ADAPTIVE SUBSPACE MODULATION IN SPATIALLY CORRELATED MIMO SYSTEMS

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Abstract—We propose an adaptive modulation scheme for wireless MIMO systems. The main goal of this scheme is to maximize the bit rate for a fixed target BER. The MIMO channel is split up into several independent channels (subspaces) by a singular value decomposition. On each of these independent channels the modulation format is adapted to the subchannel SNR. The proposed algorithm allocates appropriate modulation schemes from 2PSK to 256QAM to each subchannel. The channel is assumed to be quasi static flat Rayleigh fading, perfectly known at both ends of the link. Mean data rates that can be obtained are presented as a function of the mean $\overline{\text{SNR}}$, keeping BER below a target value of e.g. 10^{-3} for spatially uncorrelated and partially correlated channels without using any error correcting code.

Keywords—MIMO, spatially correlated channels, adaptive modulation

I. Introduction

Recent research exploiting MIMO channel capacity mostly concentrates on BLAST [1] and Space Time Coding (ST) [2]. BLAST transmits different data over each transmit antenna. If high modulation schemes are applied on each antenna, very high data rates can be achieved. Space Time Coding introduces not only temporal correlation between the transmit data but also spatial correlation is exploited to combat fading. On one hand this allows to transmit with very low power, on the other hand ST coding can improve the bit error ratio (BER). Applying BLAST or ST-Coding the channel has to be known only at the receiver to perform coherent detection.

Our method makes use of the full channel knowledge at both ends of the communication link. The channel is assumed to be quasi static and flat in frequency domain. The proposed approach is simple but very efficient. Our algorithm achieves high bit rates and low BER. First of all the MIMO channel is split into several independent SISO channels by means of singular value decomposition of the channel matrix. Then an appropriate number of the subchannels is used for data transmission. The subchannels provide different power gains and therefore we adapt the modulation format to each of these virtual links. The higher the power gain of a subspace channel is, the more complex the modulation scheme can be chosen. This scheme is an alternative to the well known waterfilling principle: Instead of allocating power to each subchannel, different modulation schemes are applied. With this adaptive subspace modulation we make use of each subchannel in an optimum way. The main goal of this method is to achieve a data rate as high as possible meeting a certain target BER.

The paper is organized as follows: The channel model is described in Section II, i.e., it is explained, how the channel is randomized and how spatial correlation is introduced. The adaptive algorithm which selects the appropriate modulation scheme is described in Section III. Simulations results are shown in Section IV. Conclusions finalize the paper.

II. Channel Model

A quasi static flat Rayleigh fading MIMO channel is assumed throughout the paper. The transmitted signal is distributed by the channel to the several receive antennas and disturbed by additive white Gaussian noise (AWGN). Only symmetric MIMO channels are considered, i.e., the numbers of receive and transmit antennas are equal and denoted by N. We consider only one MIMO link, i.e., no multiuser interference is considered. With these assumptions the received signal can be presented as:

$$\boldsymbol{y} = \boldsymbol{H}\ \boldsymbol{x} + \boldsymbol{n}\ . \tag{1}$$

$\boldsymbol{H}$ is a $(N \times N)$ matrix with complex entries which account for the random fluctuations and the random phase shifts of the channel transfer characteristics. $\boldsymbol{x}$ denotes the $(N \times 1)$ transmit signal vector. The $(N \times 1)$ vector $\boldsymbol{y}$ is the received signal at the antenna array at the receiver. $\boldsymbol{n}$ is the $(N \times 1)$ complex noise vector. The elements of $\boldsymbol{n}$ are independently identically $\mathcal{N}_C\left(0, \sigma_n^2\right)$ distributed.

A. Spatially uncorrelated channel

An uncorrelated channel matrix can be used to describe an indoor or picocell scenario where no LOS component exists. For this special case $\boldsymbol{H}$ can be characterized by a purely random matrix with independently identically Gaussian distributed complex entries of variance 1 and zero mean:

$$\boldsymbol{H} \in \mathcal{N}_C^{N \times N}\left(0, 1\right)\ . \tag{2}$$

B. Spatially correlated channel

In practice a spatially correlated channel is more likely than an uncorrelated one. In most cases the mobile station (MS) is surrounded by many local scatterers and therefore the received signals at the antenna array of the mobile station are approximately uncorrelated. At the base station (BS) different scenarios can be observed. Because of the outstanding position of the antenna array at the BS there are a few dominant scatterers and therefore a spatially correlated channel is obtained. More about spatial channel correlations can be found in [3] [4] [5] .

To model a spatially correlated channel we make use of the channel model introduced in [6] with

$$\boldsymbol{H} = \boldsymbol{G}\,\boldsymbol{A}^T \sqrt{\frac{N}{tr\left(\boldsymbol{A}^*\,\boldsymbol{A}^T\right)}}\,. \tag{3}$$

Here, $\boldsymbol{G}$ is an uncorrelated random matrix with $\boldsymbol{G} \in \mathcal{N}_C^{N\times K}(0,1)$, K denotes the number of impinging waves, tr denotes the trace of a matrix and $\boldsymbol{A}$ is a so-called steering matrix and consists of K weighted steering vectors of length N.

With this model various scenarios can be simulated. Especially a certain number of scattering clusters with different values of Angular Spread (AS) can be modeled in this way. This scenario can be handled by choosing a specific Angular Power Spectrum (APS) e.g. taken from [3]. A large enough number of angles of incidence is randomly selected. For these angles the wave powers according to a predetermined APS are calculated and the corresponding steering matrix $\boldsymbol{A}$ is built.

In this paper we focus on a spatial channel model with only one dominant scattering cluster nearby the BS. Different spatial correlation values are adjusted by varying the AS. Comprehensive measurement campaigns [7] have shown that most APS-configurations can be modeled by a truncated Laplacian distribution. The tails of the Laplace functions are cut off at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

For each simulation run the AS is fixed. Then a certain number of uniformly distributed angles are chosen randomly. The wave powers are determined according to the APS and the steering matrix $\boldsymbol{A}$ is evaluated.

III. Subspace Modulation Scheme

The flat fading MIMO channel is completely described by the channel matrix $\boldsymbol{H}$. Using singular value decomposition the MIMO channel can be transformed into independent subchannels:

$$\boldsymbol{H} = \boldsymbol{U}\;\boldsymbol{D}\;\boldsymbol{V}^H\,. \tag{4}$$

Here, $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices and $\boldsymbol{D}$ is a diagonal matrix with r non-zero singular values λ_i. r is the rank of the channel which is less than or equal to N. Using the excitation vectors $\boldsymbol{v}_1$ to $\boldsymbol{v}_N$ at the transmitter side and the corresponding weight vectors $\boldsymbol{u}_i^H$ at the receiver side, we obtain r independently fading virtual SISO channels in parallel with gain factors λ_i. If the channel matrix has full rank then $r = N$ and we get N independent channels. This idea can be formally expressed as:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{H}\boldsymbol{V}\boldsymbol{s} + \boldsymbol{n} \\ \boldsymbol{y}' &= \boldsymbol{U}^H\boldsymbol{y} = \boldsymbol{U}^H\boldsymbol{H}\boldsymbol{V}\boldsymbol{s} + \boldsymbol{U}^H\boldsymbol{n} = \boldsymbol{D}\boldsymbol{s} + \boldsymbol{U}^H\boldsymbol{n}\,, \end{aligned} \tag{5}$$

where $\boldsymbol{s}$ is the $(N \times 1)$ source vector consisting of N data symbols. The transmitted signal at the transmit antenna array is: $\boldsymbol{x} = \boldsymbol{V}\;\boldsymbol{s}$.

Transforming $\boldsymbol{s}$ and $\boldsymbol{y}$ by the unitary matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ the channel equations for the N independent subchannels can be written explicitly as:

$$y_i' = \boldsymbol{u}_i^H\;\boldsymbol{y} = \lambda_i\;s_i + \boldsymbol{u}_i^H\;\boldsymbol{n} \qquad i = 1\ldots N\,. \tag{6}$$

Hence, y_i' is an estimate of s_i. Figure 1 shows the corresponding signal processing building blocks. b and b' are the corresponding bit streams at the input and at the output of the MIMO system.

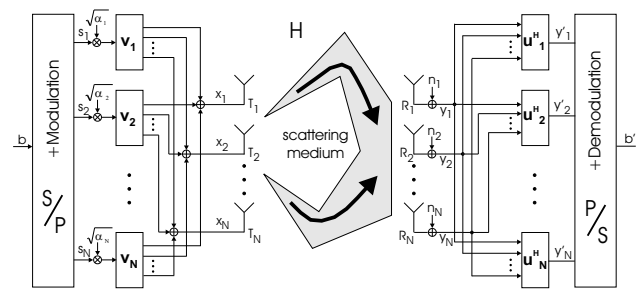

Fig. 1. Transceiver structure.

A. Singular Value Distribution

As mentioned above the squared singular values λ_i^2 determine the power gain of each subchannel and thus determine the specific SNR. Hence, the statistics of the squared singular values determines the performance of our system. In Figure 2 the Cumulative Distribution Functions (CDFs) of the squared singular values for the uncorrelated MIMO channel are shown. For the case of a correlated MIMO channel with a Laplacian APS and AS=5°, the CDFs of the squared singular values are depicted in Figure 3. From Figure 2 and 3 it can be seen, that for the case of correlated channels the first singular value is stronger and the other singular values are weaker than the corresponding singular values in the uncorrelated case. Therefore the first singular value plays a dominating role in the case of strong spatial correlation. The fourth singular value in case of AS=5° is so small, that it cannot be seen in Figure 3.

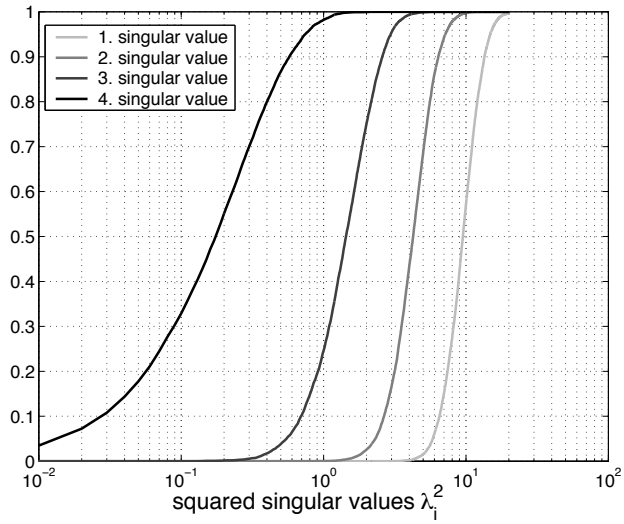

Fig. 2. CDFs of the 4 singular values of the uncorrelated MIMO channel.

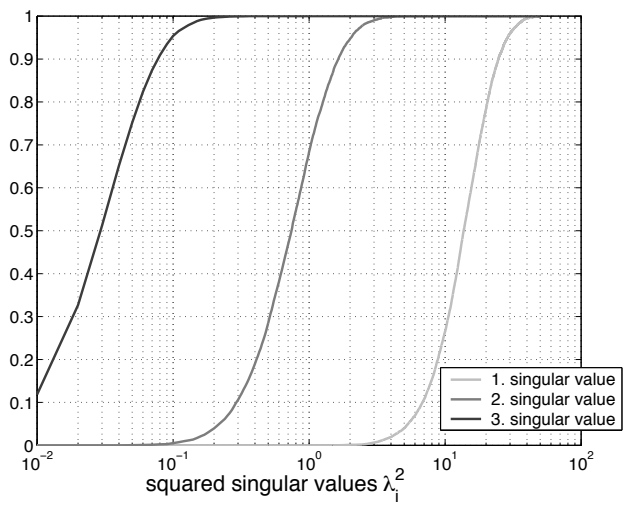

Fig. 3. CDFs of the 4 squared singular values of a MIMO channel with Laplacian APS and AS=5°.

B. Adaptive Modulation

The key idea of this paper is to adapt the modulation formats to the different SNR - values at the various subspace channels keeping the individual BERs below a fixed target BER and meeting a total power constraint. Our approach is comparable with the method in [8]. The difference is that data rates are allocated to distinct subspaces and not to different subcarriers of an OFDM system.

The basic structure of the adaptive algorithm is shown in Figure 4.

The actual SNR-values at the subchannels depend on the singular values λ_i. Obviously, it is reasonable to modulate a channel with high SNR with a high M-QAM and a channel with low SNR with low M-QAM. Given a certain target BER it is

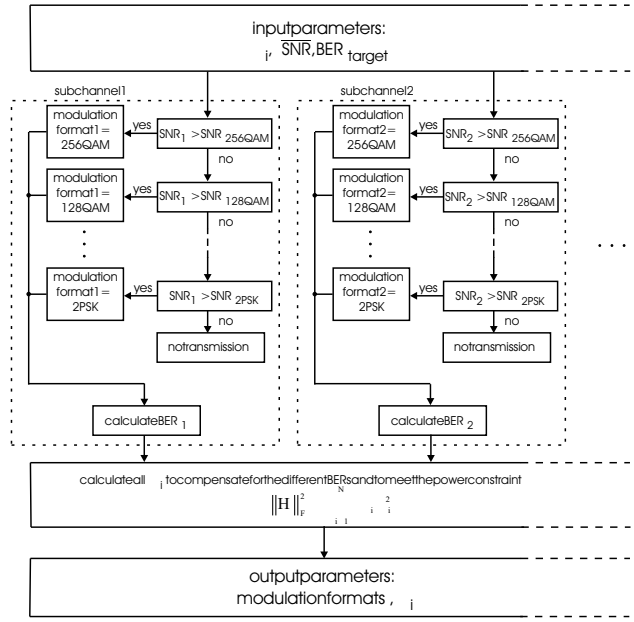

Fig. 4. The basic structure of the adaptive algorithm.

not too hard to find an appropriate modulation scheme for each subchannel such that the resulting BER is below the given target BER.

We consider a set of modulation formats from 2PSK to 8PSK and from 16QAM to 256QAM. As derived in [9], the BERs are given by the following expressions.

The BER for M-PSK is approximated by:

$$\mathrm{BER} \approx \frac{2}{ld(\mathrm{M})}\, Q\left(\sqrt{2\,\mathrm{SNR}}\,\sin\frac{\pi}{\mathrm{M}}\right) \tag{7}$$

and for M-QAM:

$$\mathrm{BER} \approx \frac{4}{ld(\mathrm{M})}\, Q\left(\sqrt{\frac{3\,ld(\mathrm{M})\,\mathrm{SNR}}{\mathrm{M}-1}}\right)\ , \tag{8}$$

where $\mathrm{SNR} = \frac{E_s}{N_o}$.

The selection of an appropriate modulation scheme for each subchannel is governed by the actual SNR at each subchannel. First of all we assume that each subchannel is driven by signals with the same signal power P_s (all gain factors α_i in Figure 1 and Figure 4 are equal to one). Our channel model is normalized such that

$$E\left\{\|\boldsymbol{H}\|_F^2\right\} = \sum_{i=1}^{N} E\left\{\lambda_i^2\right\} = N^2\ . \tag{9}$$

Then the SNR_i of subchannel number i is given by

$$\mathrm{SNR}_i = \frac{\lambda_i^2\ P_s}{\sigma_n^2} \tag{10}$$

and the mean $\overline{\text{SNR}}$ is

$$\overline{\text{SNR}} = E\left\{\frac{1}{N}\sum_{i=1}^{N}\text{SNR}_i\right\} = \frac{N\,P_s}{\sigma_n^2}\ . \tag{11}$$

Fixing the mean $\overline{\text{SNR}}$ of our data transmission the individual SNRs of subchannel number i can be expressed as

$$\text{SNR}_i = \overline{\text{SNR}}\frac{\lambda_i^2}{N}\ . \tag{12}$$

Then the algorithm for selecting the optimum modulation format works as follows (see also Figure 4): Given the target BER, the mean $\overline{\text{SNR}}$ and the gain factors λ_i we predetermine the threshold SNR_M for each modulation format (M-PSK or M-QAM) necessary to obtain an actual BER below our target BER. Afterwards we select the highest modulation format M for each subchannel such that $\text{SNR}_i > \text{SNR}_\text{M}$. At last, optionally we can optimize our system by adjusting the individual signal power of each subchannel in such a way that all subchannels achieve approximately the same BER, in spite of their possibly different modulation formats and power gains λ_i^2. This is done by means of the individual gain factors α_i which are chosen in such a way that on one hand the BERs of all subchannels are equal and below our target BER and on the other hand the overall power constraint of our system $P_{total} = N\,P_s$ is met.

IV. Simulations Results

In the following two sets of simulations are presented. The first one focuses on the behaviour of the proposed transceiver for spatially uncorrelated channels and various mean $\overline{\text{SNR}}$ values ($\overline{\text{SNR}}$=5dB to 25dB in steps of 2.5dB). The second set of simulations illustrates the performance of the transceiver for several amounts of spatial correlations (uncorrelated channel, AS=20°, AS=15°, AS=10°, AS=5° and AS=0°) and a mean $\overline{\text{SNR}}$ of 15dB. The results of our simulations cover the mean BER, the mean information bit rate, information load on each subchannel and histograms showing how often a specific modulation scheme is used on a certain subchannel.

In [10] it is shown that in the high SNR domain the channel capacity with channel knowledge at the transmitter (using waterfilling power distribution) is almost as high as the channel capacity without channel knowledge at the transmitter (applying equal power distribution). The difference in performance, the so-called waterfilling gain, is less than 20% . For this reason the measured channel capacity without channel knowledge at the transmitter is used for the comparison with the actually achieved information bit rate. The dashed line in Figure 5 shows this capacity as a function of the mean $\overline{\text{SNR}}$. The solid line in Figure 5 shows the actually achieved information bit rate of our system keeping the BER below 10^{-3}. The SNR gap between these two curves is approximately 6dB and almost constant over the whole SNR range. Obviously, the SNR gap is a function of the target BER (the higher the target BER, the lower is the SNR gap and vice versa).

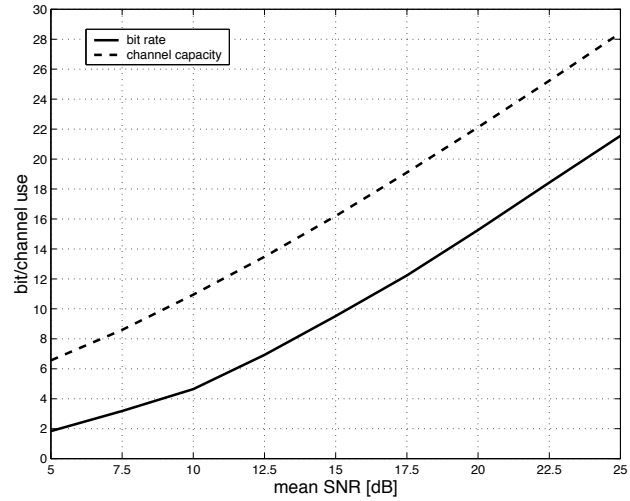

Fig. 5. Bit rate and channel capacity versus mean SNR; spatially uncorrelated channel, four transmit and receive antennas (N=4), 100000 channel realizations, $\overline{\text{BER}} < 10^{-3}$.

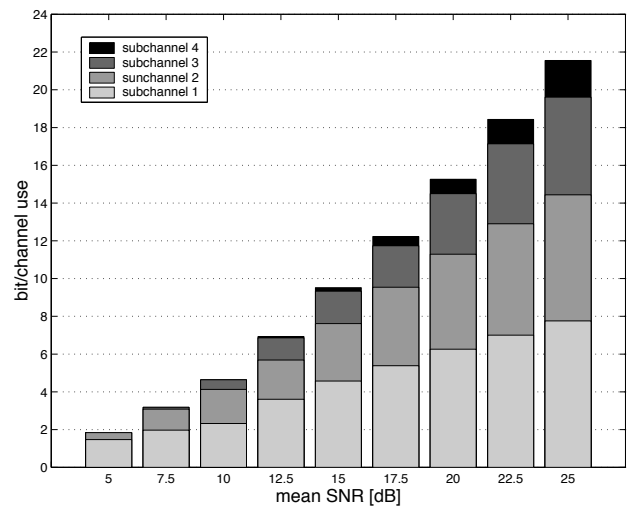

Fig. 6. Information bit rate on each subchannel for several mean SNR values; spatially uncorrelated channel, four transmit and receive antennas (N=4), 100000 channel realizations, $\overline{\text{BER}} < 10^{-3}$, $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$.

In Figure 6 it is shown that the portion of information sent over the weaker subchannels grows with increasing mean $\overline{\text{SNR}}$, i.e., for high mean $\overline{\text{SNR}}$ even the subchannels with small singular values (gains) λ_i contribute substantially to the overall information bit rate. Subchannel number one corresponds to the subchannel with the highest power gain λ_1^2, subchannel number two is the subchannel with the next highest gain λ_2^2, and so on. In Figure 7 we present histograms showing how frequently different modulation schemes are used on each subchannel. The higher the SNR, the more frequently high modulation formats are used. Note, that quite frequently no data transmission at all is performed at subchannel 4 due to the low SNR values.

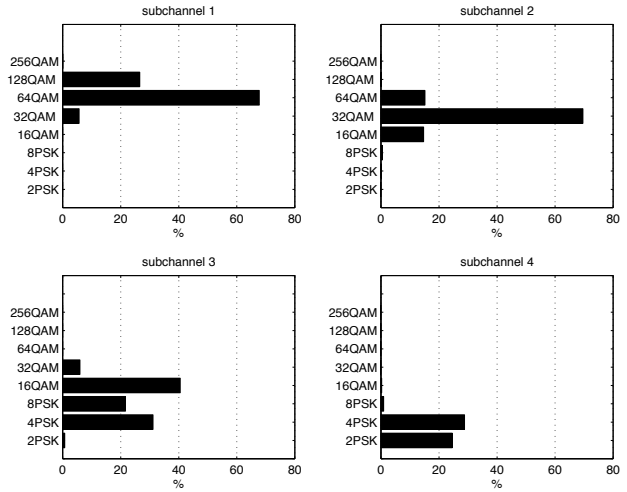

Fig. 7. Frequency of occurrence of the modulation schemes; spatially uncorrelated channel, mean SNR=20dB, four transmit and receive antennas (N=4), 100000 channel realizations, $\overline{\text{BER}} < 10^{-3}$.

Therefore the black colored percentages in the histogram of subchannel 4 do not sum up to 100%.

In Figure 8 results are shown for spatially correlated channels. The singular value distributions of spatially correlated channels are characterized by a stronger first singular value λ_1 compared to an uncorrelated channel, whereas the other singular values are smaller compared to the corresponding singular values of the uncorrelated channel. Because of the dominance of the first singular value almost all information is carried over subchannel one. Other subchannels are more or less useless. In the case of

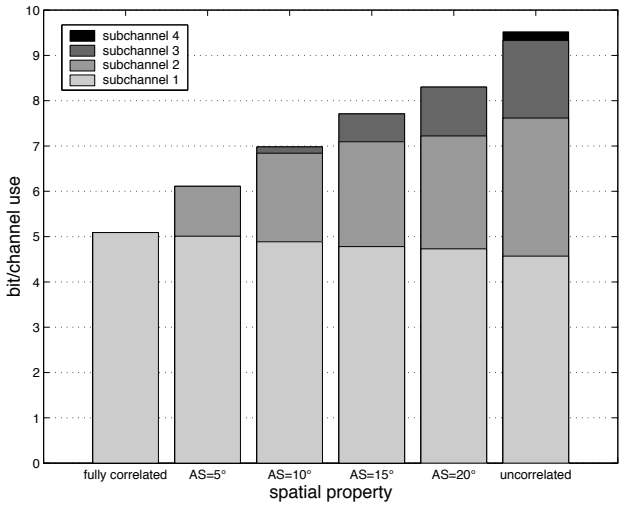

Fig. 8. Portion of bit rate for several correlation conditions; mean SNR=15dB, four transmit and receive antennas (N=4), 100000 channel realizations, $\overline{\text{BER}} < 10^{-3}$.

spatially correlated channels only one typical result is shown ($\overline{\text{SNR}}$=15dB). Figure 8 shows the information bit rates for different correlation scenarios. Obviously, in the case of full correlation all the load is carried by subchannel one. The weaker the spatial correlation, the more load is transferred to the other subchannels. Obviously, the uncorrelated channel outperforms correlated channels with respect to the achievable data rate!

V. Conclusions

We have shown how to achieve very high transmission rates by the efficient use of a MIMO channel, if we have full channel knowledge at both ends of the link and additionally use some feedback information (modulation format, power distribution factors α_i). Our paper supports the assumption, that uncorrelated MIMO channels are better than correlated channels with respect to high data rates ([11] [12]) in the scenario considered in this paper. For indoor scenarios, where uncorrelated or weakly correlated channels dominate, it is reasonable to use all subchannels for data transmission. In other realistic scenarios, where the signals at the base station antenna array are spatially correlated, it is not really necessary to use all subchannels. In most correlated cases it is sufficient and appropriate to use only the two strongest subchannels.

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