AFFINE PROJECTION ALGORITHM FOR BLIND MULTIUSER EQUALISATION OF DOWNLINK DS-CDMA SYSTEM

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ABSTRACT

In this paper, we derive an affine projection algorithm (APA) for blind multiuser equalisation, suitable for downlink DS-CDMA systems. Adaptation is performed by minimising a cost function based on the constant modulus (CM) criterion for all users. Computer simulations are used to assess the performance of the algorithm.

Keywords: blind equalisation, affine projection algorithm, constant modulus criterion, downlink DS-CDMA.

1. INTRODUCTION

In a DS-CDMA downlink scenario, transmission over a dispersive channel destroys the mutual orthogonality of the codes which are used to multiplex the various users in the system. As a result, the received code-demultiplexed user signals are subject not only to inter-symbol interference (ISI) due to channel dispersion but also to multiple access interference (MAI) due to the loss of code orthogonality.

A popular blind approach to suppress MAI and ISI is the minimum output energy (MOE) algorithm, cancelling MAI and ISI terms but passing the desired user by constraints [1]. Other blind schemes have been performed using the CM criterion [2, 3], whereby additional mutual decorrelation of the recovered user sequences is required. Alternatively, in [4, 5] a blind scheme, the so called filtered-R muliple error (FIRMER) CMA has been developed, which is similar to [2, 3] but requires neither constraints nor mutual deccorelation. Since FIRMER-CMA algorithm is only suitable for fully loaded systems, a hybrid CM/MSE algorithm, suitable for partial loading scenarios, has been derived in [6]. However, the typical slow convergence of such approaches [4, 5, 6] limits the tracking performance of the receiver.

In this paper, we aim to speed up FIRMER-CMA's convergence by adopting the concept of the affine projection algorithm (APA). The APA has been developed to ameliorate the slow convergence of the NLMS scheme by Ozeki and Umeda in 1984 [7]. A detailed quantitative analysis of the convergence behaviour of the APA can be found in a number of articles [8, 9]. In the literature, APA refers to an entire class of algorithms such as the partial rank algorithm (PRA) [10], the generalised optimal block algorithm(GOBA) [11], and NLMS with orthogonal correction factors (NLMS-OCF) [12]. The distinguishing attributes of these algorithms, which have been derived independently from different perspectives, is that they update the taps on the basis of multiple, delayed, and re-used input signal vectors [8].

All previously mentioned APA based algorithms are only suitable for trained systems. Consequently, Papadias [13] has derived a blind APA scheme, the so called normalised sliding window constant modulus algorithm (NSWCMA), by performing some insightful changes on the CM update [13]. Initially designed for a single user system, the latter algorithm provides a faster convergence speed than the basic CMA and shows a good ability to avoid local minima in its cost function [13]. Motivated by the performance achieved in [13], in the following we derive an affine projection blind multiuser equaliser similar in structure to the NSWCMA but suitable for downlink DS-CDMA systems. Based on the definition of a signal model in Sec. 2, a suitable cost function for a blind adaptation equaliser is given in Sec. 3. In Sec. 4 we derive the affine projection algorithm for the latter cost function. Simulations of the proposed algorithm are presented in Sec. 5, and conclusions drawn in Sec. 6.

2. SIGNAL MODEL

We consider the DS-CDMA downlink system in Fig. 1 with multiple symbol-synchronous users, which for simplicity are assumed to have the same rate. The system is fully loaded with N user signals $u_l[n]$, $l = 0(1)N-1$, which are code multiplexed using Walsh sequences of length N extracted from a Hadamard matrix $\mathbf{H}$. The resulting chip rate signal, running at N times the symbol rate, is further

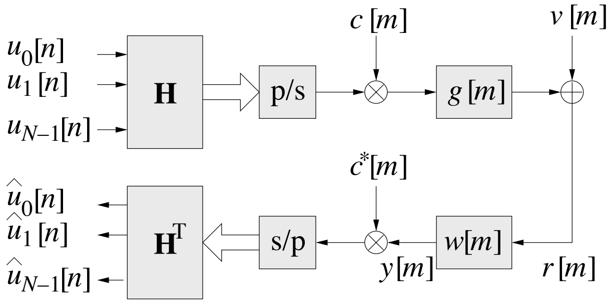

Fig. 1. DS-CDMA downlink signal model.

scrambled by $c[m]$ prior to transmission over a channel with dispersive impulse response $g[m]$ and corrupted by additive white Gaussian noise $v[m]$, which is assumed to be independent of the transmitted signal.

The dispersive channel $g[m]$ destroys the orthogonality of the Walsh codes, such that direct decoding of the received signal $r[m]$ with descrambling by $c^*[m]$ and code-matched filtering by $\mathbf{H}^{\mathrm{T}}$ will lead to MAI and ISI corruption of the decoded user signals $\hat{u}_l[n]$, $l = 0(1)N-1$. In order to re-establish orthogonality of the codes, a chip rate equaliser $\mathbf{w}$ can be utilised [15, 14]. In the following, we are concerned with amalgamating this scheme with an affine projection algorithm in order to blindly update $\mathbf{w}$.

3. MULTIUSER EQUALISATION CRITERION

We first derive the detected users' signals $\hat{u}_l[n]$ as a function of the chip-rate equaliser $\mathbf{w}$. Based on this, we state a suitable cost function on which the equaliser adaptation relies.

3.1. Demultiplexed User Signals

For the decoding, Walsh sequences are used as matched filters. The Walsh sequence for decoding the lth user, contained in a vector $\mathbf{h}_l$, can be taken from an $N \times N$ Hadamard matrix,

$$\mathbf{H}^{\mathrm{T}} = [\mathbf{h}_0 \;\; \mathbf{h}_1 \;\; \cdots \;\; \mathbf{h}_{N-1}]^{\mathrm{T}} . \qquad (1)$$

The lth user is thus decoded as

$$\begin{aligned}
\hat{u}_l[n] &= \mathbf{h}_l^{\mathrm{T}} \cdot \begin{bmatrix} c^*[nN] & & & \mathbf{0} \\ & c^*[nN-1] & & \\ & & \ddots & \\ \mathbf{0} & & & c^*[nN-N+1] \end{bmatrix} \cdot \begin{bmatrix} y[nN] \\ y[nN-1] \\ \vdots \\ y[nN-N+1] \end{bmatrix} \\
&= \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \cdot \begin{bmatrix} \mathbf{w}^{\mathrm{H}} & & \mathbf{0} \\ & \mathbf{w}^{\mathrm{H}} & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{w}^{\mathrm{H}} \end{bmatrix} \cdot \begin{bmatrix} r[nN] \\ r[nN-1] \\ \vdots \\ r[nN-L-N+2] \end{bmatrix}
\end{aligned}$$

whereby the descrambling code $c^*[m]$ has been absorbed into a modified and now time-varying code vector $\tilde{\mathbf{h}}_l[nN]$, and $\mathbf{w} \in \mathbb{C}^L$ contains the equaliser's L chip-spaced complex conjugate weights. Rearranging $\mathbf{w}$ and $\tilde{\mathbf{h}}_l[nN]$ yields

$$\begin{aligned}
\hat{u}_l[n] &= \mathbf{w}^{\mathrm{H}} \cdot \begin{bmatrix} \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & & \mathbf{0} \\ & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & \\ & & \ddots \\ \mathbf{0} & & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \end{bmatrix} \cdot \begin{bmatrix} r[nN] \\ r[nN-1] \\ \vdots \\ r[nN-L-N+2] \end{bmatrix} \\
&= \mathbf{w}^{\mathrm{H}} \, \mathbf{H}_l[nN] \, \mathbf{r}_{nN} , \qquad (2)
\end{aligned}$$

with $\mathbf{H}_l[nN] \in \mathbb{Z}^{L \times (N+L-1)}$ being a convolutional matrix comprising the lth user's modified code vector $\tilde{\mathbf{h}}^{\mathrm{T}}[n]$ and $\mathbf{r}_{nN} \in \mathbb{C}^{N+L-1}$.

3.2. Cost Function

We assume that the user signals $u_l[n]$, $l = 0(1)N-1$, consist of symbols with a constant modulus γ, such as BPSK, QPSK, or PAM. Therefore, by forcing all decoded users $\hat{u}_l[n]$ onto a constant modulus γ, a cost function ξ,

$$\xi = \mathcal{E}\left\{ \sum_{l=0}^{N-1} (\gamma^2 - |\hat{u}_l[n]|^2)^2 \right\} , \qquad (3)$$

can be formulated, whereby $\mathcal{E}\{\cdot\}$ denotes the expectation operator. For sake of simplicity, similarly to [13], the above CM cost function can be denoted as

$$\xi = \mathcal{E}\left\{ \sum_{l=0}^{N-1} |d_l[n] - \hat{u}_l[n]|^2 \right\} \qquad (4)$$

$$\text{with} \qquad d_l[n] = \gamma \frac{\hat{u}_l[n]}{|\hat{u}_l[n]|} . \qquad (5)$$

The latter alternative CM philosophy utilises the decoded symbol normalised to a constant modulus γ as a desired symbol value $d_l[n]$, as illustrated in Fig. 2. This new form results in a cost function which appear to be only of second order in the filter coefficients. While this gives the algorithm a structure similar to the LMS, DD, or MOE algorithms and seems to contradict the need of higher order statistics to enable the functioning of the CM algorithm, the answer lies hidden in the non-linearity of the normalisation operation. This results in a interesting interpretation, whereby Tab. 1 shows the variously employed $d_l[n]$ for each of the above mentioned algorithms.

criterion	MSE	DD	CM	MOE
$d_l[n]$	$u_l[n]$	dec($\hat{u}_l[n]$)	$\gamma\frac{\hat{u}_l[n]}{\lvert\hat{u}_l[n]\rvert}$	0

Table 1. Different criteria by choice of $d_l[n]$.

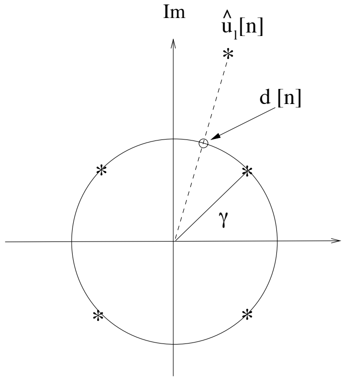

Fig. 2. Configuration of the desired response for the CM criterion.

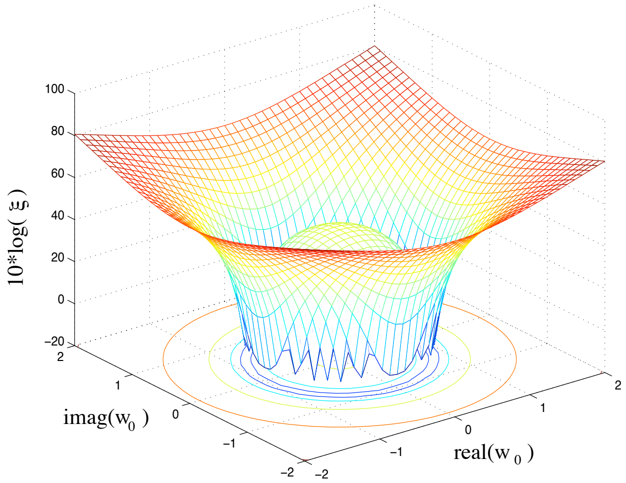

Fig. 3. Cost function ξ in dependency of a single complex valued coefficient w_0.

The optimum equaliser coefficient vector $\mathbf{w}_{\text{opt}}$ is therefore given by

$$\mathbf{w}_{\text{opt}} = \arg\min_{\mathbf{w}} \xi \quad . \tag{6}$$

There is no unique solution to (6), since minimising (5) is ambiguous with a manifold of solutions due to an indeterminism in phase rotation. However, any member of this manifold is a suitable solution for the equaliser $\mathbf{w}$, and can be used in combination with differential modulation schemes to recover $u_l[n]$.

Example. An example for ξ with $N = 4$ users employing QPSK with $\gamma = 1$ over a distortion-less and delay-less noise-free channel is given in Fig. 3. It shows ξ in dependency of an equaliser $\mathbf{w}$ with a single complex coefficient w_0. The cost function shows that there is a manifold of solutions $w_0 e^{j\varphi}$ with arbitrary angle φ.

4. AFFINE PROJECTION ADAPTATION

In this section, we derive the affine projection FIRMER-CMA (AP-FIRMER-CMA) which updates the blind multiuser equaliser weight vector $\mathbf{w}$. In the Pth order AP-FIRMER-CMA, the P most recent data vectors are explicitly taken into account. Firstly, we split (2) for the lth user's decoded symbol $\hat{u}_l[n]$ into a scalar product between the weight vector and an input vector,

$$\hat{u}_l[n] = \mathbf{w}^{\text{H}} \mathbf{x}_l[n] \quad , \tag{7}$$

whereby $\mathbf{x}_l[n]$ represents a vector of filtered received signal samples,

$$\mathbf{x}_l[n] = \mathbf{H}_l[nN]\mathbf{r}_{nN} \quad . \tag{8}$$

Secondly, we retain a record of P such past input vectors $\mathbf{x}_l[n-p]$, and the corresponding desired signal values $d_l[n-p]$, $p = 0 \cdots (P-1)$, which are collocated such that

$$\begin{aligned} \mathbf{X}_l[n] &= [\mathbf{x}_l[n]\ \mathbf{x}_l[n-1]\ \cdots\ \mathbf{x}_l[n-P+1]] \,, & (9)\\ \mathbf{d}_l[n] &= [d_l[n]\ d_l[n-1]\ \cdots\ d_l[n-P+1]]^{\text{T}} \ . & (10) \end{aligned}$$

Based on these records, we define an error vector $\mathbf{e}_l[n]$ at time instance n,

$$\mathbf{e}_l^*[n] = \mathbf{d}_l^*[n] - \mathbf{X}_l[n]^{\text{H}} \mathbf{w}_n, \quad l = 0(1)N-1. \tag{11}$$

Based on this error vector, we want to perform a weight update such that

$$\mathbf{X}_l[n]^{\text{H}} \mathbf{w}_{n+1} = \mathbf{d}_l^*[n], \quad l = 0(1)N-1, \tag{12}$$

is fulfilled. Inserting (11) into (12), we obtain for the lth user

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mathbf{X}_l^\dagger[n]\mathbf{e}_l^*[n] \tag{13}$$

with $\mathbf{X}_l^\dagger[n]$ being the pseudo-inverse of the data matrix $\mathbf{X}_l[n]$,

$$\mathbf{X}_l^\dagger[n] = \mathbf{X}_l[n](\mathbf{X}_l[n]^H \mathbf{X}_l[n] + \alpha \mathbf{I})^{-1}, \tag{14}$$

whereby the inversion is regularised by small diagonal term α.

Finally, by taking the contributions of all N users to the weight update in (13) into account, we arrive at the update rule

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \sum_{l=0}^{N-1} \mathbf{X}_l^\dagger[n]\mathbf{e}_l^*[n] \tag{15}$$

where μ is a relaxation factor. A numerically efficient implementation of the AP-FIRMER-CMA is listed in Tab. 2; this algorithm is virtually identical to the standard APA, but is application-specific in the way the data matrix and desired signal vectors are set up in step 1 according to (7) and (8).

It can be noted that for $P = 1$ we obtain the normalised FIRMER-CMA which updates the equaliser weights by exploiting only the actual filtered input vectors $\mathbf{x}_l[n]$. A similar derivation for the multiuser decision-directed (DD) criterion can be readily obtained by replacing the desired data

Pth order AP-FIRMER-CMA Algorithm	
1:	update $\mathbf{X}_l[n]$ and $\mathbf{d}_l[n]$, for $l = 0(1)N-1$
2:	$\mathbf{e}_l[n] = \mathbf{d}_l[n] - \mathbf{X}_l[n]^{\mathrm{T}}\mathbf{w}_n^*$ for $l = 0(1)N-1$
3:	$\mathbf{X}_l^{\dagger}[n] = \mathbf{X}_l[n](\mathbf{X}_l[n]^H\mathbf{X}_l[n] + \alpha\mathbf{I})^{-1}$
4:	$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu\sum_{l=0}^{N-1}\mathbf{X}_l^{\dagger}[n]\mathbf{e}_l^*[n]$

Table 2. Affine projection algorithm for multiuser equalisation.

$d_l[n]$ described in (5) by $q(\hat{u}_l[n])$, where $q(\cdot)$ is the decision function which returns the symbol in the alphabet that is closest to $\hat{u}_l[n]$.

5. SIMULATION RESULTS

For the simulations below, we transmit $N = 16$ QPSK user signals over a dispersive channel $g[m]$, represented by its transfer function

$$G(z) = 0.93 \ + \ (0.28j + 0.19)z^{-1} \ + \ 0.1z^{-2} \quad . \quad (16)$$

We apply the proposed AP-FIRMER-CMA with a length $L = 10$ for different projection orders $P \in \{1, 2, 3, 5\}$. The adaptation is initialised with the second coefficient in the weight vector set to unity. The systems converge to various solutions such that the convolution of $g[m]$ and the adapted equaliser $\mathbf{w}$ is a complex rotation of delayed Kronecker delta functions $e^{j\phi}\delta(m - \Delta)$. Both phase rotation ϕ and delay Δ are corrected prior to mean square error (MSE) and bit error ratio (BER) measurements.

Experiment 1. In order to demonstrate the convergence behaviour of the proposed algorithm, we apply the previously described systems in the absence of channel noise. The relaxation is set to a fixed value $\mu = 0.03$ for the four systems of various order P. The MSE curves of the four AP-FIRMER-CMAs are shown in Fig. 4. By increasing the projection order P, a faster convergence than the normalised FIRMER-CMA algorithm ($P = 1$) can be achieved. Note that the convergence rate improvement does not come at a detriment of thet steady-state MSE in this noiseless simulation.

Experiment 2. In a second experiment we evaluate the impact of the choice of the relaxation factor μ on the convergence speed of the above systems under the same previously described conditions. Fig. 5 shows the minmum number of symbol rate iterations required to achieve an MSE level of 10^{-4} in dependency of the relaxation μ for different orders P. As we can see, the range and values of μ for which fasted convergence is achieved varies with the projection

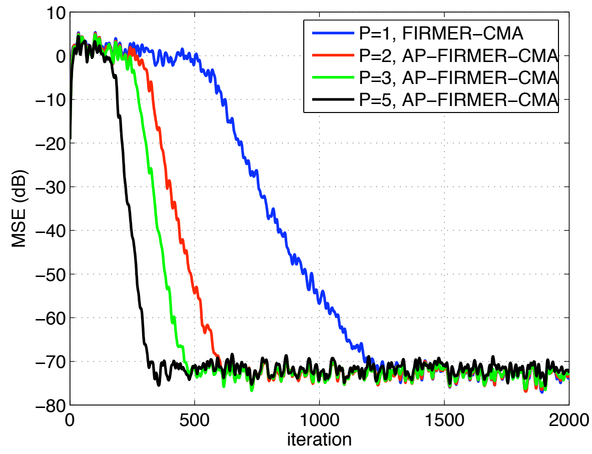

Fig. 4. Comparison of convergence speed for different projection order p values; the curves represent the ensemble MSE averaged over all 16 users.

order P. Moreover, curves with smaller orders P have a slightly lower minima at large values of μ which means, in contrast to the first experiment, slightly faster or similar convergence speed can be achieved by decreasing p and using relatively large value of relaxation factor. However, large values of μ degrades the BER performance of the system.

Experiment 3. The BER results of the above four AP-FIRMER-CMA systems are given in Fig. 6 in comparison to the normalised FIRMER-CMA ($p = 1$) with two higher step sizes $\mu_1 = 0.5$ and $\mu_2 = 0.6$. All systems have been given sufficient time to converge. Note that all four AP-FIRMER-CMAs corresponding to the step size $\mu = 0.03$ and with different orders $P \in \{1, 2, 3, 5\}$ exhibit a very similar BER performance. However, degradation in BER

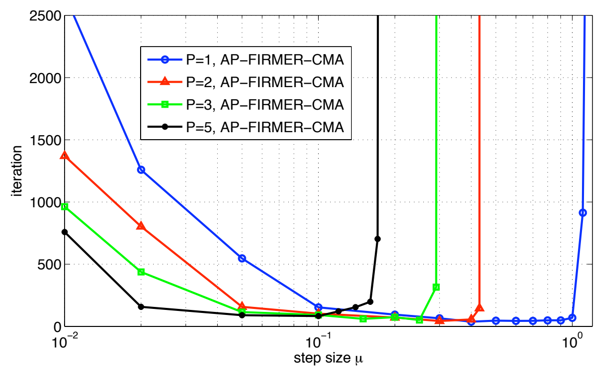

Fig. 5. number of symbol rate iterations required to achieve a MSE level of 0.0001 on dependency of step size of different algorithm's orders p.

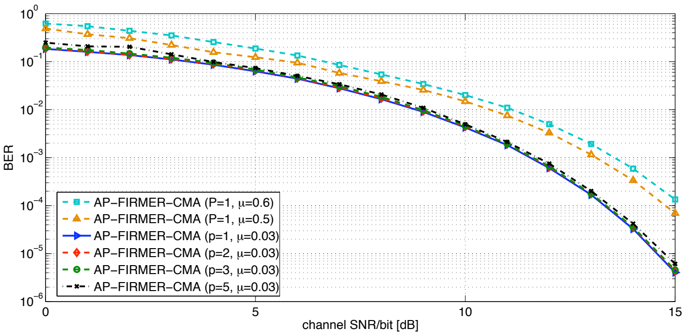

Fig. 6. Comparison of BER for different projection orders p values and various step size μ values.

can be seen by increasing the relaxation factor μ. Therefore, increasing the projection order P of AP-FIRMER-CMA appears to be a viable route to speed up the convergence rate without incurring a loss in BER, although at tha cost of a soemwhat increased computational complexity.

6. CONCLUSIONS

An affine projection algorithm AP-FIRMER-CMA for blind multiuser equalisation, suitable for DS-CDMA downlink scenario, has been derived. This algorithm is very similar to other APA schemes, but differs in its specific application through the code filtering of the data matrix. Properties of the proposed algorithm have been investigated in simulations, and a faster convergence over the normalised FIRMER-CMA approach with with no BER loss has been noted when using higher projection orders. At the expense of a somewhat increased complexity, this blind scheme offers considerably enhanced convergence speed over previous work which makes it an attractive candidate for the DS-CDMA downlink.

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