Influence of the Characteristic Length on the Strength Properties of a Material Model for Spruce Wood

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Abstract

In this contribution a brief overview over a numerical scheme for the crack modelling of wood under tensile loading is given. A material model for biaxially stressed spruce wood with consideration of the effect of knots on the strength properties of spruce wood has been developed. A necessary feature of this material model is its ability to treat cracks by means of the so-called smeared crack concept.

1. INTRODUCTION

In wood the development of cracks due to tensile loading can be observed. Cracking is described as local damage causing a loss of continuity between material points of an area where the tensile strength of the material is exceeded.

The finite element method is based on the algebraic form of a weak formulation of the underlying boundary value problem. In the case of cracking, the displacement fields exhibits discontinuities in the analytical solution. To preserve the continuity of the displacement field in the finite element solution, a discrete crack is homogenised and simulated by plastic strains distributed over a finite width.

In the present material model the so-called smeared crack concept is used. This fictitious crack concept uses the entire element domain for the representation of a discrete crack. The crack behaviour is considered in terms of stress-strain-relationships. The advantage of this concept is that the knowledge of the directions of the cracks is not required. According to this, the used finite element mesh of the investigated structure needs not to be modified.

2. CRACK MODELLING BY MEANS OF THE CHARACTERISTIC LENGTH

One disadvantage of softening cracking models is the influence of the size of the finite elements on the numerical results. In this content OLIVER [2] proposed to introduce a characteristic length and to consider the crack band width in form of a stress-strain-relationship. Following this concept the displacements are continuous and the appropiate gradients are discontinuous along a singular line in a two dimensional domain which can be defined as material line (see Figure 1 (a)).

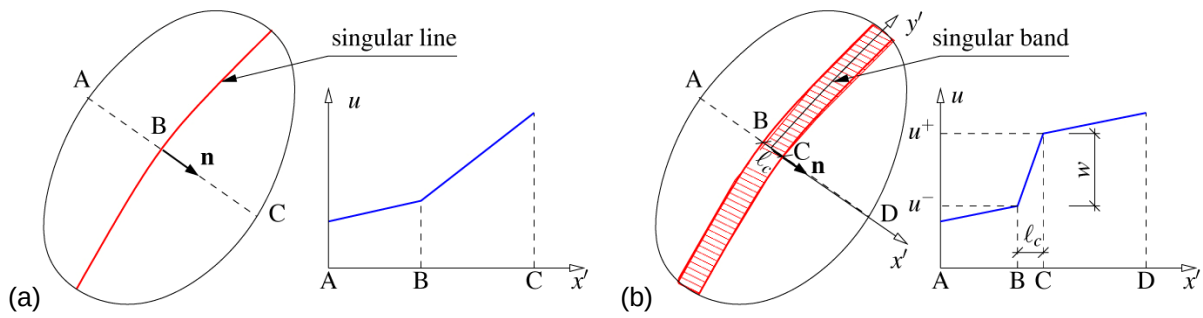

Figure 1 - Crack as (a) singular line in a continuous medium and (b) singular band between two singular lines

A singular line representing a discontinuity in the displacements can only be modelled by the edge of a standard finite element using C^0 continuity, because on these nodes only displacements results are available. However, a crack produces discontinuous in displacements and in displacement gradients. Thus, a crack can be modelled as the distance of two singular lines (see Figure 1 (b)). The area limited by these two singular lines is called singular band. The width of the singular band is called the characteristic length ℓ_c.

OLIVER [2] contemplated displacement and traction vectors as well as energy dissipation in the singular band and identified an equation for the determination of the characteristic length ℓ_c in a finite element mesh using the standard shape functions for an element. For further details see OLIVER [2].

3. CONSIDERATION IN THE MATERIAL MODEL FOR SPRUCE WOOD

The properties and the concept of the used single-surface plasticity model are summarised in MÜLLNER et al. [1]. Within this material model the characteristic length ℓ_c is considered in the dimensionless material parameter $k_{i,j}$ ($i \in \{t,c\}$, $j \in \{L,R\}$) in every evolution law which affects softening by means of a crack (e.g. tensile strength in longitudinal direction see (1)).

$$\max \beta_{t,L} = \max \beta_{t,L}{}^{0} e^{-k_{t,L}\alpha_{t,L}} \quad \text{with} \quad k_{t,L} = \frac{\max \beta_{t,L}{}^{0}\, \ell_c}{G_{f,L}{}^{I}} \qquad (1)$$

Figure 2 (a) shows the influence of ℓ_c on the softening behaviour of the tensile strength in longitudinal direction L. One can define a limit with a critical $\ell_{c,crit}$ between a unique relationship in the stress-strain-relationship and a non-unique solution (see Figure 2 (b)), which leads to an alternation between the two possible solutions. Assuming that the fracture toughness $G_{f,L}{}^{I}$ can be used as material parameter, this limit also defines the transition from a stable to an unstable crack propagation.

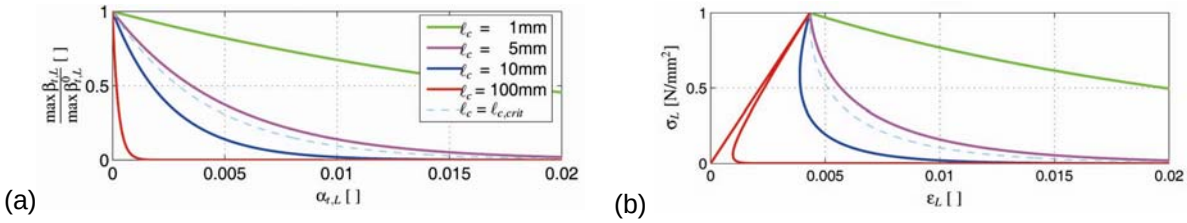

Figure 2 - Softening of tensile strength (a) ratio of tensile strength (b) stress-strain-relationship

4. NUMERICAL EXAMPLE

An example shall show the influence of the characteristic length on the results of numerical simulations. Therefore, a beam is divided into two zones. The properties of the first part are modelled by the mentioned material model, the second part is considered to behave linear elastic (see Figure 3 (a)). By increasing the horizontal displacements $\bar{u}$ a crack will appear in the first zone. Variation of the dimension a leads to different values for the characteristic lengths ℓ_c and to different progresses of the stress-strain-relationship in the post failure behaviour.

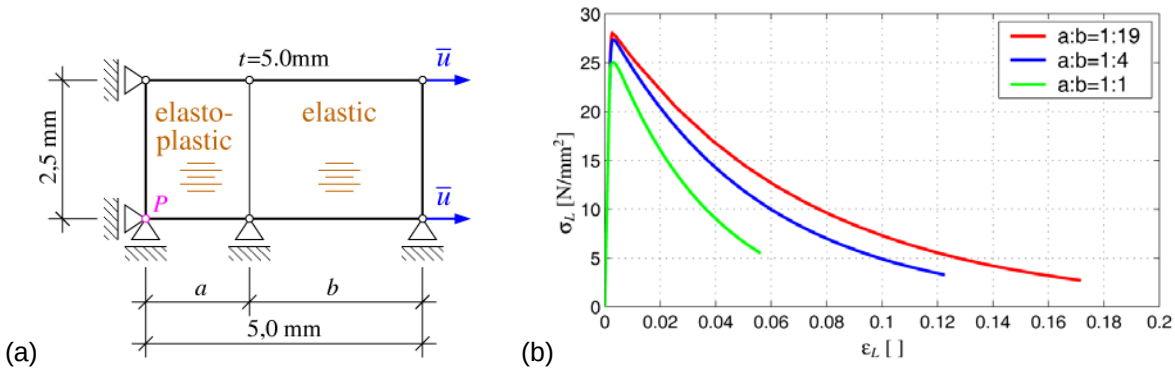

Figure 3 - Beam strained with horizontal displacements (a) dimensions (b) stress-strain-diagram for point P

Figure 3 (b) shows a stress-strain-diagram for the point P. The faster the strength decreases the bigger the numerical problems characterised by finer load increments and small dimensions of the finite elements. To work out solutions for these problems will be one goal of the ongoing research at the Institute for Mechanics of Materials and Structures.

5. REFERENCES

[1] H.W. Müllner, P. Mackenzie-Helnwein, J. Eberhardsteiner: Constitutive Modelling of Clear Spruce Wood under Biaxial Loading by Means of an Orthotropic Single-Surface Model under Consideration of Hardening and Softening Mechanisms. Proceedings of the 2nd International Symposium on Wood Machining (2004): 83 - 90.

[2] J. Oliver: A Consistent Characteristic Length for Smeared Cracking Models. International Journal for Numerical Methods in Engineering, Vol. 28 (1989): 461-474.