Iterative Joint Time-Variant Channel Estimation and Multi-User Detection for MC-CDMA

Thomas Zemen, *Member, IEEE*, Christoph F. Mecklenbräuker, *Member, IEEE*, Joachim Wehinger, *Member, IEEE*, and Ralf R. Müller, *Member, IEEE*

***Abstract*— Joint time-variant channel estimation and multi-user detection are key building-blocks for wireless broadband communication for mobile users at vehicular speed. We propose an iterative receiver for a multi-carrier (MC) code division multiple access (CDMA) system in the uplink. Multi-user detection is implemented through iterative parallel interference cancelation and conditional linear minimum mean square error (MMSE) filtering. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM), thus time-variant channel estimation can be performed for every subcarrier individually. The variation of a subcarrier over the duration of a data block is upper bounded by the maximum Doppler bandwidth which is determined by the maximum velocity of the users. We exploit results from the theory of time-concentrated and bandlimited sequences and apply a Slepian basis expansion for time-variant subcarrier estimation. This approach enables time-variant channel estimation without *complete* knowledge of the second-order statistics of the fading process. The square bias of the Slepian basis expansion is one order of magnitude smaller compared to the Fourier basis expansion. The square bias of the basis expansion is the determining factor for the performance of the iterative joint channel estimation and data detection. We present an iterative linear MMSE estimation algorithm for the basis expansion coefficients in a multi-user system. The consistent performance of the iterative receiver using the Slepian basis expansion is validated by simulations for a wide range of velocities.**

***Index Terms*— Discrete prolate spheroidal sequence, MC-CDMA, multi-user detection, OFDM, parallel interference cancelation, Slepian basis expansion, time-variant channel estimation.**

I. INTRODUCTION

WIRELESS broadband communications for mobile users at vehicular speed is the cornerstone of future 4th generation systems. This paper deals with joint iterative time-variant channel estimation and multi-user detection for the uplink of a multi-carrier (MC) code division multiple access system (CDMA).

In current direct sequence (DS) CDMA systems, like UMTS, single user RAKE receivers are used and the interference due to other users is treated as noise. Thus, the system load, defined as the ratio between the number of users K and the spreading sequence length N,

$$\beta = K/N\,, \tag{1}$$

is practically limited to values $\beta < 0.5$.

Higher system loads $\beta > 0.5$ can be achieved by iterative multi-user receivers. In such receivers the soft information gained about the transmitted data symbols after the decoding stage is used to enhance the channel estimation and to reduce the interference for data detection in consecutive iterations. It was shown [1]–[4] that iterative receivers achieve performance close to the single user bound in fully loaded systems $\beta = 1$.

The channel estimation in current UMTS systems is based on the block fading assumption which leads to performance degradation at higher user velocities [5], [6] and for longer block length. For data blocks that are longer than the coherence time of the channel the block fading assumption does not hold. Iterative receivers are more sensitive to time-variant channel conditions, since a certain minimum code block length is needed in order to achieve fast convergence. In the present paper we extend the iterative receiver concept to time-variant channels by introducing a new time-variant channel description based on the Slepian basis expansion [7].

The variation of a wireless channel over the duration of a long coded data block is caused by user mobility and multipath propagation. The Doppler shifts on the individual paths depend on the user's direction and its velocity v, the carrier frequency f_C, and the scattering environment. The maximum variation in time of the wireless channel is upper bounded by the maximum (one sided) normalized Doppler bandwidth

$$\nu_{\mathrm{Dmax}} = \frac{v_{\max} f_C}{c_0} T_{\mathrm{S}}\,, \tag{2}$$

where $v_{\max}$ is the maximum supported velocity, T_{S} is the symbol duration, and c_0 denotes the speed of light.

We apply orthogonal frequency division multiplexing (OFDM) in order to transform the time-variant frequency-selective channel into a set of time-variant frequency-flat channels, the so called subcarriers. We deal with time-variant channels which vary significantly over the duration of a long block of OFDM symbols. Each OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference.

Under the assumption of small inter-carrier interference, each time-variant frequency-flat subcarrier is fully described through a sequence of complex scalars at the OFDM symbol rate $1/T_{\mathrm{S}}$. This sequence is bandlimited by ν_{Dmax}. In order to perform coherent multi-user detection we need to estimate a time limited snapshot of this bandlimited sequence at the

Manuscript received May 12, 2004; revised April 1, 2005; accepted July 5, 2005. The associate editor coordinating the review of this paper and approving it for publication was M. Sawahashi. This work was funded by Kplus in the I0 project of the ftw. Forschungszentrum Telekommunikation Wien. Parts of this work have been presented as invited paper at the Fifth International Conference on 3G Mobile Communication Technologies, London, United Kingdom, 2004.

Thomas Zemen, Christoph F. Mecklenbräuker and Joachim Wehinger are with the Forschungszentrum Telekommunikation Wien (ftw.), Donau-City-Str. 1/3, 1220 Vienna, Austria (e-mail: thomas.zemen@ftw.at).

Ralf R. Müller was with ftw. and is now with the Norwegian University of Science and Technology (NTNU).

Digital Object Identifier 10.1109/TWC.2006.04323.

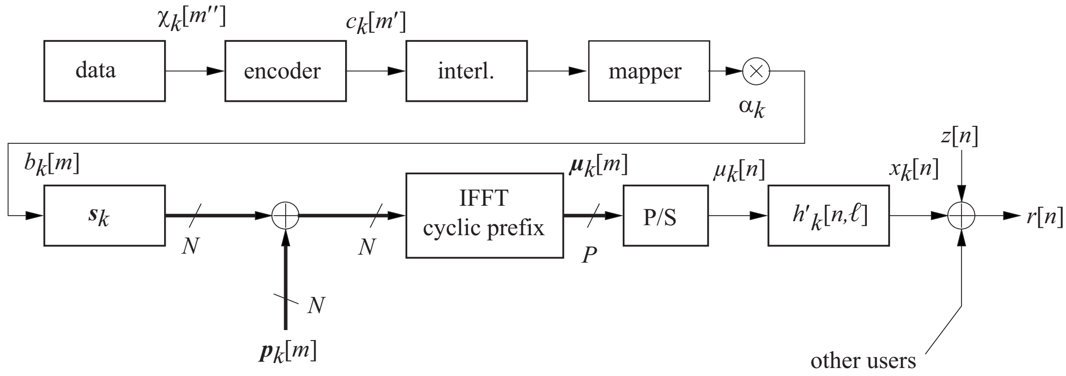

Fig. 1. Model for the MC-CDMA transmitter and the channel in the uplink.

receiver side. The length of these snapshots is equal to the length of a data block consisting of OFDM data symbols with interleaved OFDM pilot symbols.

We take advantage of Slepian's basic result that time-limited parts (snapshots) of band-limited sequences span a low dimensional subspace [8]. The basis functions of this subspace are the discrete prolate spheroidal sequences. Using these results from the theory of time-concentrated and bandlimited sequences we represent a time-variant subcarrier through a Slepian basis expansion of low dimensionality [7], [9]. It was shown in [7], [9] that the square bias of the Slepian basis expansion is more than one order of magnitude smaller compared to the square bias of the Fourier basis expansion [10] (i.e. a truncated discrete Fourier transform). The main reason for this improvement is that the frequency leakage effect of the Fourier transform is avoided. Channel estimation for a single user OFDM system using DPS sequences was also investigated independently in [11].

For iterative time-variant channel estimation we combine the pilot symbols with soft symbols which are supplied by a soft-in soft-out decoder, implemented by the BCJR algorithm [12]. For iterative multi-user detection we apply parallel interference cancelation (PIC), using feed back soft symbols, and individual linear MMSE filtering.

In [13], [14] it is stated that MC-CDMA has several drawbacks which hinders its application in the uplink. This drawbacks are namely the destroyed orthogonality of the spreading codes due to the frequency selective channels and the not adequate channel estimation quality. However, the analysis in [13], [14] was conducted for an receiver based on a single user matched filter. We show in this paper that by applying iterative multi-user detection and iterative time-variant channel estimation MC-CDMA becomes a very interesting candidate for future 4G systems. We show that MC-CDMA can achieve excellent performance for time-variant channels in the uplink.

Contributions:

- We derive an iterative linear MMSE estimator for the Slepian basis expansion coefficients in the uplink of a multi-user MC-CDMA system using feedback soft symbols.
- A lower bound for the mean square channel estimation error per subcarrier is presented and validated by simulations.
- We provide simulation results for a fully loaded MC-CDMA uplink. The bit error rate performance using the Slepian basis expansion and the Fourier basis expansion for time-variant channel estimation are compared and analyzed.

The rest of the paper is organized as follows:

We define the notation and introduce the signal model for the multi-user MC-CDMA uplink in Section II. In Section III, the iterative multi-user detector for time-variant channels is presented. The Slepian basis expansion is explained in Section IV. Based on these results the iterative multi-user channel estimator is derived. In Section V an analytic lower bound for the mean square channel estimation error per subcarrier is provided. Simulation results are given in Section VI and conclusions are drawn in Section VII.

II. Signal Model for Time-Variant Frequency-Selective Channels

A. Notation

In this paper we use the following notation: A column vector is denoted by $\boldsymbol{a}$ and its i-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$ its i,ℓ-th element by $[\boldsymbol{A}]_{i,\ell}$. Its transpose is given by $\boldsymbol{A}^{\mathrm{T}}$, its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$ and its upper left part with dimension $P \times Q$ by $\boldsymbol{A}_{P\times Q}$. A diagonal matrix with elements $a[i]$ is written as $\operatorname{diag}(\boldsymbol{a})$ and the $Q\times Q$ identity matrix as $\boldsymbol{I}_Q$. The absolute value of a is denoted through $|a|$ and its complex conjugate by a^*. The largest (smallest) integer, lower (greater) or equal than $a \in \mathbb{R}$ is denoted by $\lfloor a \rfloor$ ($\lceil a \rceil$).

B. Signal Model

The transmitter for the MC-CDMA uplink is shown schematically in Fig. 1. The transmission is block oriented, a data block consists of $M - J$ OFDM data symbols and J OFDM pilot symbols. Every OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference. Each user transmits symbols $b_k[m]$ with symbol rate $1/T_{\mathrm{S}}$. Discrete time is denoted by m. There are K users in the system, the user index is denoted by k. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k \in \mathbb{C}^N$ with independent identically distributed (i.i.d.) elements chosen from the set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$. The data symbols $b_k[m]$ result from the binary information sequence $\chi_k[m'']$ of length $2(M-J)R_C$

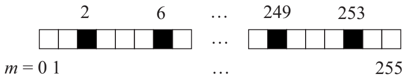

Fig. 2. Pilot pattern $\mathcal{P} = \left\{ \left\lfloor i\frac{M}{J} + \frac{M}{2J} \right\rfloor \mid i \in \{0, \ldots, J-1\} \right\}$ for block length $M = 256$ and $J = 60$ OFDM pilot symbols.

by convolutional encoding with code rate R_C, random bit-interleaving and quadrature phase shift keying (QPSK) modulation with Gray labeling.

The $M - J$ data symbols are distributed over a block of length M fulfilling

$$b_k[m] \in \{\pm 1 \pm \mathrm{j}\}/\sqrt{2} \quad \text{for} \quad m \notin \mathcal{P} \tag{3}$$

and $b_k[m] = 0$ for $m \in \mathcal{P}$ allowing for pilot symbol insertion. The pilot placement is defined through the index set

$$\mathcal{P} = \left\{ \left\lfloor i\frac{M}{J} + \frac{M}{2J} \right\rfloor \mid i \in \{0, \ldots, J-1\} \right\}, \tag{4}$$

see Fig. 2. After spreading, pilot symbols $\boldsymbol{p}_k[m] \in \mathbb{C}^N$ with elements $p_k[m,q]$ are added

$$\boldsymbol{d}_k[m] = \boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\,. \tag{5}$$

The elements of the pilot symbols $p_k[m,q]$ for $m \in \mathcal{P}$ and $q \in \{0, \ldots, N-1\}$ are randomly chosen from the QPSK symbol set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$, otherwise $\boldsymbol{p}_k[m] = \boldsymbol{0}_N$ for $m \notin \mathcal{P}$.

Then, an N point inverse discrete Fourier transform (DFT) is performed and a cyclic prefix of length G is inserted. A single OFDM symbol together with the cyclic prefix is represented by $\boldsymbol{\mu}_k[m] \in \mathbb{C}^P$ and has length $P = N + G$ chips. We write $\boldsymbol{\mu}_k[m] = \boldsymbol{T}_{\mathrm{CP}} \boldsymbol{F}_N^{\mathrm{H}} \boldsymbol{d}_k[m]$.

The cyclic prefix operation is carried out by $\boldsymbol{T}_{\mathrm{CP}} = \left[\boldsymbol{I}_{\mathrm{CP}}^{\mathrm{T}} \boldsymbol{I}_N \right]^{\mathrm{T}} \in \mathbb{R}^{P \times N}$. It replicates the last G chips of each OFDM symbol to the front. $\boldsymbol{I}_{\mathrm{CP}} \in \mathbb{R}^{G \times N}$ denotes the last G rows of the identity matrix $\boldsymbol{I}_N \in \mathbb{R}^{N \times N}$. The unitary DFT matrix $\boldsymbol{F}_N \in \mathbb{C}^{N \times N}$ has elements $[\boldsymbol{F}_N]_{i,\ell} = 1/\sqrt{N}\, \mathrm{e}^{\frac{-\mathrm{j}2\pi i \ell}{N}}$ for $i, \ell = 0, \ldots, N-1$.

After parallel to serial conversion according to $\boldsymbol{\mu}_k[m] = [\mu_k[mP], \ldots, \mu_k[mP + P - 1]]^{\mathrm{T}}$, the chip stream $\mu_k[n]$ with chip rate $1/T_{\mathrm{C}} = P/T_{\mathrm{S}}$ is transmitted over a time-variant multipath fading channel with L resolvable paths. The transmit filter, the time-variant channel and the matched receive filter together are represented by $h_k(t, \tau)$. We denote the time-variant impulse response sampled at the chip-rate by $h'_k[n, \ell] = h_k(nT_{\mathrm{C}}, \ell T_{\mathrm{C}})$.

A time-variant channel impulse response generally introduces inter-carrier interference in an OFDM system. However, if the channel variation in time, measured by the normalized Doppler bandwidth, stays below a certain threshold the inter-carrier interference is small enough to be neglected for the receiver side processing [15]. This condition is fulfilled if the one-sided normalized Doppler bandwidth ν_D is much smaller than the normalized subcarrier bandwidth P/N,

$$\frac{\nu_D N}{P} < \varepsilon\,. \tag{6}$$

For $\epsilon = 10^{-2}$ the DFT is still applicable [16] although the channel is time-variant.

For the processing at the receiver side we are able to treat the time-variant channel as constant for the duration of each single OFDM symbol if (6) is fulfilled. Hence, $h_k[m, \ell] = h'_k[mP, \ell]$, corresponding to

$$\boldsymbol{h}_k[m] = [h_k[m, 0], \ldots, h_k[m, L-1]]^{\mathrm{T}} \in \mathbb{C}^{L \times 1} \tag{7}$$

in vector notation. The time-variant frequency response $\boldsymbol{g}_k[m] \in \mathbb{C}^N$ with elements $g_k[m, q]$ is defined as the DFT of the time-variant impulse response $\boldsymbol{g}_k[m] = \sqrt{N} \boldsymbol{F}_{N \times L} \boldsymbol{h}_k[m]$.

At the receive antenna the signals of all K users add up. The receiver removes the cyclic prefix and performs a DFT. The received signal vector after these two operations is given by

$$\boldsymbol{y}[m] = \sum_{k=1}^{K} \mathrm{diag}\left(\boldsymbol{g}_k[m]\right) \left(\boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\right) + \boldsymbol{z}[m]\,, \tag{8}$$

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2 \boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m] \in \mathbb{C}^N$ with elements $z[m, q]$.

III. Iterative Data Detection

We define the time-variant effective spreading sequences

$$\tilde{\boldsymbol{s}}_k[m] = \mathrm{diag}\left(\boldsymbol{g}_k[m]\right) \boldsymbol{s}_k\,, \tag{9}$$

and the time-variant effective spreading matrix $\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \ldots, \tilde{\boldsymbol{s}}_K[m]] \in \mathbb{C}^{N \times K}$. Using these definitions we write the signal model for data detection as

$$\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m] \boldsymbol{b}[m] + \boldsymbol{z}[m] \quad \text{for} \quad m \notin \mathcal{P} \tag{10}$$

where $\boldsymbol{b}[m] = [b_1[m], \ldots, b_K[m]]^{\mathrm{T}} \in \mathbb{C}^K$ contains the stacked data symbols for K users.

Figure 3 shows the structure of the iterative receiver. The receiver detects the data $\boldsymbol{b}[m]$ using the received symbol vector $\boldsymbol{y}[m]$, the spreading matrix $\tilde{\boldsymbol{S}}^{(i)}[m]$, and the feedback extrinsic probability $\mathrm{EXT}(c_k^{(i)}[m'])$ on the code symbols at iteration i. The time-variant frequency-selective (multipath) nature of the channel implies to build a filter which is matched to the effective time-variant spreading sequence $\tilde{\boldsymbol{s}}_k^{(i)}[m]$. For the moment, it is only of interest that the channel estimator supplies an estimate $\hat{\boldsymbol{g}}_k[m]$ of the time-variant frequency response for every user. The general optimization problem is therefore reduced to the estimation of $\boldsymbol{b}[m]$ only (for the time invariant case see [17], [18]).

A. Time-Variant Parallel Interference Cancelation

In order to cancel the multi-access interference, we perform soft cancelation for user k

$$\tilde{\boldsymbol{y}}_k^{(i)}[m] = \boldsymbol{y}[m] + \tilde{\boldsymbol{s}}_k^{(i)}[m] \tilde{b}_k^{(i)}[m] - \tilde{\boldsymbol{S}}^{(i)}[m] \tilde{\boldsymbol{b}}^{(i)}[m]. \tag{11}$$

Vector $\tilde{\boldsymbol{b}}^{(i)}[m]$ contains the soft bit estimates that are computed from the extrinsic probability (EXT) supplied by the decoding stage (see Section III-C). The mapping for the QPSK alphabet is given by

$$\begin{aligned} \tilde{b}_k[m] &= \mathop{\mathbb{E}}_{b}^{(\mathrm{EXT})}\{b_k[m]\} = \\ &= \frac{1}{\sqrt{2}} \left(\mathop{\mathbb{E}}_{c}^{(\mathrm{EXT})}\{c_k[2m]\} + \mathrm{j} \mathop{\mathbb{E}}_{c}^{(\mathrm{EXT})}\{c_k[2m+1]\} \right) \end{aligned} \tag{12}$$

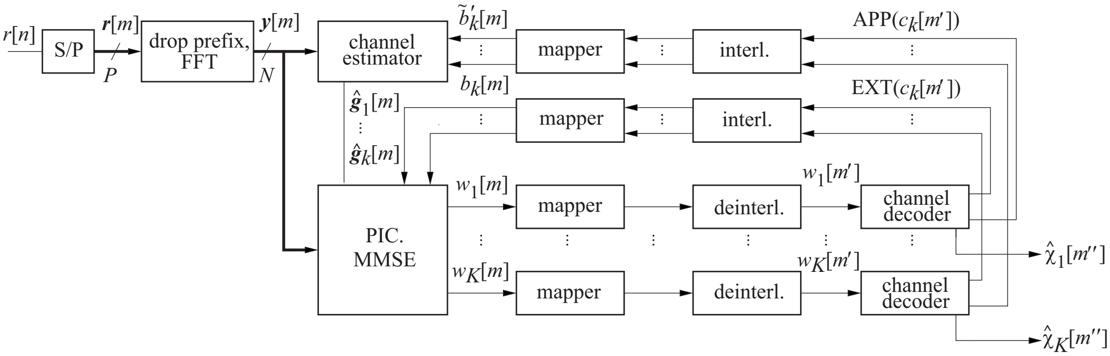

Fig. 3. Model for the MC-CDMA receiver. It performs joint iterative time-variant channel estimation and multi-user detection.

where $\mathbb{E}_c^{(\mathrm{EXT})}\{c_k[m']\} = 2\mathrm{EXT}\{c_k[m'] = +1\} - 1$ calculates the expectation over the alphabet of c which is $\{-1, +1\}$ and $\mathrm{EXT}\{c_k[m'] = +1\}$ is the EXT supplied by the BCJR decoder. The notation in (12) explicitly shows that the expectation is calculated using EXT. Later in Section IV-B we will use soft symbols derived from a-posteriori probability (APP) for iterative channel estimation.

B. Time-Variant Unbiased Conditional Linear MMSE Filter

The output of the interference canceler $\tilde{\boldsymbol{y}}_k^{(i)}[m]$ is further cleaned from noise and multi-access interference with a successive linear MMSE filter in order to obtain a code symbol estimate,

$$w_k^{(i)}[m] = \left(\boldsymbol{f}_k^{(i)}[m]\right)^{\mathrm{H}} \tilde{\boldsymbol{y}}_k^{(i)}[m]\,. \tag{13}$$

A time-variant unbiased conditional linear MMSE filter for the MC-CDMA system can be found similarly to the linear MMSE detector given in [17]. To simplify the notation we omit the iteration index i for the filter. It has the form

$$\boldsymbol{f}_k^{\mathrm{H}}[m] = \frac{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}[m]\left(\sigma_z^2 \boldsymbol{I}_N + \tilde{\boldsymbol{S}}[m]\boldsymbol{V}[m]\tilde{\boldsymbol{S}}^{\mathrm{H}}[m]\right)^{-1}}{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}[m]\left(\sigma_z^2 \boldsymbol{I}_N + \tilde{\boldsymbol{S}}[m]\boldsymbol{V}[m]\tilde{\boldsymbol{S}}^{\mathrm{H}}[m]\right)^{-1}\tilde{\boldsymbol{s}}_k[m]}\,. \tag{14}$$

Matrix $\boldsymbol{V}[m]$ denotes the error covariance matrix $\boldsymbol{V}[m] = \mathbb{E}_b\left\{(\boldsymbol{b}[m] - \tilde{\boldsymbol{b}}[m])(\boldsymbol{b}[m] - \tilde{\boldsymbol{b}}[m])^{\mathrm{H}}\right\}$ with diagonal elements $[\boldsymbol{V}]_{k,k} = 1 - |\tilde{b}_k^{(i)}[m]|^2$, the other elements are assumed to be zero. In this case we calculate the variance for the symbol at time instant m belonging to user k and call the filter conditional since it is conditioned on the feedback soft-symbols [17].

C. Decoder

The iterative receiver feeds back soft values on code bits $c_k[m']$ in order to get better detection results and better channel estimates. The soft feedback values are computed from the APP and the EXT of the code bits through mapping to QPSK symbols (29), (12), see also [18]. A soft-input soft-output decoder for binary convolutional codes, implemented using the BCJR algorithm [12], supplies these measures. The input values to the decoder are the so-called channel values $w'_k[m']$ derived from the MMSE-filter output after demapping and deinterleaving.

The noise variance is estimated as $\hat{\sigma}_{z,k}^2 = \frac{1}{2M}\sum_{m'=0}^{2M-1} |w'_k[m'] - \hat{\mu}_{w',k}|^2$ modeling the residual multi access interference after the linear MMSE filter as additional Gaussian noise. The mean value of the absolute channel values is estimated by $\hat{\mu}_{w',k} = \frac{1}{2M}\sum_{m'=0}^{2M-1} |w'_k[m']|$. The explicit estimation of $\hat{\mu}_{w',k}$ is necessary because during the first iterations the channel estimates are not accurate and thus the linear MMSE filter (14) is not truly unbiased.

The APP for the code symbol being $+1$ if $w'_k[m']$ is observed is given by $\mathrm{APP}\{c_k[m']\} = \Pr\{c_k[m'] = +1 \mid w'_k[m']\}$. The link between APP and EXT is established via $\mathrm{APP}\{c_k[m']\} \propto \mathrm{EXT}\{c_k[m']\}\Pr\{w'_k[m'] \mid c_k[m'] = +1\}$ where the last expression denotes the channel transition function, which is formulated as conditional Gaussian probability density function

$$\Pr\{w'_k[m'] \mid c_k[m'] = +1\} = \\ = \frac{1}{\sqrt{2\pi\hat{\sigma}_{z,k}^2}}\exp\left(-\frac{|w'_k[m'] - \hat{\mu}_{w',k}|^2}{2\hat{\sigma}_{z,k}^2}\right). \tag{15}$$

IV. Iterative Time-Variant Channel Estimation

The performance of the iterative receiver crucially depends on the channel estimates for the time-variant frequency response $\boldsymbol{g}_k[m]$ since the effective spreading sequence (9) directly depends on the actual channel realization. The MC-CDMA transmission takes place over N (essentially) orthogonal frequency-flat time-variant subcarriers. Reflecting this we rewrite (8) as a set of equations for every subcarrier $q \in \{0, \ldots, N-1\}$,

$$y[m,q] = \sum_{k=1}^{K} g_k[m,q]\left(s_k[q]b_k[m] + p_k[m,q]\right) + z[m,q]\,. \tag{16}$$

Please note that we aim at estimating the time-variant channel for a data block consisting of M OFDM symbols, typically $M\nu_{\mathrm{Dmax}} \geq 1$. We split the channel estimation task into two parts:

- First, we find a suitable basis expansion which describes the time-variation of $g_k[m,q]$ for the duration of a data block $m \in \{0,\dots,M-1\}$.
- In a second step the basis expansion coefficients are estimated individually for every subcarrier but jointly for all users. The estimation is performed in an iterative manner using soft feedback symbols.

A. Slepian Basis Expansion

The coefficients of the time-variant impulse response $h_k[m,n]$ are bandlimited by ν_{Dmax} (2). Under assumption (6) the same is true for $g_k[m,q]$. The Doppler spectrum for subcarrier q and user k is defined as

$$G_k(\nu,q) = \sum_{m=-\infty}^{\infty} g_k[m,q]\mathrm{e}^{-\mathrm{j}2\pi\nu m}\,. \tag{17}$$

where $\frac{1}{2} \le \nu < \frac{1}{2}$. The band limitation of $G_k(\nu,q)$ to ν_{Dmax} can be expressed by

$$g_k[m,q] = \int_{-\nu_{\mathrm{Dmax}}}^{\nu_{\mathrm{Dmax}}} G_k(\nu,q)\mathrm{e}^{\mathrm{j}2\pi\nu m}\mathrm{d}\,\nu\,. \tag{18}$$

Limiting the infinite sum in (17) to an interval of length M results in the discrete Fourier transform (DFT). Truncating the DFT leads to the Fourier basis expansion described in [10] for time-variant channel estimation. However, the time windowing with length M causes spectral leakage and the truncation of the DFT gives rise to the Gibbs phenomenon. Both effects together imply that the Fourier basis expansion suffers from high square bias [9].

The theory of time-concentrated and bandlimited sequences developed by Slepian in [8] enables a better suited approach for the time-variant estimation problem, as shown in [7] for a MC-CDMA downlink. Slepian asked which sequence is most concentrated in a given frequency range ν_{Dmax} and simultaneously in a certain time interval of length M. This optimization problem was solved for discrete time in [8].

The sequences bandlimited to ν_{Dmax} and mostly concentrated in an interval of length M are the discrete prolate spheroidal (DPS) sequences. The DPS sequences $u_i[m,\nu_{\mathrm{Dmax}},M]$ are defined as the real solution to

$$\sum_{\ell=0}^{M-1} \frac{\sin(2\pi\nu_{\mathrm{Dmax}}(\ell-m))}{\pi(\ell-m)} u_i[\ell,\nu_{\mathrm{Dmax}},M] = \\ = \lambda_i(\nu_{\mathrm{Dmax}},M)u_i[m,\nu_{\mathrm{Dmax}},M] \tag{19}$$

for $i \in \{0,\dots,M-1\}$ and $m \in \{-\infty,\infty\}$ [8]. We drop the explicit dependence of $u_i[m]$ on ν_{Dmax} and M which we consider fixed system parameters for the remainder of this paper. The DPS sequences are doubly orthogonal on the interval $[-\infty,\infty]$ and $[0,M-1]$. The eigenvalues λ_i are clustered near 1 for $i < \lceil 2\nu_{\mathrm{Dmax}}M\rceil$ and rapidly decay to zero for $i > \lceil 2\nu_{\mathrm{Dmax}}M\rceil$. Therefore, the approximate signal space dimension of time-limited snapshots of a band-limited signal is given by [8, Sec. 3.3]

$$D' = \lceil 2\nu_{\mathrm{Dmax}}M\rceil + 1\,. \tag{20}$$

For our application we are interested at $u_i[m]$ for the interval $[0,M-1]$ only. We introduce the term Slepian sequences for the index limited DPS sequences and define the vector $\boldsymbol{u}_i \in \mathbb{R}^M$ with elements $u_i[m]$ for $m \in \{0,\dots,M-1\}$. The Slepian sequences $\boldsymbol{u}_i$ are eigenvectors of the matrix $\boldsymbol{C} \in \mathbb{R}^{M\times M}$ fulfilling

$$\boldsymbol{C}\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i\,. \tag{21}$$

The eigenvalues λ_i are identical to those in (19) and matrix $\boldsymbol{C}$ is defined as $[\boldsymbol{C}]_{i,\ell} = \frac{\sin[2\pi(i-\ell)\nu_{\mathrm{Dmax}}]}{\pi(i-\ell)}$, where $i,\ell = 0,1,\dots,M-1$.

Summarizing, the Slepian sequences span an orthogonal basis which allows to represent time-limited snapshots of band-limited sequences. We expand the sequence $g_k[m,q]$ in terms of Slepian sequences $u_i[m]$

$$g_k[m,q] \approx \tilde{g}_k[m,q] = \sum_{i=0}^{D-1} u_i[m]\psi_k[i,q]\,, \tag{22}$$

where $m \in \{0,\dots,M-1\}$ and $q \in \{0,\dots,N-1\}$. The dimension D of this basis expansion fulfills $D' \le D \le M-1$. By choosing D we control the mean square error

$$\mathrm{MSE}_M = \frac{1}{M}\sum_{m=0}^{M-1} \mathbb{E}\left\{|g[m]-\tilde{g}[m]|^2\right\}\,, \tag{23}$$

where the indices k and q are omitted.

We emphasize that the selection of a suitable Slepian basis, defined by M and ν_{Dmax}, exploits solely the band-limitation of the Doppler spectrum to ν_{Dmax}. The details of the Doppler spectrum for $|\nu| < \nu_{\mathrm{Dmax}}$ are irrelevant for the design of $u_i[m]$. Our approach therefore differs from a Karhunen-Loève transform [19] which requires *complete* knowledge of the second-order statistics of the fading process.

B. Signal Model for Time-Variant Multi-User Channel Estimation

Substituting the basis expansion (22) for the time-variant subcarrier coefficients $g_k[m,q]$ into the system model (16) we obtain

$$y[m,q] = \sum_{k=1}^{K}\sum_{i=0}^{D-1} u_i[m]\psi_k[i,q]d_k[m,q] + z[m,q]\,, \tag{24}$$

where $d_k[m,q] = s_k[q]b_k[m] + p_k[m,q]$.

Thus, the subcarrier coefficient estimates $\hat{\psi}_k[i,q]$ can be obtained jointly for all K users but individually for every subcarrier q. We define the vector

$$\boldsymbol{\psi}_q = [\psi_1[0,q],\dots,\psi_K[0,q],\dots,\\ \psi_1[D-1,q],\dots,\psi_K[D-1,q]]^{\mathrm{T}} \in \mathbb{C}^{KD} \tag{25}$$

containing the basis expansion coefficients of all K users for subcarrier q. Furthermore, we introduce $\boldsymbol{y}_q = [y[0,q],\dots,y[M-1,q]]^{\mathrm{T}} \in \mathbb{C}^M$ for the received symbol sequence of each single data block on subcarrier q. Using these definitions we write

$$\boldsymbol{y}_q = \boldsymbol{\mathcal{D}}_q\boldsymbol{\psi}_q + \boldsymbol{z}_q\,, \tag{26}$$

where

$$\boldsymbol{\mathcal{D}}_q = [\mathrm{diag}(\boldsymbol{u}_0)\,\boldsymbol{D}_q,\dots,\mathrm{diag}(\boldsymbol{u}_{D-1})\,\boldsymbol{D}_q] \in \mathbb{C}^{M\times KD}\,, \tag{27}$$

and $\boldsymbol{D}_q \in \mathbb{C}^{M\times K}$ contains the transmitted symbols for all K users on subcarrier q

$$\boldsymbol{D}_q = \begin{bmatrix} d_1[0,q] & \dots & d_K[0,q] \\ \vdots & \ddots & \vdots \\ d_1[M-1,q] & \dots & d_K[M-1,q] \end{bmatrix}. \qquad (28)$$

For channel estimation, J pilot symbols in (24) are known. The remaining $M-J$ symbols are not known. We replace them by soft symbols that are calculated from the APP obtained in the previous iteration. This enables us to obtain refined channel estimates if the soft symbols get more reliable from iteration to iteration. For the first iteration the soft symbols $\tilde{b}'_k[m]$ for $m \notin \mathcal{P}$ are set to zero.

We define the soft symbol matrix $\tilde{\boldsymbol{D}}_q \in \mathbb{C}^{M\times K}$ according to (28) by replacing $d_k[m,q]$ with $\tilde{d}_k[m,q] = s_k[q]\tilde{b}'_k[m] + p_k[m,q]$. The soft symbols $\tilde{b}'_k[m]$ are defined according to

$$\begin{aligned}\tilde{b'}_k[m] &= \mathop{\mathbb{E}}_{b}^{(\mathrm{APP})}\{b_k[m]\} = \\ &= \frac{1}{\sqrt{2}}\left(\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[2m]\} + \mathrm{j}\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[2m+1]\}\right) \end{aligned} \qquad (29)$$

where $\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[m']\} = 2\mathrm{APP}\{c_k[m']\} - 1$ and $\mathrm{APP}\{c_k[m']\}$ is the APP supplied by the BCJR decoder.

Finally we define $\tilde{\boldsymbol{\mathcal{D}}}_q \in \mathbb{C}^{M\times KD}$ according to (27) by replacing $\boldsymbol{D}_q$ with $\tilde{\boldsymbol{D}}_q$. Thus, matrix $\tilde{\boldsymbol{\mathcal{D}}}_q$ contains deterministic pilot symbols and statistical information about the transmitted data symbols.

C. Linear MMSE Estimation of Basis Expansion Coefficients

We constrain the time-variant channel estimator to be linear in $\boldsymbol{y}_q$ (see [4] for the block fading case). We will omit the index q in the following derivations to simplify the notation. The linear estimator can be expressed as $\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \boldsymbol{A}\boldsymbol{y}$ where the matrix $\boldsymbol{A}$ satisfies the Wiener-Hopf equation $\boldsymbol{C}_{\boldsymbol{yy}}\boldsymbol{A}^{\mathrm{H}} = \boldsymbol{C}_{\boldsymbol{y\psi}}$. The covariance matrices are given by ($\boldsymbol{y}$, $\boldsymbol{\psi}$ and $\boldsymbol{z}$ are zero-mean and statistically independent)

$$\boldsymbol{C}_{\boldsymbol{yy}} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\mathop{\mathbb{E}}_{\boldsymbol{\psi}}\mathop{\mathbb{E}}_{\boldsymbol{z}}\left\{\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\right\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} + \sigma_z^2\boldsymbol{I}_M \qquad (30)$$

$$\boldsymbol{C}_{\boldsymbol{y\psi}} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\mathop{\mathbb{E}}_{\boldsymbol{\psi}}\mathop{\mathbb{E}}_{\boldsymbol{z}}\left\{\boldsymbol{y}\boldsymbol{\psi}^{\mathrm{H}}\right\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\right\} \triangleq \tilde{\boldsymbol{\mathcal{D}}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}, \qquad (31)$$

where the index below the expectation operator denotes the random variable with respect to which the expectation is taken. The covariance matrix $\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}$ for $\boldsymbol{\psi}$ is given by

$$\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}} = \frac{1}{2\nu_{\mathrm{Dmax}}}\boldsymbol{I}_K \otimes \mathrm{diag}([\lambda_0, \dots, \lambda_{D-1}]) \qquad (32)$$

where $\otimes$ denotes the Kronecker matrix product. We note that the diagonal structure of $\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}$ is exact for a flat Doppler spectrum with Doppler bandwidth ν_{Dmax}. The estimator is then similar to a reduced rank Wiener filter [20]. However, for other Doppler spectra the square bias of the Slepian basis expansion is still very small due to the strongly reduced dimensionality of the time-concentrated and band-limited subspace. We show analytic and numerical results in Section V and VI.

Expectations with respect to $\boldsymbol{b}$ are computed using APPs of data symbols, see (29). The linear MMSE estimator is then:

$$\begin{aligned}\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} &= \boldsymbol{C}_{\boldsymbol{y\psi}}{}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{yy}}{}^{-1}\boldsymbol{y} = \\ &= \boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\left(\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} + \sigma_z^2\boldsymbol{I}_M\right)^{-1}\boldsymbol{y}. \end{aligned} \qquad (33)$$

we note that due to the independence of the users and the data symbols within one block, it holds:

$$\mathop{\mathbb{E}}_{\boldsymbol{b}}\{b_{k'}[m']b_k^*[m]\} = \begin{cases} \tilde{b}_{k'}[m']\tilde{b}_k^*[m], & k' \neq k, m' \neq m \\ 1, & k' = k, m' = m \end{cases} \qquad (34)$$

for $k, k' \in \{1, ..., K\}$ and for $m, m' \in \{0, 1, ..., M-1\}$.

With (34) we are able to write the expectation of the product $\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\}$ as product of expectations plus a correcting diagonal matrix $\boldsymbol{\Lambda}$ which takes (34) into account

$$\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\{\boldsymbol{\mathcal{D}}\}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} + \boldsymbol{\Lambda} = \tilde{\boldsymbol{\mathcal{D}}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} + \boldsymbol{\Lambda}. \qquad (35)$$

The elements of the diagonal matrix $\boldsymbol{\Lambda}$ are defined as:

$$[\boldsymbol{\Lambda}]_{mm} = \frac{1}{N}\sum_{k=1}^{K}\sum_{i=0}^{D-1}\lambda_i u_i^2[m]\mathrm{var}\{b_k[m]\}, \qquad (36)$$

where the symbol variance

$$\mathrm{var}\{b_k[m]\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\left|b_k[m] - \mathop{\mathbb{E}}_{\boldsymbol{b}}\{b_k[m]\}\right|^2\right\} = 1 - |\tilde{b}_k[m]|^2. \qquad (37)$$

Inserting (35) into (33) yields:

$$\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\left(\tilde{\boldsymbol{\mathcal{D}}}\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} + \underbrace{\boldsymbol{\Lambda} + \sigma_z^2\boldsymbol{I}_M}_{\triangleq\boldsymbol{\Delta}}\right)^{-1}\boldsymbol{y}. \qquad (38)$$

For evaluation of this estimator it is necessary to invert an M-dimensional matrix, which is computationally expensive. Therefore, we apply the matrix inversion lemma to (38) and the final expression becomes:

$$\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \left(\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{\mathcal{D}}} + \boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}^{-1}\right)^{-1}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\boldsymbol{y}. \qquad (39)$$

The rows of matrix $\tilde{\boldsymbol{\mathcal{D}}}$ are scaled by the diagonal matrix $\boldsymbol{\Delta}$, taking into account the variances of the noise and of the soft symbol estimates.

After estimating $\hat{\boldsymbol{\psi}}_q$ for all $q \in \{0, \dots, N-1\}$ an estimate for the time-variant frequency response is given by $\hat{g}'_k[m,q] = \sum_{i=0}^{D-1} u_i[m]\hat{\psi}_k[i,q]$. Further noise suppression is achieved if we exploit the correlation between the subcarriers $\hat{\boldsymbol{g}}_k[m] = \boldsymbol{F}_{N\times L}\boldsymbol{F}_{N\times L}^{\mathrm{H}}\hat{\boldsymbol{g}}'_k[m]$. Finally, this allows to perform data detection by inserting the channel estimates $\hat{\boldsymbol{g}}_k[m]$ into (9).

V. Lower Bounds for the Iterative Time-Variant Channel Estimation Error per Subcarrier

In [7] analytic results for the mean square time-variant channel estimation error for a single subcarrier are derived and compared with simulation results for the MC-CDMA downlink. It is shown that the square bias of the Slepian basis expansion is more than one order of magnitude better compared to the Fourier basis expansion. We will use the

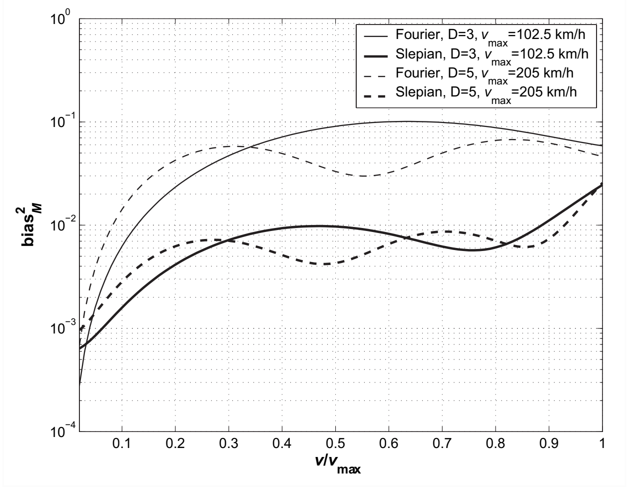

Fig. 4. bias_M^2 for the Slepian and the Fourier basis expansion. We use a Jakes' Doppler spectrum and vary the Doppler bandwidth in the range $0 \leq \nu_{\mathrm{D}} \leq 3.9 \cdot 10^{-3}$ which corresponds to a velocity of $0 \leq v \leq 102.5\,\mathrm{km/h}=\ 28.5\,\mathrm{m/s}$. The basis expansions have dimension $D = 3$ and the Slepian sequences are designed according to $\nu_{\mathrm{Dmax}} = 3.9 \cdot 10^{-3}$ and block length $M = 256$. Additionally we show bias_M^2 for a velocity range of $0 \leq v \leq 205\,\mathrm{km/h}=\ 56.9\,\mathrm{m/s}$ ($\nu_{\mathrm{Dmax}} = 7.8 \cdot 10^{-3}$).

results from [7] in order to obtain a lower bound for the iterative time-variant channel estimation error in the uplink. In the uplink K individual channels must be estimated.

The mean square error (23) of the basis expansion can be described by the sum of two terms

$$\mathrm{MSE}_M = \mathrm{bias}_M^2 + \mathrm{var}_M(\sigma_z^2, K, J, \tilde{b}'), \qquad (40)$$

where bias_M^2 is independent of σ_z^2. A lower bound is given by

$$\mathrm{MSE}_M \geq \mathrm{bias}_M^2 \qquad (41)$$

which gets tight with decreasing σ_z^2 (increasing signal to noise ratio) and with increasing number of iterations. With consecutive iterations the feedback soft symbols $\tilde{b}'$ get more reliable and act as additional pilot symbols.

In order to obtain bias_M^2 we define the instantaneous frequency response of the basis expansion [7]

$$H(m,\nu) = \boldsymbol{f}^{\mathrm{T}}[m]\boldsymbol{G}^{-1}\sum_{\ell=0}^{M-1}\boldsymbol{f}^*[\ell]\mathrm{e}^{-\mathrm{j}2\pi\nu(m-\ell)}, \qquad (42)$$

where $m \in \{0,\ldots,M-1\}$, $|\nu| < 1/2$, and

$$\boldsymbol{f}[m] = [u_0[m],\ldots,u_{D-1}[m]]^{\mathrm{T}} \in \mathbb{C}^D, \qquad (43)$$

contains the instantaneous values of the basis functions. The ideal basis expansion would have $H(m,\nu) = 1$. Matrix $\boldsymbol{G}$ is defined as

$$\boldsymbol{G} = \sum_{\ell\in\mathcal{P}}\boldsymbol{f}[\ell]\boldsymbol{f}^{\mathrm{H}}[\ell], \qquad (44)$$

taking into account the number of pilot symbols J and their placement according to the index set $\mathcal{P}$.

We define the instantaneous error characteristic as [21]

$$E(m,\nu) = |1 - H(m,\nu)|^2\ ,$$

and express the square bias of the basis expansion estimator through

$$\mathrm{bias}^2[m] = \int_{-\frac{1}{2}}^{\frac{1}{2}} E(m,\nu)S(\nu)\mathrm{d}\,\nu\,. \qquad (45)$$

For the purpose of performance *analysis* for a *nominal* ensemble of channel realizations, we specify a Doppler power spectrum $S(\nu)$. Please note that $S(\nu)$ is *not* used for the derivation of the basis functions $u_i[m]$.

The square bias for a block of length M is denoted by

$$\mathrm{bias}_M^2 = \frac{1}{M}\sum_{m=0}^{M-1}\mathrm{bias}^2[m] = \int_{-\frac{1}{2}}^{\frac{1}{2}} E_M(\nu)S(\nu)\mathrm{d}\,\nu\,, \qquad (46)$$

where the mean error characteristic is defined as

$$E_M(\nu) = \frac{1}{M}\sum_{m=0}^{M-1}E(m,\nu)\,. \qquad (47)$$

In order to highlight the constant performance gains of the Slepian basis expansion over the Fourier basis expansion we plot bias_M^2 for two different scenarios in Fig. 4. We plot bias_M^2 for the Jakes' Doppler spectrum

$$S_{gg}(\nu) = \frac{1}{\pi\nu_{\mathrm{D}}\sqrt{1-\left(\frac{\nu}{\nu_{\mathrm{D}}}\right)^2}} \quad \text{for } |\nu| \leq \nu_{\mathrm{D}}\,, \qquad (48)$$

and $S_{gg}(\nu) = 0$ for $|\nu| > \nu_{\mathrm{D}}$

We consider a transmission with carrier frequency $f_{\mathrm{C}} = 2\,\mathrm{GHz}$, each OFDM symbol has $N = 64$ subcarriers and the cyclic prefix has length $G = 15$. The chip rate is $1/T_{\mathrm{C}} = 3.84 \cdot 10^6\,\mathrm{s}^{-1}$ resulting in an OFDM symbol rate of $1/T_{\mathrm{S}} = 48.6 \cdot 10^3$ 1/s. In the first scenario the maximum speed of the user is $v_{\max} = 102.5\,\mathrm{km/h}=\ 28.5\,\mathrm{m/s}$ which results in the maximum normalized Doppler frequency $\nu_{\mathrm{Dmax}} = 3.9 \cdot 10^{-3}$. The data block has length of $M = 256$ symbols. With these system parameters the approximate dimension of the signal space becomes $D' = \lceil 2\nu_{\mathrm{Dmax}}M\rceil + 1 = 3$. We choose the number of basis functions $D = D'$.

In the second scenario the maximum speed of the user is $v_{\max} = 205\,\mathrm{km/h}=\ 56.9\,\mathrm{m/s}$ which results in $\nu_{\mathrm{Dmax}} = 7.8 \cdot 10^{-3}$ and $D = D' = 5$. Figure 4 shows that the performance advantage of the Slepian basis expansion is not dependent on the velocity range.

In order to obtain the results for the Fourier basis expansion the Slepian sequences need to be replaced by

$$u_i^{(\mathrm{F})}[m] = \mathrm{e}^{\frac{\mathrm{j}2\pi(i-(D-1)/2)m}{M}} \qquad (49)$$

and

$$\boldsymbol{\mathcal{C}}_{\boldsymbol{\psi}}^{(\mathrm{F})} = \frac{M}{D}\boldsymbol{I}_{KD}\,. \qquad (50)$$

VI. Simulation Results

The realizations of the time-variant frequency-selective channel $h_k'[n,\ell]$, sampled at the chip rate $1/T_{\mathrm{C}}$, are generated using an exponentially decaying power delay profile

$$\eta^2[\ell] = \mathrm{e}^{-\frac{\ell}{4}} / \sum_{\ell'=0}^{L-1}\mathrm{e}^{-\frac{\ell'}{4}} \qquad (51)$$

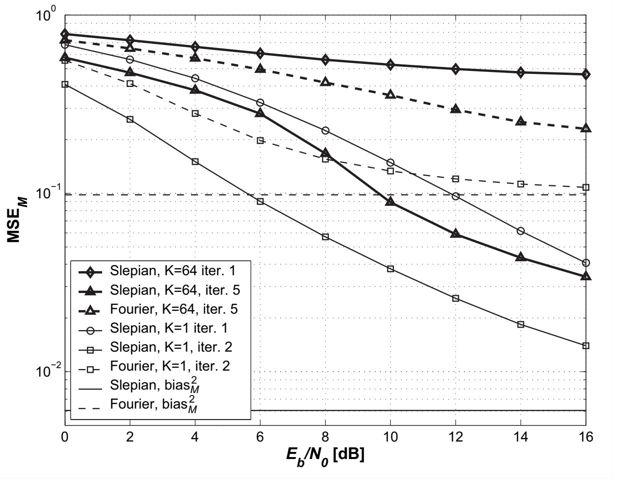

Fig. 5. Mean square channel estimation error per subcarrier MSE_M versus E_b/N_0 for the Slepian basis expansion and the Fourier basis expansions. We show MSE_M for $K=64$ users after the first and fifth iteration and for $K=1$ user after the first and second iteration. The users move with $v=70\,\mathrm{km/h}=19.4\,\mathrm{m/s}$ ($v_\mathrm{max}=102.5\,\mathrm{km/h}$, $v/v_\mathrm{max}=0.68$). For increasing E_b/N_0 and increasing number of iterations MSE_M approaches bias^2_M.

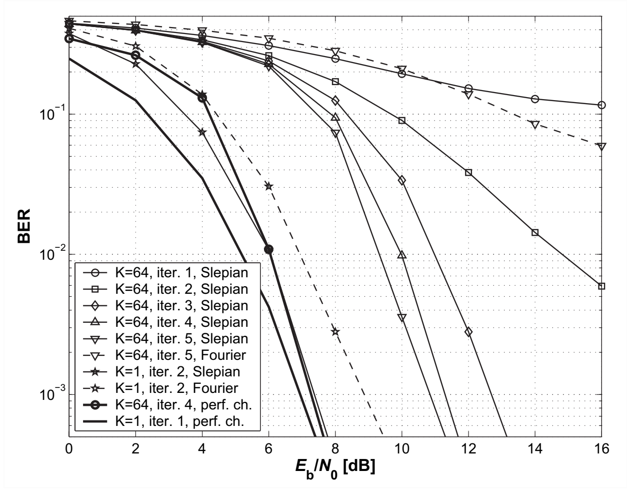

Fig. 6. MC-CDMA uplink performance in terms of BER versus SNR for iteration 1 to 5. The performance for the Slepian and the Fourier basis expansion is compared. Both use $D=3$ basis functions. The $K=64$ users move with $v=70\,\mathrm{km/h}$, the upper speed limit is $v_\mathrm{max}=102{,}5\,\mathrm{km/h}$. Additionally we provide the single user bound and the performance for $K=64$ users with perfect channel knowledge (perf. ch.).

with $L=15$ resolvable paths, $\ell=0,\ldots,L-1$ [22]. The discrete time indices n and ℓ denote sampling at rate $1/T_\mathrm{C}$. The power-delay profile corresponds to a root mean square delay spread $T_\mathrm{D}=4T_\mathrm{C}=1\mu\mathrm{s}$ for a chip rate of $1/T_\mathrm{C}=3.84\cdot10^6\,\mathrm{s}^{-1}$. The autocorrelation for every channel tap is given by $R_{h'h'}[n,\ell]=\eta^2[\ell]J_0(2\pi\nu_D Pn)$ which results in the classical Jakes' spectrum. We simulate the Jakes' spectrum using a model presented in [23] and add some enhancements to achieve correct Rayleigh fading statistics for the full range of velocity $0\le v\le v_\mathrm{max}$ [7], [24]. We emphasize that the simulation uses a time-variant channel sampled at the chip rate. Any possible effect from residual inter-carrier interference would be visible in the simulation results.

We simulate an isolated-cell scenario since no scrambling code is applied. We assume chip synchronization for the simulations and no frequency offset between the individual users. In practical systems the cyclic prefix can be enlarged $G>L$ in order to relax the time synchronization. A residual frequency offsets smaller than ν_Dmax can be handled by the time-variant channel estimation. However, time and frequency synchronization are not the scope of this paper.

The system operates at carrier frequency $f_C{=}2\,\mathrm{GHz}$. The number of subcarriers $N=64$ and the OFDM symbol with cyclic prefix has length of $P=G+N=79$. The data block consists of $M=256$ OFDM symbols. The system is fully loaded, there are $K{=}N{=}64$ users in the system, $\beta{=}K/N{=}1$.

For data transmission, a convolutional, non-systematic, non-recursive, 4 state, rate $R_C{=}1/2$ code with generator polynomial $(5,7)_8$ is used. The illustrated results are obtained by averaging over 100 independent channel realizations. The QPSK symbol energy is normalized to 1 and we defined $E_b/N_0=\frac{1}{2R_C\sigma_z^2}\frac{P}{N}\frac{M}{M-J}$ taking into account the rate loss due to coding, pilots and cyclic prefix.

We analyze two scenarios:

- The first scenario assumes $v_\mathrm{max}=102.5\,\mathrm{km/h}=28.5\,\mathrm{m/s}$ which results in $D{=}D'{=}3$ for the Slepian basis expansion. This is a typical maximum velocity for urban environments. The maximum Doppler bandwidth $B_\mathrm{Dmax}=190\,\mathrm{Hz}$ and $\nu_\mathrm{Dmax}=3.9\cdot10^{-3}$. $J=60$ OFDM pilot symbols are used per OFDM data block.
- In the second scenario $v_\mathrm{max}{=}205\,\mathrm{km/h}{=}56.9\,\mathrm{m/s}$ which results in $\nu_\mathrm{Dmax}{=}7.8\cdot10^{-3}$ and $D{=}D'{=}5$. This is a typical maximum velocity for rural environments. Due to the enlarged velocity range the subspace dimension increases. Thus, more parameters need to be estimated per subcarrier compared to the first scenario. $J=100$ OFDM pilot symbols are used per OFDM data block.

A. Channel Estimation Error

In Fig. 5 we show the MSE_M per subcarrier versus E_b/N_0. The lower bound for the MSE_M is given by bias^2_M and plotted for comparison. bias^2_M for the Slepian basis expansion is one order of magnitude smaller compared to the Fourier basis expansion. We compare the MSE_M after the first and the fifth iteration. The channel estimation error decreases with consecutive iterations due to the feedback soft symbols which serve as additional pilots. The feedback soft symbols become more reliable from iteration to iteration. The described behavior is documented in Fig. 5 for the single user case $K=1$ as well as under full load $K=N=64$.

With increasing number of users K the channel estimation performance degrades. This is due to the fact, that in the uplink the number of parameters that needs to be estimated increases linearly with the number of users while the block length M stays constant.

Comparing results for both the Slepian and Fourier basis expansion we see that the MSE_M for the Slepian basis expansion is reduced by one order of magnitude at $E_\mathrm{b}/N_0=16\,\mathrm{dB}$.

The simulations in Fig. 5 use a basis functions design for scenario one ($0\le v\le102.5\,\mathrm{km/h}=28.5\,\mathrm{m/s}$). The users move at velocity $v=70\mathrm{km/h}$.

B. Bit Error Rate

In Fig. 6 we illustrate the MC-CDMA uplink performance with iterative time-variant channel estimation in terms of bit error rate (BER) versus E_b/N_0. Scenario one is assumed. The users move with $v = 70$km/h.

The results for the Slepian basis functions and $K = 64$ users show convergence to smaller bit error rates from iteration 1 to iteration 5. For comparison the results for the Fourier basis expansion with $K = 64$ users after 5 iterations is shown too. The distance of more than 8 dB to the Slepian basis expansion is due to the higher square bias of the Fourier basis expansion (cf. Fig. 4).

Fig. 6 also shows the single user bound (SUB) which is defined as the performance for one user $K = 1$ and a perfectly known channel $\boldsymbol{g}_k[m]$. Additionally, we plot the performance for $K = 64$ users and perfect channel knowledge. It can be seen that for perfect channel knowledge the single user bound is reached at the fourth iteration.

The 1 dB penalty between the SUB and the results for $K = 1$ user including channel estimation (after 2 iterations) is mainly due to the square bias of the basis expansion for $D = 3$. The 4 dB penalty between the SUB and the results for $K = 64$ users including channel estimation (after 5 iterations) is due to the increased variance var_M in (40) (see also [7]). For every subcarrier, KD coefficients must be estimated. Under full load $KD = 192$ comes near to the overall number of symbols in a single data block $M = 256$. Thus, the variance of the channel estimates increases as the number of users approaches full load since the number of parameters increases with K while the block length M stays constant.

In Fig. 7 we show simulation results for $K = 64$ users after iteration 5. For the first scenario ($v_{\max} = 102.5$ km/h= 28.5 m/s) we show examples for $v \in \{0, 70, 100\}$ km/h. The advantage of the Slepian basis expansion over the Fourier basis expansion can be explained by the lower bias_M^2 of the Slepian basis expansion as shown in Fig. 4. For $v = 0$ km/h both basis expansion show the same square bias and thus also the bit error rate performance in Fig. 7 is comparable. For $v = 70$ km/h= 19.4 m/s the square bias distance between the two basis expansion is largest as is also the difference in the bit error rate performance. For $v = 100$ km/h= 27.8 m/s the square bias of the Slepian basis expansion is highest, thus the bit error rate performance difference to the Fourier basis expansion is reduced compared to $v = 70$ km/h= 19.4 m/s.

For the second scenario with $v_{\max} = 205$ km/h= 56.9 m/s we show an example for $v = 150$ km/h where again the performance gain of the Slepian basis expansion is documented. Due to the larger number of basis functions $D = 5$ the performance is reduced over scenario one, where only $D = 3$ parameters need to be estimated per subcarrier and user.

VII. Conclusion

We presented an iterative multi-user receiver for the uplink of a MC-CDMA system. By exploiting basic results from the theory of time-concentrated and bandlimited sequences we were able to obtain a Slepian basis expansion for iterative time-variant channel estimation. The selection of a suitable Slepian basis, defined by M and ν_{Dmax}, solely exploits the band-limitation of the Doppler spectrum to ν_{Dmax}. The details of the Doppler spectrum for $|\nu| < \nu_{\mathrm{Dmax}}$ are irrelevant. This approach enables iterative time-variant channel estimation *with almost no* knowledge of the second-order statistics of the fading process. We established an analytic lower bound for the MSE per subcarrier that enable easy comparisons between the Slepian and the Fourier basis functions. We presented an iterative linear MMSE estimation algorithm for the basis expansion coefficients using feedback soft symbols.

We showed that the distance in square bias of the Slepian and the Fourier basis expansion is the determining factor for the iterative receiver performance in terms of bit error rate versus signal to noise ratio. The consistent performance gain of the Slepian basis expansion over the Fourier basis expansion was shown by simulations for a wide range of velocities.

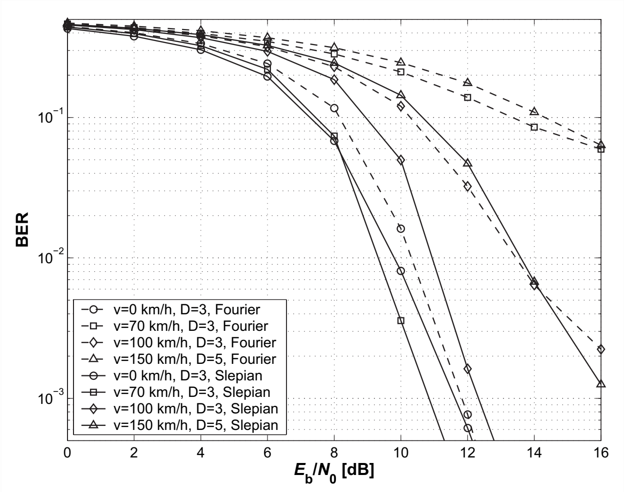

Fig. 7. MC-CDMA uplink performance in terms of BER versus SNR after 5 iterations. We compare the performance of the Slepian and the Fourier basis expansion for $K = 64$ users. We use $D = 3$ basis functions for $v \in \{0, 70, 100\}$km/h and $D = 5$ basis functions for $v = 150$ km/h.

References

[1] K. Okawa, K. Higuchi, and M. Sawahashi, "Parallel-type coherent multi-stage interference canceller with iterative channel estimation using both pilot and decision-feedback data symbols for W–CDMA mobile radio," in *11th IEEE International Symposiom on Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 1, 15-18 May 2000, pp. 709–714.

[2] A. Lampe and J. Huber, "Iterative interference cancellation for DS-CDMA systems with high system loads using reliability-dependent feedback," *IEEE Trans. Veh. Technol.*, vol. 51, no. 3, pp. 445–452, May 2002.

[3] V. Kühn, "Iterative interference cancellation and channel estimation for coded OFDM-CDMA," in *IEEE International Conference on Communications (ICC), Anchorage (AK), USA*, vol. 4, May 2003, pp. 2465–2469.

[4] T. Zemen, M. Lončar, J. Wehinger, C. F. Mecklenbräuker, and R. R. Müller, "Improved channel estimation for iterative receivers," in *IEEE Global Communications Conference (GLOBECOM)*, vol. 1, San Francisco (CA), USA, Dec. 2003, pp. 257–261.

[5] H. Andoh, M. Sawahashi, and F. Adachi, "Channel estimation using time multiplexed pilot symbols for coherent rake combining for DS-CDMA mobile radio," in *8th International Symposion on Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 3, 1-4 Sept. 1997, pp. 954–958.

[6] B. Sklar, "Rayleigh fading channels in mobile digital communication systems part II: Mitigation," *IEEE Commun. Mag.*, pp. 148–155, Sept. 1997.

[7] T. Zemen and C. F. Mecklenbräuker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. Signal Processing*, vol. 53, no. 9, September 2005.

[8] D. Slepian, "Prolate spheroidal wave functions, Fourier analysis, and uncertainty - V: The discrete case," *The Bell System Technical Journal*, vol. 57, no. 5, pp. 1371–1430, May-June 1978.

[9] T. Zemen and C. F. Mecklenbräuker, "Time-variant channel equalization via discrete prolate spheroidal sequences," in *37th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove (CA), USA, Nov. 2003, pp. 1288–1292, invited.

[10] A. M. Sayeed, A. Sendonaris, and B. Aazhang, "Multiuser detection in fast-fading multipath environment," *IEEE J. Select. Areas Commun.*, vol. 16, no. 9, pp. 1691–1701, Dec. 1998.

[11] J.-P. Javaudin, D. Lacroix, and A. Rouxel, "Pilot-aided channel estimation for OFDM/OQAM," in *Vehicular Technology Conference VTC 2003-Spring*, vol. 3, Apr. 2003, pp. 1581–1585.

[12] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, no. 2, pp. 284–287, Mar. 1974.

[13] H. Atarashi, S. Abeta, and M. Sawahashi, "Broadband packet wireless access appropriate for high-speed and high-capacity throughput," in *IEEE Vehicular Technology Conference (VTC)*, vol. 1, May 2001, pp. 566–570.

[14] S. Suwa, H. Atarashi, and M. Sawahashi, "Performance comparison between MC/DS-CDMA and MC-CDMA for reverse link broadband packet wireless access," in *56th IEEE Vehicular Technology Conference (VTC)*, vol. 4, 24-28 Sept. 2002, pp. 2076–2080.

[15] Y. G. Li and L. J. Cimini, "Bounds on the interchannel interference of OFDM in time-varying impairments," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 401–404, Mar. 2001.

[16] G. Matz and F. Hlawatsch, "Time-frequency transfer function calculus (symbolic calculus) of linear time-varying systems (linear operators) based on generalized underspread theory," *J. Math. Phys.*, vol. 39, pp. 4041–4071, Aug. 1998.

[17] R. R. Müller and G. Caire, "The optimal received power distribution for IC-based iterative multiuser joint decoders," in *39th Annual Allerton Conference on Communication, Control and Computing*, Monticello (IL), USA, Aug. 2001.

[18] J. Wehinger, R. R. Müller, M. Lončar, and C. F. Mecklenbräuker, "Performance of iterative CDMA receivers with channel estimation in multipath environments," in *36th Asilomar Conference on Signals, Systems and Computers*, vol. 2, Pacific Grove (CA), USA, 2002, pp. 1439 – 1443.

[19] M. Siala, "Maximum a posteriori semi-blind channel estimation for OFDM systems operating on highly frequency selective channels," *Annals of telecommunications*, vol. 57, no. 9/10, pp. 873–924, Sept./Oct. 2002.

[20] F. A. Dietrich and W. Utschik, "Pilot-assisted channel estimation based on second-order statistics," *IEEE Trans. Signal Processing*, vol. 53, no. 3, pp. 1178–1193, Mar. 2005.

[21] M. Niedzwiecki, *Identification of Time-Varying Processes*. John Wiley & Sons, 2000.

[22] L. M. Correia, *Wireless Flexible Personalised Communications*. Wiley, 2001.

[23] Y. R. Zheng and C. Xiao, "Simulation models with correct statistical properties for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 51, no. 6, pp. 920–928, June 2003.

[24] T. Zemen and C. F. Mecklenbräuker, "Doppler diversity in MC-CDMA using the Slepian basis expansion model," in *12th European Signal Processing Conference (EUSIPCO)*, Vienna, Austria, Sept. 2004.

Thomas Zemen (S'03–M'05) was born in Mödling, Austria, in 1970. He received the Dipl.-Ing. degree in electrical engineering and the Dr.techn. degree with distinction from Vienna University of Technology in 1998 and 2004, respectively. He joined Siemens Austria in 1998 where he worked as hardware engineer and project manager for the radio communication devices department. From October 2001 to September 2003 Mr. Zemen was delegated by Siemens Austria as a researcher to the mobile communications group at the Telecommunications Research Center Vienna (ftw.). Since October 2003 Thomas Zemen has been with the telecommunications Research Center Vienna, working as researcher in the strategic I0 project. His research interests include orthogonal frequency division multiplexing (OFDM), multi-user detection, time-variant channel estimation, iterative receiver structures and software defined radio concepts. Since May 2005 Thomas Zemen leads the project "Future Mobile Communications Systems - Mathematical Modeling, Analysis, and Algorithms for Multi Antenna Systems" which is funded by the Vienna Science and Technology Fund (Wiener Wissenschafts-, Forschungs- und Technologiefonds, WWTF). Dr. Zemen teaches "MIMO Communications" as an external lecturer at the Vienna University of Technology.

Christoph Mecklenbräuker was born in Darmstadt, Germany, in 1967. He received the Dipl-Ing. degree in Electrical Engineering from Vienna University of Technology in 1992 and the Dr.-Ing. degree from Ruhr-University of Bochum in 1998, respectively. His doctoral thesis on matched field processing was awarded with the Gert Massenberg Prize. He worked for the Mobile Networks Radio department of Siemens AG Austria where he participated in the European framework of ACTS 90 FRAMES. He was a delegate to the Third Generation Partnership Project (3GPP) and engaged in the standardisation of the radio access network for UMTS. Since 2000, he holds a senior research position at the Telecommunications Research Center Vienna (ftw.) in the field of mobile communications. His current research interests include antenna array- and MIMO-signal processing for mobile communications and ultra-wideband radio.

Joachim Wehinger (S00, M06) was born in Hohenems, Austria, in 1975. He studied electrical engineering at Technische Universitt Wien, Austria, and Chalmers Tekniska Hgskola, Gteborg, Sweden. He received the Dipl.-Ing., MSc, and PhD degrees in 2001, 2002, and 2005, all with distinction. Mr. Wehinger has been with the Forschungszentrum Telekommunikation Wien (ftw.) since September 2001 where he is currently employed as senior researcher. Since December 2005 he has been leading the application-oriented project Wireless Evolution Beyond 3G. Mr. Wehinger has (co-)authored more than 20 papers in international journals and conferences. He serves as reviewer for IEEE Trans. on Signal Processing, IEEE Trans. on Wireless Communications, and EURASIP Journal on Signal Processing. His interests are iterative receivers, channel estimation, MIMO communications, and multiuser detection.

Ralf R. Müller (S'96–M'03–SM'05) was born in Schwabach, Germany, 1970. He received the Dipl.-Ing. and Dr.-Ing. degree with distinction from University of Erlangen-Nuremberg in 1996 and 1999, respectively. From 2000 to 2004, he was with Forschungszentrum Telekommunikation Wien (Vienna Telecommunications Research Center) in Vienna, Austria. Since 2005 he has been a full professor at the Department of Electronics and Telecommunications at the Norwegian University of Science and Technology (NTNU) in Trondheim, Norway. He held visiting appointments at Princeton University, U.S.A., Institute Eurecom, France, and the University of Melbourne, Australia and was an external lecturer at Vienna University of Technology. Dr. Müller received the Leonard G. Abraham Prize (jointly with Sergio Verdú) for the paper "Design and Analysis of Low-Complexity Interference Mitigation on Vector Channels" from the IEEE Communications Society. He was presented awards for his dissertation "Power and Bandwidth Efficiency of Multiuser Systems with Random Spreading" by the Mannesmann Foundation for Mobile Communications and the German Information Technology Society (ITG). He also received the ITG award for the paper "A Random Matrix Model for Communication Via Antenna Arrays" as well as the Johann-Philipp-Reis Award. Dr. Müller is currently serving as an associate editor for the IEEE Transactions on Information Theory.