Characterizing Resonating Cantilevers for Liquid Property Sensing

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***Abstract*— Miniaturized liquid sensors are essential devices in online process or condition monitoring. In case of viscosity and density sensing, microacoustic sensors such as quartz crystal resonators or SAW devices have proved particularly useful. However, these devices basically measure a thin-film viscosity, which is often not comparable to macroscopic measurements. Miniaturized cantilever-based devices are interesting alternatives for such applications, but here the interaction between the liquid and the oscillating beam is more involved. In our contribution we describe a measurement setup, which allows the investigation of this interaction for different beam cross-sections. We present an analytical model based on an approximation of the immersed cantilever as an oscillating sphere comprising the effective mass and the intrinsic damping of the cantilever and additional mass and damping due to the liquid loading. The model parameters are obtained by a curve fitting procedure.**

I. Introduction

For many applications like online process or condition monitoring, the parameters liquid viscosity and density are of high relevance. Microacoustic sensors like quartz thickness shear mode (TSM) resonators [1] and surface acoustic wave (SAW) devices [2] have proved particularly useful alternatives to traditional viscometers [3].

However, these devices measure viscosities at comparatively high shear rates and small amplitudes. For non-Newtonian liquids, the measurement results are therefore sometimes not compareable to those obtained from conventional laboratory viscometers. Micromachined vibrating structures usually feature lower resonant frequencies and higher shear amplitudes, making them more suitable for non-Newtonian and complex liquids [4]. Micromachined cantilevers commonly used in atomic force microscopy [5] and membrane structures [6] have been successfully used as liquid sensors.

For complex liquids such as emulsions it has also been shown, that microacoustic devices may not be sufficient to detect rheological effects which are present on the macroscopic scale only [7].

In our contribution, we characterize resonating cantilevers sensing in a rheological domain which is more comparable to that probed by conventional laboratory instruments. The cantilevers are based on commercially available piezoelectric bending actuators. Different tips of well-defined geometries (rectangular cross-sections) have been attached to the cantilevers. Both resonance frequency and damping of the cantilever are influenced by the viscosity and density of the liquid in which the tip is immersed. However, the cantilevers do not show a simple relationship between sensor and liquid parameters. In our work, we aim to clarify this relation using an experimental approach combined with an analytical model.

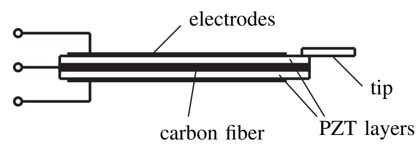

Fig. 1. PZT bending actuator with tip glued on

II. Sensor fabrication

Commercially available PZT (lead zirconate titanate) bimorph bending actuators consist of two piezoelectric PZT layers on both sides of a carbon fiber substrate (Fig. 1). The PZT layers are polarized in thickness direction. Electrodes on both sides of the PZT layers allow for excitation of the actuator. The bending actuators used in this work were supplied by Argillon GmbH, Redwitz, Germany. The center electrode is used as ground electrode (Fig. 2). A sinusoidal voltage is applied to the driving electrode, causing transversal contraction in the upper PZT layer but not in the substrate. This leads to bending vibrations of the beam. The actual beam deflection is determined by measuring the voltage at the sensing electrode.

In our setup (Fig. 2), the bending actuator is clamped at one end, whereas different tips of well-defined cross-sections have been attached to the free end of the cantilever. These tips are immersed in the respective liquid. The tip geometries

TABLE I

TIP GEOMETRIES (RECTANGULAR CROSS-SECTION)

Tip	Width	Thickness
Tip A	7 mm	0.5 mm
Tip B	5 mm	0.5 mm
Tip C	4 mm	0.3 mm
Tip D	2 mm	0.5 mm

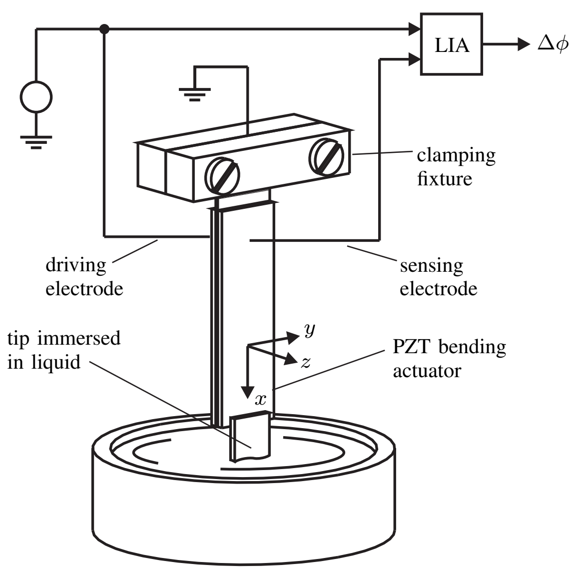

Fig. 2. Measurement setup

are given in Tab. I. The clamping fixture is mounted on a rigid frame allowing for vertical positioning of the sensor and preventing vibrations of the entire setup. A lock-in amplifier (Stanford Research SR830) is used to drive the cantilever and to measure the sensor voltage, resulting in the cantilever's frequency response.

The cantilever exhibits several resonant vibration modes. The frequency spectrum obtained for the cantilever with tip A below $5\,\mathrm{kHz}$ shows four such modes at $100\,\mathrm{Hz}$ (mode 1), $851\,\mathrm{Hz}$ (mode 2), $2456\,\mathrm{Hz}$ (mode 3), and $4870\,\mathrm{Hz}$ (mode 4).

III. THEORETICAL MODEL

The interaction with the liquid surrounding the cantilever tip changes both the cantilever's resonance frequency and damping. In particular, the resonance frequency ω_n of the vibration mode n and the damping factor D_n depend on the liquid density and viscosity and the tip geometry. Shih et al. [8] model the relationships between ω_n, D_n and η, ρ by approximating the oscillating cantilevers as an oscillating sphere immersed in a liquid. The force F acting on such a sphere is given by [9]

$$F = \underbrace{6\pi\eta\left(1+\frac{R}{\delta}\right)}_{b_\mathrm{i}} u + \underbrace{3\pi R^2\sqrt{\frac{2\eta\rho}{\omega}}\left(1-\frac{2R}{9\delta}\right)}_{M_\mathrm{i}} \frac{du}{dt}, \quad (1)$$

where R is the sphere radius, ω the angular oscillation frequency, and δ the depth of penetration of the acoustic wave, given by $\delta = \sqrt{2\eta/(\omega\rho)}$. Considering the bending actuator as an oscillator immersed in a liquid and driven by a harmonic force, the differential equation for the motion u in z-direction (Fig. 2) is [8]

$$(M_\mathrm{e}+M_\mathrm{i})\frac{d^2u}{dt^2} + (b_\mathrm{e}+b_\mathrm{i})\frac{du}{dt} + Ku = F_0 e^{j\omega t}, \quad (2)$$

where M_e and b_e are the effective mass and the intrinsic damping of the cantilever, M_i and b_i are the additional mass and damping due to the liquid loading, given by (1), K is the spring constant and F_0 and ω are the driving force's amplitude and angular frequency. Solving the differential equation using Laplace transform respectively yields

$$U(s) = \frac{1}{1+\frac{2D_n}{\omega_n}s+\frac{1}{\omega_n^2}s^2}F(s), \quad (3)$$

where $U(s)$ and $F(s)$ are the Laplace transforms of the motion u and the driving force,

$$\frac{1}{\omega_n^2} = \frac{M_\mathrm{e}+M_\mathrm{i}}{K}, \quad (4)$$

and

$$\frac{2D_n}{\omega_n} = \frac{b_\mathrm{e}+b_\mathrm{i}}{K}, \quad (5)$$

where ω_n is the resonance frequency and D_n the damping factor of the respective vibration mode. From (1), (4), and (5), we have

$$\omega_n^2 = \omega_{n,\mathrm{air}}^2 \frac{1}{1+\frac{2\pi R^3}{3M_\mathrm{e}}\rho+\frac{6\pi R^2}{\sqrt{2}M_\mathrm{e}}\frac{1}{\sqrt{\omega_n}}\sqrt{\eta\rho}}, \quad (6)$$

and

$$\frac{2D_n}{\omega_n} = \frac{2D_{n,\mathrm{air}}}{\omega_{n,\mathrm{air}}}\left(1+\frac{6\pi R}{b_\mathrm{e}}\eta+\frac{6\pi R^2}{\sqrt{2}b_\mathrm{e}}\sqrt{\omega_n}\sqrt{\eta\rho}\right), \quad (7)$$

where $\omega_{n,\mathrm{air}}$ and $D_{n,\mathrm{air}}$ are the resonance frequency and damping factor of the cantilever without liquid loading. The oscillating sphere model shows that the cantilever's resonance frequency is affected by a term of liquid density, and a second term of the viscosity-density product, whereas the time constant $T_n = 2D_n/\omega_n$ depends on the viscosity and the viscosity-density product.

For the interpretation of our measurement results involving rectangular cross-sections, we have used a more generalized model. Based on equations (6) and (7) and by introducing four independent coefficients c_1, c_2, c_3, and c_4 we have

$$\omega_n^2 = \omega_{n,\mathrm{air}}^2 \frac{1}{1+c_1\rho+c_2\frac{1}{\sqrt{\omega_n}}\sqrt{\eta\rho}}, \quad (8)$$

and

$$T_n = T_{n,\mathrm{air}}\left(1+c_3\eta+c_4\sqrt{\omega_n}\sqrt{\eta\rho}\right), \quad (9)$$

where $T_{n,\mathrm{air}}$ is the in-air time constant $2D_{n,\mathrm{air}}/\omega_{n,\mathrm{air}}$.

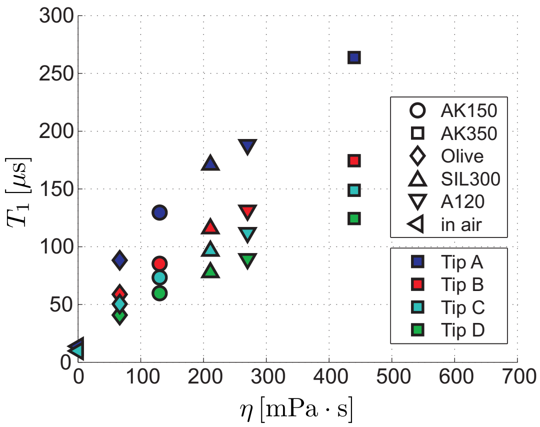

Fig. 3. Measured time constant T_1 versus liquid viscosity η. The results indicate that T_1 is dominantly influenced by the viscosity.

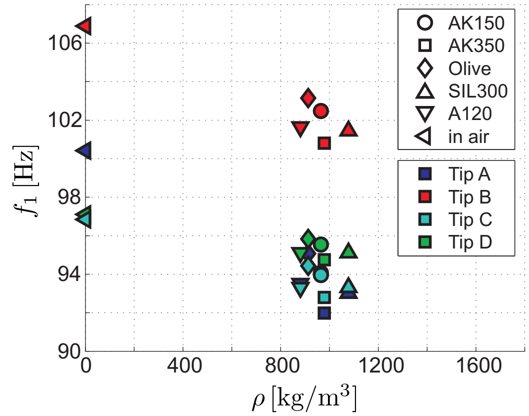

Fig. 4. Measured first mode resonance frequency f_1 versus liquid density ρ. The spread in the results indicates that f_1 not only depends on the density, but also on the liquid's viscosity.

IV. Measurements

With the described setup the frequency response of the cantilevers is examined. The lock-in amplifier is used to drive the cantilever and to measure the phase shift $\phi(j\omega)$ between the driving voltage, i. e., the driving force, and the sensor voltage, i. e., the actual cantilever deflection. The measurements are carried out with a sinusoidal driving voltage of $200\,\mathrm{mV}$ rms and within a frequency range from 70 to $110\,\mathrm{Hz}$. At the resonance frequency of the first mode, a maximum tip deflection of 38 $\mu\mathrm{m}$ was measured in air by means of a Polytec laser vibrometer.

As test liquids, a variety of oils is used: AK150, AK350 (Wacker Chemie), and SIL300 are silicone oils, an olive oil sample, and Alcatel 120 (A120) oil. They exhibit liquid densities in the range from 880 to $1080\,\mathrm{kg/m^3}$ and viscosities from 63 to $440\,\mathrm{mPa\cdot s}$. The liquid parameters are obtained from data sheets and measurements by means of a Brookfield LVDV+II-CP cone/plate rheometer.

From the cantilever's frequency response we extract the first mode resonance frequency $\omega_1 = 2\pi f_1$ and the damping factor D_1 by fitting a second order transfer function

$$\phi(j\omega) = \arg\left\{\frac{1}{1 + j\omega\frac{2D_1}{\omega_1} - \frac{\omega^2}{\omega_1^2}}\right\} \tag{10}$$

to the measurement results with respect to the parameters ω_1 and D_1.

Figure 3 shows the measured time constant T_1 versus the dynamic viscosity η of the sample liquids. The results for each cantilever appear to be lying on a single trend curve, indicating that T_1 is dominantly influenced by the liquid's dynamic viscosity. According to model eq. (9), the value of c_4 must be small compared to c_3.

Figure 4 depicts the resonance frequency f_1 versus the liquid density. Obviously, the relationship is not as straightforward as in the previous diagram, the results are widely spread in the ρ–f_1 plane. The resonance frequency of the cantilever tip immersed in the liquid is not only influenced by the liquid's density, but also by its viscosity. In our model this relationship is described by (8).

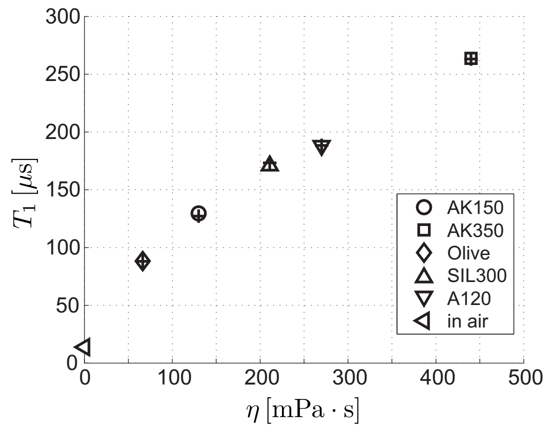

Fig. 5. Measurement results for Tip A, time constant T_1 versus liquid viscosity. Crosses indicate fitted values.

To verify the applicability of the model given in (8) and (9) to our PZT cantilevers, the model equations are fitted to the measurement results with respect to the parameters c_1, c_2, c_3, and c_4. These coefficients determine the relationships between ω_1, T_1, and ρ, η, and $\sqrt{\rho\eta}$, respectively. Figures 5 and 6 depict the measurement results for tip *A*, and a dipping depth of $2\,\mathrm{mm}$. The crosses indicate the fit results for (9) in Fig. 5, and for (8) in Fig. 6, respectively, and show good agreement between the experimental results and the model.

The fit results for (8) are depicted in Fig. 7. The diagram shows the dependence of the fit parameters c_1 and c_2 on the kind of tip used. It turns out, that the sensitivity of the resonance frequency to the density (determined by c_1) can be steadily increased by increasing the tip width, whereas its sensitivity to the viscosity-density product tends to saturate for increasing widths. This can be explained by the fact that larger amounts of liquid must be moved by the oscillating tip with increasing tip width, whereas the influence of the viscosity is concentrated to the edges of the tip, where major shear

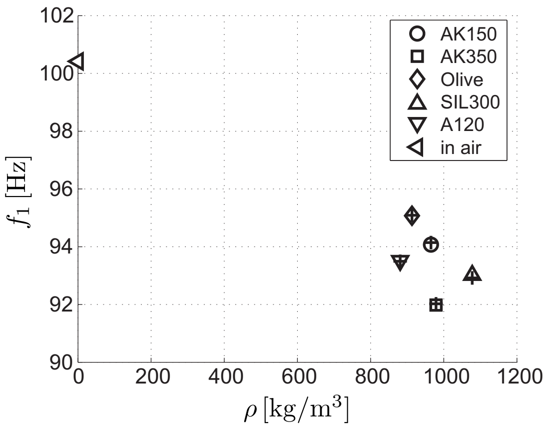

Fig. 6. Measurement results for Tip *A*, resonance frequency versus liquid density. Crosses indicate fitted values.

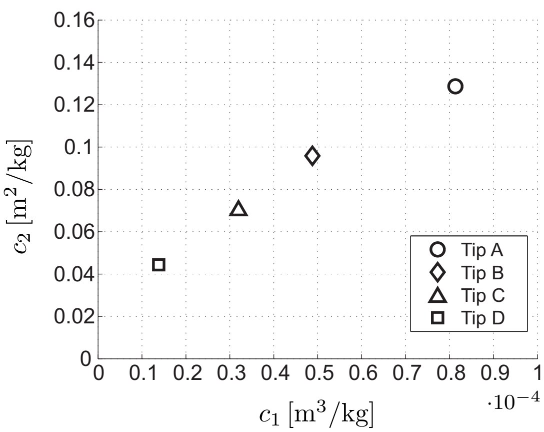

Fig. 7. Fitted parameter values c_1 and c_2 of (8) for different tip geometries. c_1 represents the sensitivity of the resonance frequency to the liquid density, whereas c_2 accounts for the influence of the viscosity-density product.

displacements occur.

V. Conclusion

The change of the dynamic behavior of a vibrating cantilever allows investigating the physical properties of liquids. Various types of small tips of different geometries are attached to the cantilever and immersed into the sample solutions. The liquid surrounding the cantilever tip changes both the resonance frequency and the damping of the entire cantilever structure. To be able to conclude from the measured frequency response to the liquid's parameters an analytical model is needed. The developed model is based on the forces acting on an oscillating sphere in liquid, but generalized model parameters are used to consider the actual geometries of the applied cantilever tips. This model is well-suited for the characterization of various cantilevers and tip geometries by measuring in liquids with known density and viscosity. To extract the model parameters from the measurement results a curve fitting procedure is performed. The obtained parameters are specific for each cantilever tip and allow now the simultaneous determination of density and viscosity of unknown liquids.

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