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CONTROL OF PEDESTRIAN-INDUCED BRIDGE VIBRATIONS BY TUNED LIQUID COLUMN DAMPERS

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In this paper tuned liquid column dampers (TLCD) are proposed for vibration control of footbridges. TLCD are innovative vibration absorbing devices applicable to the low frequency range ($< 3.5 - 4.0\,Hz$) of bridges. Oblique bending and torsional motions of a continuous bridge with several TLCD installed are described mathematically and analyzed numerically. The dynamic behavior due to pedestrian excitation of the original Millennium Bridge in London is considered and it is shown that the installation of three TLCD to the main-span would have been necessary to secure its serviceability. The numerical simulations indicate that the TLCD increase the natural material damping of the bridge, which suppresses the instability phenomenon of synchronization, as discussed by Newland [1]. It is concluded that TLCD are effective and economic damping devices for the control of pedestrian-induced vibrations of footbridges.

Keywords: *passive vibration control, liquid motion, effective damping, synchronization*

1 Introduction

In recently constructed footbridges, such as the Millennium Bridge in London and the Toda Park Bridge in Japan, unexpected lateral vibrations occurred when a large number of pedestrians were crossing the bridges. In both cases of the mentioned bridges secondary tuned structures were installed in order to increase the system damping and to secure their serviceability. Especially the undesired instability phenomenon of synchronization, by which people respond naturally to a vibrating bridge when one of its natural frequencies is close to the walking or running frequency, was found to be responsible for the increasing of the lateral response of footbridges. Newland [1] has presented a required damping level to ensure that synchronization does not lead to high vibration amplitudes of bridges,

$$\zeta_{S,0} = \frac{1}{2}\frac{m_P^*}{M^*} \quad . \tag{1}$$

In Eq. (1) m_P^* and M^* denote a modal pedestrian mass and a modal bridge mass, respectively, see Reiterer [2]. In the present investigation it is proposed to apply the more efficient and more economic TLCD for suppressing pedestrian-induced vibrations of footbridges and to increase the effective damping to the required level. The TLCD is a damping device, which relies on the motion of liquid mass in a sealed rigid U-tube. The external motion of the main system (bridge) induces a phase-delayed motion of the liquid mass and hence, interaction forces and moments that counteract the external forces. A built-in orifice plate induces turbulent damping forces that dissipate kinetic energy. Due to the considered sealed U-tube a non negligible pressure difference can built up on either side of the inclined pipe sections. The arising air-spring effectively increases the frequency range of applicability above $0.5\,Hz$. For optimal tuning of the TLCD the natural circular frequency ω_A and the equivalent linear damping term ζ_A have to be chosen optimal, likewise to the conventional tuned mass damper (TMD), as discussed by Den Hartog [3]. In case of the TLCD ω_A and ζ_A can be simple controlled by the air-spring height H_L, which defines the length of the air column above the free liquid surface and by the opening angle of the built-in orifice, as illustrated in Fig. 1.

In this study a mathematical model of a coupled bridge/TLCD system is presented and analyzed numerically. A mechanical model of the original Millennium Bridge in London is analyzed numerically with and without TLCD. Three sealed TLCD are installed to the main-span. The performed numerical simulations indicate that the TLCD secure the serviceability of the main-span, requiring just a fraction of the actual installation costs.

2 Mathematical Model

The response of footbridges due to pedestrian excitation is a combination of horizontal, vertical and rotational motions. The equations of motion for the complex hybrid bridge/TLCD system are derived by a substructure synthesis method, which splits the problem into two parts. In a first part the sealed TLCD is

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separated from the bridge and considered under a combined horizontal w_t, vertical v_t and rotational φ_t excitation, as illustrated in Fig. 1. Thereby, the TLCD parameters are given by the horizontal length of the liquid column B, the length of the liquid column in the inclined pipe section at rest H (i.e. its total length is $B+2H$), the inclined and horizontal cross-sectional areas A_H and A_B, respectively, the air-spring height H_L, and the opening angle of the inclined pipe section $\pi/6<\beta<\pi/2$. The relative and incompressible flow of the liquid inside the pipe is described by the liquid surface displacement u.

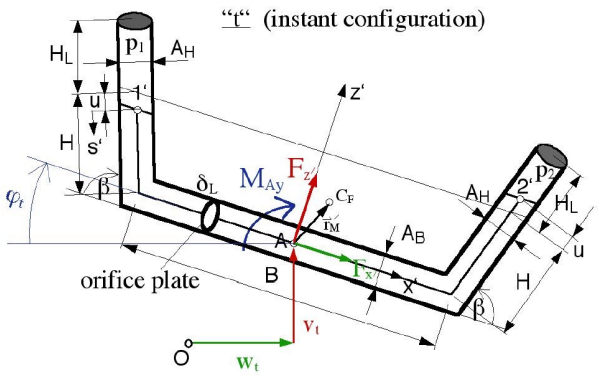

Figure 1: Separated TLCD, considered under a general plane motion

In Fig. 1 the reference point A is chosen symmetrically, whereby the absolute acceleration $\vec{a}_A$ is projected into the moving reference frame ($\varphi_t << 1$),

$$\vec{a}_A^{\,'} = \begin{pmatrix} a_{Ax}^{'} \\ a_{Az}^{'} \end{pmatrix} = \begin{pmatrix} \ddot{w}_t - \ddot{v}_t\,\varphi_t \\ \ddot{w}_t\,\varphi_t + \ddot{v}_t \end{pmatrix} . \tag{2}$$

Applying the modified Bernoulli equation along the relative non-stationary streamline in the moving frame and in an instant configuration, Ziegler [4], yields the nonlinear parametric excited equation of motion of the separated TLCD ($\varphi_t << 1$), see Reiterer [2],

$$\begin{aligned} \ddot{u} + \delta_L |\dot{u}|\dot{u} + \omega_A^2 \left[1 + \frac{2\,a_{Az}^{'}}{L_{eff}\,\omega_A^2} \sin\beta - \kappa_2 \frac{\Omega^2}{\omega_A^2} \right] u &= \\ = -\kappa\left(a_{Ax}^{'} - g\,\varphi_t\right) + \kappa_1 \frac{B}{2}\,\dot{\Omega} &\ . \end{aligned} \tag{3}$$

In Eq. (3) $\Omega = \dot{\varphi}_t$ and $\dot{\Omega} = \ddot{\varphi}_t$ denote the angular velocity and angular acceleration, respectively. The geometry coefficients $\kappa, \kappa_1, \kappa_2$ and the effective length L_{eff} of the liquid column are defined by

$$\begin{aligned} \kappa &= \frac{B + 2H\cos\beta}{L_{eff}} , \quad \kappa_1 = \frac{2H\sin\beta}{L_{eff}} , \\ \kappa_2 &= \frac{B\cos\beta + 2H}{L_{eff}} , \quad L_{eff} = 2H + \frac{A_H}{A_B} B \ . \end{aligned} \tag{4}$$

The undamped circular natural frequency of the sealed TLCD is given by

$$\omega_A = \sqrt{\frac{2g}{L_{eff}} \left(\sin\beta + \frac{n\,p_0/\rho g}{H_L} \right)} \quad , \tag{5}$$

where n denotes the polytropic index, which is determined by the type of state change of the gas (mean value: $n = 1.2$). Furthermore, p_0, ρ and g denote the initial pressure, the liquid density (for water $\rho = 1000\,kg/m^3$), and the gravity constant, respectively. The air-spring height H_L is an important design variable because it directly influences the TLCD undamped circular natural frequency. On the left hand side of Eq. (3) the turbulent damping term δ_L has been introduced and can be controlled by the opening angle of the built-in orifice plate. For the optimal tuning process of the TLCD the turbulent damping term δ_L has to be transformed into its equivalent linear expression ζ_A. Applying the method of harmonic balance, see e.g. Ziegler [4], yields,

$$\zeta_A = 4\pi U_0 / 3\pi , \tag{6}$$

where U_0 denotes the relative stationary vibration amplitude of the liquid surface. Thereby, the value of U_0 is determined by numerical simulations of the linearized system and commonly chosen as $U_0 = U_{\max}$. Optimal tuning of the TLCD adjusts only the undamped circular natural frequency, Eq. (5), and the equivalent linear viscous damping coefficient ζ_A, Eq. (6).

The time variant stiffness parameter in Eq. (3) leads to parametric excited oscillations and under special conditions of the lightly damped TLCD to the undesired instability phenomenon of parametric resonance, which also exists for the classical pendulum type tuned mass damper (TMD). However, Reiterer [2] has presented a sufficient condition for the required equivalent linear damping of the fluid motion to prevent any worsening effects on the optimal damping behavior of the attached TLCD even under the most critical conditions,

$$\zeta_{A,0} = \frac{v_{t0}}{g\left(1 + \dfrac{np_0/\rho g}{H_L \sin\beta}\right)}\,\omega_A^2 = \frac{2\,v_{t0}}{L_{eff}} \sin\beta \ . \tag{7}$$

In Eq. (7) v_{t0} denotes the maximum vertical response of the bridge at the most critical vertical excitation frequency $\nu_z = 2\,\omega_A$.

In the next step the interaction force and the interaction moment caused by the TLCD dynamics have to be determined. Applying conservation of momentum and moment of momentum with respect to the accelerated reference point A to the separated TLCD, as shown in Fig. 1, yields the nonlinear resultant force $\vec{F}$ in x' and z' direction, and the nonlinear resultant

moment $\vec{M}_A$, (acting on the fluid body), see Reiterer and Ziegler [5].

In the second part of the substructure synthesis method the free body diagram of the main system (the continuous bridge) is considered at a discrete cross section, whereby the nonlinear interaction force and nonlinear moment from the TLCD dynamics have to be assigned, as illustrated in Fig. 2. The deformation of the bridge is given by its horizontal displacement $v(x,t)$, its vertical displacement $w(x,t)$ and its rotation $\vartheta(x,t)$. Applying conservation of momentum and moment of momentum to the free body diagram of the main system, Fig. 2, yields three nonlinear coupled partial differential equations, which are given in Reiterer [2].

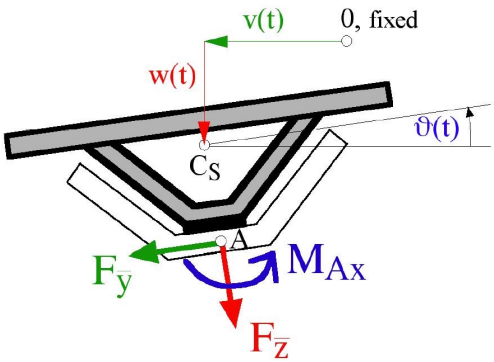

Figure 2: Separated continuous bridge with assigned interaction force and moment from the TLCD dynamics

The solution of these, altogether four equations of motion, can be obtained by means of truncated modal expansion and subsequent numerical integration by means of Matlab 6.5 in combination with Simulink.

3 Numerical Simulation of a mechanical model of the original Millennium Bridge in London

In order to verify the effectiveness of TLCD a numerical simulation of a mechanical model of the original Millennium Bridge in London (main-span) is presented. All bridge parameters are given in a detailed report by Dallard et al. [6]. The length of the main-span is $l = 144\,m$, and the mass per unit length is $\rho A = 1500\,kg/m$. Because the mode shapes are close to sinusoidal, all modal masses of the bridge are given by $M^* = \rho A l / 2 = 108000\,kg$. The natural frequencies and damping coefficients are listed in Tab.1.

Table 1: Natural frequencies and damping coefficients of the main-span (ms); L(lateral); V(vertical)

	Natural frequencies	Damping coefficients
msL 1	$0.48\,Hz$	0.6%
msL 2	$0.95\,Hz$	0.65%
msV 3	$1.15\,Hz$	0.70%
msV 4	$1.54\,Hz$	0.75%
msV 5	$1.89\,Hz$	0.80%
msV 6	$2.32\,Hz$	0.85%

The symmetric and asymmetric mode of the first and the second natural frequency, listed in Tab. 1, have dominant lateral response characteristics, and all higher modes have a dominant vertical response. Altogether three TLCD are attached to the main-span, whereby the TLCD 1 is tuned to the fundamental frequency $f_{S1} = 0.48\,Hz$ (located in the center) and the TLCD 2 and 3 are tuned to the second frequency $f_{S2} = 0.95\,Hz$ (located in the quarter points). Two pedestrian excitation states are considered: 1) symmetric excitation in lateral and vertical direction, 2) asymmetric excitation in lateral and symmetric excitation in vertical direction. The time periodic excitation forces are a combination of three harmonics, see Reiterer [2]. The number of pedestrians moving in synchronization and the mean mass of one pedestrian is assumed to be $n_P = 450$ and $m_P = 80\,kg$, respectively. Furthermore, the most critical first harmonic acceleration part of the lateral excitation force with corresponding excitation frequency $f_{y1} = f_{S1}$ and $f_{y1} = f_{S2}$, respectively, is chosen as $a_{1,y} = 0.05\,g$. Hence, the modal pedestrian mass becomes $m_P^* = n_P\, m_P\, a_{1,y} / g = 1800\,kg$, see Reiterer [2]. Evaluation of the required damping, according to Eq. (1), in order to prevent synchronization of the moving pedestrians gives $\zeta_{S,0} = 0.83\%$.

3.1 Optimal tuning of the TLCD

In a first step the liquid masses m_{fi} of each TLCD have to be determined. Assuming mass ratios of $\mu_i = m_{fi} / M^* \approx 1\%$ yield fluid masses of $m_{f1} = m_{f2} = m_{f3} = 1000\,kg$. In a second step the geometries of the TLCD have to be chosen, as shown in Tab. 2.

Table 2: Geometries of the TLCD

	TLCD 1	TLCD 2	TLCD 3
B [m]	2.00	2.00	2.00
H [m]	2.00	1.50	1.50
A_B=A_H [m^2]	0.17	0.20	0.20
L_{eff}=2H+B [m]	6.00	5.00	5.00
β [rad]	π/4	π/4	π/4
κ	0.80	0.82	0.82
κ_1	0.47	0.42	0.42

Then it the optimal tuning parameters $f_{Ai} = \omega_{Ai} / 2\pi$ and ζ_{Ai} are computed. Thereby, in a first step an analogy of the TLCD to the classical mechanical TMD is considered. The post optimization is performed in the state space by minimizing the performance Index $J = J(\omega_{Ai}, \zeta_{Ai})$, see Reiterer [2], and yields the following optimal tuning parameters of the three attached TLCD,

$$f_{A1} = 0.47\,Hz, \quad f_{A2} = 0.98\,Hz, \quad f_{A3} = 0.91\,Hz,$$

$$\zeta_{A1} = 3.8\%, \quad \zeta_{A2} = 3.5\%, \quad \zeta_{A3} = 3.1\% \;.$$

Note that f_{A2} is larger and f_{A3} is smaller than the considered natural frequency $f_{S2} = 0.95\,Hz$. Hence, the robustness of the attached damping device in view of changes of the bridge parameters (mass, stiffness) during the operating life is increased substantially. The optimized frequencies f_{Ai} of the TLCD have to be realized by adjusting the air-spring heights $H_{L,i}$. Evaluating Eq. (5) yields for $n = 1.2$ and the initial pressures $p_{0,i} = 10^5\,Pa$ together with the geometry parameters listed in Tab. 2 the following air-spring heights: $H_{L1} = 6.24\,m$, $H_{L2} = 1.37\,m$ and $H_{L3} = 1.60\,m$.

The considered combined lateral and vertical excitation of the coupled bridge/TLCD system requires to verify the sufficient stability condition given in Eq. (6): $\zeta_{A1,0} = 0.005 < \zeta_{A1} = 0.038$, $\zeta_{A2,0} = 0.004 < \zeta_{A2} = 0.035$ and $\zeta_{A3,0} = 0.004 < \zeta_{A3} = 0.031$. Thereby the maximum vertical vibration amplitudes at the critical excitation frequency $\nu_z = 2\omega_A$ are derived from numerical simulations of the linearized coupled system: $w_1 = 20\,mm$, $w_2 = 15\,mm$ and $w_3 = 15\,mm$. In all cases the optimized damping coefficients of the TLCD are larger than the required ones and hence, no undesired worsening effects on the optimal damping behavior are expected.

3.2 Numerical results of the main-span

The numerically obtained dynamic magnification factor (DMF) of the lateral displacement at the center of the span $v(t, x = l/2)$, with and without TLCD in nonlinear modeling, where both the lateral and vertical excitation force are symmetric, is shown in Fig. 3.

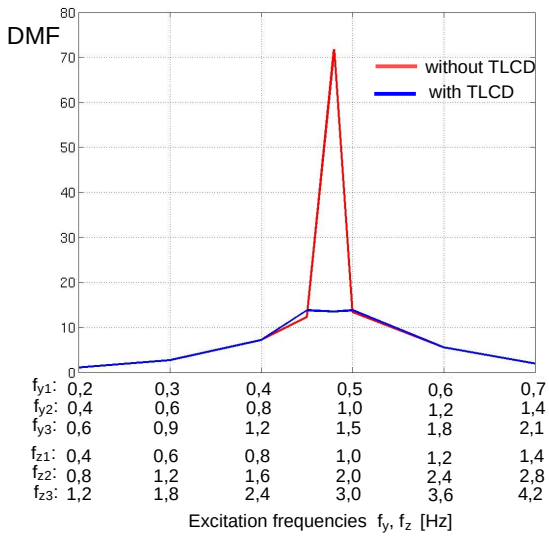

Figure 3: DMF of the main-span, lateral displacement $v(t, x = l/2)$, nonlinear modeling, symmetric pedestrian excitation in lateral and vertical direction

Fig. 3 indicates a large reduction of the DMF with TLCD. From Fig. 3 the effective damping is determined to be $\zeta_{eff} = 3.6\%$, which is much higher than the natural material damping of the bridge, $\zeta_S = 0.6\%$. Furthermore, Fig. 4 illustrates the numerically obtained DMF of the lateral displacement at the quarter point $v(t, x = l/4)$ with and without TLCD in nonlinear modeling, where the lateral and vertical excitation forces are asymmetric and symmetric, respectively.

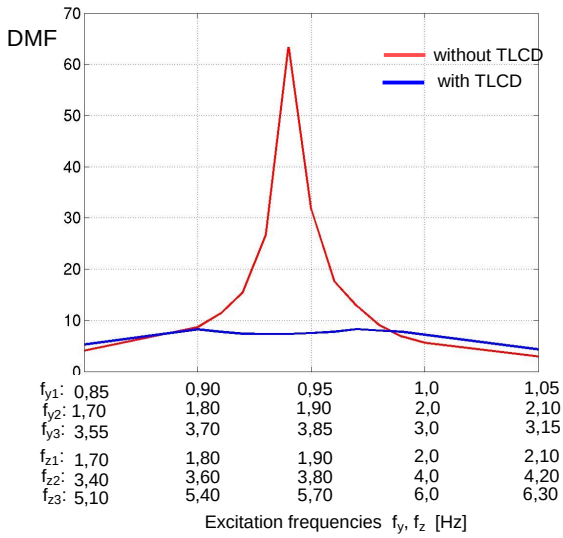

Figure 4: DMF of the main-span, lateral displacement $v(t, x = l/4)$, nonlinear modeling, asymmetric lateral and symmetric vertical pedestrian excitation

Again, Fig. 4 indicates a large reduction of the DMF with TLCD, with and effective damping of $\zeta_{eff} = 6.3\%$. Due to gain of the natural material damping to ζ_{eff} with attached TLCD it is possible to prevent the undesired synchronization effect of pedestrians, because ζ_{eff} is in all cases larger than the required one, $\zeta_S = 0.83\%$.

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