AN ORTHOTROPIC SINGLE-SURFACE PLASTICITY MODEL FOR SPRUCE WOOD UNDER CONSIDERATION OF KNOT EFFECTS

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1. Introduction

For the design of timber structures and its structural details, a suitable material model is required. As regards basic wood characteristics, there is still a lack of such constitutive material models in timber engineering. This was one of the motivations for starting a research project in the field of material modelling in timber engineering.
The project was started in the 1990's by Eberhardsteiner [1] with the development of a suitable testing equipment, followed by a comprehensive series of biaxial experiments, characterised by a combined compressive and tensile loading oblique to grain. In addition, experiments quantifying the influence of knots and the deviation of the fibre direction around the knots on the strength properties were performed by the first author. An implementation of the developed material model [2] in a finite element code for more realsitic two- and three-dimensional numerical simulations of timber structures was done. The poster presents the results of a numerical analysis of a timber-shell structure.

2. Experimental Investigation

All tests were performed on a macroscopic level and are divided in three categories:

a) investigation of stress states with their principal directions being oblique to the principal material directions L (longitudinal) and R (radial), i.e. in the LR-plane (423 experiments, clear spruce wood).
b) additional experiments in the LT-plane (12 experiments, clear spruce wood).
c) specimens with selected knots, $L\overline{R}\overline{T}$-plane (52 experiments).

The notation $L\overline{R}\overline{T}$ indicates that the fibre orientations of the specimens are mixed between the LR- and LT-plane (see Figure 1).
All tests were performed under displacement control on a biaxial servohydraulic testing apparatus equipped with a full-field measuring system (ESPI – Electronic Speckle Pattern Interferometry). Before testing, the specimens were stored at 20 °C and 65 % relative humidity until an equilibrium moisture content of u=12 % was reached. Due to the size of the specimen it can be assumed that the average moisture content did not change during the comparable short testing procedure at room climate without special climate control.

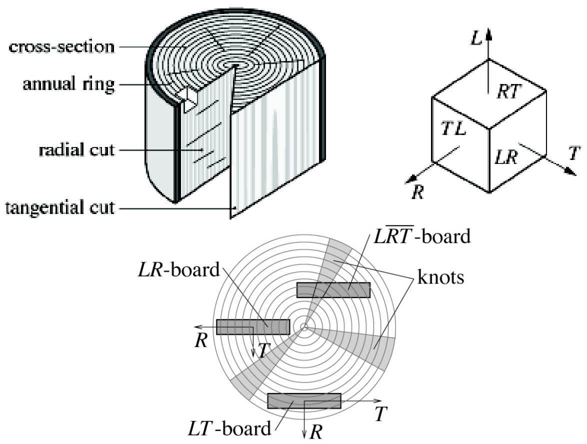

Fig. 1: Section of a stem, principal material directions

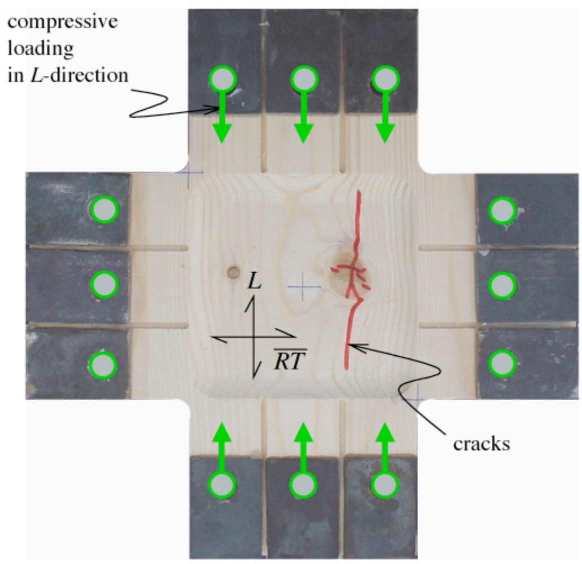

Fig. 2: Cruziform flat specimen with selected knots at test end (compressive loading in L-direction).

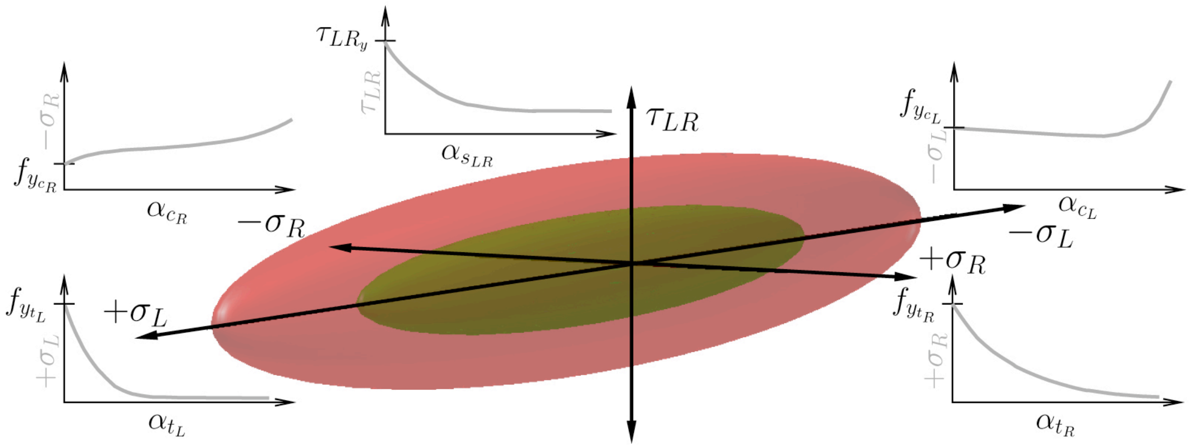

Fig. 3: Initial elliptical yield surface by Tsai & Wu for clear spruce wood (red) and for spruce wood with a knot factor of *ksa* = 0.15 (green), evolution laws for hardening and softening, respectively.

3. Development of an Orthotropic Elasto-plastic Constitutive Material Model

According to the experiments, the development of a 2D orthotropic plasticity model was done in three steps:
i) Based on a multi-surface plasticity model [3], a single-surface plasticity model including hardening and softening was fitted by Mackenzie-Helnwein et al. [2]. The Tsai & Wu failure criterion [4] is used as an initial yield surface to fit the test data available for the *LR*-plane. The plastic strains $\varepsilon^p \equiv \alpha$ are described by means of nonlinear evolution laws (hardening for compression loading and softening for tension loading, see Fig. 3).
ii) The cross section of boards and scantlings, which are widely used in timber engineering, is characterised by fibre orientations in *R*- as well as in *T*-direction. Compared to the *L*-direction, the material behaviour in *R*- and *T*-direction is similar. Therefore, it is possible to reduce the mechanical behaviour in *R*- and *T*-direction to a $\overline{RT}$-equivalent. This fact leads to use the same transversely isotropic material model as for the *LR*-plane in the elastic region for spruce wood.
iii) Knots are a commanding criterion of wood. This fact is inseparably combined with the deviation of the fibre direction around the knots. These influences on the failure strength (for tension loading) and on the yield stress (for compression loading), respectively, will be included in the Tsai & Wu-yield surface using a knot factor *ksa*:

$$ksa = \frac{\sum_{i=1}^{m} k_i + s \cdot \sum_{j=1}^{n} ek_j}{2\,b}$$

k_i ... width of a knot
ek_j ... width of an edge knot
s ... increase factor for edge knots
b ... width of a board

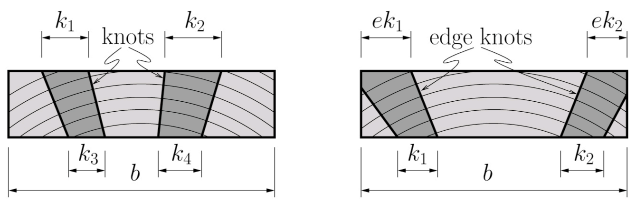

Fig. 4: Cross sections of boards without edge knots (left) and with edge knots (right).

4. Numerical Example

The presented material model was used for the analysis of a single-curved layered wooden shell with an opening in the centre. One result of the FE-simulation of the ton shell with a length of 10.0 m, a span of 7.7 m and a thickness of 63 mm (3 layers) is given in Figure 5.

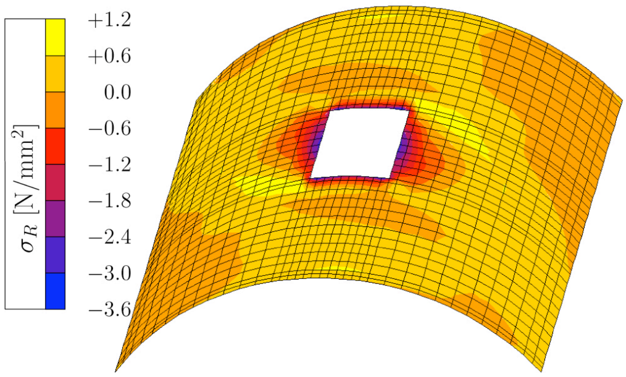

Fig. 5: Cylindrical shell - distribution of the stress component in *R*-direction of the upper (=outer) layer.

5. References

1. J. Eberhardsteiner: *Mechanisches Verhalten von Fichtenholz – Experimentelle Bestimmumg der biaxialen Festigkeitseigenschaften*, Springer-Verlag, 2002.
2. P. Mackenzie-Helnwein, H.W. Müllner, J. Eberhardsteiner, H.A. Mang: *Analysis of Layered Wooden Shells using an Orthotropic Elasto-Plastic Model for Multiaxial Loading of Clear Spruce Wood*, Computer Methods in Applied Mechanics and Engineering, 2004, in print.
3. P. Mackenzie-Helnwein, J. Eberhardsteiner, H.A. Mang: *A Multi-Surface Plasticity Model for Clear Wood and its Application to the Finite Element Analysis of Structural Details*, Computational Mechanics, Vol. 31, 2003, pp. 204-218.
4. S.W. Tsai, E.M. Wu: *A General Theory of Strength for Anisotropic Materials*, Journal of Composite Materials, 5:58-80, 1971.