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Improved Design of Uniform and Nonuniform Modulated Filter Banks

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ABSTRACT

In this paper, we describe an improved design method for multirate modulated filter banks with uniform or nonuniform frequency spacing. Compared to other design techniques, our method converges very rapidly and allows for the design of excellent prototype filters. The optimization procedure avoids nonlinear function minimization and requires linear equation solving only. We demonstrate our method by design examples of filter banks for critical-band analysis/synthesis.

1 INTRODUCTION

In the past few years the design of nonuniform filter banks has gained much attention for applications in multiresolution spectral analysis and speech coding systems. The analysis of multirate nonuniform filter banks has been intensively studied and various design theories have been proposed [5, 8, 9, 10, 11]. Although the principal design issues like perfect reconstruction properties and selection of subsampling factors have been investigated in detail, the problem of finding the coefficients of the analysis and synthesis filter bank stages is still under debate. This is due to the fact that nonlinear optimization techniques with a large number of variables are involved. Consequently, convergence problems and the occurrence of local minima may hamper the finding of an adequate solution which must show a high stopband attenuation for the individual channel filters and a small overall reconstruction error.

In this paper, we present an improved design for near-perfect reconstruction (NPR) critically sampled modulated filter banks. The design technique is less prone regarding convergence to local minima than conventional methods. The improvement is obtained by a special optimization procedure which is discussed in Section 2.1. NPR modulated filter banks offer significant advantages for design and implementation. First of all, the design freedom is largely reduced because only a small number of different prototype filters is needed. In the case of uniform filter banks only a single prototype filter has to be designed. In addition, computationally efficient and memory saving implementations are possible [1, 3, 4].

The extension of our design procedure to nonuniform NPR modulated filter banks is outlined in Section 2.2. The basis for our design technique is the transition filter concept as proposed in [10]. However, we use a different formulation and solution of the optimization problem involved in this method.

In Section 3, we present design examples of nonuniform filter banks for critical-band analysis/synthesis. Such filter banks are needed in a wide variety of applications like speech coding and speech enhancement.

2 DESIGN PROCEDURE

In this paper, the design of uniform NPR modulated filter banks is based on the concept of cosine-modulating a lowpass filter to obtain the analysis and synthesis stage filters (see e.g. [1]). As a consequence, the design of these filter banks is reduced to the design of a single prototype lowpass filter which normally has linear phase and a finite length impulse response (FIR-filter).

In the case of nonuniform NPR filter banks the concept of modulating lowpass filters to get the channel bandpass filters can also be applied [10]. In order to limit the number of prototype filters needed, however, a piecewise approximation of the nonuniform frequency spacing by means of separate uniform filter bank sections is employed. Transition filters are used to join the individual sections.

2.1 Uniform Filter Banks

The zero-phase response $H(\mathbf{h},\theta)$ of the linear phase prototype FIR-filter may be represented by

$$H(\mathbf{h},\theta) = \sum_{n=0}^{\frac{L}{2}-1} 2h(n)\cos\left[\left(n-\tfrac{L-1}{2}\right)\theta\right] \qquad (1)$$

for an even filter length L, and

$$H(\mathbf{h},\theta) = h\left(\tfrac{L-1}{2}\right) + \sum_{n=0}^{\frac{L-1}{2}-1} 2h(n)\cos\left[\left(n-\tfrac{L-1}{2}\right)\theta\right] \qquad (2)$$

for L odd, respectively. The first half of the symmetrical FIR filter impulse response is stored in the coefficient vector $\mathbf{h} = (h(0), h(1), \ldots, h(\lfloor \frac{L-1}{2} \rfloor))^T$. The prototype lowpass filter of N-channel uniform NPR modulated filter banks must show a high stopband attenuation for $\theta > \theta_s$, with $\frac{\pi}{2N} < \theta_s \leq \frac{\pi}{N}$. In addition, $H(\mathbf{h}, \theta)$ must obey the following relationship:

$$T(\mathbf{h}, \theta) = [H(\mathbf{h}, \theta)]^2 + \left[H\left(\mathbf{h}, \theta - \tfrac{\pi}{N}\right)\right]^2 = 1, \quad (3)$$

with $\theta \in [0, \frac{\pi}{2N}]$ (see e.g. [1]). The filter may be designed by minimizing the error measure

$$\mathcal{E}(\mathbf{h}) = \int_0^{\pi/2N} [T(\mathbf{h}, \theta) - 1]^2 d\theta + \alpha \int_{\theta_s}^{\pi} [H(\mathbf{h}, \theta)]^2 d\theta. \quad (4)$$

The influence of the stopband energy is controlled by the weighting factor α. Instead of using nonlinear function minimization to find the optimum prototype filter impulse response, we use a linearization technique which reduces the computational effort to linear equation solving. Our method is an extension of the design of 2-channel non-modulated filter banks as outlined in [7] (frequency domain approach) and [2] (time domain approach), respectively. The linearization is obtained by the following splitting of the squared zero-phase response:

$$[H(\mathbf{h}, \theta)]^2 \approx H(\mathbf{h}_{k-1}, \theta) H(\mathbf{h}_k, \theta), \quad (5)$$

where $k = 0, 1, 2, 3, \ldots$ is an iteration index. If Eq. 5 is used and the minimization of $\mathcal{E}(\mathbf{h})$ in Eq. 4 is carried out with respect to $\mathbf{h}_k$ (with $\mathbf{h}_{k-1}$ fixed), then $\mathbf{h}_k$ can be found by solving a linear set of equations for each iteration. The extension of the method given in [7] to the design of an N-channel filter bank is straightforward and is summarized in Table 1. Independent from our work, the same extension has been reported recently in [12].

Table 1: Algorithm to compute the prototype filter coefficient vector $\mathbf{h}$.

compute matrices $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{Q}$ and starting vector $\mathbf{h}_{-1}$
for each iteration index k **do**
$\mathbf{U} = \mathrm{diag}(\mathbf{U}_1 \mathbf{h}_{k-1}) \mathbf{U}_1 + \mathrm{diag}(\mathbf{U}_2 \mathbf{h}_{k-1}) \mathbf{U}_2$
$\mathbf{g} = (\mathbf{U}^T \mathbf{U} + \alpha \mathbf{Q})^{-1} \mathbf{U}^T \mathbf{1}$
$\mathbf{h}_k = (1 - \tau) \mathbf{h}_{k-1} + \tau \mathbf{g}$
$E = \lvert\lvert \mathbf{h}_k - \mathbf{g} \rvert\rvert_2 / \lvert\lvert \mathbf{h}_k \rvert\rvert_2$
terminate, if $E < 10^{-5}$.

$\mathrm{diag}(\cdot)$ is a diagonal matrix, $\mathbf{1}$ is a $N_f \times 1$ vector of ones, $\lvert\lvert \cdot \rvert\rvert_2$ denotes the L_2 vector norm, $\tau = 0.5 \ldots 0.8$.

The cosine-matrices $\mathbf{U}_1$ and $\mathbf{U}_2$ are easily obtained by writing Eq. 1 (Eq. 2) as a vector inner product $H(\mathbf{h}, \theta) = \mathbf{c}^T(\theta)\mathbf{h}$:

$$\mathbf{U}_1 = \left[\mathbf{c}(\theta_1)\, \mathbf{c}(\theta_2) \cdots \mathbf{c}(\theta_{N_f})\right]^T \quad (6)$$

$$\mathbf{U}_2 = \left[\mathbf{c}(\theta_1 - \tfrac{\pi}{N})\, \mathbf{c}(\theta_2 - \tfrac{\pi}{N}) \cdots \mathbf{c}(\theta_{N_f} - \tfrac{\pi}{N})\right]^T \quad (7)$$

with a selection of frequency points $\theta_i \in [0, \frac{\pi}{2N}]$, $i = 1, 2, \ldots, N_f$.

In contrast to [7], we derive the elements q_{ik} of the symmetrical matrix $\mathbf{Q}$ in Table 1 by analytically evaluating the second integral in Eq. 4:

$$q_{ik} = \mathrm{si}(i - k) + \mathrm{si}(L + 1 - i - k) \quad (8)$$

with $i, k = 1, 2, \ldots, \lceil \frac{L-1}{2} \rceil$ and

$$\mathrm{si}(n) = \begin{cases} -2\dfrac{\sin \theta_s n}{n} & n = \pm 1, \pm 2, \ldots \\ 2(\pi - \theta_s) & n = 0 \end{cases}. \quad (9)$$

For an odd filter length L we get an additional row and column with elements

$$q_{i, \frac{L+1}{2}} = \begin{cases} \mathrm{si}\left(\frac{L+1}{2} - i\right) & i \neq \frac{L+1}{2} \\ \pi - \theta_s & i = \frac{L+1}{2} \end{cases}. \quad (10)$$

It should be noted that the first integral in Eq. 4 can also be solved by hand. In Table 1, only the computation of matrix $\mathbf{U}$ must be replaced. However, the build-up of the new matrix is quite involved and leads to an inefficient program (especially when using MATLAB).

Typically, 15 iterations are needed for $\mathcal{E}(\mathbf{h})$ to converge to a relative error less than 10^{-5}. By designing filter banks with $N = 2$ to 128 channels and prototype FIR filter lengths L up to 1536, we got excellent results and did not encounter local minima problems which can be observed with standard optimization techniques.

2.2 Nonuniform Filter Banks

Our design of nonuniform NPR modulated filter banks is based on the transition filter concept as presented in [10]. Transition filters link filter bank channels with different bandwidths allowing near-perfect reconstruction similar to uniform NPR modulated filter banks. A simple method to approximate a given frequency spacing by means of a nonuniform filter bank is to design a set of uniform filter banks. From each of these filter banks we then select a proper number of channels to obtain the desired frequency scale partitioning. Care must be taken, however, to get a feasible set of subsampling factors for the resulting nonuniform filter bank [5, 11]. The transition filters needed are symmetrical bandpass filters and may be derived from real-valued lowpass prototype filters.

A more flexible design is possible by the use of unsymmetrical bandpass transition filters which link

channel filters with different bandwidths and transition widths. Now, the transition filter is a modulated version of a complex lowpass prototype filter. The frequency response $H_t(\theta)$ of this prototype filter and its connections to the adjacent channel frequency responses $H_1(\theta)$, $H_2(\theta)$ are illustrated in Fig. 1. Note that H_1 (H_2) is part of a N_1 (N_2) - channel uniform filter bank section. Within the frequency intervals marked in Fig. 1 the zero-phase response $H_t(\theta)$ of the prototype transition filter must satisfy the following reconstruction conditions:
for θ in **interval I**

$$\frac{1}{N_1}H_1\big(\theta+\tfrac{\pi}{2N_1}+\tfrac{\pi}{2N_2}\big)H_1\big(-\theta+\tfrac{\pi}{2N_1}-\tfrac{\pi}{2N_2}\big) = \frac{1}{N_2}H_t(\theta)H_t\big(-\theta-\tfrac{\pi}{N_2}\big), \tag{11}$$

$$\frac{1}{N_1}\Big[H_1\Big(\theta+\frac{\pi}{2N_1}+\frac{\pi}{2N_2}\Big)\Big]^2+\frac{1}{N_2}[H_t(\theta)]^2=1, \tag{12}$$

for θ in **interval II**

$$\frac{1}{N_2}[H_t(\theta)]^2=1, \tag{13}$$

and for θ in **interval III**

$$H_2\Big(\theta-\frac{\pi}{N_2}\Big)H_2(-\theta)=H_t(\theta)H_t\Big(-\theta+\frac{\pi}{N_2}\Big), \tag{14}$$

$$\frac{1}{N_2}\Big[H_2\Big(\theta-\frac{\pi}{N_2}\Big)\Big]^2+\frac{1}{N_2}[H_t(\theta)]^2=1. \tag{15}$$

The dependence of $H(\cdot)$ on the filter coefficient vector $\mathbf{h}$ has been omitted for clarity. Equations 11 and 14 are needed for aliasing cancellation and are derived in [10]. The remaining equations are equivalent to Eq. 3.

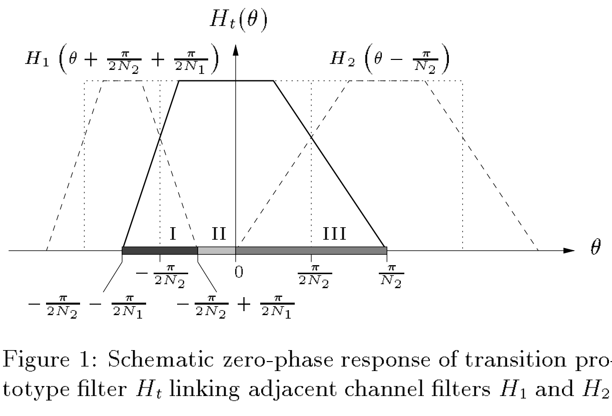

Figure 1: Schematic zero-phase response of transition prototype filter H_t linking adjacent channel filters H_1 and H_2.

Note that the coefficients of the adjacent filters H_1, H_2 are fixed and only the prototype filter coefficient vector $\mathbf{h}_t$ is a variable in the design procedure. By forming an error function similar to Eq. 4, we can use nonlinear function minimization to find $\mathbf{h}_t$. The optimization using the constraints Eq. 11 - Eq. 15 is completely different to that one applied in [10] where reconstruction conditions based on [6] are used.

We implemented the optimization method proposed in [10] and compared it with our approach using the same design examples. As a result, our method converges much faster and is less sensitive to different starting solutions. Additionally, the set of constraints given in Eq. 11 - Eq. 15 can be linearized with respect to $\mathbf{h}_t$ resulting in a similar iterative algorithm as outlined in Section 2.1.

Using vector notation, we can incorporate the (linearized) 5 equations 11 – 15 in the error measure

$$\mathcal{E}(\mathbf{h}_t)=\sum_{i=1}^{5}\alpha_i(\mathbf{a}_i-\mathbf{B}_i\mathbf{h}_t)^T(\mathbf{a}_i-\mathbf{B}_i\mathbf{h}_t)+\alpha\mathbf{h}_t^T\mathbf{Q}_t\mathbf{h}_t \tag{16}$$

where the last term represents the stopband energy. Vectors $\mathbf{a}_i$ and matrices $\mathbf{B}_i$ can be easily deduced from Eq. 11 – Eq. 15. The zero-phase response $H_t(\theta)$ of the complex-valued transition prototype filter (with even filter length L) is given by

$$H_t(\theta) = \sum_{n=0}^{\frac{L}{2}-1} 2\,\Re e\{h_t(n)\}\cos\left[\left(n-\tfrac{L-1}{2}\right)\theta\right] + \sum_{n=0}^{\frac{L}{2}-1} 2\,\Im m\{h_t(n)\}\sin\left[\left(n-\tfrac{L-1}{2}\right)\theta\right] \tag{17}$$

and the coefficient vector may be written as

$$\mathbf{h}_t = \big(\Re e\{h_t(0)\},\ldots,\Re e\{h_t(\tfrac{L}{2}-1)\}, \Im m\{h_t(0)\},\ldots,\Im m\{h_t(\tfrac{L}{2}-1)\}\big)^T. \tag{18}$$

Setting the gradient $\frac{\partial\mathcal{E}}{\partial\mathbf{h}_t}$ of the error measure defined by Eq. 16 to zero, we get the optimum coefficient vector

$$\mathbf{h}_t=\left(\sum_{i=1}^{5}\alpha_i\mathbf{B}_i^T\mathbf{B}_i+\alpha\mathbf{Q}_t\right)^{-1}\sum_{i=1}^{5}\alpha_i\mathbf{B}_i^T\mathbf{a}_i. \tag{19}$$

Aside from the different matrix computations, the same iterative algorithm as in Table 1 can be applied to the transition filter design. According to our experience, results found with the iterative algorithm are slightly better and are less sensitive to the selection of a starting solution as compared to results obtained by nonlinear function minimization. However, the iterative algorithm needs more computation time mainly due to the matrix computations involved.

3 DESIGN EXAMPLES

To illustrate our filter bank design method, we present two design examples of nonuniform multirate filter banks for critical band analysis/synthesis. In the first example, we show a 25 channel filter bank which was designed by proper channel merging of 8 uniform filter banks with $N = 96, 64, 48, 32, 24, 16, 12, 8$ channels (Fig. 2). From each of these 8 uniform filter banks we select a certain number of channels to approximate the frequency spacing of the Bark scale. All channel filters have the same filter length and transition width between passbands and stopbands.

An alternative nonuniform filter bank using the same frequency spacing as in Fig. 2 but with different transition bandwidths is shown in Fig. 3. Three unsymmetrical transition filters are needed to link channels with different transition bandwidths.

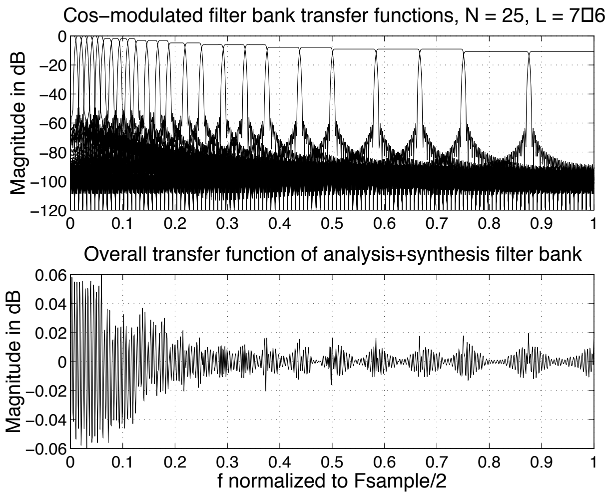

Figure 2: 25 channel nonuniform multirate filter bank for critical band analysis/synthesis (reconstruction error shown in lower diagram, FIR filter length $L = 768$, 16 kHz sampling frequency).

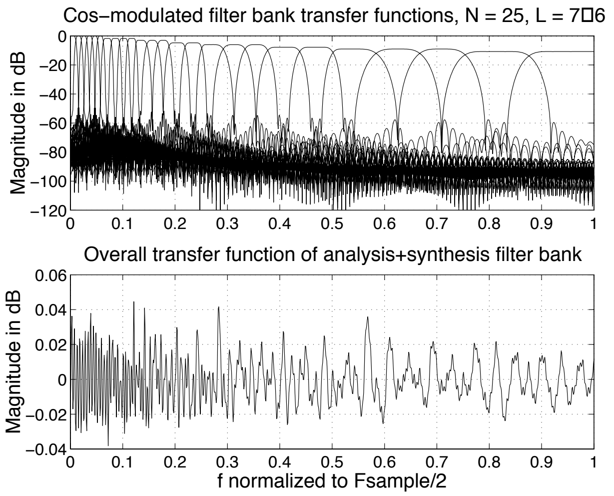

Figure 3: 25 channel nonuniform multirate filter bank for critical-band analysis/synthesis including 3 unsymmetrical transition filters and 22 symmetrical filters (FIR filter length $L = 769$ at low frequencies decreasing to $L = 97$ at high frequencies, 16 kHz sampling frequency).

Compared with Fig. 2, we need to design and store only two prototype lowpass filters (for $N = 96$ and $N = 64$) and one prototype transition filter (for $N_1 = 96$, $N_2 = 64$). All other filters are obtained by decimating the impulse responses of these three filters.

4 CONCLUSIONS

In this paper, a design procedure for uniform and nonuniform NPR modulated filter banks has been developed which uses iterative linear equation solving to find the optimum prototype filters. The advantage of this method is its rapid convergence and robustness in regard to local minima problems even for large prototype FIR-filter lengths.

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