Shear strength model of reinforced concrete circular cross-section members

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ABSTRACT: A shear capacity model of reinforced concrete members with circular cross section transversely reinforced with circular hoops has been developed. The proposed model is based on the truss analogy by adding a concrete contribution term to the capacity of the shear reinforcement. An additional *deviatoric* shear resisting mechanism of hoops - present solely in members with curved transverse reinforcement - was identified and expressed analytically. It is explained by the fact that a curved reinforcing bar under tension induces compression in radial direction as well. The component of this compressive force in the direction of external shear could thus be considered as an additional shear enhancing mechanism of the hoops. Its magnitude is expressed through the friction force that is present between the concrete and steel after the section is cracked and the bond partially destroyed. The concrete shear capacity has been derived by a parameter study. The proposed model has been compared with recently proposed models and it was found that it predicts reasonably well the shear capacity of circular sections. By applying the strength reduction factors a sufficiently conservative design equation could be obtained suitable for incorporation in codes.

1 INTRODUCTION

Reinforced concrete (RC) structural elements of circular cross-section are preferred as columns in high rise buildings or bridges, as secant pilings forming diaphragm walls or in the foundations of buildings. Columns are basically axial load carrying elements. However, as a result of lateral loads due to wind pressure or earthquake, they are subjected to considerable shear load and should thus inevitably be designed to suppress a possible shear failure.

Generally, the majority of design codes do not distinguish between the design of a rectangular section and the one of a circular section under shear. It is simply assumed that the shear capacity of a circular section equals the capacity of an equivalent rectangular section.

A very limited number of shear models for circular section members exist in literature. Earlier experiments on circular RC members loaded monotonically in shear aimed for verification of the use of design equations developed for rectangular sections on circular sections (Capon and Cossio 1965, Khalifa and Collins 1981). Clarke and Bir-jandi (1993) proposed to use for circular sections the same shear design approach as given by the British Codes of Practice, BS 5400 for rectangular sections with a modification of the section's effective depth as the distance from the extreme compression fibre to the centroid of the tension reinforcement. The effective shear area is then defined as the area corresponding to the effective depth. The shear capacity of the member is added together from the shear carried by concrete and transverse reinforcement

$$V = 0.27\alpha\left(\frac{100A_s}{b_w d}\right)^{1/3}\left(\frac{500}{d}\right)^{1/4}(f_{cu})^{1/3}b_w d + \frac{A_{sv} f_{yv} d}{s} \tag{1}$$

where A_s is the area of longitudinal steel, b_w the section's width, d the effective depth, f_{cu} the concrete cube strength, A_{sv} the cross-sectional area of the link's both leg at the section's neutral axis and $\mathrm{f_{cu}}$ is the link's yield strength. In a member with axial compression load, N, the shear capacity should be multiplied by $(1 + 0.05N/A_c)$, where A_c is the area of concrete.

Ang et al. (1989) proposed a model for shear capacity of circular sections under seismic load later extended by Priestley et al. (1994) and by Kowalsky and Priestley (2000). According to the model the member's shear capacity was thus added together from three components, i.e., the concrete contribution term, the transverse reinforcement capacity calculated from the 45-degree truss mechanism and the axial load enhancement

$$V = k\sqrt{f_c{}'}A_e + \frac{\pi(2A_{sh}f_{yh})D'}{4s} + \frac{D-c}{2a}p \tag{2}$$

where k is a coefficient describing the degradation of concrete shear strength with increasing displacement ductility, $f_c{}'$ the concrete compressive strength, A_e the effective shear area proposed as 0.8 times the gross section area, A_{sh} the area of the shear reinforcement (one leg), s the hoops' spacing, f_{yh} their yield strength, *D'* the distance between the centers of the peripheral hoop or the spiral, D is the overall section diameter, c the depth of the compression zone and a the shear span.

Recently about a deficiency of the model was reported (Dancygier 2001, Kim and Mander 2005). The applied integral averaging by the reinforcement term restricts its use only to members with diameter at least four times the spacing of the shear reinforcement. For all other ratios the formula could be even more than 50% nonconservative.

The objective of this work was to develop a simple shear capacity model of RC circular sections. It is based on the truss analogy by adding an empirical concrete contribution term to the capacity of the shear reinforcement. Resulting from the continuous improvements of the recently proposed model (Merta 2004a, Merta 2004b) a shear capacity model for monotonic load has been developed.

2 THE CONCRETE SHEAR CAPACITY

The concrete shear capacity, taken as the capacity of the section without shear reinforcement, has been proposed as follows

$$V_c = \left[3.7\rho_l + 0.18 + 0.08(\frac{P}{A_g})0.3\right] \times k \cdot \sqrt{f_c{}'} \cdot 0.7A_g \tag{3}$$

The influence of the main variables on the shear capacity, such as the longitudinal reinforcement ratio ρ_l, axial load level P/A_g and the shear span-to-depth ratio a/D have been determined empirically, based on a total of 44 data of circular cross section specimens without shear reinforcement under monotonic load (Merta 2006). The term $07A_g$ represents the section's effective shear area and its derivation has been published elsewhere (Merta and Kolbitsch 2006). The shear enhancement coefficient k due to the shear span-to-depth ratio a/D is proposed as follows

$$k = 1.00 \text{ if } a/D > 2.5;\ \ 1.25 \text{ if } a/D \leq 2.5 \tag{4}$$

3 THE SHEAR REINFORCEMENT CAPACITY

The capacity of the shear reinforcement is based on the truss analogy. With the appearance of the first diagonal cracks the shear reinforcement is mobilized in resisting shear by tension in it. At the ultimate state a major diagonal crack forms and the section fails by rupture of the shear reinforcement. In the cracked region the bond between concrete and shear reinforcement is gradually destroyed. As a result of the hoops' curved shape the change of its direction induces lateral pressure (*deviation stresses*) on the concrete. The component of the deviation stresses in the direction of the external shear represents an additional shear resisting mechanism of hoops. The entire shear capacity of shear reinforcement is thus added together from: the *tension component* - resulting from the tension in shear reinforcement and the *deviation component* - resulting from the shear enhancement of deviation stresses.

3.1 *Tension component*

In the limit state it is assumed that all transverse reinforcement crossed by a diagonal crack yields and resists a tension force of $Af_{sw}yw$. A_{sw} is the area of the hoop's one leg and f_{yw} is the hoop's yield strength (Fig. 1). The tension force acts in the hoop's tangential direction and only its component in the direction of external shear is active in resisting shear. Instead of a smeared distribution of shear reinforcement a discrete number of bars crossing the diagonal crack have been considered. The resultant tension force resisting external shear is obtained by summing up the tension force components traversing the crack plane

$$V_{st} = 2A_{sw}f_{yw}\sum_{i=1}^{n_t}\cos\varphi_{y,i} \tag{5}$$

where i is the index of the hoop that is crossed by a crack, cp, $\varphi_{y,i}$ is the pertaining central angle (Fig. 1) and n_t is the number of hoops across the crack. Accord-

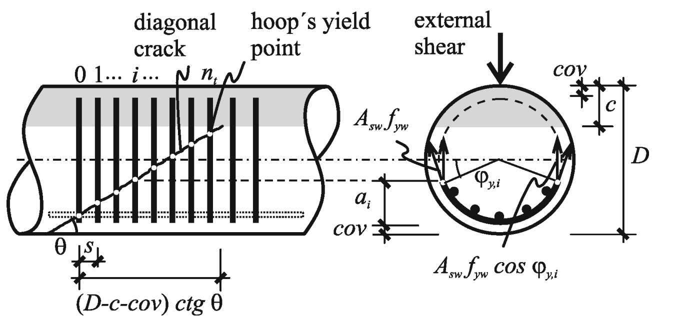

Figure 1. Shear carried by tension in hoops crossed by a diagonal crack.

ing to Kowalsky and Priestley 2000 only the hoops outside the compression zone have been considered, since shear cannot be transferred across the sections compression zone by tension in hoops. The number of hops is $n_t = INT[((D - c - cov)ctg\theta)/s]$, where D is the section's diameter, c the compression zone's depth, cov the concrete cover and s the spacing of the hoops. The central angle is $sin\varphi_{y,i} = (D/2 - cov - a_i)l(D/2 - cov)$ where a_t is the ordinate of the particular yield point defined as $a_i = i{\cdot}s{\cdot}tg\theta$. Substituting these expressions into the Eq. 5 as well as taking into account a typical value of $c/D = 0.3$ proposed by Kowalsky and Priestley 2000 and the typical values of cov/D of 1% to 20%, the resultant shear resisting force of hoops crossed by a crack is obtained

$$V_{st} = 2A_{sw}f_{yw}\sum_{i=1}^{n_t}\sqrt{1 - \left(1 - 1.5\frac{i}{n_t}\right)^2} \tag{6}$$

The summation term has been replaced by the continuous function $k_t = 0.9n_t$. The shear resisting force in hoops is obtained as $V_{st} = 1.8A_{sw}f_{yw} \cdot n_t$.

3.2 *Relation between the hoop's tension and the deviation function*

Generally, shear reinforcement is active in carrying shear by tension in it. But in the case of curved reinforcement the change of direction of the bar's tension force induces lateral pressure on the concrete core, the so called deviation stresses, f_d. Because of action-reaction, the concrete section acts on hoops with normal pressure, f_n, of the same intensity as the deviation stresses, f_d, but in the opposite direction (Fig. 2a,b). From the equilibrium in radial and

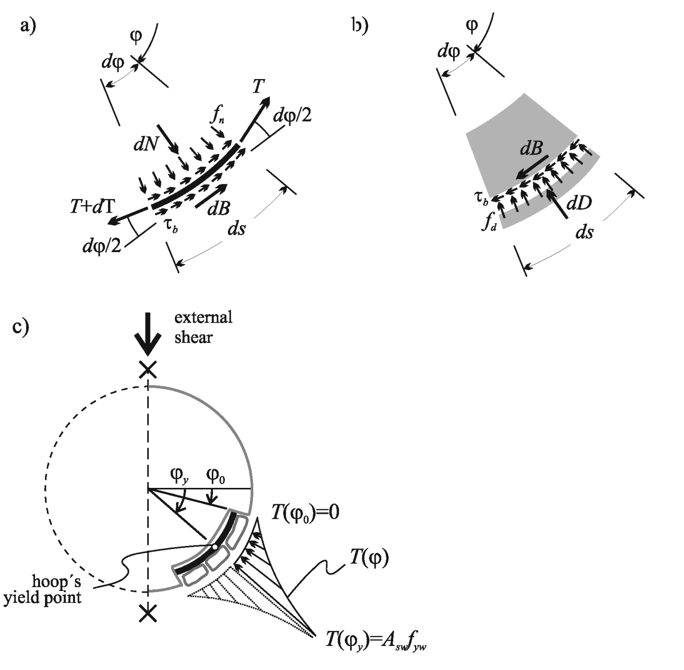

Figure 2. Forces acting (a) on an infinitesimal hoop element and (b) on concrete (c) the hoop's tension function, T(φ).

tangential direction follows that

$$Td\varphi = dN \tag{7}$$

$$dT = dB \tag{8}$$

respectively, where T is the tension force in hoop, dB the bond force, defined as the shear stress τ_b that develops along the lateral surface of the bar of length ds, and dN is the concrete normal force, defined as the normal stress f_n along ds. The second expression defines the fundamental stress transfer mechanism between concrete and steel, whereby, the change of bond forces is proportional to the change of tension force in hoops. Simplifying, it could be assumed that bond is mainly governed by friction between steel and concrete and could be captured by the "frictional concept", where the friction force is related to the normal force through the friction coefficient. The bond force, dB, will be related to the normal force, dN, through the friction coefficient μ and $dB = \mu dN$. Applying the frictional concept, Eq. 8 becomes

$$dT = \mu dN \tag{9}$$

where the hoops tension force is expressed through the normal force. From the two equilibrium Equations (7. and 9.) by eliminating dN, a differential relation of the hoop's tension force is obtained $dT/T = \mu d\varphi$. By integrating both sides, assuming that the friction coefficient is constant, the hoop's tension force function is obtained

$$T(\varphi) = A_{sw}f_{yw}e^{-\mu(\varphi_y - \varphi)} \tag{10}$$

where as it has been assumed that the tension force reaches its maximal value $A_{sw}f_{yw}$ at the hoop's yield point, where f_{yw} is the hoop's yield stress. Substituting further Equation 10 into the Equation 9 the hoop's deviation function is obtain

$$D(\varphi) = \frac{A_{sw}f_{yw}}{\mu}e^{-\mu(\varphi_y - \varphi)} \tag{11}$$

The deviation force has its maximum, $D_{max} = A_{sw}f_{yw}/\mu$, at the yield point, $\varphi = \varphi_y$, and decreases with the distance from the yield point, reaching zero at the boundary point of the cracked region, where $\varphi = \varphi_0$. In order to simplify, in this work a constant friction coefficient of 1.5 has been assumed along the whole length of one hoop.

3.3 *Deviation component*

Each hoop in the cracked region acts with a resultant deviation force on concrete of different magnitude, resulting from a different arc length within the cracked region. To obtain the hoop's total shear capacity the deviation function along the hoop's arc length, within

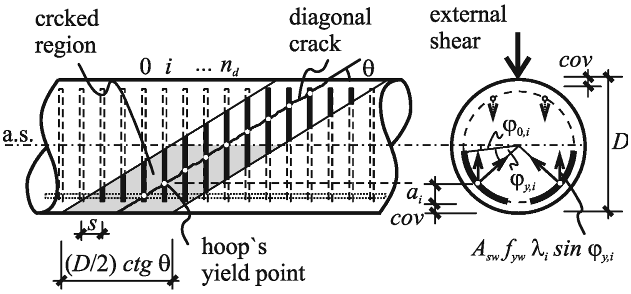

Figure 3. Hoop's deviatoric shear carrying capacity.

the cracked region, should be integrated and then the component of the resultant deviation force of each hoop in the direction of the external shear summed up along the number of all hoops in the cracked region. The resultant deviation force is obtained by integrating the deviation function (Eq. 11) from angle $\varphi_{0,i}$ – denoting the cracked region's boundary point - to $\varphi_{y,i}$ and then multiplying it by two to take into account the function's descending and ascending branch. Thus, $D_{h,i} = (2A_{sw}f_{yw}/\mu^2)(1 - e^{-\mu(\varphi_{y,i}-\varphi_{0,i})})$. The hoop's total deviation force is obtained by summing up the components of the resultant deviation force of a particular hoop in the direction of the external shear along the cracked region (Fig. 3).

$$V_{sd} = 2A_{sw}f_{yw}\sum_{i=0}^{n_d}\lambda_i \cdot \sin\varphi_{y,i} \tag{12}$$

where $\lambda_i = 2(1 - e^{-\mu(\varphi_{y,i}-\varphi_{0,i})})/u^2$. The coefficient 2 in Eq. 12 denotes that each hoop corresponds with two resultant deviation forces symmetrically. *nd* is the number of hoops crossed by the diagonal crack. Only by hoops with a yield point under the symmetry axis (shaded area on Fig. 3) the component of the deviation force counteracts to the external shear and enhances the member's shear capacity.

The magnitude of the influence coefficient of a particular hoop depends on the value of the central angle $\varphi_{y,i} - \varphi_{0,i}$. However, all hoops within the inclined cracked region have a different arc length and consequently a different central angle. Employing a simplification derived elsewhere (Merta 2006) the central angle is $\varphi_{max} = 34^\circ - 38^\circ \cong 5$, and for assumed friction coefficient of 1.5 the influence coefficient obtained as $\lambda = \lambda_i(\pi/5) = 0.53$. The hoop's total deviatoric component is thus simplified as

$$V_{sd} = 2A_{sw}f_{yw}\sum_{i=0}^{n_d}\sin\varphi_{y,i} \tag{13}$$

The number of hoops under the symmetry axis crossed by the diagonal crack is $n_d = \mathrm{INT}[(D/2 - cov)ctg\theta/s]$ and the central angle $sin\varphi_{y,i} = (D/2 - cov - a_i)/(D/2 - cov)$ with $a_i = i\cdot(D/2 - cov)/n_d$.

Substituting these expressions into the Eq. 13 the total deviatoric shear capacity of hoops is obtained

$$V_{sd} = 2A_{sw}f_{yw}\lambda\sum_{i=0}^{n_d}\left(1 - \frac{i}{n_d}\right) \tag{14}$$

The summation term has been replaced with the continuous function $k_d \cong 0.5n_d + 0.5$ and the deviatoric shear capacity of hoops is obtained $V_{sd} = 2A_{sw}f_{yw}\cdot\lambda\cdot(0.5n_d + 0.5)$.

4 VERIFICATION OF THE PROPOSED MODEL FOR MONOTONIC LOAD

The proposed shear capacity model of RC circular cross-section members is

$$V = \left[3.7\rho_l + 0.18 + 0.08(\frac{P}{A_g})0.3\right]\cdot k \cdot \sqrt{f_c{}'}\cdot 0.7A_g + A_{sw}f_{yw}(1.8\cdot n_t + \lambda\cdot n_d + 1) \tag{15}$$

with

$$k = \begin{cases} 1.00 & \text{for} \quad a/D > 2.5 \\ 1.25 & \text{for} a/D \le 2.5 \end{cases} \qquad \lambda = 0.53$$

$$n_t = \frac{(D - c - cov)}{s}ctg\ \theta$$

$$n_d = INT\left[\frac{(D/2 - cov)}{s}ctg\ \theta\right]$$

ρ_l is the longitudinal reinforcement ratio, P the axial load, A_g the sections gross area, $f_c{}'$ the concrete compressive strength, A_{sw} the cross section of the hoop, f_{yw} its yield strength, a the shear-span, D is the section's diameter, c the compression zone's depth, *cov*

Table 1. Statistical comparison of models in terms of experimental/theoretical shear strength ratio

	Clarke and Birjandi (1993)	Kowalsky and Priestley (2000)	Proposed
Mean value	1.23	1.25	1.01
Standard deviation	0.13	0.21	0.11
Coefficient of variation [%]	10	16	10
Coefficient of determination	0.851	0.66	0.88

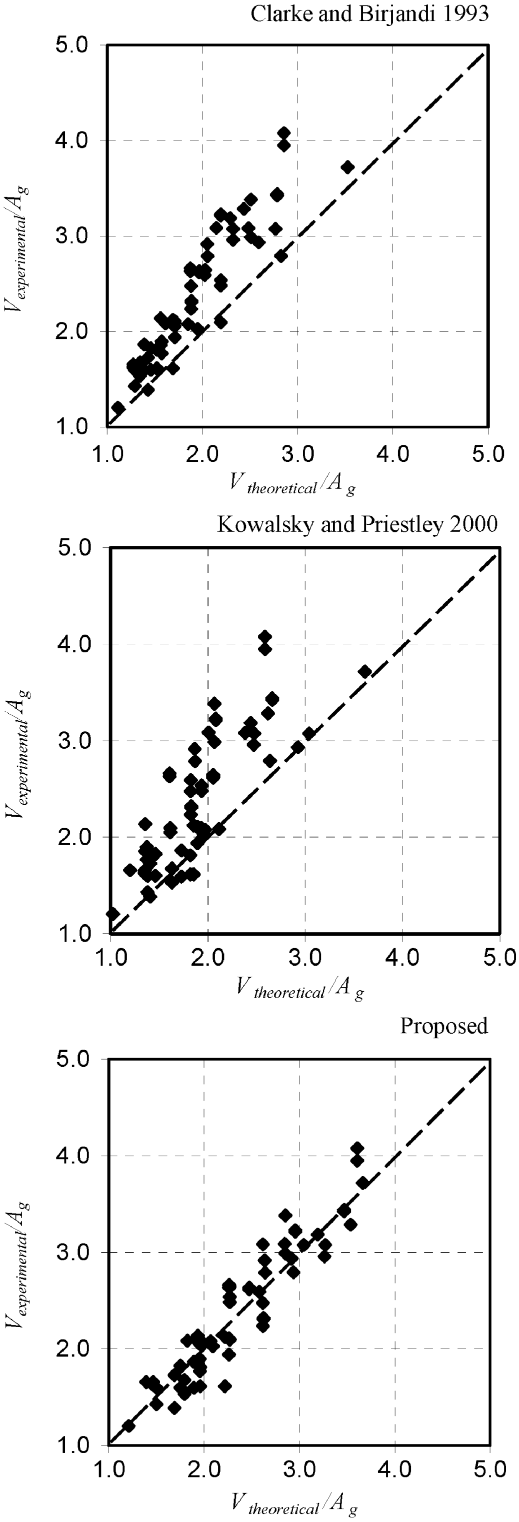

Figure 4. Ultimate shear strength of circular section members with shear reinforcement.

the concrete cover, s the spacing of the hoops and θ the crack's inclination angle.

The model has been verified on the database of 62 circular members with shear reinforcement (Capon and Cossio 1965, Khalifa and Collins 1981, Clarke and Birjandi 1993, Collins et al. 2002) by comparing it to the two existing models (Clarke and Birjandi 1993, Kowalsky and Priestley 2000). The smaller scatter in the plot of the experimental/theoretical ultimate shear strength of specimens (Fig. 4) indicates that the proposed model clearly improves the prediction

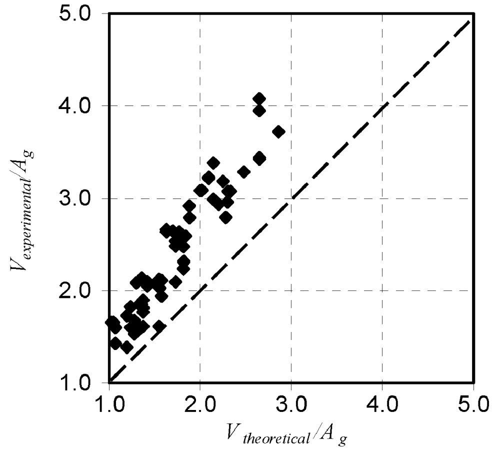

Figure 5. Design shear strength of circular section members with shear reinforcement.

of the shear capacity. The statistical comparison of experimental/theoretical shear strength of the models (Table 1) indicates that the proposed model provides the closest correlation with experimental data.

The proposed formula is a predictive and not a design equation, representing therefore the characteristic shear resistance of the member. It follows that higher average values of the strength ratios are inevitable. In the model measured concrete compression strength and reinforcement yield strength was used. In the design situations, however, nominal material strengths would be used.

By applying the strength reduction factors – $\gamma_c = 1.5$ for concrete and $\gamma_s = 1.15$ for steel - a reasonable lower bound of data is obtained, resulting in an adequately conservative design equation suitable for incorporation into design codes (Fig. 5).

5 CONCLUSION

In the paper, an analytical model for the prediction of the shear capacity of reinforced concrete members with circular cross-section transversely reinforced with circular hoops has been developed. The proposed shear capacity model is a semi-empirical equation based on the truss analogy by adding an empirical concrete shear capacity term to the capacity of the shear reinforcement. To overcome the deficiencies of the recently proposed seismic design equation (Ang et al. 1989; Kowalsky and Priestley 2000) instead of a smeared distribution a discrete distribution of the shear reinforcement has been applied. Thus the equation is not any more restricted to members with diameter at least four times the spacing of the shear reinforcement.

The validity of the proposed model has been compared to other recently proposed models for monotonic load and it has been shown that the shear capacity predicted by the proposed model is in stronger correlation with the experimental values than by other existing proposals.

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