

# DISTRIBUTED ENERGY RESOURCE ALLOCATION AND DISPATCH: AN ECONOMIC AND TECHNOLOGICAL PERCEPTION

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## ABSTRACT

Despite the recent easing of electricity wholesale prices, the absolute level of on-peak electricity prices for most markets is tremendously high. The German on-peak electricity wholesale price is about 290% higher than six years ago, which has resulted in tariff hikes. These tariff hikes burden economies worldwide and result in higher inflation or economic cool down. The first part of this paper focuses on market power and the amplified price spikes during on-peak hours especially. A simple model is presented that is able to describe the strategic behaviour of similar market players during on-peak and off-peak hours. Furthermore, the work shows in an easy way how consumers can mitigate market power by creating a short-term demand curve due to load-management programs. We conclude that for a sustainable electricity system without unusually high price spikes, a consideration of the short-term demand curve by using automated systems is important. It is necessary to introduce a technical infrastructure that makes unused load shift potential accessible and gives consumers the possibility to respond to price spikes easily in the short term without sacrificing comfort or services. We present a new automated approach to create such a short-term demand curve. The proposed Integral Resource Optimization Network (IRON) indicates a robust and distributed automation network for the optimization of distributed energy supply and usage. We describe a basic generic model for load shifting, which allows describing a collective storage management in an easy way. Furthermore, the first real implementation result of the research is presented—the so-called IRON-Box, a hardware interface that realises the interface between load resources and the IT infrastructure.

**Keywords:** Demand Response, Distributed Generation (DG), Information Systems, Load Management, Market Issues and Strategic Pricing, Market Power, Real-Time-Pricing

## 1. INTRODUCTION

Energy consumers worldwide have been concerned for the past three years about dramatically increasing energy prices. Before electricity market restructuring in Europe, politicians always emphasized the advantages of liberalization. A very important objective was to provide “cheap” electricity for the European Union and its economy. Now, it looks like the exact opposite happened (see also [7])

Within the last six years, the average electricity wholesale prices in Germany and Austria increased by 285%. The average off-peak prices for the German/Austrian market increased by 270% compared to an increase of 290% for the average

on-peak prices. On-peak prices are increasing faster than the off-peak prices, which indicates market problems especially during on-peak hours. Empirical investigations of several other spot markets show the same pattern. For example the Nord Pool system price shows average on-peak spikes in the 80€/MWh range. Furthermore, the investigations show that many markets get more and more volatile, which increases the uncertainties for market participants (see also [2]). These higher prices on the wholesale markets translate into higher consumer prices (see also [7]). Within the last six years, the average electricity price (including taxes) for industrial consumers in the European Union increased by 31%

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(source: [7]), which burdens the European economy considerably.

Furthermore, these price jumps are not only in the electricity sector, but also in the natural gas sector. Natural gas and oil are used as energy inputs in many electric power plants worldwide, and therefore, the jump in primary energy explains partly the skyrocketing electricity prices. Additionally, the new CO<sub>2</sub> emissions trading system may contribute partly to these higher prices. However, the question is: is it possible that the missing short-term demand curve contributes to the tight volatile market and high prices we see now?

Of course, we might run out of oil and peak in the production very soon, but in the short to mid term the problem certainly does not lie in the problem of peak oil, directly. The problem is constituted by the fact that consumers have no or only limited capability to react to price signals during times with limited supply capacities and high demand. For example, if you live in the outskirts of Los Angeles and you have to get to the city for work, the only possibility you have is to drive. This means your response to high gasoline prices will be almost zero, but this behaviour creates economic inefficiency for the whole society.

In the electricity sector, we have similar conditions. Which consumer in the European Union can respond to real-time prices and change his / her consumption behaviour? Almost no short-term elastic demand curve exists in most of the electricity markets. This problem is basically constituted by the fact that no information flows between suppliers and consumers (in terms of real-time price signals): the market is in an unstable diverging condition.

On the other hand, research reveals that it would be possible to shed peak demand with enhanced automation technologies without loss in comfort. Such systems can create elastic demand curves in a simple way. [3] shows the Albertson enhanced lighting control system, which allows 300 Albertson stores to reduce peak demand up to 7.5 MW. For more information on large facilities and demand-response please refer to [11].

In this work, we present a new approach to create a long- and short-term demand curve that stabilizes the electricity market, which then contributes to CO<sub>2</sub> emissions reduction. The proposed Integral Resource Optimization Network (IRON) indicates a robust and distributed automation network for the optimization of distributed energy supply and usage. Networked consumers, producers, and storage services (e.g. refrigerators) have the (technical) capability and the right to manage – within certain limits – their supply and consumption over time. This pattern will enable previously unused and inaccessible shiftable potentials and directly result in a long- and short-term elastic demand curve that contributes to sustainable energy market equilibria.

## 2. THE THREAT OF AMPLIFIED PRICE SPIKES

Figure 1 shows for the German / Austrian electricity market an average daily wholesale price pattern. The figure denotes the disproportionate increase in on-peak<sup>2</sup> prices compared to off-peak prices.

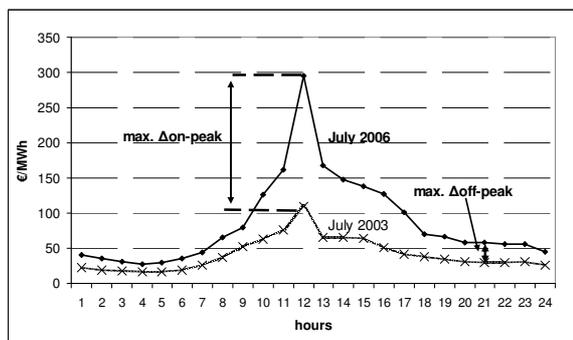


Figure 1: Average German/Austrian daily wholesale prices for selected months. Source: Own database.

The missing possibility of consumers to respond to price signals may provoke the threat of strategic on-peak pricing. The natural gas price – as reference for the marginal power plant - increased by 167% from July 2003 to July 2006, but the average on-peak price increased by 269% in the same period, and therefore, we focus on the threat of amplified price spikes due to market power<sup>3</sup>. To demonstrate this behaviour we will show a simple gaming approach for the Nordic Power market. Additional information according market power in the Nordic power market and German market can be found at [5] and [10]. The theory is based on the assumption that some companies monitor the market conditions and withdraw one or more power plant units to shift the supply curve (see Figure 2) to the left. In this way the intersection between supply and demand can be modified. The suppliers try to control the market price due to faked maintenance. During off-peak conditions, the withdrawal of some power plant units

<sup>2</sup> In practice the definition of on-peak and off-peak depends on the country considered. On-Peak: 08.00 hours to 20.00 hours for Germany and Austria.

<sup>3</sup> We do not postulate that strategic pricing is the one and only reason for these high prices. Other reasons for the disproportional increase can be the CO<sub>2</sub> emissions trading system, different input fuels during on-peak hours as well as lower power plant efficiencies during on-peak hours. We do not investigate the impact of the CO<sub>2</sub> emissions trading system, different input fuels or lower power plant efficiencies in this work.

will not change much. The marginal<sup>4</sup> power plant remains constant, and therefore, no market price change can be created<sup>5</sup>. However, if market players realize that the market is in an on-peak condition and about to use the next type of power plant very soon, the players can provoke a jump to the next marginal power plant (e.g. 62 GW on-peak demand according Figure 2 and Table 1).

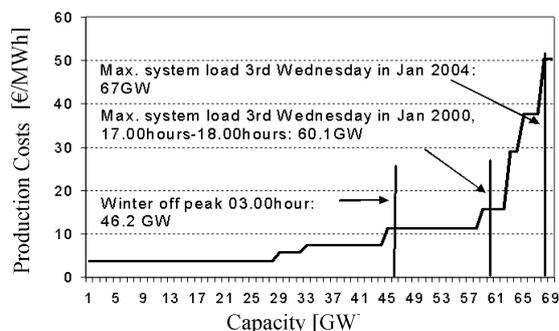


Figure 2: Modified winter supply curve for the Nordic<sup>6</sup> power market, 60 per cent water reservoirs (Supply curve based on installed capacity in 2000 and price data for 2000). Source: [17].

During on-peak conditions the commanding market players need to withhold only few units to provoke a jump to the next market price level. However, this behaviour is supported by the fact that consumers have no or only restricted possibility to react to higher prices in the short term. First of all, almost all consumers have no possibilities to see real-time market prices. How should a consumer know about the conditions on the market when no price signal is received? Furthermore, if the consumer would see a price, then how should he / she respond? Most of the actions would require time and potentially lead to a loss in comfort (e.g. indoor temperature). This means all actions are burdened by transaction costs<sup>7</sup>, and therefore, the reaction to market price changes would be minor in the short-term. Due to this inelastic demand curve, the strategic price setting is an easy game for the commanding market players. However, the creation of an elastic demand curve is exactly the objective of the IRON project and will be discussed in the following chapters.

<sup>4</sup> The market price in a market system always represents the costs of the last used unit (in our case power plant), which is the marginal unit.

<sup>5</sup> Unless the foul-playing supplier withdraws 12 GW to reach the next marginal power plant (according to Table 1 coal).

<sup>6</sup> Denmark, Finland, Norway, Sweden.

<sup>7</sup> For example, transaction costs can be search and information costs incurred in determining price or bargaining required to come to an acceptable agreement with the other party.

In the following sections 2.2 and 2.3, we want to demonstrate the underlying gaming theory in a simple way. To show a simple mathematical model, it is necessary to create two similar players (= suppliers or consortium of suppliers): player 1 and player 2 with similar market shares and power plants. This means, the term “player” refers to the company. The capacity of one power plant unit is assumed to be 0.5 GW.

Table 1: Modified<sup>8</sup> winter supply curve for the following winter examples. Source: [17] and own calculations.

	System Capacity [GW]	Number of Units for Each Player ( $n$ )	Costs in 2000 [€/MWh]	Estimated Costs <sup>9</sup> in 2004 [€/MWh] <sup>10</sup>
Hydro	0-28	0-28	3.78	3.78
CHP Industry	28-32	28-32	5.79	6.00
Nuclear	32-44	32-44	7.55	7.55
CHP district heating	44-58	44-58	11.32	11.50
Coal	58-62	58-62	15.73	18.71
Oil	62-64	62-64	28.93	40.00
Gas	64-67	64-67	37.74	52.80
Others	67-	67-	50.32	50.32

## 2.1 Abbreviations

$d$	Demand [GW]
$i$	Number of player (1,2)
$m$	Power plant units, it is assumed that the units either run at full power or are off-line
$n$	Number of power plant units for each player, $n = m/2$ , integer value (each player operates units with 0.5 GW)
$j$	Number of removed system capacity during off-peak conditions
$\Delta c$	Cost difference if the marginal power plant changes
$\pi_c$	Specific profit for full-supplying player
$\pi_0$	Specific profit for full-supplying player without any gaming activities in the market
$\Delta\pi_f$	Additional specific profit for full-supplying player
$\pi_w$	Additional specific profit for supplier that withdraws a power unit (supplier which acts unlawfully)

<sup>8</sup> The modified supply curve is distinguished from the theoretical supply curve by maintenance and water supply. In this, case we assume 60% water reservoir availability.

<sup>9</sup> Without emissions trading. In our approach we consider only cost differences and therefore emission trading can be neglected for the considered units.

<sup>10</sup> 2004 values are estimated according data from the Energy Information administration. See also[8].

Table 2: Payoff matrix: Specific additional gains during winter on-peak conditions, chicken game

Chicken Game ( $\Delta\pi_{f,w}$ )		$i=2$	
		Unit Available	Unit Not Available
$i=1$	Unit Available	$0_{i=1}/0_{i=2}$	$1320_{i=1}/1299_{i=2}$
	Unit Not Available	$1299_{i=1}/1320_{i=2}$	$1299_{i=1}/1299_{i=2}$

## 2.2 Winter On-Peak, Incremental Plant Not Available

If we assume a peak demand ( $d$ ) of 60.1 GW, then  $m$  has to be 121 (see also Table 1). However, to reach the next incremental power plant it is necessary to withhold more than two units with 0.5 GW (see Table 1). To show the underlying theory, a typical winter situation of the last years (e.g. 2004) is assumed. We assume a demand greater than 61.5 GW. According to this assumption it is sufficient to withhold only one unit to reach the next incremental plant.

$$61.5 \text{ GW} < d < 62 \text{ GW}, n = 62 \quad (1)$$

This means the cost difference ( $\Delta c$ ) can be calculated from Table 1 according to Equation 2.

$$\Delta c = c_{n+1} - c_n = 40 \text{ €/MWh} - 18.71 \text{ €/MWh} = 21.29 \text{ €/MWh} \quad (2)$$

The full-supplying player that does not withhold any unit collects an additional specific gain according to Equation 3.

$$\Delta\pi_f = \pi_c - \pi_0 = (c_{n+1} - c_n) * n = 1320 / \text{MWh}^{11} \quad (3)$$

The player that withholds one unit collects a lower additional gain than the full supplying player.

$$\Delta\pi_w = (c_{n+1} - c_n) * (n - 1) = 1299 / \text{MWh}^{11} \quad (4)$$

This means we obtain a chicken game with the best strategy to cooperate if the other player withdraws a unit and the information about this action is available. There is no reason to fight the other supplier, because the maximum gain is constituted by cooperation (see also Table 2). In this game, one player always wants to do the opposite of whatever the other player is doing. However, if we assume that the action of one party is made without knowledge of the others action, then the safe strategy is always to

<sup>11</sup> To calculate the net gain [€/h],  $\Delta\pi$  has to be multiplied by 0.5 GW.

withhold a unit. In any case, the market price is manipulated and lifted (see Table 2).

## 2.3 Winter Off-Peak

We assume a typical off-peak situation with a demand ( $d$ ) of 46.2 GW.

$$d = 46.2 \text{ GW}, n = 47, m = 94 \quad (5)$$

Normally, the players would withhold the expensive power plant units to allocate high profits for each power plant. However, in this case this would mean withdrawing Combined Heat and Power (CHP) district heating units, which seems difficult during winter months, and therefore, the player has to remove a nuclear power plant unit<sup>12</sup>.

$$32 < j < 44 \quad (6)$$

$$\Delta c = c_{n+1} - c_n = 11.5 \text{ €/MWh} - 11.5 \text{ €/MWh} = 0 \text{ €/MWh} \quad (7)$$

The full-supplying player that does not withhold any unit collects an additional specific gain according to Equation 8 of 0 €/MWh.

$$\Delta\pi_f = \pi_c - \pi_0 = (c_{n+1} - c_n) * n = 0 / \text{MWh}^{13} \quad (8)$$

The player that withholds one unit collects a loss according to Equation 9.

$$\Delta c_j = c_n - c_j = (11.5 \text{ €/MWh} - 7.55 \text{ €/MWh}) = 3.95 \text{ €/MWh} \quad (9)$$

$$\Delta\pi_w = n * \Delta c - \Delta c_j = -3.95 / \text{MWh} \quad (10)$$

The best strategy for both players is to provide the market with all units; all other possible options result in losses for either one or both players

This means market power is suppressed if Equation 11 is true.

$$\Delta\pi_w = n\Delta c - \Delta c_j < 0 \quad (11)$$

<sup>12</sup> If the player withholds the incremental plant then all payoffs in all cells of Table 3 would be zero. This means no additional gains, because of the withdrawal, can be collected.

<sup>13</sup> The type of marginal plant does not change.

Table 3: Payoff matrix: Specific additional gains during winter off-peak conditions

Payoffs for Winter Off-Peak ( $\Delta\pi_{f,w}$ )		$i=2$	
		Unit Available	Unit Not Available
$i=1$	Unit Available	$0_{i=1}/0_{i=2}$	$0_{i=1}/-3.95_{i=2}$
	Unit Not Available	$-3.95_{i=1}/0_{i=2}$	$-3.95_{i=1}/-3.95_{i=2}$

On basis of the criterion above, the following conclusions can be obtained:

- For low demand (e.g. summer) there is minor market power. However, the additional gain is very small (respectively zero), and therefore, the additional gaming risk is higher than the additional gain, so if this risk is internalized, then market power is suppressed according Equation 11 and no price manipulation takes place (a detailed analysis for summer months can be found at [17]).
- There is one real situation with  $\Delta\pi_w < 0$ , if the incremental plant does not change when a unit is withheld (demand curve is far away from the next higher cost level at supply curve, e.g. winter off-peak).

Therefore, a further reason for the observed increase in electricity on-peak prices seems the strategic behaviour of few utilities. However, the incentive to withhold capacities during off-peak hours is minor or not given.

The threat of strategic prices is supported by the fact that consumers have only restricted possibilities to react to price spikes. Therefore, it seems necessary that consumers have the possibility to react to high prices and can reduce the on-peak demand to contribute to the increase of market performance. An elastic demand curve created by e.g. an IRON system can turn a profitable chicken game (withhold units) into a non-profitable game (offer all units) if the consumers shift the demand curve depending on the seen price. This means, the IRON system shifts the demand also to the left and stabilizes the intersection point with the supply curve on a lower level (see also Figure 4). In any case, consumers need to see market prices (tariffs) otherwise there is no information flow and no demand response.

### 3. DISTRIBUTED GENERATION AND LOAD MANAGEMENT

#### 3.1 Definition of the Term Demand-Side Management (DSM)

Investigations performed in course of this work revealed that researchers use different definitions of DSM frequently, and therefore, we want to clarify our definition of DSM.

The term demand-side management (DSM) includes following measures:

- Efficiency-increasing measures (high-efficiency bulbs, efficiency increases in heating systems, low-energy buildings, distributed generation using combined heat and power). The term efficiency-increasing measures is equivalent to the term demand-side measures and these measures lead to real energy reductions during certain periods (e.g. year) and consequently to CO<sub>2</sub> emissions reduction.
- Load-management measures (time-of-use tariffs<sup>14</sup>, real-time-pricing, interruptible loads, internet-controllable loads).

In contrast to efficiency measures, load management measures are merely load-shifting measures because the shifted on-peak power (or energy) is consumed during off-peak hours. These measures are used to manage shortfalls in supply during on-peak hours. No major CO<sub>2</sub> reduction is achieved<sup>15</sup>. Loads that allow such alterations in their consumption patterns without loss in consumer comfort are discussed in Section 4.4.

#### 3.2 Synergy between Efficiency and Load-Management Measures

Important is that distributed generation (DG) can be an efficiency measure and/or a load-reduction<sup>16</sup> measure depending on the specific usage pattern of the DG technology and depending on the use of combined heat and power (CHP). If DG with CHP is installed, then the overall system efficiency increases in general, and therefore, the emissions are reduced. However, some DG technologies can be used during times of high electricity prices to reduce the electricity demand. In other words, low-efficiency DG technologies such as reciprocating engines without CHP can be used to create an elastic demand curve and to reduce utility-delivered electricity by operating during times of high electricity prices. The usage of such inefficient technologies might decrease the overall system (e.g. micro-grid) efficiency. Figure 3 illustrates these two different basic options for a generic healthcare facility in San Francisco.

Pacific Gas & Electric (PG&E)-the utility serving San Francisco-charges an additional demand

<sup>14</sup> Time-of-use tariffs and real-time-pricing can result in efficiency increasing measures. Without any price signal no incentive for the customer would exist.

<sup>15</sup> Please note that a shift in the load curve increases the off-peak load, and therefore, it can increase the off-peak power plant efficiency by using more efficient generators. This effect can also result in CO<sub>2</sub> reductions, but this effect is not the main focus of this paper.

<sup>16</sup> Load reduction is seen by the utility.

charge<sup>17</sup> of 2.65 \$/(kW month) during mid peak hours<sup>18</sup> on winter days<sup>19</sup> and 11.8 \$/(kW month) for summer months<sup>20</sup> during on-peak<sup>21</sup> hours, and therefore, the optimal economic decision for the healthcare facility is to install a 500 kW reciprocating engine without CHP and run it during mid-peak and on-peak hours to reduce the demand charge related costs (see also Figure 3). In this example, the 500 kW engine creates an elastic demand curve – seen by the utility - without increasing the system efficiency.

Additionally, the healthcare facility has to install a 500 kW reciprocating CHP system and to run it 24 hours a day to increase the overall system efficiency and to decrease the energy-related costs. The 500 kW internal combustion CHP system is a demand-side measure that increases the efficiency. The increase in efficiency is realized by using waste heat for hot water and absorption cooling. Please note that prior to the DG installation electric chillers were used in the building. The optimal investment uses waste heat for absorption cooling and therefore cooling (electricity) is being offset by waste heat (see also Figure 3<sup>22</sup>).

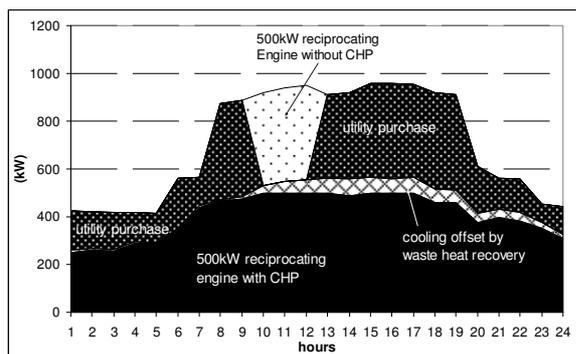


Figure 3: Optimal<sup>23</sup> electricity supply structure for a healthcare facility in San Francisco on a January weekday. Source [17].

<sup>17</sup> They are proportional to the maximum rate of electricity consumption (regardless of the duration or frequency of such consumption).

<sup>18</sup> 9 am to 12 pm

<sup>19</sup> October to May

<sup>20</sup> June to September

<sup>21</sup> 12 pm to 6 pm

<sup>22</sup> Please note that there is also a heating offset due to waste heat, but this offset is not shown in Figure 3.

<sup>23</sup> Such optimal investment and operation decisions can be found with DER-CAM (Distributed Energy Resources Customer Adoption Model) from Lawrence Berkeley National Laboratory. It is a mixed-integer linear optimization program (MILP) written and executed in the General Algebraic Modeling System (GAMS). The objective is to minimize annual energy costs for the modeled site, including utility electricity and natural gas costs, amortized capital costs for DG investments, and maintenance costs for installed DG equipment.

In this way, the overall system efficiency can increase to 80%. A detailed description of the shown healthcare facility can be found at [17].

However, instead of the inefficient 500 kW reciprocating engine, a load-management system can kick in and reduce / shift parts of the load without losses in system efficiency. This means the combination of CHP-enabled technologies and load-management measures (e.g. interruptible loads) to reduce on-peak demand seems the most favourable option for a distributed automation network as the IRON System. The combination of load management and CHP-enabled DG results in amplified benefits for the considered site. This means, there is a synergetic relationship between efficiency measures and load-management measures (for more information see also [4] and [17]).

With the IRON System, we want to establish a real optimization algorithm for distributed supply and demand. This new algorithm can be applied to a single site or a combination of distributed sites.

#### 4. THE DERIVATION OF THE DEMAND CURVE

##### 4.1 Long-Term Elastic Demand Curve V.S. Short-Term Elastic Demand Curve

Based on the description of efficiency measures and load-management measures from Section 3 long-term and short-term demand curves can be created.

The short-term demand curve reflects reactions to price changes without any investment in demand-side measures. In contrast to the short-term demand curve, the long-term demand curve represents all reactions to price changes with investments in demand-side-measures.

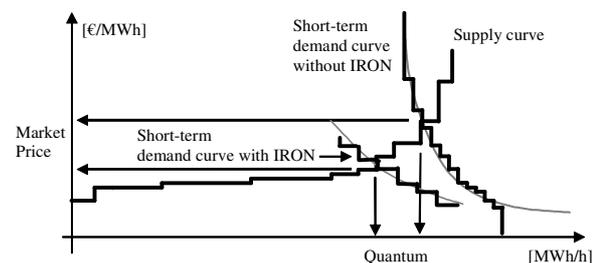


Figure 4: Principal short-term demand curve with and without an IRON system

Naturally, without any automatic system (e.g. IRON system), the short-term demand curve is very steep due to transaction costs and the loss in comfort. Figure 4 shows a principal short-term demand curve. Each small horizontal step indicates a certain measure (e.g. decrease in heating set-point). However, the level (costs in €/MWh) indicates the associated transaction costs and defines the loss in comfort for

each short-term action. For example, simple measures as turning-off lights can be very easily achieved without much loss in comfort and without high transaction costs. If the demand reduction is not enough and additional measures are required, then more inconvenient actions are needed. Such actions can be reduction in heating set-point temperature, which results in a loss in comfort, and therefore, higher costs. Additionally, most of the "expensive" actions result in minor reductions, and in this way, we get a very steep short-term demand curve.

Investigations performed in other projects show very limited potentials for load shifting or reduction because of such simple measures – without an IRON (see also [15]). Equation 12 estimates<sup>24</sup>-based on empirical investigations in Germany–the short-term demand curve for household costumers (without an IRON system).

$$\text{Load reduction} = 3.87 \times \ln\left(\frac{p_{\max}}{p_{\min}}\right) + 4.08 \quad [\%] \quad (12)$$

$p_{\max}$  maximal charged tariff during a day  
 $p_{\min}$  minimal charged tariff during a day

From this, a 10% reduction in demand is reachable only if the customer recognizes a 500% higher electricity tariff than normal (off-peak). If enough consumers react to high prices a feedback loop will be created that is reducing the electricity price due to lower demand. A detailed analysis about the reachable price reduction due to load reduction can be found at [15] and [16].

To maximize the short-term effect of high prices, more flexible consumer behaviour has to be created by using automated systems (e.g. IRON) for load shifting / reduction that minimize the loss in comfort.

For more information on the reachable load reduction for large costumers through automated systems, please look at [3] and [11].

#### 4.2 Achieving a Short-Term Elastic Demand Curve Due to Load-Management

Increased elasticity of the consumer demand implies that consumers are able to adapt their consumption in accordance to the respective electricity price. In order to achieve a demand curve with a maximum of elasticity, two conditions have to

be fulfilled. First of all, the on-site potentials to shift load in time or even perform load shedding have to be utilized optimally, i.e. potential resources are used as much as possible, but only to that extend that the comfort of the user is not negatively influenced. Secondly, since this optimal utilization is demanding the process should be self-controlled. However, the fulfillment of these two conditions imposes transaction costs. This means a maximum of flexibility in the electricity demand is not necessarily equal to the social-economic optimum.

The key concepts leading towards a flexible short term demand curve are load shedding and load shifting / energy storages. In case of load shedding, loads are simply switched off. Performing load shedding during on-peak consumption periods is a simple measure. Energy storage in general can be realized by actual (i.e. "real") storage or by conceptual storage, i.e. load shifting. Here, the demand-side flexibility is constituted in the possibility to schedule a consumption process freely within certain restrictions defined by the considered application. Still, a reduction of the consumption is performed (e.g. during on-peak times), but the consumption is delayed only until more supply is available. For more information on the economics of load shifting, please look at [16].

#### 4.3 Utilizing Consumption Processes

Load shifting cannot be applied to every type of load. For example, the electricity demand of an elevator cannot be shifted in time. However, for many loads a delay in operation of few minutes does not matter, e.g. an electric hot-water boiler. For some loads, even longer delays up to few hours can be acceptable, e.g. refrigerators and domestic dishwashers or washing machines. However, the load-shifting capabilities for individual loads vary a lot and also depend on real-time factors such as changing hot-water demand. Additionally, loads with delay times of only few minutes were considered as inappropriate for effective load shifting in the past.

Another group of electrical loads that can also be used to achieve a more elastic demand curve are inert thermal process loads. These processes can be categorized into heating applications (electrical heaters, domestic and industrial ovens, etc.) and cooling applications (air-conditioning, refrigerators, freezers, etc.). All these thermal processes have in common that they are able to store thermal energy due to the heat capacity of a room or a thermal-isolated box. This heat capacity can be utilized for shifting electrical energy consumption if the temperature set-points of the process allow a certain amount of variation. When the temperature set-point for a cooling process is decreased, the system consumes more energy  $E'$  than in average ( $E$ ) for the time needed to reach the lower temperature

<sup>24</sup> Please note that the estimation of such load reductions is very difficult and is subject to high variations. Equation 12 is a rough estimation of the principle logarithmic shape based on 5 case studies performed in Germany. Investigations performed for industrial costumers show the same principle logarithmic shape, but show also high variations within the industrial sector. For more information, please look at [15].

set-point. This circumstance is depicted in Figure 5 (middle) for the case of a two-point regulated cooling system. When the original set-point is restored, the system consumes less energy ( $E''$ ) than in average for a certain time period (see Figure 5, bottom). Thus, a load shift is performed by employing thermal energy storages. The advantage of thermal storage systems is that the possible storage time is normally very long or even unrestricted.

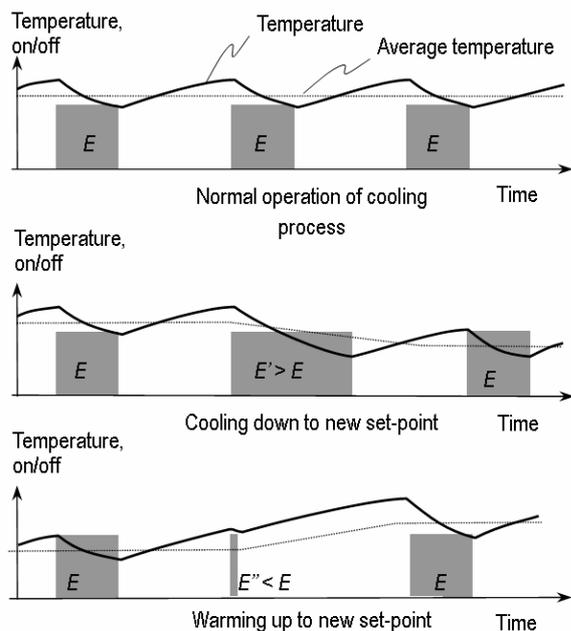


Figure 5: Storing into and releasing energy from a two-point-regulated cooling process

When changing temperature set-points, also the thermal losses of the system are influenced. These losses depend on the difference of inside and outside temperature and the quality of thermal isolation. Lower temperature differences result in a more efficient system, higher differences in a less efficient system. This means for the model of thermal storage devices that the storage should only be charged when needed. Further, it can be seen from Figure 5 that the switching activity of the “thermal pump” (which can be realised in many different ways) is generally not changed by the measures described here. Only the on and off times are changed. Although not included in the simplified example discussed here, costs of switching activities or set-point deviation can be also easily incorporated into the optimisation problem.

Although the described measures are not directly electrical energy storage techniques, they can potentially be used to operate in the same way or provide the same service as real electrical energy storages such as pumped storage schemes. Such direct electrical energy storages can be also Vanadium-Redox batteries or even flywheels (see also [12] and [6]). The costs for setting up such direct storages can be significantly higher than making use

of existing consumption processes. Moreover, pumped storage schemes cannot be realised everywhere because of geographical restrictions.

The overall consumption pattern of the load shift action can be described as the superimposition of the unmodified process and a storage pattern. The fact that load shifting can conceptually be described as storing and releasing energy is exploited by the generic description model presented in the next section.

#### 4.4 A Generic Model for Load Shifting

When implementing a self-controlled system performing the previously discussed measures of load shedding and load shifting, all (distributed) participating loads can be seen as resources. Operating the system does basically mean to solve the problem of optimal resource allocation and dispatch. A preferably simple and consistent description of the resources involved is crucial for the dispatch algorithm to be efficient and flexible.

The first step towards such a generic model is to describe both techniques of load shedding and load shifting / utilisation of thermal energy storage as special cases of a general class of conceptual electricity storages. For thermal processes, this has been discussed already.

Figure 6 gives an overview of basic storage characteristics. Having mapped the different options of load management to conceptual storage characteristics, each activity can be modelled with an individual set of parameters

$\{P_0, T_{charge}, T_{uncharge}, T_{store}, T_{nostore}\}$ , where

- $P_0$  the power amplitude,
- $T_{charge}$  the time to charge the conceptual storage
- $T_{uncharge}$  the time to discharge the conceptual storage (in some cases this time is smaller than  $T_{charge}$  due to losses in the process)
- $T_{store}$  the storage time
- $T_{nostore}$  the minimum time between two storage operations (not depicted in Figure 6)

More complex procedures, e.g. those where different power amplitudes are involved for the conceptual charging and discharging can be described by superposing multiple scaled instances of the basic prototypes<sup>25</sup>.

<sup>25</sup> Please note that a pre-charge storage basic concept also exists, but is not used in this work. Basic load shifting, load interruption, and load shedding can be described with a post-charge storage only.

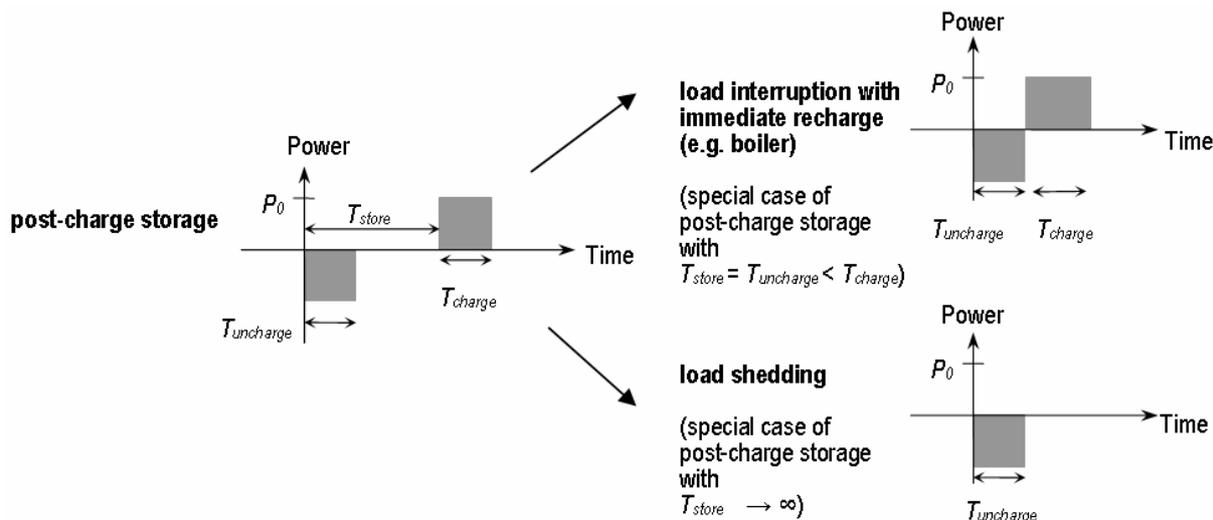


Figure 6: Basic Storage Characteristics

**4.5 One More Degree of Freedom**

The major application for load management in our perspective is peak-load reduction due to load shifting. Peak-load reduction will be taken as an example for the following discussion, but the general concept can also be applied to other demand-side applications, such imbalance energy and other auxiliary services.

Storage should hold energy in times of low electricity prices and release it in times of peak demand and prices. This is valid for real storage such as pumped storage schemes as well as conceptual storages as described in the previous section. Conceptual storages are available in large numbers, but they are distributed and have restricted individual storage capacities. Furthermore, many distributed storages have very short storage times, which appears to be one reason that utilization of distributed storages was regarded as ineffective in the past. However, the collective (because networked) storage capability is considerable. Each conceptual storage  $i$  can hold the energy;

$$E_i = P_{0,i} \cdot T_{charge,i} \tag{13}$$

If  $E_i$  is seen as an "energy packet", which can reside in a conceptual storage for a certain time  $T_{store,i}$ , then the objective of peak-load reduction can be re-formulated: shift as many (and as large) energy packets as possible from off-peak to on-peak times. This rather sophisticated description of on-peak demand reduction leads to a new insight, which is simple, but allows an additional degree of freedom in resource allocation: if a single (conceptual) storage is not able to hold an energy packet long enough, then it can be transferred to other storages after the storage time – of the initial storage – has expired. Figure 7 shows a simplified example of four different duty-cycled consumption processes (e.g. refrigerators

or air-condition systems). The first process stores additional thermal energy to release it (by reducing its power consumption) after  $T_{store,1}$ . At the same time, the second process loads its thermal storage to release the energy after  $T_{store,2}$ . This is repeated for processes 3 and 4. The total effect of this schedule is the same as that of a single storage with

$$T_{store,total} = T_{store,1} + T_{store,2} + T_{store,3} + T_{store,4} \tag{14}$$

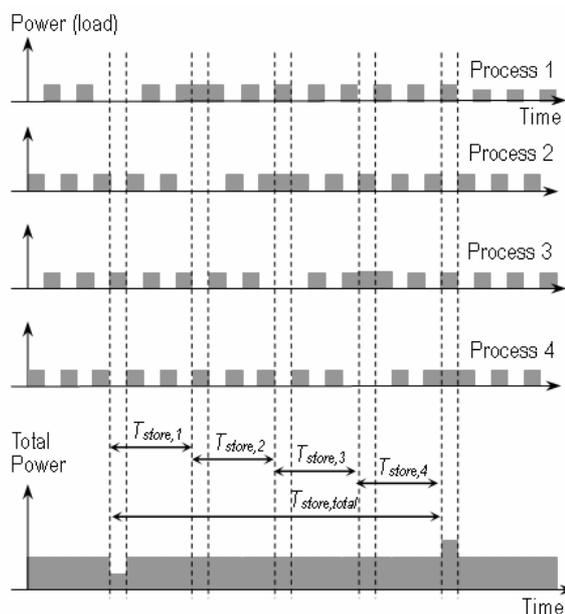


Figure 7: Increasing the storage time by utilizing multiple conceptual storages: four individual load shifts (top) accumulate to one long shift operation (bottom)

Losses (not shown in Figure 7) occur in all consumption processes, but the handoff of energy packets from one process to the next does not cause additional losses. This is due to the fact that energy is not actually flowing from one process to the other

process nor is electrical energy factually stored in a releasable way. In fact, just a subtle coordination of DSM resources is applied in order to achieve a modulation of the overall power consumption that is equal to that achieved by a real storage.

With this new approach, we can achieve larger capacities by means of parallel operation and longer storage times by means of serial operation. Since a large number of single resources can be utilized, any combination of both options is possible. If all resources have the same properties, then all achievable combinations would be given by Equation 15,

$$E \cdot T_{store} = const. \quad (15)$$

meaning that the total energy stored in the system  $E$  and the duration it can be stored  $T_{store}$  are two parameters that can be chosen freely as long as their product stays constant. This constant is system-specific.

But since each individual resource  $i$  has its own set of  $\{E_i, T_{store,i}, T_{nostore,i}\}$ , only the summation of all individual terms is constant;

$$\sum_{\forall i} \underbrace{E_i \cdot T_{store,i}}_{K_i} = \sum_{\forall i} K_i = const. \quad (16)$$

$K_i$  describes the abstract storage potential of one individual conceptual storage  $i$ . This can be clarified with the following example: The objective of shifting the energy  $E$  from  $t$  to  $t+T$  can be achieved in two different ways using two different basic resources with different  $E_i$  and  $T_{Store,i}$ . First, each resource is able to store  $E_i = 1/2E$  for  $T_{Store,i} = T$ , or secondly, each resource is each able to store  $E_i = E$  for  $T_{Store,i} = 1/2T$ . For both cases,  $K_i = 1/2E \cdot T$  is the same.

#### 4.6 Collective Storage Management

Having described a unified description of DSM resource behaviour, this can be used to generate optimal schedules for the distributed storage system. The term "optimal schedule" is used in a more specific way here. We assume an objective that shall be achieved by the DSM resource collective. The collective will not be able to meet the objective perfectly under any circumstances. For an optimal schedule, the distance to the objective (according to a certain distance metric) is minimized. The objective function can be individual energy cost reduction. In this case, each individual storage does its best to reduce consumption in high-price times. A storage with arbitrary storage time examines the anticipated price curve and searches for the lowest and highest point. Then, it charges at the minimum and discharge at the maximum of the price curve.

Alternatively, a resource with restricted storage time searches for the highest slope in the prospected price curve, since here the maximum gain can be achieved.

More sophisticated resource scheduling is needed for another kind of objective: load curve following. The idea here is to control all distributed resources as they were one large single storage<sup>26</sup>. A large collective can have considerable capabilities and can e.g. be used for peak-shaving. The collective behaviour is controlled by defining a load curve that is followed (as good as possible) by the distributed resources. This is particularly interesting in the context of demand charges, where a large percentage of the energy bill is constituted by costs for the largest monthly or yearly peak demand (see also [17]).

In Figure 8 an example for optimal load curve following is depicted. Twenty thermal processes, A01 to A20, with similar properties (arbitrary storage time, equal storage energy) were assumed in this example. Nevertheless, similar results can be achieved with other storage types, i.e. with restricted storage time. The comparison of specified and achieved load curve is shown at the top. Below that, the charge / discharge schedule for some of the resources is depicted. Not shown in Figure 8 is the actual power consumption of the loads used as DSM resources, which would simply add up to a constant offset.

This schedule is the result of an offline optimization. A precise mathematical description of the resource behaviour constrains the problem of minimizing the integral difference to the target curve<sup>27</sup>. In a real-world application, a more sophisticated algorithm is needed that still uses offline results as a guideline but is able to react on real-time events such as resource outage due to e.g. communication failure or unexpected discharge. The detailed algorithm can be found at appendix A.

## 5. THE IRON INFRASTRUCTURE

### 5.1 Communication within the Electricity System

The key to the Integral Resource Optimization Network is the self-controlled operation of DSM resources, and therefore, it heavily relies on a communication infrastructure. The control strategies

<sup>26</sup> This concept is somewhat similar to the concept of virtual power plants, where (primarily) distributed generators are controlled as a single unit. However, in our approach the focus is on storage and not on generation.

<sup>27</sup> The mixed-integer optimization problem was described using ZIMPL [9] and solved using SCIP [1].

can contain energy price broadcasting or precisely coordinated operation of distributed storages (and supply capacities). The rapid growth of information technology (IT) related services and equipment over the past decades went along with a strong decline in costs for IT equipment. Hence, communication systems needed for coordinated DSM were already considered to be affordable in the 1990s (see also [14]). However, by now there is merely no information flow in the electricity systems besides remote control of transformer stations and data accounting. We see the reasons for that in the highly diversified interests of different players such as distribution grid operators, energy providers and

consumers, and in the lack of standardization of remote metering and control systems. In the frame of the IRON project, a common information infrastructure is specified that is able to meet the needs of distributed DSM algorithms and can handle the vast growing number of connected sites. The challenge is to identify a suitable and flexible network topology together with a protocol that is open for future enhancements. Furthermore, the connection interface between the IRON and the electrical equipment has to be specified (see also Figure 9).

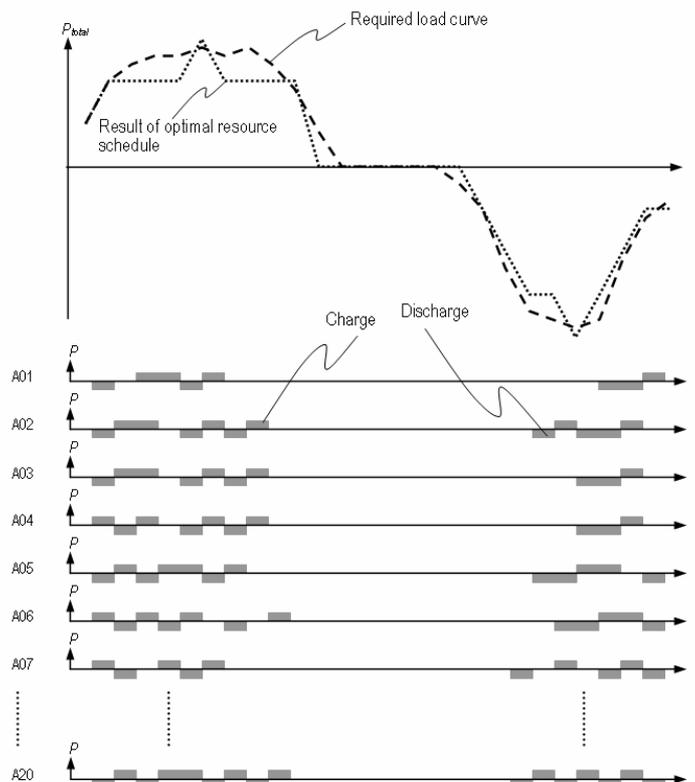


Figure 8: Example of twenty similar DSM resources optimally scheduled so that that the sum of charging and discharging powers follow a given load curve as close as possible. Only the differential power consumption is shown

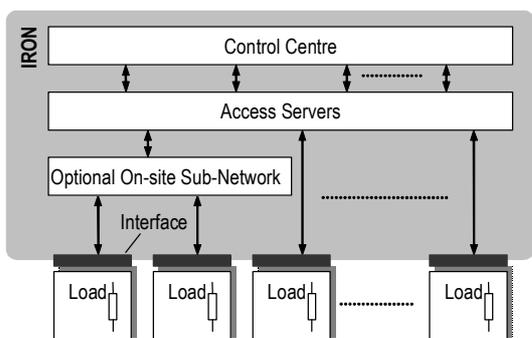


Figure 9: Structure of the prospected IRON communication system.

Given the possibly large number of single communication nodes that are going to be connected by the IRON communication infrastructure, only a hierarchical network structure appears useful (see Figure 9).

Due to cost restrictions, the top-level (long-distance) infrastructure can be implemented only by using existing communication networks, predominantly the Internet and Telecom networks. Different types of end-user equipment and on-site sub-networks are connected to the access servers of this top-level network. The size and structure of these sub-networks depend on the kind of entity: for an industrial plant an already present automation

infrastructure can be used, while for an office building an already present building automation network can be used. In case that no other automation infrastructure is present, the interface between load and IRON is realized by using an “IRON-Box”.

## 5.2 The IRON-Box

The IRON-Box realises the interface between the DSM resource (e.g. load) and the IRON. Since freely configurable equipment for load management is not available as COTS (commercially off-the-shelf) products, the IRON team has been developed a prototype, which is depicted in Figure 10. The prototype serves as hardware equipment for experiments as well as a basis for estimating the costs for a commercial production. The device features optional WLAN (Wireless Local Area Networks) or mobile network (General Packet Radio Switched Network, GPRS) data connection on the infrastructure side. A small on-board processor manages the communication flow and local control loops. On the load side, two channels with one relay output, three digital inputs and one power sensor each are available. Using the power sensor, the IRON-Box can measure the power consumption patterns of the connected load and generate statistics about the load utilization, which can be used to optimize load schedules. The relays allow switching on / off the connected loads according to the optimization algorithm used.

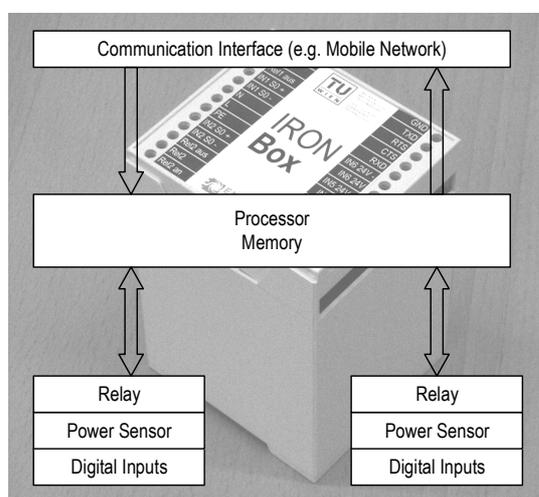


Figure 10: Main components of the IRON-Box

Additionally, measurement equipment (e.g. energy meters or temperature sensors) can be connected to the digital inputs. The IRON-Box can work together with (or even replace simple) controllers for air conditioning systems or refrigerators.

The experimental unit has a size of 75 x 70 x 110 mm. It consists of 120 components that account

for material costs of 230 Euro (single unit in very low volume). Still, large potentials for optimization in terms of size, complexity and costs exist. Considering prices for comparable mass-market products, we anticipate that the device could be manufactured for costs well below 100 Euro.

## 6. CONCLUSIONS

Load-management measures, used to manage lack in supply during on-peak hours, allow alteration in the consumption pattern without loss in consumer comfort and can create a short-term elastic demand curve. Such measures can be coordinated by an Integral Resource Optimisation Network—“IRON”, which indicates a robust and distributed automation network for the optimization of distributed usage and supply.

Because of the current situation on the electricity markets and, particularly, due to the lack of an elastic demand curve in the short term, the threat of strategic prices is increasing. It seems necessary that consumers have the possibility to respond to high prices and can reduce the on-peak demand in order to contribute to the increase of market performance. An elastic demand curve, which can be achieved by using the discussed IRON System, can turn a profitable chicken game (withhold units) into a non-profitable game (offer all units) if the consumers shift the demand depending on the charged price (e.g. real-time pricing). In any case, consumers need to see tariffs based on the market prices otherwise there is no information flow and no demand response.

Additionally, the combination of CHP-enabled technologies and load-management measures to reduce on-peak demand seems the most favourable option for a distributed automation network as the IRON. With the IRON system, we want to establish a holistic optimization algorithm for distributed supply and demand. This new algorithm can be applied to a single site or a combination of distributed sites.

The IRON System offers a new solution to integrate participants which have been considered unreachable in conventional structures of contemporary electricity markets. It has been shown that the potential of distributed resources can be exploited by applying the concepts of storage modelling on load shift measures as well as serial and parallel operation of such real or conceptual storages. Despite strong restrictions in individual resources, the collective system is very flexible.

The fact that all participants are equally capable of exerting influence on the consumption and use of the electrical “resources” means a great economic and socio-economic benefit. This reduces the impact of disadvantages and problems caused by the insufficiently liberalized contemporary electricity markets without a short term elastic demand curves.

Unlike traditional means of demand side control the described IRON System exploits unused optimization potentials without negatively influencing the customers' processes or comfort. The system acts in the background, in a distributed, discrete and flexible manner. The implemented hardware prototype (IRON-Box) that interfaces an individual load with the IRON System allows cost estimates for the IRON infrastructure. Given the current situation of rapidly decreasing costs for information and communication technology on one hand and steadily rising energy prices on the other hand it is just a matter of time when the benefit of coordinated short-term DSM measures is broadly realized.

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**Michael Stadler** joined Berkeley Lab as a student in 2002, returned as Post-Doctoral Fellow in 2005, and is currently a Visiting Scholar. In December 2006 he founded the Center for Energy and innovative Technologies, which he has been heading since then. He also supported the University of California, Berkeley's Pacific Region CHP Application Center, where he conducted site analyses of varied commercial, agricultural, and industrial CHP projects. Previously, he worked with the Energy Economics Group at the Vienna University of Technology, from which he holds a Masters degree in electrical engineering and a Ph.D. summa cum laude in energy economics. His fields of research are distributed energy, electricity markets, and demand response.

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## APPENDIX: MODEL USED FOR OPTIMIZATIONS

As outlined in Section 4.4, DSM resources can be modelled by the linear superposition of basic storage characteristics. Consequently, it is possible to translate the scheduling problem of DSM resources into a linear optimization problem. In order to avoid the following discussion becoming unnecessarily complex, the optimization problem shall only be discussed for resources with arbitrary storage time  $T_{store}$ .

DSM resources can either be charged or discharged, resulting in a positive or negative contribution to the load profile. Two series  $c_{n,i}$  and  $d_{n,i}$  can be defined, so that

$$c_{n,i} = 1 \text{ for } n \text{ is the centre time slot of a charging process for resource } i, \text{ or } 0 \text{ otherwise,} \quad (17)$$

and

$$d_{n,i} = 1 \text{ for } n \text{ is the centre time slot of a discharging process for resource } i, \text{ or } 0 \text{ otherwise.} \quad (18)$$

It is useful to split charging and discharging into two series, so that the total number of charging processes per resource can easily be restricted by expressions like

$$\sum_{\forall n} c_{n,i} < m \quad (19)$$

without having to operate with absolute values. Nevertheless, a single series

$$a_{n,i} = c_{n,i} - d_{n,i} \quad (20)$$

can also be defined, that includes both charging and discharging information. In  $a_{n,i}$ ,  $c_{n,i}$ , and  $d_{n,i}$ , only one element is non-zero for each charging or discharging process.

Each resource has its individual charge and discharge profile. These profiles can also be described as series. The convolution of the impulse series  $c_{n,i}$  and  $d_{n,i}$  with these time-discrete load profiles  $f_{n,i}$  and  $g_{n,i}$  result in the actual (time-discrete) power consumption  $p_{n,i}$  of the resource  $n$  as described in (21). This equation is actually a linear constraint equation to the optimization problem since it constraints  $p_{n,i}$  in regard to the previously defined variables.

$$p_{n,i} = c_{n,i} * f_{n,i} - d_{n,i} * g_{n,i} = \sum_{k=-\infty}^{+\infty} c_{k,i} f_{n-k,i} - \sum_{l=-\infty}^{+\infty} d_{l,i} g_{n-l,i} \quad (21)$$

When calculating the convolution, the sum index has only to cover those terms that are non-zero. In the following discussion it will always assumed that all processes are fully scheduled in the time interval  $[0, T_{end}]$  with no process overlapping the borders of this interval. In this case, the convolution can be calculated as shown in (22) and thus has a finite number of terms.

$$a_n * b_n = \sum_{k=0}^{T_{end}} a_k b_{n-k} \quad (22)$$

Furthermore, the energy level  $s_{n,i}$  of resource  $n$  can be defined as

$$s_{n,i} = \sum_{t=0}^n p_{t,i}. \quad (23)$$

To sum up the previous variable definitions, the impulse series  $c_{n,i}$  and  $d_{n,i}$  are the actual unknown variables defining the schedule of charging and discharging processes that shall be determined by the optimization. Power  $p_{n,i}$  and energy level  $s_{n,i}$  are derived from them and are needed in the objective function and further constraints respectively.

The objective function subtracts the achieved overall load profile  $p_{tot,n}$  from the required profile  $p_{req,n}$ :

$$\sum_{n=0}^{T_{end}} |p_{tot,n} - p_{req,n}| = \sum_{n=0}^{T_{end}} \left( \sum_{i=0}^N p_{n,i} \right) - p_{req,n} \rightarrow \min, \quad (24)$$

(where  $N$  is the total number of resources). The absolute value of the difference is used in order to keep the problem piecewise linear (it is possible to translate (24) into a set of linear equations with

restricted index ranges [9]). From an application perspective, the square of the difference would be preferable, but in that case the problem cannot be solved by solver for linear problems.

Further constraints of the problem are restricted storage capacity:

$$S_{\min,i} \leq s_{n,i} \leq S_{\max,i} \quad (25)$$

restricted charge and discharge frequency: the impulses in  $c_{n,i}$  and  $d_{n,i}$  must not come too close to each other (refer to  $T_{\text{nostore}}$  in Section 4.4). The resource may need some time to settle on a certain energy level until the next set-point change can occur. A possibility to express this in a linear equation is to make use of a distance series

$$\delta_{n,i} = 1 \quad \text{for } n = 0 \dots T_{\text{charge}} + T_{\text{nostore}}, \quad (26)$$

$$0 \text{ otherwise,}$$

and to restrict the amplitude of the convolution of distance series and impulse series. Depending on the restrictions of actual resources it may be necessary to define separate  $\delta$  series for charging and discharging or even to formulate more complex constraints.

$$t_{n,i} = (c_{n,i} + d_{n,i}) * \delta_{n,i} \leq 1 \quad (27)$$