MODEL ORDER SELECTION FOR MULTIPATH MIMO CHANNELS USING THE BENJAMINI-HOCHBERG PROCEDURE

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ABSTRACT

We solve the model order selection problem for a multiple-input multiple-output (MIMO) channel in analogy to the detection problem for multiple signals embedded in noisy observations. Our approach uses a multiple hypothesis test based on log-likelihood ratios. This multiple test detects the number of propagation paths in a multipath channel iteratively. To control the global level of the multiple test, we apply the false discovery rate (FDR) criterion proposed by Benjamini and Hochberg. The power of this multiple test has been investigated through narrow- and broadband simulations in previous studies.

We apply the multiple hypothesis test to real MIMO antenna array measurements. MIMO measurement data were recorded with the RUSK-ATM vector channel sounder. The sounder operated at a center frequency of 2000 MHz with an output power of 2 W and a transmitted signal bandwidth of 120 MHz. Maximum-likelihood estimates for direction of arrival are obtained for all model orders $p = 0, 1, \dots, P_{\max}$. Finally, the Benjamini-Hochberg procedure is invoked to select the model order $\in \{0, 1, \dots, P_{\max}\}$ for a given FDR level q.

1. INTRODUCTION

In MIMO communication systems, it is costly to over– or under–estimate the model order of the MIMO channel. Typical effects of model order mismatch in MIMO systems include bit error rate floors, SNR penalties, and numerical instabilities. We propose to solve the model order selection problem for a frequency-selective multiple-input multiple-output (MIMO) channel by a multiple hypothesis test. This problem is strongly related to estimating the number of signals embedded in noisy observations in array processing. Maiwald and Böhme proposed a multiple testing procedure for determining the number of signals from array observations [1, 2]. Their procedure often leads to conservative results, i.e. the procedure tends to under-estimate the true model order. To overcome this drawback, Chung et al. adopted the false discovery rate (FDR) criterion [3] to keep balance between type-one error control and power of the test [4]. They proved the validity of the required statistical independence condition in [5]. In the present contribution, we apply the procedure for selecting MIMO channel model orders from vector channel sounder data.

2. MEASUREMENT DATA

The measurement data used for the numerical experiments in this paper were recorded during a measurement run in Weikendorf (see `http://www.ftw.at/measurements`), a suburban area in a small town approximately 50 km north–east of Vienna, Austria, in autumn 2001 [6, 7]. The measurement area covers one-family houses with private gardens around them. The houses are typically one floor high. A rail-road track is present in the environment which breaks the structure of single placed houses. A small pedestrian tunnel passes below the railway. An aerial photo of the environment with the position of the receiver and transmitter is shown in Figure 1, cf. [7]. Measurement data were recorded with the RUSK-ATM vector channel sounder, manufactured by MEDAV (see `http://www.channelsounder.de/`). The sounder operated at a center frequency of 2000 MHz with an output power of 2 Watt and a transmitted signal bandwidth of 120 MHz. The transmitter emitted a periodically repeated signal composed of $K = 384$ subcarriers in the band 1940—2060 MHz. The repetition period was 3.2 microseconds. The transmitter was the mobile station and the receiver was at a fixed location. The transmit array had a uniform circular geometry (UCA-15) composed of $M = 15$ monopoles arranged on a ground plane at an inter-element spacing of $d_{\mathrm{T}} = 0.43\lambda \approx 6.45$ cm. The receive array was a uniform linear patch array (mounted above rooftop) composed of $L = 8$ elements (ULA-8) with spacing $d_{\mathrm{R}} = \lambda/2$. The location of the ULA-8 is indicated in Figure 1 by the marker with the label Rx3. The mobile transmitter was mounted on top of a small trolley together with the uniform circular array at a height of approximately 1.5m above ground level. During the measurement, the transmitter moves from location Tx7 to Tx26.

3. SIGNAL MODEL

The 4D parameter estimation problem for MIMO measurements applies to the following double-directional MIMO channel model in which the signal is assumed to propagate from the transmitter to the receiver over P discrete propagation paths. Each path $(p = 1, \ldots, P)$ is characterized by the following parameters: complex path gain w_p, direction of departure (DOD) θ_p, direction of arrival (DOA) ϕ_p, propagation delay τ_p, and Doppler shift ν_p. We repeat N MIMO snapshot measurements consecutively in time and assemble a 4-way array of dimensions $K \times L \times M \times N$ which we refer to as a "Doppler block." Our idealizing assumptions on data acquisition are described in further detail in Ref. [9]. After a discrete Fourier transform over time, we obtain the following data model in the frequency domain

$$y_{k,l,m,n}(t) = \sum_{p=1}^{P} w_p(t) a_p^k b_p^l c_{m,p} d_p^n + \text{noise} . \tag{1}$$

Here, $(a_p)^k$ depends on propagation delay τ_p, $(b_p)^l$ defines the Rx array manifold for the ULA-8, $c_{m,p}$ defines the Tx array manifold for the UCA-15, and $(d_p)^n$ depends on the Doppler shift ν_p. To be more specific, we have

$$\begin{aligned} a_p &= \exp(-j2\pi\tau_p/K) , &(2)\\ b_p &= \exp(-j2\pi(d_\mathrm{R}/\lambda)\cos\phi_p) , &(3)\\ c_{m,p} &= \exp(-j\pi(d_\mathrm{T}/\lambda)\gamma_m(\theta_p)) , &(4)\\ \gamma_m(\theta_p) &= \cos\left(\frac{2\pi(m-1)}{M} - \theta_p\right) \Big/ \sin(\pi/M), &(5)\\ d_p &= \exp(-j2\pi\nu_p/N) . &(6) \end{aligned}$$

The noise is assumed to be independent identically distributed zero-mean complex circular Gaussian, i.e. temporally and spatially white.

3.1. Simplified Rx array model

In this first application to measurement data, we specialise to the narrowband case ($K := 1$) by selecting the single frequency bin for $k = 1$ and ignore the Doppler effects ($N := 1$). The receive data at the lth receive array element is modeled as

$$x_{l,m}(t) = \sum_{p=1}^{P} \tilde{w}_{m,p}(t) b_p^l + n_{l,m}(t) , \tag{7}$$

where $\tilde{w}_{m,p} = w_p a_p c_{m,p} d_p$. We define the receive data vector as $\boldsymbol{x}_m(t) = (x_{1,m}(t), \ldots, x_{L,m}(t))^T$. We estimate the $L \times L$ receive covariance matrix $\boldsymbol{R}(t) = \mathrm{E}\{\boldsymbol{x}_m(t)[\boldsymbol{x}_m(t)]^H\}$ by using the Tx array for generating $M = 15$ snapshots of the receive data,

$$\hat{\boldsymbol{R}}(t) = \frac{1}{M} \sum_{m=1}^{M} \boldsymbol{x}_m(t)[\boldsymbol{x}_m(t)]^H . \tag{8}$$

4. TEST PROCEDURE

We formulate the problem of detecting the number of observable propagation paths at the Rx array as a multiple hypothesis test in analogy to Refs. [4, 5]. Let $P_{\max} = 7$ denote the maximum number of propagation paths. The following procedure detects one propagation path after another. For $p = 1$, we define the hypothesis H_1 and corresponding alternative as

$$\begin{aligned} H_1 &: \text{Data contains only noise,}\\ A_1 &: \text{Data contains at least one path.} \end{aligned}$$

For $p = 2, \ldots, P_{\max}$, we define the pth hypothesis and alternative as

$$\begin{aligned} H_p &: \text{Data contains at most } (p-1) \text{ paths,}\\ A_p &: \text{Data contains at least } p \text{ paths.} \end{aligned}$$

Based on the likelihood ratio (LR) principle, we obtain the test statistics $\hat{T}_p(t)$, $(p = 1, \ldots, P_{\max})$ at time t as follows.

$$\begin{aligned} \hat{T}_p(t) &= \log\left(\frac{\mathrm{tr}[(\boldsymbol{I} - \boldsymbol{P}_{p-1})\hat{\boldsymbol{R}}(t)]}{\mathrm{tr}[(\boldsymbol{I} - \boldsymbol{P}_p)\hat{\boldsymbol{R}}(t)]}\right)\\ &= \log\left(1 + \frac{\mathrm{tr}[(\boldsymbol{P}_p - \boldsymbol{P}_{p-1})\hat{\boldsymbol{R}}(t)]}{\mathrm{tr}[(\boldsymbol{I} - \boldsymbol{P}_p)\hat{\boldsymbol{R}}(t)]}\right) . \end{aligned} \tag{9}$$

Here, $\boldsymbol{P}_p$ is the projection matrix onto the subspace spanned by the first p estimated propagation paths. We initialise the null-projection $\boldsymbol{P}_0 = \boldsymbol{0}$. The distribution of

$$T_p(t) = \log\left(1 + \frac{\mathrm{tr}[(\boldsymbol{P}_p - \boldsymbol{P}_{p-1})\boldsymbol{R}(t)]}{\mathrm{tr}[(\boldsymbol{I} - \boldsymbol{P}_p)\boldsymbol{R}(t)]}\right) . \tag{10}$$

under hypothesis H_p is known analytically [2].

In the following, we assume that exactly P_0 hypotheses are true among all $P_{\max}$ tested hypotheses ($P_0 \leq P_{\max}$). Let

$$\{p_1(t), p_2(t), \ldots, p_{P_{\max}}(t)\}$$

be the p-values (observed significance values) corresponding to the test statistics (9)

$$\{\hat{T}_1(t), \hat{T}_2(t), \ldots, \hat{T}_{P_{\max}}(t)\} .$$

By definition,

$$p_m(t) = \mathrm{P}\{T_m(t) > \hat{T}_m(t)\} = 1 - P_{H_m}(\hat{T}_m(t)) \tag{11}$$

where P_{H_m} is the distribution of $T_m(t)$ under H_m.

Let $\{p_{(1)}, p_{(2)}, \ldots, p_{(P_{\max})}\}$ be the *sorted* p-values and $p_{(s)}$ corresponds to the hypothesis $H_{(s)}$ for all $1 \leq s \leq P_{\max}$.

4.1. Benjamini–Hochberg Procedure

Benjamini and Hochberg [3] showed that when the test statistics corresponding to the true null hypotheses are independent, the following procedure controls the False Discovery Rate (FDR) at level $qP_0/P_{\max} \leq q$ (see [3]). Define

$$\hat{P}_{\mathrm{FDR}} := \max_{1 \leq r \leq P_{\max}} \left\{ r \middle| p_{(r)} \leq \frac{r}{P_{\max}} q \right\} \tag{12}$$

and reject $H_{(1)}, \ldots, H_{(\hat{P}_{\mathrm{FDR}})}$. If no such $\hat{P}_{\mathrm{FDR}}$ exists, reject no hypothesis.

5. RESULTS

We apply three different methods for determining the number of propagation paths in the data set:

1. The Benjamini–Hochberg procedure to control the FDR at level $q = 0.1$,
2. the Bonferroni–Holm procedure [8] to control the FWE at level $\alpha = 0.1$, and
3. the Minimum Description Length (MDL) criterion by Wax and Ziskind [9].

The detection results are shown in Figure 2. In the data set, we observe that the number of propagation paths is sharply reduced during measurement times t between 29 s and 34 s. This observation is consistent with results reported in [7]. During these measurement times, the transmitter entered the small pedestrian tunnel underneath the railway. The tunnel blocks most propagation paths which are present during other times in the data.

6. CONCLUSIONS

We clearly see from these experiments that the FWE controlling procedure [8] is more conservative than the FDR controlling procedure [3]: The FWE controlling procedure never detects more paths than the FDR controlling procedure and the FWE controlling procedure tends to detect less paths than the FDR controlling procedure. On the other hand, nothing of the kind can be said of the MDL criterion: it is *neither more nor less* conservative than the FWE and FDR controlling procedures. For these reasons, we would like to note that the MDL criterion is not suitable for validating MIMO channel models. For validating channel models, multiple hypotheses tests (such as the FWE or FDR controlling procedures) should be used.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

[1] J.F. Böhme, “Statistical array signal processing of measured sonar and seismic data,” in *Proc. SPIE 2563 Advanced Signal Processing Algorithms*, San Diego, July 1995.

[2] D. Maiwald, *Breitbandverfahren zur Signalentdeckung und –ortung mit Sensorgruppen in Seismik– und Sonaranwendungen*, Ph.D. thesis, Ruhr-Universität Bochum, May 1995.

[3] Y. Benjamini and Y. Hochberg, “Controlling the false discovery rate: a practical and powerful approach to multiple testing,” *J. Roy. Stat. Soc., Ser. B*, vol. 57, pp. 289—300, 1995.

[4] P.-J. Chung, J.F. Böhme, A.O. Hero, and C.F. Mecklenbräuker, “Signal detection using a multiple hypothesis test,” in *Third IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM 2004)*, Barcelona, Spain, July 18–21 2004.

[5] P.-J. Chung, J.F. Böhme, C.F. Mecklenbräuker, and A.O. Hero, “On signal detection using the benjamini-hochberg procedure,” in *IEEE Workshop on Statistical and Signal Processing (SSP 2005)*, Bordeaux, France, July 2005.

[6] H. Hofstetter, C.F. Mecklenbräuker, H. Anegg, E. Bonek, R.R. Müller, and H. Kunczier, “The FTW wireless MIMO measurement campaign at 2GHz: documentation of the downloadable data sets,” Temporary Document COST 273 TD(02)-135, Lisboa, Portugal, September 19–20 2002, `http://www.lx.it.pt/cost273`.

[7] M. Pesavento, C.F. Mecklenbräuker, and J.F. Böhme, “Multi-dimensional rank reduction estimator for parametric MIMO channel models,” *EURASIP J. Appl. Sig. Proc.*, vol. 2004, no. 9, pp. 1354–1363, 2004, Special Issue on Advances in Smart Antennas.

[8] S. Holm, “A simple sequentially rejective multiple test procedure,” *Scand. J. Statist.*, vol. 6, pp. 65–70, 1979.

[9] M. Wax and I. Ziskind, “Detection of the number of coherent signals by the MDL principle,” *IEEE Trans. Acoust. Speech Sig. Proc. (ASSP)*, vol. 37, no. 8, pp. 1190–1196, August 1989.

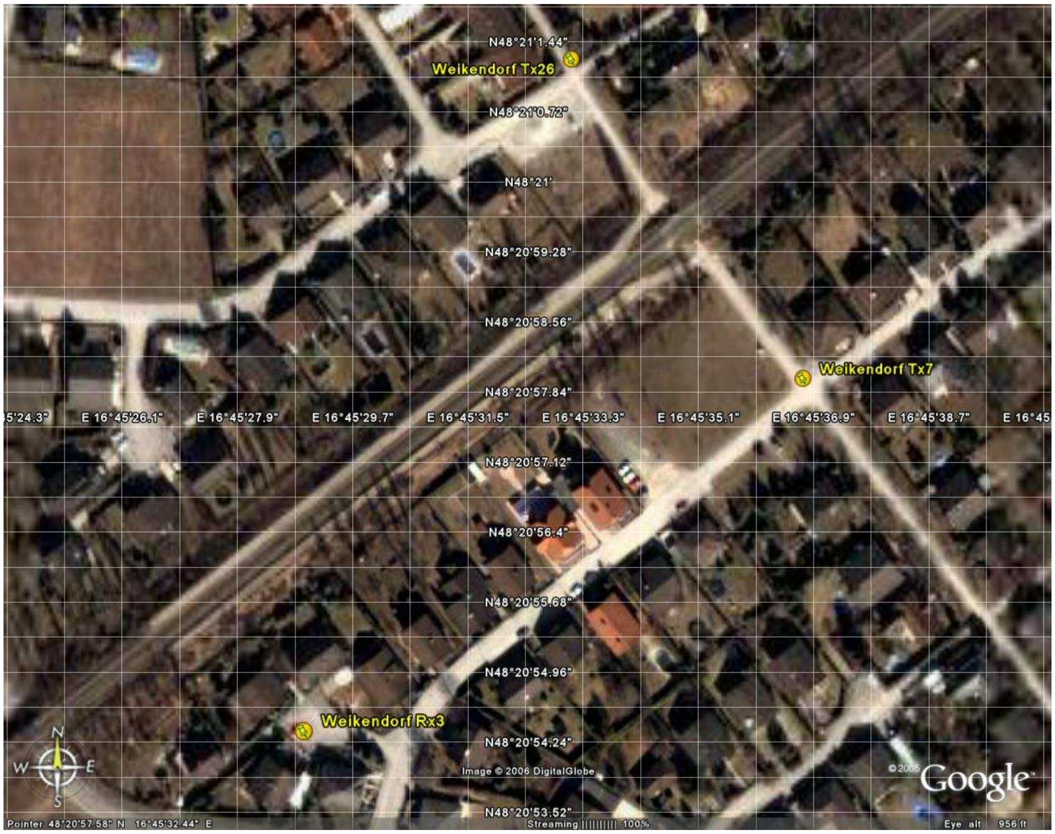

Fig. 1. Tx and Rx locations in Weikendorf (source: Google Earth)

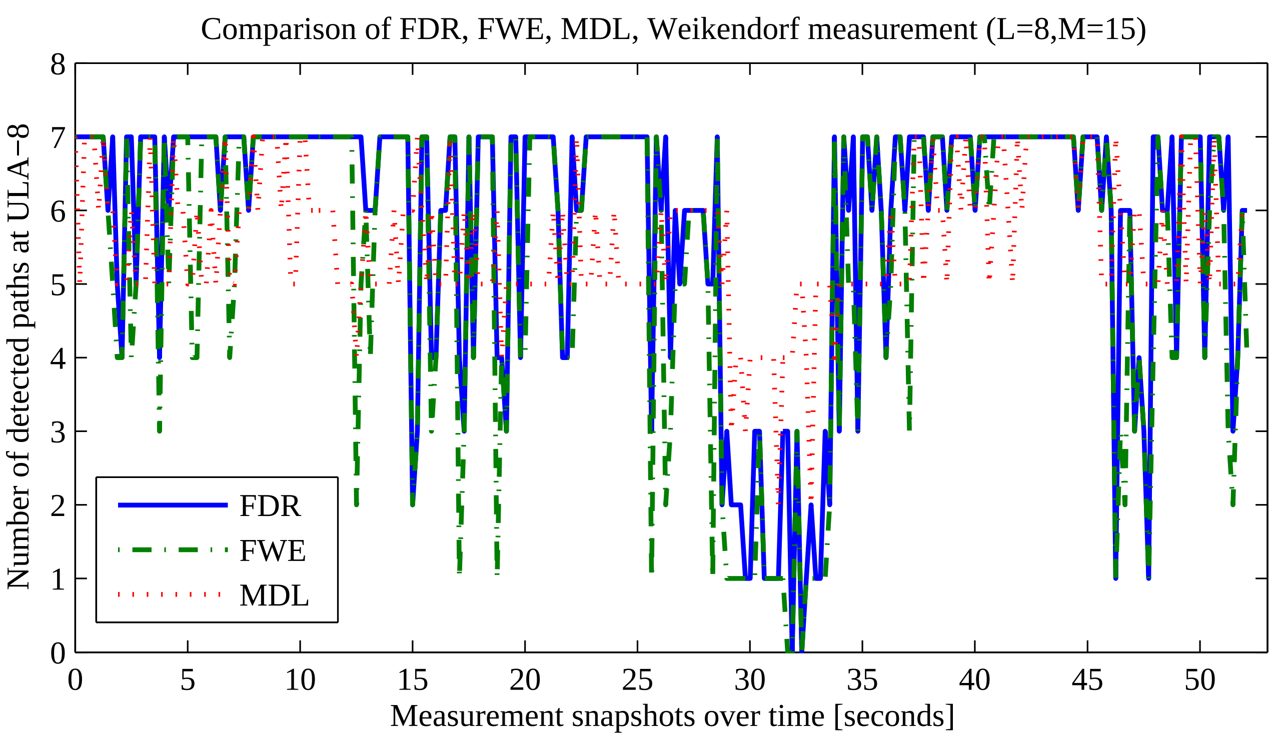

Fig. 2. Test results over time for FDR and FWE procedures in comparison to MDL