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TIME-VARYING COMMUNICATION CHANNELS: FUNDAMENTALS, RECENT DEVELOPMENTS, AND OPEN PROBLEMS

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ABSTRACT

In many modern wireless communication systems, the assumption of a locally time-invariant (block-fading) channel breaks down due to increased user mobility, data rates, and carrier frequencies. Fast time-varying channels feature significant Doppler spread in addition to delay spread. In this tutorial paper, we review some characterizations and sparse models of time-varying channels. We then discuss several models and methods recently proposed for communications over time-varying wireless channels, and we point out some related open problems and potential research directions.

1. INTRODUCTION

Wireless communications with mobility of transmitter, receiver, and/or reflecting (refracting, diffracting) objects have become ubiquitous. This is why *linear time-varying (LTV) channels* have recently attracted considerable interest in the signal processing, communications, propagation, and information theory communities. LTV channels are furthermore important in underwater acoustic communications. They are also called *time and frequency (TF) dispersive* or *doubly dispersive*, as well as *TF selective* or *doubly selective*.

Here, we discuss some fundamentals, recent developments, and open problems regarding LTV wireless (mobile radio) channels. In Section 2, we review basic deterministic and stochastic characterizations of LTV channels. Some low-dimensional (sparse) parametric representations are discussed in Section 3. Section 4 presents a survey of several models and methods recently proposed for communications over LTV channels. In addition, some open problems are outlined and suggestions for future research are given.

2. FUNDAMENTALS

First, we review some basic characterizations of LTV channels. We consider the complex baseband representation of a wireless system operating at carrier frequency f_c.

2.1 Deterministic Channel Characterization

Delay-Doppler Domain. An intuitive and physically meaningful characterization of LTV channels is in terms of delays and Dopper shifts. Delays are due to multipath propagation and time dispersion, while Doppler shifts are caused by mobility as well as carrier frequency offsets and oscillator drift. Let us first assume an LTV channel $\mathbf{H}$ with P propagation paths and receiver movement only. The channel output signal (receive signal) $r(t) = (\mathbf{H}s)(t)$ for a channel input signal (transmit signal) $s(t)$ is here given by

$$r(t) = \sum_{p=1}^{P} a_p\, s(t-\tau_p)\, e^{j2\pi\nu_p t}, \tag{1}$$

where a_p, τ_p, and ν_p denote, respectively, the complex attenuation, delay, and Doppler frequency associated with the pth path. We have $\tau_p = d_p/c$ and $\nu_p = v\cos(\phi_p)\, f_c/c$, where d_p is the distance travelled, ϕ_p is the angle of arrival, v is the receiver velocity, and c is the wave propagation speed.

Eq. (1) models the effect of P discrete specular scatterers (ideal point scatterers). This expression can be generalized to a continuum of scatterers as (e.g., [1–3])

$$r(t) = \int_\tau \int_\nu S_{\mathbf{H}}(\tau,\nu)\, s(t-\tau)\, e^{j2\pi\nu t}\, d\tau\, d\nu\,, \tag{2}$$

where integration is from $-\infty$ to ∞. The weight function $S_{\mathbf{H}}(\tau,\nu)$ is termed the *(delay-Doppler) spreading function*. It characterizes the attenuation and scatterer reflectivity associated with paths of delay τ and Doppler ν. Thus, the spreading function describes the channel's *TF dispersion characteristics*. We emphasize that (2) can be used to characterize *any* LTV channel. In fact, it is equivalent to the generic time-delay domain relation

$$r(t) = \int_\tau h(t,\tau)\, s(t-\tau)\, d\tau\,, \tag{3}$$

in which $h(t,\tau) = \int_\nu S_{\mathbf{H}}(\tau,\nu)\, e^{j2\pi\nu t}\, d\nu$ is the impulse response of the LTV channel $\mathbf{H}$.

TF Domain. Time dispersiveness corresponds to frequency selectivity and frequency dispersiveness corresponds to time selectivity. The joint TF selectivity of an LTV channel is characterized by the *TF transfer function* [1, 4]

$$L_{\mathbf{H}}(t,f) = \int_\tau \int_\nu S_{\mathbf{H}}(\tau,\nu)\, e^{j2\pi(t\nu - f\tau)}\, d\tau\, d\nu\,. \tag{4}$$

This extends the relation $H(f) = \int_\tau h(\tau)\, e^{-j2\pi f\tau} d\tau$ valid for a time-invariant channel to the time-varying case. According to (4), the TF echoes described by $S_{\mathbf{H}}(\tau,\nu)$ correspond to TF fluctuations of $L_{\mathbf{H}}(t,f)$, which are known as (small scale) *fading*. For *underspread* channels (to be defined presently), the TF transfer function $L_{\mathbf{H}}(t,f)$ can be interpreted as the channel's complex gain at time t and frequency f.

2.2 Statistical Channel Characterization

Often, statistical channel characterizations have to be used because a deterministic description is not feasible. We restrict to the case where the channel's system functions $S_{\mathbf{H}}(\tau,\nu)$, $L_{\mathbf{H}}(t,f)$, and $h(t,\tau)$ are 2-D complex Gaussian random processes with zero mean (Rayleigh fading).

WSSUS Channels. In general, the correlation function $\mathcal{E}\{S_{\mathbf{H}}(\tau,\nu)\, S^*_{\mathbf{H}}(\tau',\nu')\}$ of the spreading function depends on four variables. A significant simplification is obtained with the assumption of *wide-sense stationary uncorrelated scattering* (WSSUS) [1–3]. For WSSUS channels, the reflectivities of any two scatterers with different delay or different Doppler are uncorrelated, i.e.,

$$\mathcal{E}\{S_{\mathbf{H}}(\tau,\nu)\, S^*_{\mathbf{H}}(\tau',\nu')\} = C_{\mathbf{H}}(\tau,\nu)\, \delta(\tau-\tau')\, \delta(\nu-\nu')\,. \tag{5}$$

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Thus, the spreading function $S_{\mathbf{H}}(\tau,\nu)$ is a 2-D *white* process with mean intensity $C_{\mathbf{H}}(\tau,\nu) \geq 0$, which is known as the channel's *scattering function* [1, 3].

Due to the Fourier transform relation (4), an equivalent TF-domain formulation of the WSSUS assumption is

$$\mathcal{E}\{L_{\mathbf{H}}(t,f)\, L^*_{\mathbf{H}}(t',f')\} = R_{\mathbf{H}}(t-t', f-f') .$$

This means that the TF transfer function $L_{\mathbf{H}}(t,f)$ is a *stationary* process. The *TF correlation function* $R_{\mathbf{H}}(\Delta t, \Delta f)$ is related to the scattering function $C_{\mathbf{H}}(\tau,\nu)$ as

$$C_{\mathbf{H}}(\tau,\nu) = \int_{\Delta t}\int_{\Delta f} R_{\mathbf{H}}(\Delta t,\Delta f)\, e^{-j2\pi(\nu\Delta t - \tau\Delta f)}\, d\Delta t\, d\Delta f. \tag{6}$$

Hence, the scattering function is the *power spectral density* of the stationary process $L_{\mathbf{H}}(t,f)$.

Dispersion and Coherence Parameters. In practice, a statistical channel description by just a few global parameters is often desired. An important parameter is the *dispersion spread* [2] $d_{\mathbf{H}} = \tau_{\max}\nu_{\max}$, where $\tau_{\max}$ and $\nu_{\max}$ are the channel's maximum delay and maximum Doppler, respectively, i.e., the largest τ and ν for which $C_{\mathbf{H}}(\tau,\nu)$ is nonzero. Other definitions of dispersion spread use moments like the root-mean-square (rms) delay spread and rms Doppler spread instead of $\tau_{\max}$ and $\nu_{\max}$. The dispersion spread $d_{\mathbf{H}}$ measures the size of the (effective) support region of $C_{\mathbf{H}}(\tau,\nu)$, and hence quantifies how much the channel smears out the transmit signal in time and frequency.

A channel is called *underspread* if $d_{\mathbf{H}}$ is (much) less than one, and *overspread* otherwise [2, 3]. "Underspread" does not necessarily mean "slowly time-varying": a rapidly varying channel (large $\nu_{\max}$) with short delays (small $\tau_{\max}$) may be underspread. Since delay *and* Doppler of any resolvable path are inversely proportional to the propagation speed c, we have $d_{\mathbf{H}} \propto 1/c^2$. Wireless channels are always underspread since $c^2 \approx 9 \cdot 10^{16}$ m/s. However, underwater channels only have $c^2 = 2.2 \cdot 10^6$ m/s and hence are potentially overspread.

For an underspread channel, it follows from (4) that the TF transfer function $L_{\mathbf{H}}(t,f)$ is a smooth function. Similarly, it follows from (6) that the TF correlation function $R_{\mathbf{H}}(\Delta t,\Delta f)$ has slow decay and, thus, $L_{\mathbf{H}}(t,f)$ features significant temporal and spectral correlations. The temporal correlations are usually quantified by the *coherence time* [3], which is the duration for which the channel is strongly correlated and thus can be considered approximately constant. In a similar way, the spectral correlations are quantified by the *coherence bandwidth* [3].

3. SPARSE CHANNEL REPRESENTATIONS

Sparse (parsimonious, low-dimensional, low-rank) representations of LTV channels are useful in many applications. We will study sparse representations in a discrete-time setting. The channel's input-output relation now reads

$$r[n] = \sum_{m=0}^{M-1} h[n,m]\, s[n-m] , \tag{7}$$

where $s[n]$, $r[n]$, and $h[n,m]$ are sampled versions of $s(t)$, $r(t)$, and $h(t,\tau)$ in (3), with sampling frequency f_{s} larger than the system bandwidth, and $M = \lceil f_{\mathrm{s}}\tau_{\max}\rceil$.

3.1 Basis Expansion Models

A popular class of low-rank channel models uses an expansion of $h[n,m]$ with respect to n into a basis $\{u_l[n]\}_{l=0,\dots L-1}$ [5, 6], i.e.,

$$h[n,m] = \sum_{l=0}^{L-1} c_l[m]\, u_l[n] . \tag{8}$$

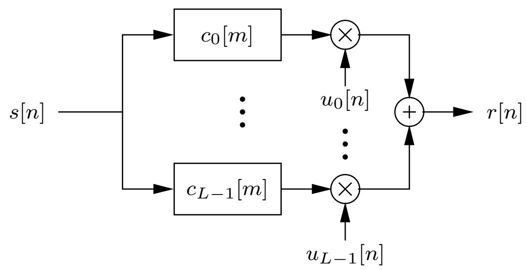

Figure 1: *Basis expansion model for an LTV channel.*

The coefficients are obtained as $c_l[m] = \langle h[\cdot,m], \tilde{u}_l\rangle = \sum_n h[n,m]\, \tilde{u}^*_l[n]$, where $\{\tilde{u}_l[n]\}_{l=0,\dots L-1}$ is the biorthogonal basis (i.e., $\langle u_l, \tilde{u}_{l'}\rangle = \delta[l-l']$). Choosing complex exponentials and polynomials for the $u_l[n]$'s results in Fourier and Taylor series, respectively. These two representations were essentially already proposed in [1].

Inserting the basis expansion (8) into (7) gives

$$r[n] = \sum_{l=0}^{L-1} u_l[n] \sum_{m=0}^{M-1} c_l[m]\, s[n-m] .$$

Hence, the LTV channel can be viewed as a bank of time-invariant filters $c_l[m]$ whose outputs are multiplied by the basis functions $u_l[n]$ and added (see Fig. 1).

Complex Exponential (Fourier) Basis. Today, the term "basis expansion model" (BEM) is mostly used when the basis consists of complex exponentials [5, 6], i.e.,

$$h[n,m] = \sum_{l=0}^{L-1} c_l[m]\, e^{j2\pi\nu_l n}.$$

For uniform Doppler spacing, i.e., $\nu_l = (l - \lceil L/2\rceil)\nu_1$, the coefficients $c_l[m]$ are essentially samples of the spreading function $S_{\mathbf{H}}(\tau,\nu)$. A BEM with nonuniform Doppler spacing is more flexible but requires estimation of the "active Doppler frequencies" ν_l, which may be difficult.

If the BEM is used within a block of length N, the Doppler resolution is given by $\nu_1 = 1/N$. However, the finite block length results in a "Doppler leakage effect" that necessitates an increased model order L. This effect can be reduced by using longer observation windows (possibly overlapping with adjacent blocks) or windowing techniques [7, 8]. Another approach is to replace the Fourier basis functions by discrete prolate spheroidal sequences [9], which are finite-length orthonormal functions of maximum spectral concentration. Although originally proposed for the (flat fading) channel coefficients of individual OFDM subcarriers, the same idea is applicable in the time-delay domain in the sense of (8).

Polynomial Basis. Polynomial basis expansions result from a Taylor series approach. The channel impulse response $h(t,\tau)$ can be approximated about any time instant t_0 as

$$h(t_0+t,\tau) \approx \sum_{l=0}^{L-1} c_l(t_0,\tau)\, t^l, \quad \text{with } c_l(t_0,\tau) = \frac{1}{l!}\frac{d^l h(t,\tau)}{dt^l}\bigg|_{t_0}.$$

For underspread channels that evolve smoothly with time, the $c_l(t_0,\tau)$'s decrease quickly with increasing l so that typically a small order L (e.g., $L=2$) is sufficient. The discrete-time version of the polynomial basis expansion is given by

$$h[n_0+n,m] = \sum_{l=0}^{L-1} c_l[n_0,m]\, n^l, \tag{9}$$

with $c_l[n_0,m] = c_l(n_0T_{\mathrm{s}}, mT_{\mathrm{s}})\, T^l_{\mathrm{s}}$ (T_{s} is the sampling period).

3.2 Sparse Statistical Models

Sparse (parsimonious) models for the channel statistics are also of interest. A frequently used model for the scattering function is given by the following separable function with Jakes Doppler profile and exponential power profile [2, 3]:

$$C_{\mathbf{H}}(\tau,\nu) = \begin{cases} 10^{-\frac{\rho_{\mathbf{H}}^2}{10}} \frac{e^{-\frac{\tau}{\tau_0}}}{\tau_0} \frac{1}{\sqrt{\nu_{\max}^2-\nu^2}}\,, & |\nu| < \nu_{\max} \\ 0\,, & |\nu| > \nu_{\max}\,. \end{cases}$$

This model is very simple—it involves only the path loss $\rho_{\mathbf{H}}^2$, rms delay τ_0, and maximum Doppler $\nu_{\max}$—but often does not agree well with real propagation environments.

More flexibility is obtained with the *autoregressive moving average (ARMA)* channel model [10–12]. For WSSUS channels, the channel taps $h[n,m]$ are stationary processes with respect to n and mutually uncorrelated for different m. The ARMA model is then given by

$$h[n,m] \,=\, -\sum_{i=1}^{r_{\mathrm{AR}}} a_m[i]\, h[n-i,m] \,+\, \sum_{i=0}^{r_{\mathrm{MA}}-1} b_m[i]\, e_m[n-i]\,,$$

where the $e_m[n]$'s are normalized white innovations processes that are uncorrelated for different m, and $a_m[i]$ and $b_m[i]$ are the (nonrandom) parameters of the AR and MA part, respectively. The scattering function follows as

$$C_{\mathbf{H}}(m,\xi) \,=\, \left|\frac{B_m(\xi)}{A_m(\xi)}\right|^2,$$

with $A_m(\xi) = 1 + \sum_{i=1}^{r_{\mathrm{AR}}} a_m[i]\, e^{-j2\pi\xi i}$, $B_m(\xi) = \sum_{i=0}^{r_{\mathrm{MA}}-1} b_m[i]\, e^{-j2\pi\xi i}$. Hence, the second-order channel statistics are characterized by $r_{\mathrm{AR}} + r_{\mathrm{MA}}$ complex parameters per channel tap.

Another reasonably sparse description of the channel statistics can be based on a BEM (8). Using a discrete version of the WSSUS assumption, the correlation of the BEM coefficients $c_l[m]$ is given by $\mathcal{E}\{c_l[m]\, c_{l'}^*[m']\} = r_c[l-l', m]\, \delta[m-m']$, with some correlation function $r_c[\Delta l, m]$, $\Delta l = -L+1, -L, \ldots, L-1$, $m = 0, \ldots, M-1$.

4. RECENT DEVELOPMENTS AND OPEN PROBLEMS

Next, we review a subjective selection of recent developments in the area of communications over LTV channels, and we indicate some related open problems bearing potential for future research. For convenience, we will freely switch between continuous-time and discrete-time formulations.

4.1 Channel Estimation and Equalization

Recent Developments. Most of the recent work on LTV channel estimation (e.g. [6, 7, 9, 13–15]) is based on sparse channel representations as discussed in Section 3. Because these models are linear in the coefficients, least-squares (LS) and minimum mean-square error (MMSE) methods can easily be applied for estimating the coefficients. The sparseness of the model tends to result in a smaller estimation variance; however, model mismatch may result in a bias. BEMs have also been used to design LTV equalizers for doubly-selective channels (e.g. [5, 14, 16]).

As an example (e.g. [9]), consider the flat fading channel

$$y[n] = h[n]\, x[n] + w[n]\,. \tag{10}$$

(This model also applies to the individual subcarriers of an OFDM system transmitting over a non-flat fading channel.) The goal is to estimate and equalize the channel coefficients $h[n]$ within the block $n = 0, \ldots, N$–1. The transmit sequence $x[n]$ contains N_p equispaced pilot symbols at positions $n_i = \lfloor N/(2N_p) \rfloor + iN/N_p$, $i = 0, \ldots, N_p-1$. With a BEM, $h[n] = \sum_{l=0}^{L-1} c_l\, u_l[n]$ and thus, at the pilot positions,

$$y[n_i] = \left(\sum_{l=0}^{L-1} c_l\, u_l[n_i]\right) x[n_i] + w[n_i]\,, \quad i = 0, \ldots, N_p-1\,.$$

The LS estimate of the coefficients c_l is hence obtained as $\hat{\mathbf{c}} = \mathbf{U}^{\#}\tilde{\mathbf{y}}$, where $[\hat{\mathbf{c}}]_l = \hat{c}_l$, $[\tilde{\mathbf{y}}]_i = y[n_i]/x[n_i]$, $[\mathbf{U}]_{i,l} = u_l[n_i]$, and $\mathbf{U}^{\#}$ is the pseudo-inverse of $\mathbf{U}$. The channel estimate then follows as $\hat{h}[n] = \sum_{l=0}^{L-1} \hat{c}_l\, u_l[n]$. Note that L can be chosen smaller when the channel varies more slowly; this results in a reduced estimator variance. If channel statistics are available, an MMSE estimator can be used instead of an LS estimator. If the noise $w[n]$ in (10) is white, then maximum likelihood (ML) detection is equivalent to simple scalar equalization followed by quantization, i.e., $\hat{x}[n] = \mathcal{Q}\{y[n]/h[n]\}$, where $\mathcal{Q}\{\cdot\}$ denotes quantization according to the symbol alphabet. In practice, $h[n]$ is replaced by the channel estimate $\hat{h}[n]$.

Several modifications of this basic channel estimation and equalization method have been proposed. For a BEM with complex exponential basis, it was proposed to improve the Doppler decay and thus reduce the BEM order L by applying a window to the received sequence $y[n_i]$ (e.g. [7, 8]). Furthermore, larger observation intervals that overlap with neighboring blocks can be used. These modifications can however be avoided by using a BEM with a basis of discrete prolate spheroidal sequences [9].

For doubly selective channels, the simple pilot structure considered above is no longer optimal. Indeed, optimal pilot placement requires that the pilot and data subspaces at the channel output are orthogonal and that the channel modes $u_l[n]$ are uniformly excited by the pilots [17, 18]. Examples of optimal pilot patterns are the above time-domain pilots augmented with guard periods or frequency-domain pilots with guard bands [18]. The first (second) option is preferable when the maximum delay in samples is larger (smaller) than the maximum Doppler in samples. With these specific training schemes, doubly selective channels can be estimated in a similar way as outlined above for the flat fading case.

Open Problems. The estimation and equalization of *multiple-input multiple-output* (MIMO) LTV channels still present many unresolved questions. For example, what is the optimal training for doubly dispersive MIMO channels? What is the best BEM? How can MMSE gains (relative to LS) be leveraged through online learning of the channel statistics? Should equalization be performed in the time domain or in the frequency domain?

In rapidly varying scenarios, LTV channel *prediction* [19, 20] is beneficial for link adaptation and precoding schemes using channel state information (CSI) at the transmitter. Some open questions in this context are: What are the ultimate limits (in terms of delay and estimation error) of channel prediction with and without training? How should the placement and power of training data be chosen? What is the relevance of predicted CSI to the capacity of rapidly varying channels with feedback? When will transmission schemes with mismatched (inaccurate) CSI at the transmitter outperform schemes without CSI?

4.2 Intercarrier Interference in OFDM Systems

Recent Developments. OFDM systems operating in highly mobile scenarios have recently gained importance. An example is DVB-T [21], which is now being studied in mobile environments with speeds >100 km/h. Here, significant channel variations within a single OFDM symbol cause strong intercarrier interference (ICI).

Consider an OFDM system with K subcarriers and a cyclic prefix that is longer than the channel length L. The transmitted OFDM symbol $\mathbf{a} = [a_1 \dots a_K]^T$ is mapped to the demodulated receive vector $\mathbf{y} = [y_1 \dots y_K]^T$ as

$$\mathbf{y} = \mathbf{H}'\mathbf{a} + \mathbf{w} \qquad \text{with } \mathbf{H}' = \mathbf{F}\mathbf{H}\mathbf{F}^H, \tag{11}$$

where $[\mathbf{H}]_{n,n'} = h[n, (n'-n) \bmod K]$ and $\mathbf{F}$ is the $K \times K$ DFT matrix. The off-diagonals of $\mathbf{H}'$ are due to ICI. ZF or MMSE equalization of (11) requires the inversion of a $K \times K$ matrix, which has complexity $\mathcal{O}(K^3)$. However, because $\mathbf{H}'$ exhibits off-diagonal decay, it can be approximated by a band matrix with b sub- and superdiagonals (b is usually chosen proportionally to $\nu_{\max}$) [8, 22]. Inverting the band matrix reduces complexity to $\mathcal{O}(b^2 K)$ operations.

An alternative to banded equalization is based on the polynomial model (9) for the channel taps $h[n,m]$ (i.e., the diagonals of the time-domain matrix $\mathbf{H}$) [23]. The receiver uses iterative ICI cancellation for data detection and multistage channel estimation where the model coefficients $c_l[n_0, m]$ are estimated one after another considering higher-order terms (larger l) as white interference.

A different approach to reducing ICI and thereby allowing for higher mobility is *pulse-shaping OFDM* (e.g. [24–26]). Pulse shaping also reduces out-of-band emissions and the sensitivity to narrowband interference and synchronization errors. In [24, 25], pulse optimization procedures for channels with a given scattering function are proposed; the transmit (TX) pulse and receive (RX) pulse are constrained to satisfy a biorthogonality condition. In [24], the TX pulse is designed such that the non-ICI term is maximized and the RX pulse is chosen to satisfy the biorthogonality condition. In contrast, [25] proposes to prescribe the TX pulse and optimize the biorthogonal RX pulse such that ICI is minimized. Alternatively, a joint nonlinear optimization of TX and RX pulses is presented that achieves even smaller ICI by omitting the biorthogonality condition. In [26], a rectangular RX pulse is used and the TX pulse is optimized such that ICI is limited to a few adjacent subcarriers.

Open Problems. Avoiding ICI is closely related to finding transmit and receive pulses that are almost eigenfunctions of the channel and thus approximately diagonalize the channel [24]. Some questions in this context include the following: How is the off-diagonal decay of the channel matrix $\mathbf{H}'$ linked to channel and pulse parameters? What is the tradeoff between equalizer complexity and system performance when varying the width of the banded channel approximation? Which type of training is optimal for ICI estimation and equalization? Which BEM is most suitable for ICI suppression? Can MIMO-OFDM transmissions be designed to suffer less from ICI than SISO systems?

4.3 Non-WSSUS Channels

Recent Developments. The WSSUS assumption simplifies the statistical characterization of LTV channels. However, it is satisfied in practice only approximately within certain time and frequency intervals. For *non-WSSUS* channels, the spreading function $S_{\mathbf{H}}(\tau, \nu)$ is no longer white and the TF transfer function $L_{\mathbf{H}}(t, f)$ is no longer stationary. A statistical characterization of non-WSSUS channels is given by the *local scattering function (LSF)* [27]

$$\mathcal{C}_{\mathbf{H}}(t, f; \tau, \nu) = \int_{\Delta t} \int_{\Delta f} \mathcal{E}\big\{L_{\mathbf{H}}(t-\Delta t, f)\, L^*_{\mathbf{H}}(t, f+\Delta f)\big\} \times e^{-j2\pi(\nu\Delta t - \tau\Delta f)}\, d\Delta t\, d\Delta f,$$

which describes the power of multipath components with delay τ and Doppler shift ν occurring at time t and frequency f. For WSSUS channels, $\mathcal{C}_{\mathbf{H}}(t, f; \tau, \nu) = C_{\mathbf{H}}(\tau, \nu)$ (cf. (6)).

A channel correlation function $\mathcal{A}(\Delta t, \Delta f; \Delta\tau, \Delta\nu)$ generalizing the TF correlation function $R_{\mathbf{H}}(\Delta t, \Delta f)$ is given by the 4-D Fourier transform of the LSF. This function characterizes the correlation of multipath components separated in time by Δt, in frequency by Δf, in delay by $\Delta\tau$, and in Doppler by $\Delta\nu$. The channel's *stationarity time* and *stationarity bandwidth* are defined as $T_{\mathrm{s}} = 1/\Delta\nu_{\max}$ and $F_{\mathrm{s}} = 1/\Delta\tau_{\max}$, respectively, where $\Delta\tau_{\max}$ and $\Delta\nu_{\max}$ are the delay and Doppler correlation widths as defined by the support of $\mathcal{A}(\Delta t, \Delta f; \Delta\tau, \Delta\nu)$. For a given TF point (t_0, f_0), we have $\mathcal{C}_{\mathbf{H}}(t, f; \tau, \nu) \approx \mathcal{C}_{\mathbf{H}}(t_0, f_0; \tau, \nu)$ in a local *stationarity region* $\mathcal{R}_{\mathrm{s}} = [t_0 - \alpha T_{\mathrm{s}}, t_0 + \alpha T_{\mathrm{s}}] \times [f_0 - \alpha F_{\mathrm{s}}, f_0 + \alpha F_{\mathrm{s}}]$ (here, the parameter α controls the accuracy of approximation).

One can also define a local *coherence region* $\mathcal{R}_{\mathrm{c}}$ within which $L_{\mathbf{H}}(t, f)$ is approximately constant. Wireless channels are *doubly underspread* [27], which means that $\mathcal{R}_s$ is much larger than $\mathcal{R}_{\mathrm{c}}$ and the area of $\mathcal{R}_{\mathrm{c}}$ is > 1. The practical relevance of this property was discussed in [27]. In particular, it was shown that the size of $\mathcal{R}_s$ is crucial for achieving ergodic capacity and that delay-Doppler correlation has a negative impact on delay-Doppler diversity schemes.

The LSF and related descriptions are 4-D functions. The high complexity of these descriptions can be avoided by parametric statistical models for non-WSSUS channels. A nonstationary vector AR model for the channel taps of a non-WSSUS channel has been proposed in [28]. Tap correlation alone was previously considered e.g. in [29].

Open Problems. Non-WSSUS channels still pose many questions. For example, what is the most parsimonious parametric LSF model? How much nonstationarity and tap correlation need to be modeled for practical propagation scenarios? Further questions concern the impact of nonstationarity and delay-Doppler correlation on system design: How much delay-Doppler diversity is supported by realistic non-WSSUS channels? How can non-WSSUS channels be decomposed into independently fading subchannels, and what are corresponding signaling schemes? What is the operational meaning of the ergodic capacity of non-WSSUS channels, and how does the outage capacity vary over time?

The extension of the LSF and channel correlation function to MIMO channels is another open issue. How are the spatial, temporal, and spectral stationarity and correlation parameters related, and what is their relevance to MIMO system designs? What are parsimonious parametric models for non-WSSUS MIMO channels? How do antenna configuration and polarization affect channel nonstationarity?

4.4 Wideband Systems

Recent Developments. While for narrowband systems the Doppler effect can be approximately represented as a frequency shift, it must be exactly characterized by a time scaling (compression/dilation) in the case of *wideband* transmit signals $s(t)$ like the signals used in ultrawideband (UWB) communications. Therefore, (1) has to be replaced by [3]

$$r(t) = \sum_{p=1}^{P} a_p \frac{1}{\sqrt{\alpha_p}}\, s\Big(\frac{t-\tau_p}{\alpha_p}\Big), \quad \text{with } \alpha_p = 1 - \cos(\phi_p)\frac{v}{c}.$$

Generalization to a continuum of scatterers yields

$$r(t) = \int_{\tau}\int_{\alpha} F_{\mathbf{H}}(\tau, \alpha)\, \frac{1}{\sqrt{\alpha}}\, s\Big(\frac{t-\tau}{\alpha}\Big)\, d\tau\, d\alpha, \tag{12}$$

where the α integration is from 0 to ∞. Here, $F_{\mathbf{H}}(\tau, \alpha)$ denotes the *delay-scale spreading function* [30, 31], which is usually supported in a small delay-scale region $[0, \tau_{\max}] \times [\alpha_{\min}, \alpha_{\max}]$. Like (2), the expression (12) can represent *any* LTV channel, but it is most appropriate for wideband systems. In analogy to WSSUS channels (cf. (5)), different delays and scales are usually assumed to be uncorrelated, i.e.,

$$\mathcal{E}\big\{F_{\mathbf{H}}(\tau, \alpha)\, F^*_{\mathbf{H}}(\tau', \alpha')\big\} = B_{\mathbf{H}}(\tau, \alpha)\, \delta(\tau-\tau')\, \delta(\alpha-\alpha').$$

Here, $B_{\mathbf{H}}(\tau, \alpha)$ is the *wideband scattering function* [30–32].

In [30], the notion of *delay-Doppler diversity* introduced in [33] was extended to the wideband regime. Consider spread spectrum transmission of a binary symbol b over a wideband channel (12) with white Gaussian noise. The transmit signal is $s(t) = bx(t)$ with some chip spreading sequence $x(t)$. The *wideband delay-scale rake receiver* correlates the receive signal $r(t)$ with delayed and scaled versions $x_{m,n}(t) = \frac{1}{\sqrt{\alpha_0^m}} x\big(\frac{t}{\alpha_0^m} - nt_c\big)$ of $x(t)$. Here, t_c is the chip duration, $\alpha_0 = 1 + 1/N_s$ with N_s denoting the spreading factor, and $m = -M, \ldots, M$, $n = 0, \ldots, N$ with M, N chosen to capture most of the energy of $r(t)$. The resulting 2-D sequence $r_{m,n} = \langle r, x_{m,n} \rangle$ is then linearly combined to obtain a symbol estimate $\hat{b} = \sum_{m,n} f_{m,n} r_{m,n}$ that is subsequently quantized. This scheme can realize significant delay and Doppler (scale) diversity gains provided that sufficient channel variations occur within a single symbol period [30, 31].

Open Problems. Without constraints on the support of $S_{\mathbf{H}}(\tau, \nu)$ and $F_{\mathbf{H}}(\tau, \alpha)$, the narrowband and wideband channel characterizations (2) and (12) are mathematically equivalent. However, up to which bandwidths is the narrowband representation preferable? Which representation is more parsimonious and/or achieves better decorrelation of multipath components for specific (measured) channels? Is the narrowband or wideband WSSUS assumption more appropriate in certain scenarios? What is the ergodic capacity of wideband channels and which type of signaling should be used (e.g., a wavelet-transform counterpart of OFDM)?

5. CONCLUSIONS

We reviewed some fundamentals and recent developments in the area of communications over time-varying channels. It was seen that sparse channel representations enable an efficient characterization of such channels and a simplified design of corresponding transceiver algorithms. In spite of the progress made in recent years, numerous unsolved problems remain to be addressed by future research.

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