becomes a special type of Hills differential equation, the so-called Mathieu equation, see *Klotter* [6, p. 281],

$$\ddot{u} + \omega_A^2 \left[ 1 - v_z^2 \, \frac{v_{t0}}{g} \cos v_z t \right] \, u = 0 \,. \tag{12}$$

The Mathieu equation is a linear ordinary differential equation with time-harmonic variant stiffness parameter. Classically, Mathieu equation is the equation of a plane pendulum with vertically moving point of suspension, see e.g. [3, p. 576] and compare with the conventional pendulum type of TMD. It is not possible to find a closed form solution of Mathieu equation, but the determination of dynamic stability, i.e. statement of stable or unstable behavior of the solution, has been studied extensively in the last century, see e.g. *Nayfeh* [7, p. 258]. In order to reduce Eq. (12) into the standard form of Mathieu equation we introduce the non dimensional time  $\tau = v_z t$  and hence, after the appropriate transformation, Eq. (12) can be rewritten as,

$$u'' + [\lambda + \gamma \cos \tau] u = 0, \quad u' = \frac{du}{d\tau},$$
(13)

where  $\lambda$  and  $\gamma$  denote the nondimensional stability parameters given by

$$\lambda = \frac{\omega_A^2}{v_z^2}, \quad \gamma = -\frac{v_{t0}}{g} \; \omega_A^2. \tag{14}$$

Obviously, depending on the choice of the stability parameters  $\lambda$  and  $\gamma$  the motion may become either stable or unstable. The domains of stability for Mathieu equation can be determined applying the perturbation method, see again [7]. The results of this method are given in form of the Ince-Strutt diagram, see Fig. 3, where the stable domains correspond to darkened areas.



Fig. 3. Domains of stability and instability given as Ince-Strutt diagram,  $\delta^2 = \lambda \zeta_A^2$ , source: [6, p. 301]

If at certain values of the parameters  $\lambda$  and  $\gamma$  instability occurs, then this phenomenon is called parametric resonance. From Fig. 3 one can see that parametric resonance may occur not only at single vertical excitation frequencies  $V_z$ , (for small values of  $\gamma$  instability occurs close to  $\lambda = n^2/4$ , where *n* is an integer), which is an essential difference to the conventional resonance problem. Furthermore, one can see that danger of parametric resonance mainly occurs at the frequency ratio  $\lambda = 1/4$ , (the domain of instability at this value is quite large) and also that damping strongly influences the occurrence of parametric resonance: If linear viscous damping of the dynamic system is taken into account, the domains of stability increases in the Ince-Strutt map, indicated by hyperbolic curves in Fig. 3.

Subsequently, referring to the problem of the model TLCD without orifice plate built-in, one may assume that the equivalent linear viscous damping  $\zeta_A$  is about 5%, (which is the case in the experimental studies). Due to this fact, the domain of stability increases and parametric resonance is suppressed until the critical value of vertical excitation amplitude  $v_{t0}$  is reached, see Eq. (14). After overshooting this critical value, parametric resonance occurs and leads to growth of the vibration amplitudes of the linear damped system beyond bounds. At this point, the nonlinear turbulent damping of the moving fluid always stabilizes the vibrating system, which is an additional favorable advantage of the TLCD over the pendulum type of TMD. The required value for equivalent linear viscous damping of the TLCD,  $\zeta_{A,requ.}$  for stable motions at the frequency ratio  $\lambda = 1/4$ , has been presented in [7, p. 301],

$$\zeta_{A,requ.} = \frac{v_{t0}}{g} \,\,\omega_A^2 \,, \tag{15}$$

which equals the absolute value of the stability parameter  $\gamma$ , given in Eq. (14).

## NUMERICAL ANALYSIS UNDER TIME-HARMONIC EXCITATION

The following numerical simulations are performed, according to the experimental investigations for a partially optimized TLCD configuration, which is tuned by frequency only. The combined and assigned base excitation in horizontal and vertical directions are assumed to be time-harmonic in the following manner,

$$w_{g} = w_{g0} \cos v_{x}t, \quad \ddot{w}_{g} = -v_{x}^{2} w_{g0} \cos v_{x}t,$$

$$v_{g} = v_{g0} \cos v_{z}t, \quad \ddot{v}_{g} = -v_{x}^{2} w_{g0} \cos v_{x}t.$$
(16)

Furthermore, the vertical excitation frequency  $v_z$  and the vertical excitation amplitude  $v_{g0}$  are kept constant within the range of numerical analysis. On the other hand the horizontal excitation frequency  $v_x$  is sweeped over the frequency range of interest. Thereby  $v_z$  is chosen twice the eigen frequency of the TLCD,  $v_z = 2\omega_A$ , i.e. stability parameter  $\lambda = 1/4$ , see Eq. (14), in order to observe parametric resonance.

#### **Partially-optimized TLCD**

The parameters of TLCD and main structure for performing the numerical analysis are chosen according to the laboratory model. However, contrary to actual problems, the natural frequency of the main system  $\Omega_S$  is subjected to optimization, together with the turbulent damping equivalent  $\zeta_A$ . The reason for this unusual way is based on the fact that the eigen frequency of the test main structure model is more easily tuned than the eigen frequency of the TLCD in the laboratory. The assumption of parameters is given in

Table 1. From Table 1 the ratio of fluid mass  $m_f$  to the total floor mass of the main structure M is given by

$$\mu = \frac{m_f}{M} = 0.071 = 7.1\%, \tag{17}$$

which is higher than the desired mass ratio  $\mu = 0.5 - 3\%$  of modal building mass in practice. In Table 1, the damping terms  $\zeta_A$  for the TLCD and  $\zeta_S$  for the main structure are determined from free vibration tests in the laboratory. Thereby  $\zeta_A$  is found as the mean value of certain free vibration tests within an amplitude range of  $U_0 = 40 - 60mm$ . The results obtained by numerical simulations are presented and discussed in the last section of this paper. Thereby it is important to emphasize that the nonlinear turbulent damping term, given in Eq. (3), must be considered in the equation of the TLCD in order to get good agreements between numerical and experimental results.

Table 1. parameters of partially optimized TLCD and primary structure; parameter for the combined and assigned time-harmonic base excitation

PARTIALLY OPTIMIZED TLCD			
$\beta = \pi / 4$	$A_H = A_B = 0.0005 m^2$	$L_{eff} = B + 2H = 0.42  r$	$m \qquad m_f = 0.21  kg$
$\omega_A = 5.65  rad  /  s$ ,	$f_A = \frac{\omega_A}{2\pi} = 0.90  Hz$	$\zeta_A = 0.045 \rightarrow \text{not subjected to optimization}$	
MAIN STRUCTURE			
$\Omega_S = \text{optimal}$ , $f_A = \text{optimal}$		$\zeta_S = 0.01$	M = 2.96  kg
HORIZONTAL AND VERTICAL EXCITATION PARAMETERS			
$v_x \rightarrow$ sweeped,	$f_x \rightarrow$ sweeped	$w_{g0} = 0.004 m$	
$v_z = 2\omega_A = 11.30 \ rad$	$/ s$ , $f_z = 2f_A = 1.80 Hz$	$v_{g0} = 0.016 m$	

#### **OPTIMAL TUNING OF TLCDs**

For optimal tuning of the attached TLCD the design parameters  $\delta^* = \omega_A^* / \Omega_S^*$  and  $\zeta_A^*$  have to be chosen suitably, where the star over them indicates that these parameters are in accordance with the conventional design parameters of a conjugate TMD problem, see *Den Hartog* [8, p. 91]. The derivation of the optimal design parameters for TLCDs and the analogy of TMD and TLCD for a SDOF-main structure have been derived by *Hochrainer* [2]. Thus, the optimal frequency ratio  $\delta^*$  of the conjugate TMD system is related to that of the TLCD by

$$\delta^* = \frac{\omega_A^*}{\Omega_S^*} = \delta \sqrt{1 + \mu \left(1 - \kappa^2\right)}, \quad \delta = \frac{\omega_A}{\Omega_S}.$$
(18)

Further the optimal equivalent linear damping term  $\zeta_A^*$  of conjugate TMD system remains unchanged

$$\zeta_A^* = \zeta_A \quad . \tag{19}$$

In case of time-harmonic base excitation, *Den Hartog's* [8] solution for optimal tuning parameters  $\delta_{opt}^*$ and  $\zeta_{opt}^*$  require

$$\delta_{opt}^{*} = \frac{\sqrt{1 - (\mu^{*}/2)}}{1 + \mu^{*}}, \qquad \zeta_{A,opt}^{*} = \sqrt{\frac{3\mu^{*}}{8(1 + \mu^{*})(1 - \mu^{*}/2)}}, \qquad (20)$$

where  $\mu^*$  denotes the conjugate mass ratio given in [2],

$$\mu^* = \frac{m_f^*}{M^*} = \frac{\kappa^2 \mu}{1 + \mu \left(1 - \kappa^2\right)}, \qquad \mu = \frac{m_f}{M}.$$
(21)

Working out Eq. (21) for the model parameters, see Table 1, and inserting in Eq. (20), result in the following values for optimal tuning parameters of the conjugate TMD problem,

$$\delta_{opt}^* = 0.95, \qquad \zeta_{A,opt}^* = 0.11.$$
 (22)

By means of these values it is possible to calculate the real frequency ratio  $\delta$  of the TLCD problem by solving Eq. (18). Hence, the optimal frequency ratio of the model and optimal frequency of the main structure becomes

$$\delta_{opt} = \frac{\delta_{opt}^*}{\sqrt{1 + \mu \left(1 - \kappa^2\right)}} = 0.94, \qquad \Omega_{S,opt} = 6.01 \, rad \, / \, s, \qquad f_{S,opt} = \frac{\Omega_{S,opt}}{2\pi} = 0.96 \, Hz \,. \tag{23}$$

It is important to note that the optimal value of linear damping coefficient  $\zeta_{A,opt} = \zeta_{A,opt}^* = 0.11$  is not considered in the numerical and experimental study. The reason for this sub-optimal assumption refers to the experimental possibilities: In order to study the phenomenon of parametric resonance, a sufficiently large vertical excitation amplitude  $v_{g0}$  is necessary. Its value is limited in the laboratory by  $v_{g0} \le 16 mm$  due to the deployed type of actuator, Brüel&Kjar 4808. On the other hand, damping strongly influences the occurrence of parametric resonance. Increasing the value of the damping coefficient  $\zeta_A$  enlarges the stability domain in the Ince-Strutt map, see again Fig. 3. To observe parametric resonance, the damping coefficient for the TLCD must be kept very small, i.e. no orifice plate is built-in into the fluid stream path. Performing free vibration tests determines the viscous damping coefficient  $\zeta_A = 0.045$  as indicated in Table 1 and thus, this value is not increased to the optimal one for these numerical and experimental investigations. Only one numerical example will be presented under consideration of  $\zeta_{A,opt}^*$  in order to illustrate that in case of optimal damping, parametric resonance and its influences to the optimal damping behavior of TLCDs are not observed.

#### **EXPERIMENTAL INVESTIGATION**

In order to get more insight into the phenomenon of parametric resonance and to compare the results of numerical simulation with experimentally obtained results, a small scale testing facility has been

developed in the laboratory of the Institute of Rational Mechanics at the Vienna University of Technology. A front view of the constructed testing facility is shown in Fig. 4.



Fig. 4. Front view of the experimental model set-up

The experimental model set-up, a plane SDOF pendulum, consists of two rigid aluminum bars, an upper and a lower one. These bars are connected with to pairs of hangers, whereby a set of roller bearings is placed at each joint in order to achieve a small damping coefficient of the structural model. The upper rigid bar is carried on a base framework, connected with a set of guiding Thomson bearings. The latter show negligible friction and due to vertical disposition of the bearings the upper rigid bar is constrained to a vertical translational motion. The lower rigid bar represents the main structure of the experimental model, comparable to the floor of an equivalent SDOF-shear frame. The TLCD made of plexiglass pipe with rectangular cross section is fixed on top of the lower bar. One end of the lower bar is linked to an actuator, an electromagnetic shaker of Brüel&Kjaer Type 4808, through a coil spring whose stiffness models the elastic columns of an equivalent SDOF-shear frame. Additionally, a second electromagnetic shaker of Brüel&Kjaer Type 4808, whose stroke amplitude is magnified by a simple lever construction, excites the upper bar vertically. The time-harmonic excitation signals are provided by the software LabView 7.0. In order to reduce the dynamic vertical forces, the structural model mass is counter-balanced by a pulley mechanism. Both the horizontal excitation displacement and the response displacement of the lower bar are measured by means of contact-less optical laser transducers, Type optoNCDT 1605. The vertical excitation acceleration is measured by means of a piezoelectric accelerometer of Brüel&Kjaer Type4367. The vertical accelerometer is connected to a charge amplifier and an implemented integrator, to transform the measured accelerations into equivalent displacements. All measured signals are recorded by means of the software BEAM-DMCplusV3.7 through the board of Digital Amplifier System DMCplus (Hottinger Baldwin Messtechnik, HBM).

For the sake of measuring the liquid surface displacement inside the TLCD a new electronic resistance measurement device has been developed. For this sake, the piping system is equipped with two pairs of wire electrodes whose resistance depends on the water level inside the pipe. To compensate for several nonlinearities, the electrodes are in series connection for the actual resistance measurements. For further processing, the electronic signal is band-pass-filtered, in order to reduce the static drift and high frequency noise. The measured signal is recorded again by means of the software BEAM-DMCplusV3.7, whereby the measured changes of resistance are transformed into the equivalent displacement of the liquid surface by means of a nonlinear transfer function, obtained by calibration of the TLCD.

Free vibration tests of primary structure and TLCD were performed to determine the natural frequency and damping coefficient. Furthermore a series of forced vibration tests were performed with a partially optimized TLCD configuration, as indicated in the previous section, sweeping the horizontal excitation frequencies to study the influence of parametric resonance on the damping behavior of TLCDs. The experimentally obtained results due to forced vibrations are presented in the following section a comparison of experimental and numerical results.

## COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

The experimentally and numerically obtained results of time history response of the main structure for the partially optimized TLCD, tuned by frequency only, with a horizontal forcing frequency close to the eigen frequency of the main structure  $f_x = 1.00 Hz$  is shown in Fig. 6.



Fig. 6. Experimental and numerical results of time history response of main structure; nonlinear turbulent damping term  $\delta_L = 1.325$ ; horizontal excitation frequency  $f_x = 1.00 Hz$ ; partially optimized TLCD with light damping (no built-in orifice plate)

The blue line represents the steady state vibrations without any vertical excitation and the red line indicates the influence of the assigned vertical excitation. It is important to emphasize that the chosen combination of the values  $v_z$  and  $v_{g0}$  leads to parametric resonance for the light damping assigned. Of course, by increasing the vertical excitation amplitude  $v_{g0}$ , which is limited in course of the experiment, parametric resonance occurs much stronger. Furthermore, one can see that the time history response due to additional vertical excitation leads to the beat phenomenon. Fig. 6 shows an excellent agreement between experimental and theoretical results, whereby it is important to mention that the nonlinear turbulent damping term  $\delta_L$  must be considered in the equation of motion for the TLCD, given in Eq. (3). The linearized equation of motion, equivalent damping term  $\zeta_A$ , for an assumed maximal displacement amplitude u of the lightly damped TLCD, grows beyond bounds and thus, is not sufficient enough to compare experimental and numerical results, as illustrated in Fig. 7. Hence, it must be pointed out that the

nonlinear equation of motion, Eq. (1), is superior and describes the physical behavior of TLCDs more closely, when the excitation renders parametric resonance.



Fig. 7. Experimental and numerical results of time history response of main structure; equivalent linear viscous damping term  $\zeta_A = 0.045$ ; horizontal excitation frequency  $f_x = 1.00 Hz$ ; partially optimized TLCD with light damping (no built-in orifice plate)

Furthermore, Fig. 8 shows the experimental and numerical results of the Dynamic Magnification Factor (DMF) of the main structure for the partially optimized TLCD in case of light damping.



Fig. 8. Experimental and numerical results of the Dynamic Magnification Factor (DMF) of the main structure; nonlinear turbulent damping term  $\delta_L$  varied over the frequency range of interest; partially optimized TLCD with light damping (no built-in orifice plate)

The DMF for the parametric excited coupled main structure/TLCD,  $v_g \neq 0$ , is determined at discrete values of the horizontal excitation frequency  $f_{x,i}$ . Thereby, the frequency dependent turbulent damping term  $\delta_{L,i}$  has to be varied over the frequency range of interest. In order to determine the appropriate value  $\delta_{L,i}$ , given in Eq. (3), one has to choose  $U_{0,i}$  by the maximum value of vibration amplitude, i.e.  $U_{0,i} = U_{\max,i}$ . The latter is found from numerical simulations of the linear system, Eq. (3), without any vertical excitation, and considering the steady state time history response. Together with the linear viscous damping term  $\zeta_A$ , which is found as mean value of certain free vibration tests within an amplitude range of  $U_0 = 40-60mm$ , the *DMF<sub>i</sub>* at the excitation frequency  $f_{x,i}$  is given as

$$DMF_i = \left| \frac{w_{max,i}}{w_g} \right|,\tag{24}$$

where  $w_{\text{max},i}$  is determined by the steady state time history response of the nonlinear, parametric excited coupled system. Out of Fig. 8 one can see that the DMF for the partially optimized TLCD turns out similar to *Den Hartog* [8, p. 98]. Den Hartog's solution yield a response function that contains two fixed-points, obtained by changing the linear viscous damping coefficient. In case of partially optimization these fixedpoints occur at the same height as illustrated in Fig. 8. Again an excellent agreement between experimental results and those derived by computational simulations are obtained. Furthermore, Fig 8 shows that the occurrence of parametric resonance approximately doubles the resonant peaks. However, when optimal damping, Eq. (22), is taken into account, no influence of parametric resonance is observed for the range of vertical excitation amplitudes considered, see Fig. 9.



Fig. 9. Numerically obtained results of the Dynamic Magnification Factor (DMF) of the main system; nonlinear turbulent damping term  $\delta_L$  varied over the frequency range of interest; optimal damping taken into account

Fig. 9 illustrates the numerically obtained DMF of the main structure with the optimal tuned TLCD, (tuned by frequency and damping) taken into account. According to Den Hartog's solution the frequency response function for the coupled system indicates two fixed-points at the same height and additional, as an effect of optimal damping of the TLCD, horizontal tangents in these two points. Hence, it can be concluded that parametric resonance only influences the lightly damped TLCD and disappears in case of optimal damping for the considered range of vertical excitation amplitudes.

#### NUMERICAL ANALYSIS UNDER REALISTIC EARTHQUAKE EXCITATION

In the following section, numerical simulations of the above mentioned mechanical model are performed considering the realistic European Friuli earthquake (north-south component, station name: Tolmezzo-Diga Ambiesta) which appeared in the year 1976 in northern Italy, downloaded from the EU funded Internet Site for *European Strong Motion Data* [9]. In Fig. 10 the original acceleration strong motion signal in time and frequency domain is presented. One can see that the duration of strong motion phase takes only a few seconds, from about t = 3 to t = 8s and further that the main quake energy occurs in the low frequency range, from about f = 2 to f = 5Hz. However, the eigen frequencies of the main structure and TLCD are given as  $f_s = 0.96Hz$  and  $f_A = 0.90Hz$ , see previous section, thus the time history of the

considered quake has to be adjusted. For further processing the given sampling rate  $\Delta t = 0.01 s$  is doubled which results in increasing the duration of the strong motion phase and decreasing the peaks location in the frequency domain.



Fig. 10. Friuli earthquake (north-south component, station name: Tolmezzo-Diga Ambiesta) in time and frequency domain, source: European Strong Motion Data [9]

The Friuli quake is assigned horizontal as well as vertical with the same strength. Again, nonlinear turbulent damping is taken into account in the equation of motion of the TLCD, which ensures a stable motion in case of parametric resonance. The simulations of the nonlinear, parametrically excited system have been performed using Simulink, a powerful tool allowing graphical programming, system analysis and simulation, which is smoothly integrated into the Matlab 6.5 scientific computing environment. Simulink calculates the response of nonlinear parametrically excited systems by time integration.



Fig. 11. Friuli earthquake (north south), main structure acceleration a) comparison of main structure without and with attached TLCD, reduction of maximum peak about 50% b) comparison of main structure acceleration without and with additional vertical excitation

The Fig. 11a shows the time history response of the main structure, whereby the blue line represents the response without TLCD and the read line indicates the damping effect of an attached optimally tuned TLCD. Thereby, the coupled main structure/TLCD is considered under combined and assigned horizontal

and vertical seismic activation, whereby equivalent optimal turbulent damping of the attached TLCD, for an assumed maximum vibration amplitude  $U_{max} = 80 mm$ , is taken into account. Furthermore in Fig 11b the time history response of the main structure with attached TLCD is presented. Thereby, the blue line represents the response in absence of any vertical excitation and the read line shows the response under consideration of additional vertical seismic activation. One can see, that the additional vertical excitation due to Friuli earthquake has no or only less influence on the optimal damping behavior of the TLCD. Both lines nearly coincide and thus confirm, that due to vertical excitation no undesired influence on the damping behavior of the TLCD is expected.

### CONCLUSION

Detailed theoretical and experimental investigations on the influence of additional vertical excitation on the damping behavior of a partially optimized TLCD, have been carried out. A mechanical model of a TLCD under combined and assigned horizontal and vertical base excitation has been developed and analyzed numerically. Furthermore, a small scale testing facility has been constructed in the laboratory of the TU-Institute to perform experimental investigations and to compare the obtained results. An excellent agreement between theoretically and experimentally derived results has been obtained. The vertical excitation frequency  $f_z$  is found most critical at the value  $f_z = 2f_A$ . At this value the domain of instability is quite large and thus the undesired phenomenon of parametric resonance is more probably encountered. Furthermore it is important to note that the equivalent linear damping term, in the linearized equation of motion of TLCD, is not sufficient for comparing experiment and computer simulation. Close agreement is obtained by using the nonlinear increasing damping term in the equation of motion of TLCD. Over-linear turbulent damping stabilizes the liquid flow and the vibration amplitude of TLCD becomes limited which contributes an important feature of superiority over TMDs. Damping strongly influences the occurrence of parametric resonance and due to optimal damping of the attached TLCD, parametric resonance is not observed within the considered range of vertical excitation amplitudes. It is further recommended to use sealed tubes and to include the over-linear air-spring effect to further counteract parametric resonance. In conclusion the vertical motion of structures should be considered in the design stage of the TLCD since in case of parametric resonance a sufficiently high value of fluid damping is crucial in order to prevent undesired worsening-effects on the damping characteristics of the attached passive TLCD.

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