Experimental validation of analytical MIMO channel models

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The promise of multiple-input multiple-output systems (MIMO) to overcome the radio bottleneck in high-speed data transmission requires detailed models of the spatio-temporal MIMO channel to come true. In this paper, popular MIMO channel models are compared with two independent measurement campaigns at 2 and 5 GHz by using four different, mostly novel performance figures (or metrics). Each of these metrics describes one or more different aspects of MIMO, such as multiplexing gain, spatial diversity, or beamforming. Of the models investigated, the Weichselberger model performs overall best, whereas the Kronecker model should be used only for limited antenna numbers, such as 2× 2, and "the virtual channel representation" only for very large antenna numbers.

Keywords: **MIMO; analytical channel models; quality metrics**

Experimentelle Validierung von MIMO-Kanalmodellen.

Die Erzielung wirklich schneller Datenübertragung über den Funkkanal mithilfe von Mehrantennensystemen (multiple-input multiple-output systems [MIMO]) wird nur bei voller Ausnützung des MIMO-Kanals gelingen. Dazu sind MIMO-Kanalmodelle erforderlich, die alle Feinheiten des räumlich-zeitlich variablen Funkkanals erfassen. Eine sorgfältige Überprüfung dreier bekannter MIMO-Kanalmodelle mittels zweier unabhängiger Messkampagnen bei zwei unterschiedlichen Frequenzen (2 bzw. 5 GHz) offenbart ihre begrenzte Gültigkeit und Eignung. Anhand vier, teilweise neuer Qualitätsmaßen wird festgestellt, welcher der unterschiedlichen Vorteile von MIMO, etwa Raummultiplex zur Erhöhung der Kanalkapazität, Diversität oder Strahlformungspotential, von den Modellen am besten wiedergegeben wird. Das nach Weichselberger benannte Modell liefert generell die genauesten Ergebnisse, während das einfache und deswegen häufig verwendete so genannte Kronecker Modell (bzw. die 'Virtuelle Kanaldarstellung') nur für wenige (bzw. sehr viele) Antennenelemente eingesetzt werden sollte.

Schlüsselwörter: **MIMO; Kanalmodelle; Qualitätsmaße**

1. Introduction

MIMO stands for multiple-input multiple-output systems. Several antennas at transmitter, n, and receiver, m, promise a multiplication of the Shannon capacity to min (m, n), thus eliminating the bottleneck that the radio channel presents for high-speed data transmission. Other possible benefits are diversity order of up to $n \times m$ or beamforming gain. MIMO has been hyped over the last years since the seminal paper of Foschini and Gans (*Foschini, Gans, 1998*)[1], with some renowned conferences boasting of more than 50 percent of the contributions dealing with transceiver algorithms, space-time codes, channel measurements and channel models for MIMO. In the meantime, quite a number of implementations of MIMO have, on the one hand, substantiated the feasibility of this concept but, on the other hand, have revealed sobering facts. Among these are the linear increase of power consumption with the number of receive trains, the fact that actual radio channels rarely provide the basis for more than, say, eight antennas at one link end at the most, that high transmit power is necessary for high SNR (which in turn is beneficial for high capacity), that multiple antennas in close vicinity suffer from so much loss in radiation efficiency that the capacity gain may be lost, and that interference in multi-user MIMO systems threatens to annihilate the gains in capacity[2]. Still, I am convinced that only with MIMO schemes the ambitious goal of Fig. 1 (*International Telecommunications Union*), aiming at data rates of 100 to 1000 Mbit/s over radio can be approached, but definitely not without. An excellent, readable overview of all aspects of MIMO can be found in (*Bölcskei et al., 2005*).

This paper addresses designers and users of MIMO channel models. It deals with the following questions for *model users*:

- What makes you sure you are using the right model?
- How close to reality is the model you are using to reality?
- Which aspect of MIMO system performance do those models capture?

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[1] At its publication, and even before so in the review process, the revolutionary idea of providing tens of bits/s per Hertz bandwidth created widespread doubt and disbelief.

[2] It is amazing to observe that still scientific papers appear with tens of antennas at one link end.

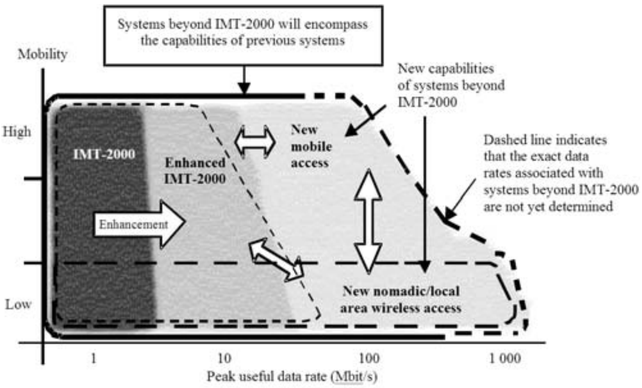

Fig. 1. The view of the International Telecommunications Union, ITU, of the mobile radio generation beyond 3G

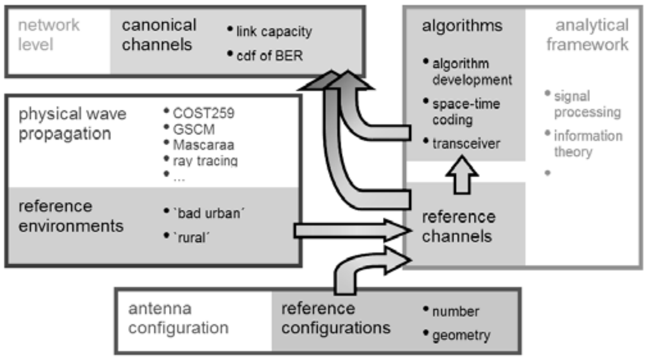

Fig. 2. MIMO channel models – an overview

- Are some models better suited than others to predict a certain aspect of MIMO system performance?

and for *model designers*:

- How did you validate your model?
- With your own measurements or with those of others as well?
- How close is your model to reality?

I will argue that too little effort has been spent so far in model validation compared to model design. Capacity as a metric for model validation is necessary, but not sufficient. For a class of MIMO models that are particularly well suited for the simulation of new MIMO transceiver algorithms and space-time codes (so-called *analytical* models), I will present four different metrics that can be used for model validation, namely

- double-directional angular power spectrum,
- mutual information ("capacity"),
- a diversity measure (by Ivrlač and Nossek),
- correlation matrix distance (by Herdin).

Three different models

- the so-called Kronecker model,
- Weichselberger's model, and
- the virtual channel representation (Sayeed)

will be compared with MIMO measurements from

- an indoor campaign at 5.2 GHz with up to 8×8 antennas (TU Wien)
- an outdoor campaign at 2 GHz with 15×8 antennas (ftw.)

by the four metrics mentioned above. The various metrics give different insights into the measured MIMO channels.

The models are not equally well suited for predicting specific MIMO performance indicators for different implementations:

- "Kronecker" should only be used for limited antenna numbers, such as 2×2;
- "Sayeed" can only be used for very large antenna numbers;
- "Weichselberger" is the overall best performing model, but has problems with the joint APS and the diversity measure for large antenna numbers.

It will become clear that only careful model validation makes one sure of simulating what one wants to simulate, and provides inspiration for improvement of MIMO models.

Figure 2 shows a hierarchy of MIMO channel models. Electromagnetic wave propagation is at the basis of any wireless communication system. Thus, physical propagation models of agreed-on reference scenarios and propagation environments, like the famous COST models, provide knowledge of the *propagation channel*. Convolution with antennas at receiver and transmitter, specified by the number of elements, their geometry and their polarizations, give *reference channels*. Information theory and signal processing require models that facilitate the analytical treatment of the MIMO channel in concise mathematical formulation, describing the spatial structure of the channel precisely by as few parameters as possible. The box on the right-hand side of Fig. 2 contains channel models that provide this analytical framework to develop MIMO transceiver algorithms, space-time codes and the like. Hence, the name *analytical* channel models. Analytical models are easier to validate – for propagation models you have to make assumptions beyond the sheer propagation model. Though very important for MIMO system deployment, we will not consider models at the network level here.

2. Theoretical or experimental validation?

Comparing MIMO channel models by theory is possible, but difficult. You have to be sure that the theory you compare your model with is proven solid. Customary, for instance, is comparison with i.i.d. uncorrelated Rayleigh fading. For this initial assumption about MIMO (*Telatar, 1995*), many results have been obtained by Monte Carlos simulations and are readily available. However, in situations with *correlated* Rayleigh fading or Rician fading, you are on less firm ground and have to make auxiliary assumptions, making the result of the validation process less reliable. Correlation has been identified early on as a showstopper for MIMO (*Shiu et al., 2000*). So, the ultimate test of any model is by experiment. But which experiment?

Experimental validation by one set of measurements -in most cases your own -is not sufficient. I think the MIMO community has been carried away by early success, but has been negligent on the hard work of thorough validation of the models that they use. Particularly I am amazed about the multitude of papers developing elaborate space-time codes with inadequate models (say, flat Rayleigh fading in spatially unstructured environments).

Though I will present evidence from two different measurement campaigns in Section V, taken at two different carrier frequencies and even by researchers from different institutions, we also could have done better.

3. How to deal with correlation

To obtain the full autocorrelation matrix, $\mathbf{R}_H$, we first stack the elements, h_{ij} , of the MIMO channel matrix $m \times n$ specifying the path gain from each transmit antenna to each receive antenna into a tall vector $[n \cdot m \times 1]$ by the vec($\cdot$) operator[3]. The autocor-

[3] The following notation will be used throughout this paper: $(\cdot)^{1/2}$ denotes the matrix square root; $(\cdot)^T$ stands for matrix transposition; $(\cdot)^*$ stands for complex conjugation; $(\cdot)^H$ stands for matrix Hermitian; $\odot$ denotes the element-wise Schur-Hadamard multiplication; $\otimes$ denotes the Kronecker multiplication; $E\{\cdot\}$ denotes the expectation operator; tr $(\cdot)$ denotes the trace of a matrix; vec $(\cdot)$ stacks a matrix into a vector, columnwise; $\|\cdot\|_F$ stands for the Frobenius norm.

relation matrix, $\mathbf{R}_H$, is then given by the expectation of the product

$$\mathbf{R_H} = \mathrm{E}\,\{\mathrm{vec}\,(\mathbf{H}) \cdot \mathrm{vec}\,(\mathbf{H})^H\}. \quad (1)$$

It fully describes channels that can be characterized by second-order statistics, but only such channels. With $\mathbf{R_H}$ given, appropriate realizations of $\mathbf{H}$ can be generated for Monte-Carlo simulations by

$$\mathrm{vec}\,(\mathbf{H}) = \mathbf{R}_\mathbf{H}^{1/2}\mathbf{g}, \quad (2)$$

where $\mathbf{g}$ is an i.i.d. random fading vector with unit-variance, circularly symmetric complex Gaussian entries.

The elements of $\mathbf{R_H}$ describe the correlation between any pair of h_{ij} elements and as such provide an ideal basis for studying correlation properties of MIMO channels. But there are hurdles. The elements of $\mathbf{R_H}$ are difficult to interpret physically, and the full correlation matrix is very large, namely $[n \cdot m \times n \cdot m]$, so researchers have strived to find meaningful approximations of $\mathbf{R_H}$.

4. Three analytical models

We will now discuss briefly three popular analytical channel models, out of a hierarchy described in detail in (*Weichselberger, 2003, chap. 6.9*).

4.1 Kronecker model (*Chuah, Kahn, Tse, 1998*)

The model derives its name from the separability of $\mathbf{R}_H$ into a Kronecker product

$$\mathbf{R}_{\mathbf{H},\,\mathrm{kron}} = \frac{1}{\mathrm{tr}\,(\mathbf{R}_{Rx})}\,\mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx}, \quad (3)$$

where $\mathbf{R}_{Tx} = \mathrm{E}\,\{\mathbf{H}^T\mathbf{H}^*\}$ and $\mathbf{R}_{Rx} = \mathrm{E}\,\{\mathbf{H}\mathbf{H}^H\}$ denote the (single-sided) correlation matrices of the transmit elements and of the receive elements, respectively[4]. The Kronecker model neglects the full spatial structure of the MIMO channel and describes it by separated link ends. As current literature is fuzzy about the necessary and sufficient condition for this model to hold, I want to repeat it here in words: Any transmit signal results in *one and the same* correlation of the receive elements (and vice versa). If and only if this condition holds, realizations of the MIMO channel matrix can be generated by

$$\mathbf{H}_{\mathrm{kron}} = \frac{1}{\sqrt{\mathrm{tr}\,(\mathbf{R}_{Rx})}}\,\mathbf{R}_{Rx}^{1/2}\mathbf{G}\,(\mathbf{R}_{Tx}^{1/2})^T, \quad (4)$$

where $\mathbf{G}$ is an i.i.d. random fading matrix with unity-variance, circularly symmetric complex Gaussian entries. The parameters for the Kronecker model are the correlation matrices of the transmit and receive elements, respectively.

The Kronecker model became popular because of its simple analytic treatment. However, the main drawback of this model is that it forces both link ends to be separable (*Özcelik et al., 2003*), irrespective of whether the channel supports this or not.

4.2 Weichselberger model

The idea of Weichselberger was to relax the separability restriction of the Kronecker model and allow coupling between the transmit and receive eigenbases, i.e. to model the correlation properties at the receiver and transmitter jointly. He did so by generalizing the MISO (multiple-input single-output) eigenmodes to MIMO. The eigenmode decomposition is unique, and the eigenmodes are orthogonal. It results also in the smallest possible number of modes, and they fade independently. The only problem with MIMO eigenmodes is that they are matrices, but a linear array of antennas can only be excited by rank-1 vector modes. Can we excite a MIMO system by something similar to eigenmodes? Yes, we can, but only approximately by MIMO vector modes. Weichselberger imposed the condition (or "structure") on these modes in the form of the eigenvalue decomposition of the Rx and Tx correlation matrices

$$\mathbf{R}_{Rx} = \mathbf{U}_{Rx}\boldsymbol{\Lambda}_{Rx}\mathbf{U}_{Rx}^H, \quad (5)$$

$$\mathbf{R}_{Tx} = \mathbf{U}_{Tx}\boldsymbol{\Lambda}_{Tx}\mathbf{U}_{Rx}^H, \quad (5)$$

to obtain the model definition (*Weichselberger et al., 2005; Weichselberger, 2003, chap. 6.4.3*)

$$\mathbf{H}_{\mathrm{weichsel}} = \mathbf{U}_{Rx}\,(\tilde{\boldsymbol{\Omega}}_{\mathrm{weichsel}} \odot \mathbf{G})\,\mathbf{U}_{Tx}^T. \quad (6)$$

Here, $\mathbf{U}_{Rx}$ ($\mathbf{U}_{Tx}$) are the eigenbases of correlation matrix of the receive (transmit) elements, respectively, and $\tilde{\boldsymbol{\Omega}}_{\mathrm{weichsel}}$ is defined as the element-wise square root of the power coupling matrix $\boldsymbol{\Omega}_{\mathrm{weichsel}}$. The elements of $\boldsymbol{\Omega}_{\mathrm{weichsel}}$, $\omega_{\mathrm{weichsel},\,ij}$, are the eigenvalues of the approximated channel correlation matrix and are calculated as the average power-coupling between the i-th transmit and the j-th receive eigenmode

$$\omega_{\mathrm{weichsel},\,ij} = (\mathbf{u}_{Tx,\,i} \otimes \mathbf{u}_{Rx,\,j})^H\,\mathbf{R_H}\,(\mathbf{u}_{Tx,\,i} \otimes \mathbf{u}_{Rx,\,j})$$
$$= \mathrm{E}\,\{\|\,\mathbf{u}_{Rx,\,i}^H\,\mathbf{H}\,\mathbf{u}_{Tx,\,j}^*\|_2^2\}, \quad (7)$$

where the $\mathbf{u}$'s are the respective elements of the $\mathbf{U}$'s. $\mathbf{G}$ is modeled as in the Kronecker model.

The eigenbases of the correlation matrices of the receive and transmit elements, and the coupling matrix constitute the parameters of the Weichselberger model.

4.3 Virtual channel representation

In contrast to the two prior models, the virtual channel representation (VCR) models the MIMO channel in beamspace instead of eigenspace. In particular, the eigenvectors are replaced by fixed and predefined steering vectors (*Sayeed, 2002*).

The VCR can be expressed as

$$\mathbf{H}_{\mathrm{virtual}} = \mathbf{A}_{Rx}\,(\tilde{\boldsymbol{\Omega}}_{\mathrm{virtual}} \odot \mathbf{G})\mathbf{A}_{Tx}^T, \quad (8)$$

where orthonormal response and steering vectors constitute the columns of the unitary response and steering matrices $\mathbf{A}_{Rx}$ and $\mathbf{A}_{Tx}$. Further, $\tilde{\boldsymbol{\Omega}}_{\mathrm{virtual}}$ is defined as the elementwise square root of the power coupling matrix $\boldsymbol{\Omega}_{\mathrm{virtual}}$, whose positive and real-valued elements $\omega_{\mathrm{virtual},\,ij}$ determine – this time – the average power-coupling between the i-th transmit and the j-th receive *directions*.

The VCR can be easily interpreted. Its angular resolution, and hence 'accuracy', depends on the actual antenna configuration. Its accuracy increases with the number of antennas, as angular bins become smaller.

The model is fully specified by the coupling matrix. Note that there still exists one degree of freedom in choosing the first direction of the unitary transmit/receive matrices $\mathbf{A}_{Tx/Rx}$.

4.4 Number of model parameters

Table 1 summarizes the number of real-valued parameters that have to be specified for modeling an $n \times m$ MIMO channel using the models previously reviewed. When only mutual information (or channel capacity) is of interest, the number of necessary parameters of the Kronecker model and the Weichselberger model reduce to $m + n$ and mn, respectively.

Table 1. Number of model parameters of considered channel models

	number of real-valued parameters
Kronecker	$m^2 + n^2$
Weichselberger	$mn + m\,(m-1) + n\,(n-1)$
VCR	mn

[4] Hans Weinrichter reminded me recently that these cannot be measured by simply looking at the antenna element correlation at one link end only.

5. Measurements

So far we have used two different measurement campaigns for model validation. The first campaign was performed in the third floor of the offices of the Institut für Nachrichtentechnik und Hochfrequenztechnik der Technischen Universität Wien, at a carrier frequency of 5.2 GHz that is envisaged for beyond 3G systems. The transmitter (Tx) consisted of a positionable sleeve antenna on a 20×10 grid with an inter-element spacing of $\lambda/2$, where only a sub-set of 12×6 Tx antenna positions was used to avoid large-scale fading effects (*Özcelik, 2004, chap. 4.3.4*). The receiver (Rx) was a directional 8-element uniform linear array of printed dipoles with $0.4\,\lambda$ inter-element spacing and $120°$ 3 dB field-of-view. The channel was probed at 193 equispaced frequencies over 120 MHz of bandwidth. The (virtual) transmit array was positioned in a hallway and the receiver assumed 24 different positions each looking into three different directions (rotated by $120°$) in several offices connected to this hallway, leading to 72 different "scenarios" (Fig. 3). As this method stipulates a static channel, the measurements were made during night time. A detailed description of the measurement campaign can be found in (*Özcelik, 2004, chap. 4*).

The investigated models assume that the channel is sufficiently described by its second-order moments, hence by the full channel correlation matrix $\mathbf{R_H}$, only. As a consequence, measurements used for the evaluations have to fulfil this requirement, too. Only a restricted set of 58 scenarios (out of 72) met this condition; the others were excluded.

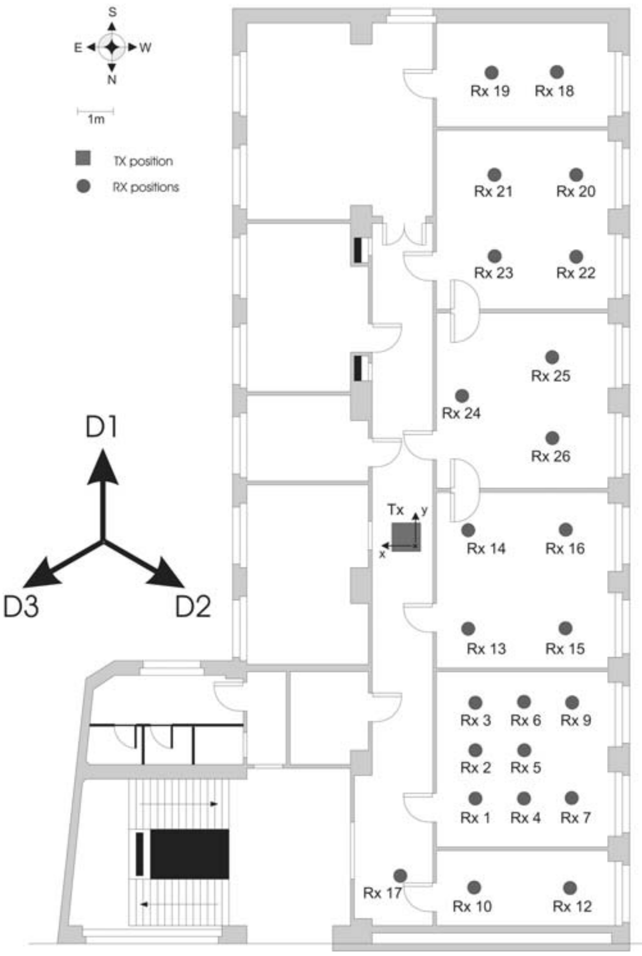

Fig. 3. Layout of MIMO measurement campaign at Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien. Circles in rooms indicate the various receiver positions, D1…3 indicate direction of 8-element receive array

The second campaign was performed by ftw. in a suburban environment with single-family homes dispersed in a garden city (Fig. 4, "Weikendorf"). The carrier frequency was ≈ 2 GHz. The transmitter with 15 sleeve antennas, circularly spaced $0.34\,\lambda$ apart, was moved on a trolley (Fig. 5) on a pathway having both LOS and NLOS to the eight-element receive antenna ($120°$ field of view, $0.5\,\lambda$ spaced linear uniform array).

Fig. 4. View from the receiver towards the Tx trajectory. Courtesy H. Hofstetter, ftw.

Fig. 5. 2 GHz transmitter on trolley, connected to 15 element circular antenna. Courtesy H. Hofstetter, ftw.

Several meters of the trajectory lead through an underground railroad passage. A detailed description of the measurement campaign can be found in (*Hofstetter, Viering, Utschik, 2002*).

6. Validation method

The actual validation of each of the three analytical MIMO channel models comprises three steps. For each measured position and antenna configuration ("scenario"): (1) extract model parameters from measurement; (2) generate synthesized channels with these parameters by Monte-Carlo simulations according to the model under consideration; (3) compare metrics calculated from synthesized channels vs. measurement.

Step 1: Model parameter extraction

First we obtain estimates (designated by the hat symbol) for $\mathbf{R}_{Tx}$ and $\mathbf{R}_{Rx}$ from the measured spatial and frequency realizations of the **H**-matrices by

$$\hat{\mathbf{R}}_{Rx} = \frac{1}{N}\sum_{r=1}^{N}\mathbf{H}(r)\,\mathbf{H}(r)^{H}, \quad (9)$$

$$\hat{\mathbf{R}}_{Tx} = \frac{1}{N}\sum_{r=1}^{N}\mathbf{H}(r)^{T}\,\mathbf{H}(r)^{*}, \quad (10)$$

where N is the number of channel realizations, while $\mathbf{H}(r)$ denotes the r-th channel realization. The number of different realizations, N, of **H** at each scenario was 5,790 by sliding eight adjacent Tx antenna positions over the relevant 12×6 grid (i.e., 30 spatial realizations; additionally all 193 frequencies were considered as realisations which yields a total number of 5,790 channel realisations per scenario).

This suffices for the Kronecker model. For the Weichselberger model we first apply the eigenvalue decomposition of the estimated correlation matrices,

$$\hat{\mathbf{R}}_{Rx} = \hat{\mathbf{U}}_{Rx}\hat{\mathbf{\Lambda}}_{Rx}\hat{\mathbf{U}}_{Rx}^{H}, \text{ and} \quad (11)$$

$$\hat{\mathbf{R}}_{Tx} = \hat{\mathbf{U}}_{Tx}\hat{\mathbf{\Lambda}}_{Rx}\hat{\mathbf{U}}_{Tx}^{H}, \quad (12)$$

from which we obtain the estimated power coupling matrix $\tilde{\mathbf{\Omega}}_{weichsel}$ of the Weichselberger model by

$$\tilde{\mathbf{\Omega}}_{weichsel} = \frac{1}{N}\sum_{r=1}^{N}(\hat{\mathbf{U}}_{Rx}^{H}\mathbf{H}(r)\hat{\mathbf{U}}_{Tx}^{*}) \odot (\hat{\mathbf{U}}_{Rx}^{T}\mathbf{H}^{*}(r)\hat{\mathbf{U}}_{Tx}). \quad (13)$$

The arrangement of entries in this matrix gives clues as to whether the channel is conducive to spatial multiplexing, to diversity at either receiver or transmitter, or to beamforming (*Weichselberger, 2003*), as will be seen later in an example.

For the VCR we get the estimated coupling matrix $\tilde{\mathbf{\Omega}}_{virtual}$ by

$$\tilde{\mathbf{\Omega}}_{virtual} = \frac{1}{N}\sum_{r=1}^{N}(\mathbf{A}_{Rx}^{H}\mathbf{H}(r)\,\mathbf{A}_{Tx}^{*}) \odot (\mathbf{A}_{Rx}^{T}\mathbf{H}^{*}(r)\,\mathbf{A}_{Tx}). \quad (14)$$

For the unitary steering/response matrices, $\mathbf{A}_{Tx}$ and $\mathbf{A}_{Rx}$, one steering/response direction was selected as the direction normal to the antenna array.

Step 2: Monte-Carlo simulations

Now we can generate synthesized channels through Monte-Carlo simulations by randomly choosing **G**, according to the three models, Kronecker (4), Weichselberger (6), and the VCR (8).

The number of realizations was chosen to be equal to the number of measured realizations.

Step 3: Calculate suitable performance figures or metrics from both the synthesized channels and the measured ones, and compare the results. First, however, we have to discuss which metrics will be useful.

Which metric?

The quality of a model has to be defined with a view toward a specific channel property or aspect we are interested in. For this we need performance figures or metrics that cover the desired channel aspects and apply these metrics to measured and modeled channels, enabling a comparison of the models investigated. Of course, it would be very helpful and advantageous to have a single metric that is capable of capturing all properties of a MIMO channel. However, this is not possible since the application of a specific metric implies a reduction of reality to some selected aspects, as modeling always does.

Capacity is the most common performance figure by which MIMO systems are measured. It is, however, a complex quantity depending, besides antenna number, on receive signal-to-noise ratio, SNR, and, above all, the spatial structure of the channel. Normalizing to a given SNR in comparing MIMO schemes, as is usually done, blurs the issue of whether high SNR in a line-of-sight situation is better or worse for MIMO capacity than a low SNR in a rich multipath environment. If a model predicts average capacity well, but the capacity value is low, the model cannot answer how to improve capacity. So, to predict capacity correctly is a necessary but not a sufficient condition for a good MIMO channel model. For our investigation we have chosen the average mutual information. Mutual information (German: “Transinformation”), as used by Telatar in his landmark paper (*Telatar, 1995*) on multi-antenna transmission does not assume prior optimization of the MIMO channel matrix (as “capacity” does). Considering a narrowband channel unknown at Tx, the mutual information of the MIMO channel with equally allocated transmit powers was calculated for each realization using (*Foschini, Gans, 1998; Telatar, 1995*) and then averaged

$$I = \mathrm{E}\{\log_2 \det(\mathbf{I}_n + \frac{\varrho}{m}\mathbf{H}\mathbf{H}^{H})\}, \quad (15)$$

where $\mathbf{I}_n$ denotes the $n\times n$ identity matrix, ϱ the average receive SNR, and **H** the normalized $n\times m$ MIMO channel matrix. The normalization was done such that for each scenario the power of the channel matrix elements h_{ij}, averaged over all realizations was set to unity (*Özcelik, 2004, chap. 5.3.1*). In the subsequent evaluations for Section 7, the average receive SNR for each scenario was fixed at 20 dB. We want to caution the reader, though, that such normalization is not always appropriate, e.g., in a MIMO system *without* transmit power control.

The spatial structure of the channel is a more difficult quantity to render. We have chosen the *joint double-directional angular power spectrum* (APS) as a metric for MIMO models. The APS is calculated using Capon’s beamformer, also known as minimum variance method (MVM) (*Capon, 1969*),

$$P_{Capon}(\varphi_{Rx}, \varphi_{Tx}) = \frac{1}{\tilde{\mathbf{a}}^{H}R_{H}^{-1}\tilde{\mathbf{a}}}, \quad (16)$$

with

$$\tilde{\mathbf{a}} = \mathbf{a}_{Tx}(\varphi_{Tx}) \otimes \mathbf{a}_{Rx}(\varphi_{Rx}), \quad (17)$$

using the normalized steering vector $\mathbf{a}_{Tx}(\varphi_{Tx})$ into direction φ_{Tx} and response vector $\mathbf{a}_{Rx}(\varphi_{Rx})$ from direction φ_{Rx}. Here, $\mathbf{R}_{\mathbf{H}}$ denotes the full MIMO channel correlation matrix (1). Capon’s beamformer was preferred over the simpler Bartlett beamformer because it provides better angular resolution. On the other hand, it is more robust and easier to implement than super-resolutional parametric estimation methods like SAGE (*Fleury et al., 1999*) or ESPRIT (*Fuhl, Rossi, Bonek, 1997*).

An alternative description of the spatial structure is by Weichselberger’s $\mathbf{\Omega}$-matrix. A nonzero entry in this matrix indicates significant power being transferred from the specific transmit eigenmode to the specific receive eigenmode. Whereas these are not equal to directions, they pertain to the spatial structure of the channel by all means (*Weichselberger, 2003, chap. 6.4.3.4*).

Transmit or receive *diversity* is another attractive feature of MIMO, making data transmission not necessarily faster but more reliable. It is well known that the diversity order achievable in a certain propagation environment is given by the number of significant eigenvalues of $\mathbf{R}_{\mathbf{H}}$, with a maximum $n\times m$. But eigenvalues don’t separate easily in significant ones

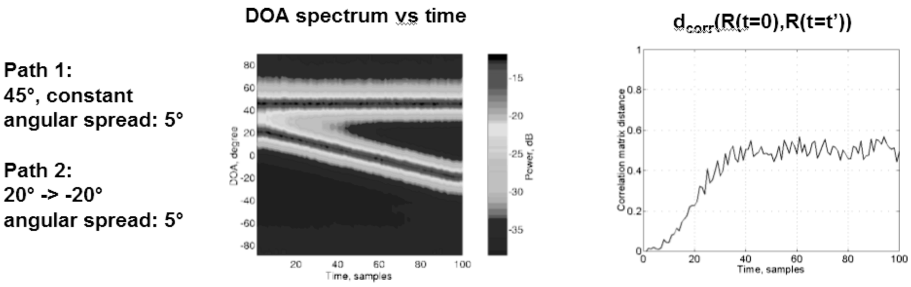

Fig. 6. Correlation matrix distance d_{corr} of a time-varying synthetic propagation scenario (a) Capon DOA spectra as a function of time; (b) corresponding CMD values (*Herdin, 2004*)

and negligible ones (noise), thus rendering this statement rather empty. So, in order to avoid the complete eigenvalue profile, a single number describing diversity order and being derived from measurements would be nice. Such a measure, a *Diversity Measure*, $\Psi(\mathbf{R_H})$, was recently introduced (*Ivrlač, Nossek, 2003*)

$$\Psi(\mathbf{R_H}) = \left(\frac{\mathrm{tr}\,(\mathbf{R_H})}{\|\mathbf{R_H}\|_F}\right)^2 = \frac{\left(\sum_{i=1}^{K}\lambda_i\right)^2}{\sum_{i=1}^{K}\lambda_i^2}. \qquad (18)$$

An interesting metric to assess the similarity between different correlation matrices was recently introduced by Herdin (*Herdin, 2004*). To obtain a metric that is equal to zero when the correlation matrices are identical (apart from at scalar factor) and equal to unity if they differ maximally, the *correlation matrix distance* (*CMD*) between $\mathbf{R}$ and $\hat{\mathbf{R}}$ is defined as

$$d_{\mathrm{corr}} = 1 - \frac{\mathrm{tr}\,(\mathbf{R}\hat{\mathbf{R}})}{\|\mathbf{R}\|_F \|\hat{\mathbf{R}}\|_F}. \qquad (19)$$

The meaning of this metric can easily be understood from a different formulation of it

$$d_{\mathrm{corr}} = 1 - \frac{(\mathrm{vec}\,(\mathbf{R}))^H \mathrm{vec}\,(\hat{\mathbf{R}})}{\|\mathrm{vec}\,(\mathbf{R})\|_2 \|\mathrm{vec}\,(\hat{\mathbf{R}})\|_2}. \qquad (20)$$

The CMD is directly related to the inner product of the vectorized correlation matrices, in the present context the measured and the modelled ones. If the correlation matrices are equal, the CMD becomes zero, the more they differ from each other, the larger the CMD becomes. Finally, if they differ to a maximum amount, the CMD becomes 1. Figure 6 shows the CMD of a synthetic, time-evolving propagation scenario with two distinct directions. The respective correlation matrices were calculated for each time instant and compared with the very first one. The CMD reflects the difference between the correlation at the beginning and the end precisely (CMD: = 0.5), as only half of the scenario changes.

The advantage of the CMD is that it measures how the *structure* of the matrices differs, independent of changes in power or system parameters like number of used eigenmodes.

7. Validation results and discussion

We now apply our four metrics to the validation approach described, using the two measurement campaigns, and have a look how the three models behave.

Figure 7 compares the average mutual information as measured with the modelled one. For each model, a specific marker corresponds to one of the 58 scenarios. The Kronecker model (crosses) underestimates the "measured" mutual information[5], i.e., the points lie below the identity line (dashed). Moreover, the mismatch increases up to more than 10 % with decreasing mutual information. A more detailed analysis in (*Özcelik et al., 2003*) showed that scenarios with low mutual information correspond to high correlation. Thus, the Kronecker model, introduced to account for correlation, fails the more, the more correlated the channel is. We want to mention that the mutual information of the Kronecker model was already investigated in several publications, e.g. in (*McNamara, Beach, Fletcher, 2002*) or (*Yu et al., 2001*), where the performance of the model was found to be satisfactory for a 2 × 2 and 3 × 3 system. This is not in contradiction to our results. When the antenna size increases, thereby improving the angular resolution, the deficiency of the Kronecker model becomes worse.

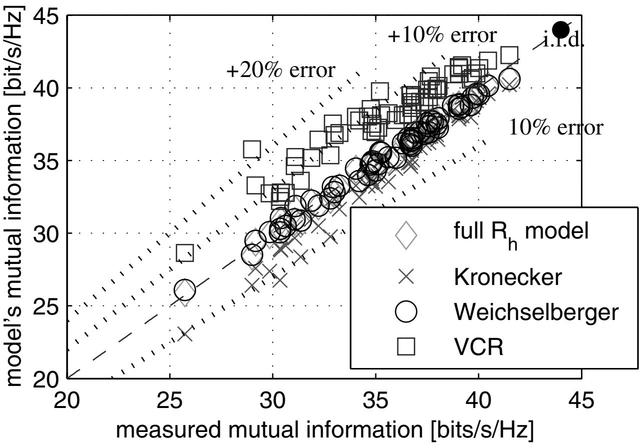

Fig. 7. Average mutual information of measured vs. modeled 8 × 8 MIMO channels at a receive SNR of 20 dB

The VCR (squares) overestimates the "measured" mutual information significantly. The reason is due to its fixed steering/response directions. Thus, it tends to model the MIMO channel with more multipath components than the underlying channel actually has, thereby reducing channel correlation and increasing the mutual information.

The Weichselberger model (circles) fits the measurements best with relative errors within a few percents.

The diamonds (full $\mathbf{R_H}$ model) are actually not a model, but a check on whether the underlying assumption – that the measured and selected channels can be completely described

[5] Monte-Carlo simulations that we have performed with completely synthetic MIMO channels showed that, although very seldom, the Kronecker model might also overestimate the "measured" mutual information. The probability of overestimation decreases with increasing antenna number.

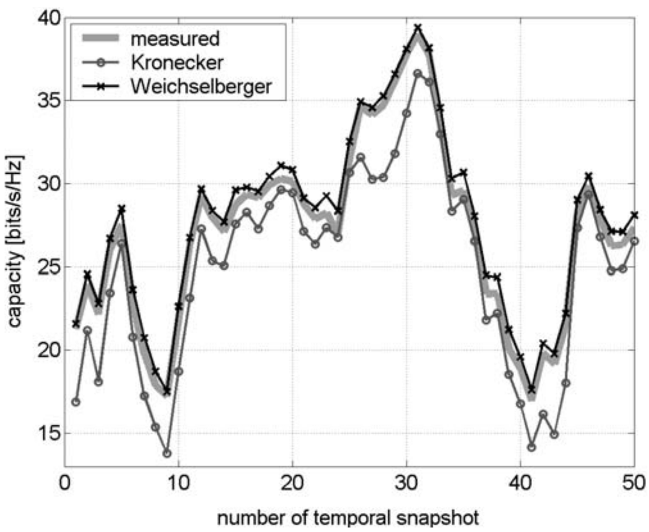

Fig. 8. Average mutual information ("capacity") measured along the Weikendorf trajectory and normalised to 20 dB receive SNR. Courtesy: Helmut Hofstetter, ftw.

by second-order statistics – is fulfilled. The correlation matrix was calculated from the measurement-derived parameters. Any major deviation from the identity line would mean that the channels violate this assumption, which – evidently – is not the case. Moreover, the near-coincidence of the full correlation matrix points with the Weichselberger points prove that the Weichselberger model is a very good approximation of reality indeed. It reflects the multiplexing gain of the measured channels best.

Figure 8 shows snapshots of the average mutual information measured along the Weikendorf trajectory (No. 1 at the Tx start, No. 50 at the Tx stop). The ups and downs in the curve, normalized again to 20 dB receive SNR, correspond to NLOS and LOS situations, respectively. It is particularly interesting to scrutinize the Weichselberger Omega-matrices at some extreme points. Position no. 9 was a clear LOS scenario, reflected both in the Omega matrix (Fig. 9a, in linear scale) as a single dominant transmit eigenmode coupling to a single receive eigenmode, as well as in the low mutual information. The latter reflects the suppression of the, certainly present, multipath by the large LOS power. This is a direct consequence of normalizing to constant receive SNR. Position no. 27 (Fig. 9b) was taken when the transmitter was just entering the railroad underpass, showing rich multipath and – consistently – high mutual information. Particular useful information can be derived from Fig. 9c, taken at position no. 34. The strongest six transmit eigenmodes couple only into one receive eigenmode, making such a channel ideally suited for transmit, but not receive diversity. From the mutual-information curve, usually being the only available source of information, such information can never be extracted.

The Kronecker model again underestimates capacity consistently, which is particularly surprising at position no. 31. This was also taken when the transmitter was in the railroad underpass, a situation where complete decoupling of Tx and Rx correlation might be conceivable. Also, the Weichselberger

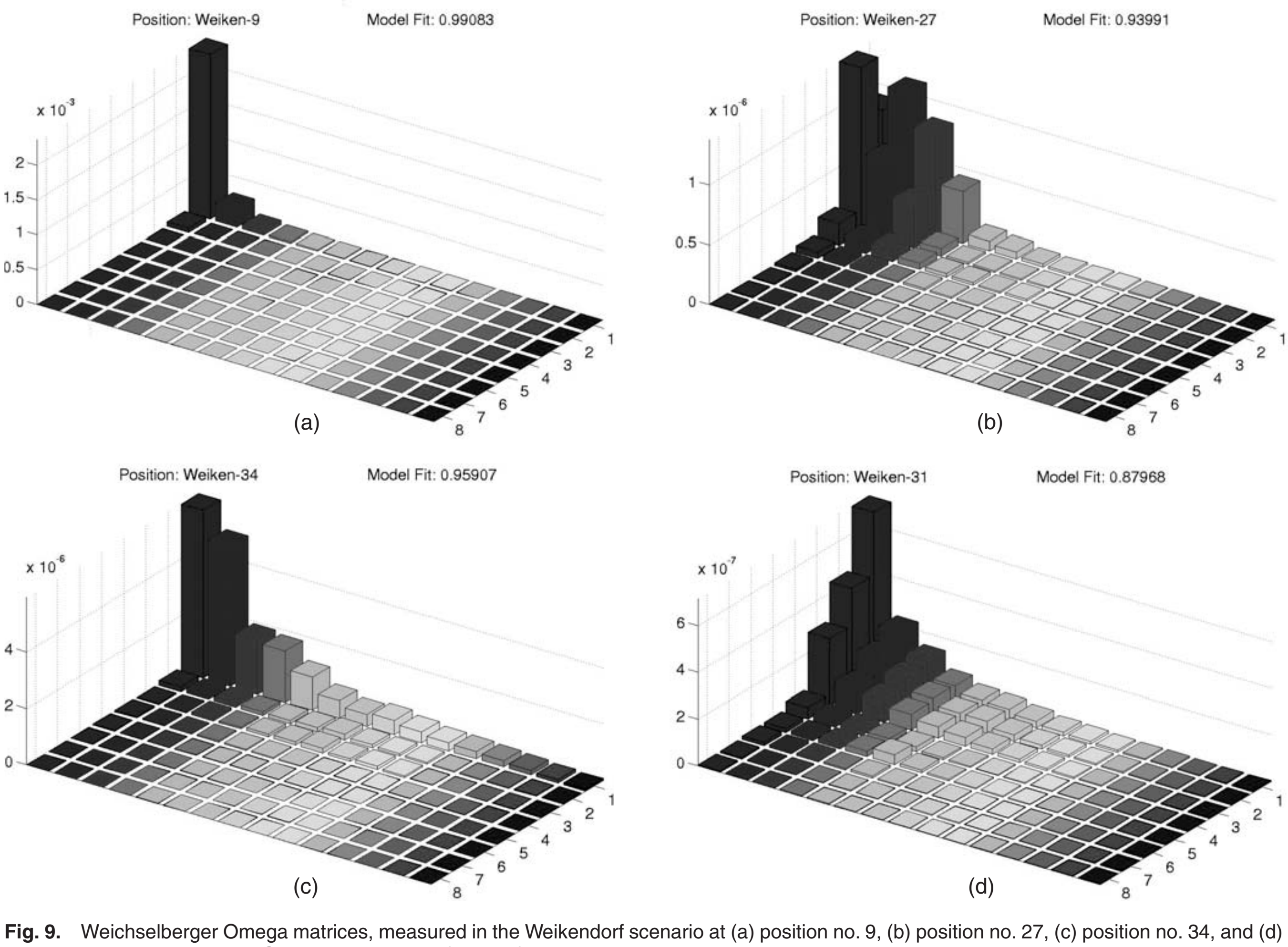

Fig. 9. Weichselberger Omega matrices, measured in the Weikendorf scenario at (a) position no. 9, (b) position no. 27, (c) position no. 34, and (d) position no. 31. Linear scale. Courtesy: Helmut Hofstetter, ftw.

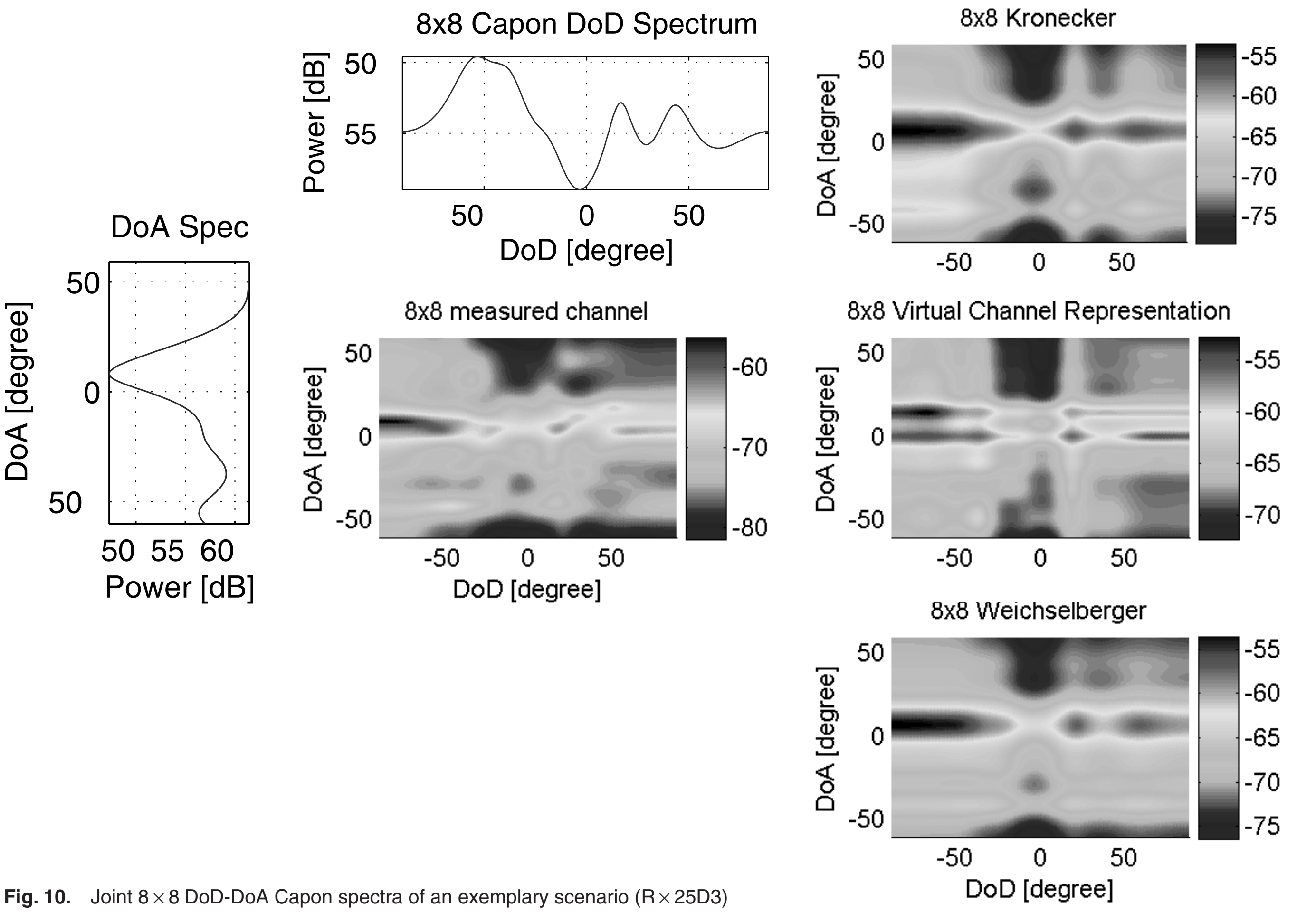

Fig. 10. Joint 8 × 8 DoD-DoA Capon spectra of an exemplary scenario (R × 25D3)

Omega-matrix, shown in (Fig. 9d), gives the impression as if it were composed from a product of two rank-1 MIMO eigenmode matrices (i.e., vectors), which is constitutive for the Kronecker model to hold.

Figure 10 compares the APS of the measured and modeled 8 × 8 MIMO channel for an exemplary scenario (Rx25D3). The

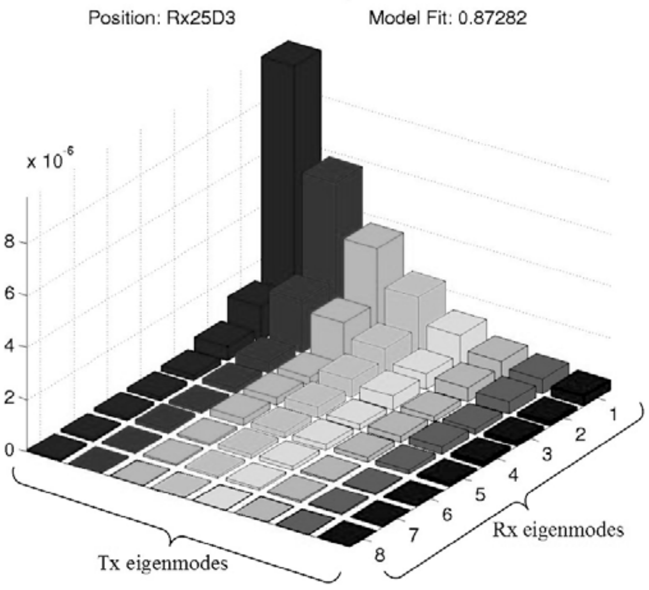

Fig. 11. Weichselberger coupling matrix of scenario Rx25D3 at INTHF, in linear scale

measured APS (joint and marginal APS) are given on the left side, whereas the right side shows the models' joint APS.

In the measured channel, a specific DoA is clearly linked to widespread but specific DoDs, such that the joint APS is not totally separable. In contrast, the Kronecker model introduces artefact paths lying at the intersections of the DoA and DoD spectral peaks. The Weichselberger model does not render the multipath structure completely correct either. It is, though, interesting to have again a closer look at the entries in the Weichselberger Omega-matrix, shown in Fig. 11 in linear scale. There are significant entries in almost all eight transmit eigenmodes that couple into the first receive eigenmode, lesser that couple into the second receive eigenmode. Though the eigenmodes are not identical to directions, as mentioned before, the analogy between Fig. 11 and 10 is striking. As a result one can conclude that, in this specific exemplary channel, transmit diversity would be advantageous, but not so receive diversity or spatial multiplexing. For a full appreciation of the various channel configurations, the interested reader is referred to (*Özcelik, 2004*). The VCR should be able to cope with any arbitrary DoD/DoA coupling. The joint APS shows that it does not because of the fixed and predefined steering vectors. It is not able to reproduce multipath components between two fixed steering vector directions properly and therefore splits the major DoA into two.

The degree of diversity offered by the measured indoor MIMO channels is shown in Fig. 12.

As can be seen, the modeled channels either match or overestimate the diversity measures of the corresponding measured channels. Although the Weichselberger model (circles) outperforms both the Kronecker model (crosses) and

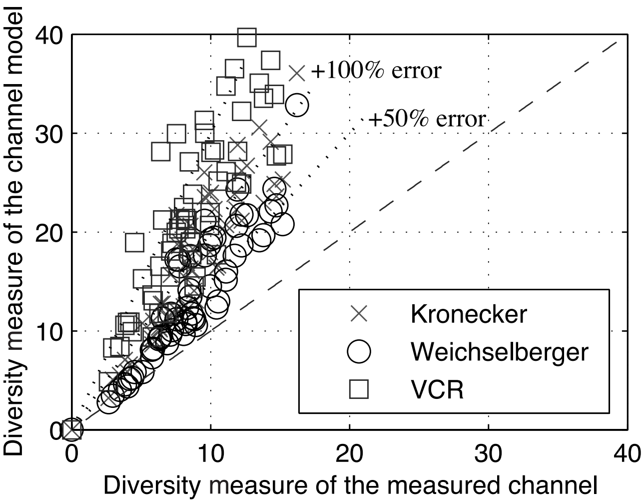

Fig. 12. Diversity measure, $\Psi(\mathbf{R_H})$, of the measured vs. modeled 8×8 MIMO channels

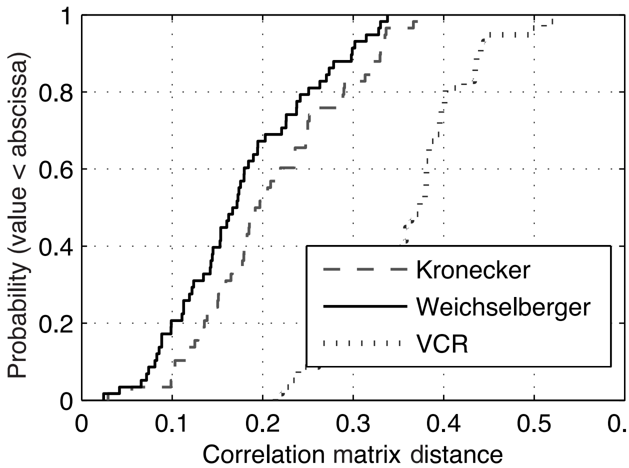

Fig. 13. Empirical cdf of the correlation matrix distance (CMD) between measured and synthesized 8×8 MIMO channels with inter-element spacings of $0.5\,\lambda$ at Tx and $0.4\,\lambda$ at Rx. Empirical CDF of the measured CMD in the scenarios of INTHF. The more left the curves are, the better the model.

the VCR (squares) clearly, it shows large relative errors for high diversity. Though not shown here, the Diversity Measure is a much more reliable metric for predicting 2×2 MIMO channels by the Kronecker and Weichselberger models (*Özcelik, 2004*). In contrast, the VCR again fails completely and overestimates the diversity measure significantly. The reasons for the poor performance of the VCR are, again, its fixed, predefined steering directions.

At this stage, we stress that the validation approach just discussed is the proper one to arrive at models that re-construct realistic MIMO channels, e.g., channels that are measured. This approach, though, is not the only one possible. Should one be interested in a single aspect of MIMO only, then models that contain proper parameters (that can be specified more or less freely) might perform better. For instance, the VCR allows for modeling channels with arbitrary multiplexing orders by choosing appropriate coupling matrices. Similarly, an appropriate choice of the Weichselberger coupling matrix enables the setting of arbitrary multiplexing and diversity orders.

To assess models by the correlation matrix distance CMD, we choose another form of representation, the empirical cumulative density function (cdf). Figure 13 shows the empirical cdf of the CMD between the measured and synthesized 8×8 channels. With a mean CMD of 0.18 and a maximum of 0.34, the Weichselberger model (full line) has the smallest deviation from the measurement. Although it renders the measured channel correlation best, it does not so sufficiently for all scenarios. Under the premise that a CMD value of 0.4 emphasizes significant deviations, as it was argued in (*Herdin, 2004*), the modeling of those scenarios that result in a CMD of 0.34 are critical with respect to the full channel correlation.

The performance of the Kronecker model (dashed line) is slightly poorer. Its mean CMD results as 0.21, whereas its maximum CMD is 0.37. Evidently, the virtual channel representation (dotted line) shows the largest deviation from the measured channel correlation. With a mean CMD of 0.36 it is not advisable for modeling correlation of 8×8 channels.

8. Conclusions

A good channel model is a model that renders correctly the relevant aspects of the MIMO system to be deployed.

Different MIMO aspects may require different models, as there are spatial multiplexing, spatial diversity or beamforming. The consequences of the model choice on bit-error-ratio simulations and on system self-interference in multipoint MIMO systems are enormous. Research in these areas has just began (*Badics et al., 2004; Gritsch, Weinrichter, Rupp, 2005*), but many more surprises are to be expected. If no specific channel property is in focus, a good MIMO channel model is one that reflects the spatial structure of the channel. MIMO channel measurements with many antenna elements, e.g, the described 8×8 and 15×8 campaigns at 5.2 and 2 GHz, reveal the strengths or weaknesses of models better than, say, model validation with 2×2 systems. Of the models investigated, the Weichselberger model performs best with respect to the analyzed metrics, even though it is inaccurate for joint APS and diversity measure. The Kronecker model should only be used for limited antenna numbers, such as 2×2, and the virtual channel representation can only be used for modeling the joint APS for very large antenna numbers.

Experimental model validation requires more than a single measurement. Much more careful and self-critical work is needed in this area if we want to fully exploit the promises of the wondrous world of MIMO.

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References

Badics, B., Herdin, M., Gritsch, G., Rupp, M., Weinrichter, H. (2004): Performance of various data transmission methods on measured MIMO channels. In: IEEE VTC Spring 2004, Milan, Italy, 2004.

Bölcskei, H. et al. (eds.) (2005): Space-time wireless systems: from array processing to MIMO communications. Cambrigde: Cambridge University Press.

Capon, J. (1969): High-resolution frequency-wavenumber spectrum analysis. Proc. of the IEEE 57 (8), Aug. 1969: 1408–1418.

Chuah, C-N., Kahn, J. M., Tse, D. (1998): Capacity of multi-antenna array systems in indoor wireless environment. In: IEEE Global Telecomm. Conf. 1998, Vol. 4, Sydney, Australia, pp. 1894–1899.

Fleury, B. H., Tschudin, M., Heddergott, R., Dahlhaus, D., Pedersen, K. I. (1999): Channel parameter estimation in mobile radio environments using the SAGE algorithm. IEEE J. Selected Areas in Comm. 3: 434–450.

Foschini, G. J., Gans, M. J. (1998): On limits of wireless communications in fading environments when using multiple antennas. Wirel. Pers. Comm. 6: 311–335.

Fuhl, J., Rossi, J.-P., Bonek, E. (1997): High-resolution 3-D direction-of-arrival determination for urban mobile radio. IEEE Trans. on Antennas and Propagation 45 (4), Apr. 1997: 672–682.

Gritsch, G., Weinrichter, H., Rupp, M. (2005): MIMO paradoxon of nonorthogonal space-time block codes. Electr. Lett. 41 (6): 343–344.

Herdin, M. (2004): Non-stationary indoor MIMO radio channels. Ph.D. dissertation, Technische Universität Wien. http://www.nt.tu-wien.ac.at/mobile/theses_finished/.

Hofstetter, H., Viering, I., Utschick, W. (2002): Evalution of suburban measurements by eigenvalue statistics. In: Proc. COST 273 Workshop 'Opportunities of the Multidimensional Propagation Channel', Espoo, Finland, 29 May 2002.

International Telecommunications Union. Recommendation ITU-RM1645. http://www.itu.int/rec/recommendation.asp/.

Ivrlač, M. T., Nossek, J. A. (2003): Quantifying diversity and correlation of Rayleigh fading MIMO channels. In: IEEE Int. Symp. on Signal Processing and Information Technology, ISSPIT'03, Darmstadt, Germany, Dec. 2003.

McNamara, D. P., Beach, M. A., Fletcher, P. N. (2002): Spatial correlation in indoor MIMO channels. In: IEEE Int. Symp. on Personal, Indoor and Mobile Radio Comm., Vol. 1, Lisbon, Portugal, 2002, pp. 290–294.

Özcelik, H. (2004): Indoor MIMO channel models. Ph.D. dissertation, Technische Universität Wien. http://www.nt.tuwien.ac.at/mobile/theses_finished/.

Özcelik, H., Herdin, M., Hofstetter, H., Bonek, E. (2003): Capacity of different MIMO systems based on indoor measurements at 5.2 GHz. In: 5th European Personal Mobile Comm. Conf., EPMCC 2003, Glasgow, Scotland, Apr. 2003, pp. 463–466.

Özcelik, H., Herdin, M., Weichselberger, W., Wallace, J., Bonek, E. (2003): Deficiencies of the Kronecker MIMO radio channel model. Electr. Lett. 39 (16): 1209–1210.

Sayeed, A. M. (2002): Deconstructing multiantenna fading channels. IEEE Trans. on Signal Proc. 50 (10), Oct. 2002: 2563–2579.

Shiu, D-S., Foschini, G. J., Gans, M. J., Kahn, J. M. (2000): Fading correlation and its effect on the capacity of multielement antenna systems. IEEE Trans. Comm. 48 (3), March 2000: 502–513.

Telatar, I. E. (1995): Capacity of multi-antenna Gaussian channels. AT & T Bell Laboratories, Tech. Rep. BL0112170-950615-07TM.

Weichselberger, W. (2003): Spatial structure of multiple antenna radio channels – a signal processing viewpoint. Ph.D. dissertation, Technische Universität Wien. http://www.nt.tuwien.ac.at/mobile/theses_finished/.

Weichselberger, W., Herdin, M., Özcelik, H., Bonek, E. (2005): A stochastic MIMO channel model with joint correlation of both link ends. IEEE Trans. on Wireless Comm. 2005 (to appear).

Yu, K. et al. (2001): Second order statistics of NLOS indoor MIMO channels based on 5.2 GHz measurements. In: IEEE Globecom 2001, Vol. 1, December 2001, pp. 156–160. ■