MODELLING AND IDENTIFICATION OF A NONLINEAR POWER-AMPLIFIER WITH MEMORY FOR NONLINEAR DIGITAL ADAPTIVE PRE-DISTORTION

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ABSTRACT

In this paper a comparison between a memoryless nonlinear power amplifier model and a special nonlinear model with memory (Wiener-model) based on measurements on a power amplifier is presented. With moderate signal bandwidths of only 2MHz the nonlinear model with memory outperforms the memoryless model. The amplifier model is intended to be used for pre-distortion purposes. Furthermore, an adaptive algorithm for finding the model parameters is presented.

1. INTRODUCTION

With the advent of broadband wireless communication systems like UMTS, subsystems in the transmission path which so far have been modelled as frequency independent (without memory effects) show memory effects due to the large signal bandwidth. A typical part of the transmission path which must be considered as frequency dependent (i.e. afflicted with memory) in a broadband communication system is the power amplifier.

Due to the tight power consumption constraints, linear power amplifiers cannot be applied since their power efficiency is very poor. The only possible solution are nonlinear amplifiers with high power efficiency but with the drawback of causing severe distortion in neighboring frequency bands. Power efficiency is essential in mobile handsets in order to consume as low energy as possible, whereas linearity is important in both sides of the communication link: on the one hand the service provider has to comply with limits of spurious radiation in adjacent frequency bands (mainly important at the base station due to the high radiated power levels), on the other hand, nonlinear distortions increase the bit error probability due to distortions of the signal point diagram, an important degradation if multilevel modulation formats like M-ary QAM formats are used.

Attempts to overcome the nonlinear effects are of manyfold nature. In order to compensate for the nonlinear distortions introduced by the nonlinear power amplifier at the transmitter, two possibilities exist:

1. Compensation at the transmitter side.
2. Compensation at the receiver side.

In the first case the baseband signal is pre-distorted in the digital domain before transferring it to the power amplifier, whereas the second approach applies a post-distortion at the receiver end. Pre-distortion at the transmitter is technically much easier since the memory effects are only due to the amplifier circuitry while post-distortion at the receiver also includes the memory effects of the channel, a system that is typically linear but time-variant in nature and thus much more complicated to deal with, see e.g. [1]. It makes also much more sense to solve the trouble of nonlinear operations at the point of their origin where only one additional circuitry is required to remove the nonlinear effects while at the receiver side as many circuits are required as receivers are used, obviously a much more cost intensive solution. Furthermore, if the distortions are compensated at the receiver, expensive analog filters are needed at the front end of the transmitter in order to remove the spurious out-of-band emissions produced by the nonlinearity.

Most distortion schemes rely on look up table techniques for the selection of the distorted signal samples, see e.g. [2], working very well for narrow-band transmission systems. If, however, broadband systems are considered, the power amplifier is afflicted with memory, the table with the pre-computed distorted signal samples becomes very large and the procedure of selecting the actual distorted signal sample becomes more and more complex. Moreover, if the power amplifier changes its characteristic a large table has to be updated with newly calculated signal samples and/or the selection strategy of the distorted signal samples has to be changed.

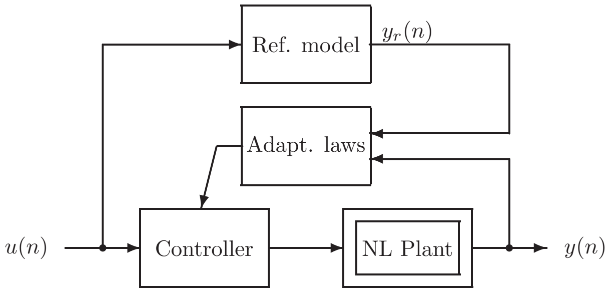

Fig. 1. Model Reference Adaptive Control (MRAC) scheme

An alternative approach is the utilization of an adaptive nonlinear control scheme, pictured in Fig.1, based on a Model Reference Adaptive Control (MRAC) scheme, see e.g. [3]. The parameters of the controller (pre-distortion unit) are adaptively adjusted in order to achieve a minimal deviation from the target behavior, in our case a linear amplification of the input signal. The difficulties with this scheme are not of trivial nature - a specific controller structure together with a robust control algorithm are to be decided upon. In order to decide for a specific control strategy it is essential to have an accurate model of the plant under control, i.e. the power amplifier.

2. NONLINEAR MODELS

The classical and most often used nonlinear model of a power amplifier is Saleh's model [4], published in 1981. It is a pure nonlinear model without memory, developed with single-tone measurements. Only two parameters, α and β, characterize the behavior of Saleh's nonlinear model of the power amplifier in terms of the AM-AM and AM-PM distortions. The equations determining this distortions are

$$\begin{aligned} A(r) &= \frac{\alpha_A r}{1+\beta_A r^2} \\ \Phi(r) &= \frac{\alpha_\Phi r^2}{1+\beta_\Phi r^2} \quad , \end{aligned} \tag{1}$$

respectively, where $r(t)$ stands for the envelope of the applied input signal $x(t) = r(t)\cos(\omega_0 t + \psi(t))$. The output of the power amplifier model is

$$y(t) = A(r(t))\cos(\omega_0 t + \psi(t) + \Phi(r(t))) \quad . \tag{2}$$

They describe typical saturation-like characteristics, with a linear behavior if the signal amplitude is small.

The common description of nonlinear systems with memory is the Volterra-series, a generalization of the classical Taylor-series, see e.g. [5]. The time variant Volterra-series of order P in discrete time form is

$$y(n) = \sum_{p=0}^{P} \overline{h}_{p,n}[u(n)] \quad . \tag{3}$$

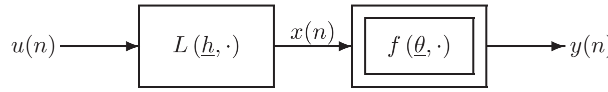

Fig. 2. Simplified Wiener-model

The output signal of the Volterra-system is $y(n)$, the input is $u(n)$. The time variant operators $\overline{h}_{p,n}[\cdot]$ represent a multidimensional convolution,

$$\begin{aligned} &\overline{h}_{p,n}[u(n)] = \\ &= \sum_{n_1=0}^{N} \cdots \sum_{n_p=0}^{N} h_{p,n}(n_1,\ldots n_p) \prod_{i=1}^{p} u(n-n_i) \, , \end{aligned} \tag{4}$$

where we assumed causality and finite, equal memory lengths N along the different "dimensions" of the individual Volterra-kernels $h_{p,n}(\cdot)$. As is the case with the Taylor-series, the Volterra-series has some significant drawbacks: slow convergence and, if the nonlinearity is pronounced, like saturation characteristics are, a high number of coefficients, appearing linear in the description, must be estimated in order to achieve good accuracy. Therefore, alternative mathematical descriptions for nonlinear systems with memory are of interest. One such description is a simplified Wiener-model, where a memoryless single input nonlinearity follows a linear filter, see Fig. 2. The memory effects lie entirely in the linear filter, whereas the nonlinear effects lie entirely in the memoryless subsystem. Mathematically this simplified Wiener-model can be described as

$$\begin{aligned} y(n) &= f(\underline{\theta}, L[u(n)]) \\ L[u(n)] &= \sum_{m=0}^{M-1} h(m)\, u(n-m) \quad , \end{aligned} \tag{5}$$

where the vector $\underline{\theta}$ was introduced to parameterize the nonlinear function $f(\cdot)$.

3. MEASUREMENT SETUP

In order to determine the parameters of the nonlinear model of the power amplifier, an input-output characteristic of a solid state amplifier (Minicircuits, Type: ZLH-42W) was measured. The complex baseband signal for the measurement setup was a sum of complex exponentials,

$$u(n) = \sum_{i=I_1}^{I_2} \exp(j(n\theta_i + \phi_i)) \quad , \tag{6}$$

whereby the phases ϕ_i were drawn from a uniform distribution $U(-\pi,\pi)$. The frequency separation of the individual tones was 20kHz, the total bandwidth of the complex

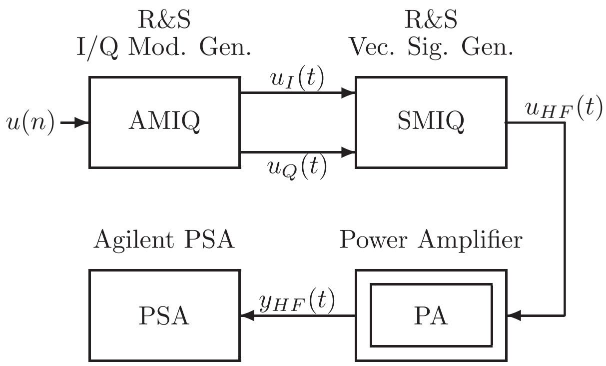

Fig. 3. Measurement setup

baseband signal was 2MHz. This signal represented the input signal for the Rhode&Schwarz-I/Q-Modulation generator AMIQ, which provided the I-Q-signals with a resolution of 14bit for the Rhode&Schwarz Vector Signal Generator SMIQ. With the signal generator the I-Q-signal was up-converted to 2GHz. The output signal of the amplifier was afterwards analyzed with a Agilent Power Spectrum Analyzer PSA. The internal sampling rate of the Spectrum Analyzer was 10.24MHz. For convenience, the complete measurement setup is represented in Fig. 3. The power of the input signal of the amplifier was set to 0dBm. Due to the nonlinearity of the amplifier large spurious parts of the signal out of the original bandwidth of 2MHz could be observed. The measured signal bandwidth was 8MHz.

4. MODELLING RESULTS

Using the sampled signals obtained with the described measurement setup, a nonlinear least squares data fit to the Wiener model (5) was performed . The memoryless nonlinearity used is

$$f\left(\theta, x\left(n\right)\right) = \frac{x(n)}{1+\theta\left|x\left(n\right)\right|^2} \quad , \tag{7}$$

resembling the first equation in (1). We denote with $x(n)$ the (complex) output of the linear filter. The only parameter θ is also complex, allowing for phase transformations in a single non-linear transformation block.

For comparison the input-output data were fitted to Saleh's model, described in (1), as well. The length of the input filter in the Wiener model was increased up to 10 taps, one tap corresponds to a time delay of $T = \frac{1}{10.24\text{MHz}} = 97.65ns$. As a measure of quality for the fit, the mean-squared error

$$\varepsilon_{MSE} = \frac{1}{N}\sum_{n=1}^{N}\left|y(n) - \hat{y}(n)\right|^2 \tag{8}$$

was used. An amount of $N = 50,000$ data samples was used for the fit. The nonlinear fit was performed using Matlab®'s nonlinear least squares data fitting procedure (*lsqnonlin()*).

Parameters	value
α_A	11.534
$\beta_A[1/V^2]$	1.6242
$\alpha_\Phi[rad/V^2]$	11.431
$\beta_\Phi[1/V^2]$	39.071
ε_{MSE}	$1.3e-3$

Table 1. Estimated parameters and mean squared error of Saleh's model

Memory length	ε_{MSE}	η_{MSE}/[dB]
0	$1.25 \cdot 10^{-3}$	0.18
1	$5.59 \cdot 10^{-4}$	3.66
2	$5.23 \cdot 10^{-4}$	3.96
3	$5.14 \cdot 10^{-4}$	4.03
4	$5.03 \cdot 10^{-4}$	4.12
5	$5.00 \cdot 10^{-4}$	4.14
6	$4.99 \cdot 10^{-4}$	4.15
7	$4.97 \cdot 10^{-4}$	4.17
8	$4.96 \cdot 10^{-4}$	4.18

Table 2. Mean squared error of Wiener model and relative improvement of the MSE with respect to Saleh's model

The optimal parameters and mean-squared error for Saleh's model are reported in Table 1. In Table 2 the mean-squared error together with the relative improvement of the mean-squared error $\eta_{MSE} = \frac{\varepsilon_{MSE,Saleh}}{\varepsilon_{MSE,Wiener}}$ using the Wiener-model is listed. The results show significant improvements up to two taps, obviously due to the rather small-band input signal. Note also that even for zero filter length, the selected nonlinear model (7) gave better results than Saleh's model (1). Measurements using input signals with higher bandwidth will be presented in a future publication.

In Table 3 the estimated parameters for the Wiener-model using a linear filter with four coefficients are presented.

filter coefficient	value
$\hat{h}_0$	$13.343 + j2.439$
$\hat{h}_1$	$0.55812 + j0.064852$
$\hat{h}_2$	$-4.8596 - j0.94393$
$\hat{h}_3$	$2.4361 + j0.46289$
$\hat{\theta}$	$0.012121 - j0.00395$

Table 3. Estimated parameters for the Wiener-model. The linear filter has four coefficients.

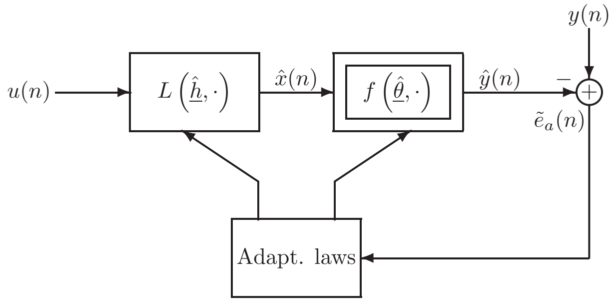

Fig. 4. Adaptive Identification of a Wiener-system

5. ADAPTIVE ALGORITHM FOR THE SYSTEM IDENTIFICATION

In a pre-distortion setup, the parameters of the power amplifier model have to be estimated continuously since the characteristic of the amplifier may change during transmission time. As a numerically simple and robust adaptation algorithm, the LMS-algorithm, see e.g. [6], for the Wiener-model was derived as explained in the following, see Fig. 4.

The disturbed a-priori error $\tilde{e}_a(n)$ is the difference between the desired signal $y(n)$ and the model output $\hat{y}(n)$,

$$\tilde{e}_a(n) = y(n) - \hat{y}(n) \quad . \tag{9}$$

The output of the Wiener model is

$$\hat{y}(n) = f\left(\hat{\underline{\theta}}, L\left[\hat{\underline{h}}(n), u(n)\right]\right) \quad , \tag{10}$$

where the nonlinear function is given by

$$f(\hat{\underline{\theta}}(n), \hat{x}(n)) = \frac{\hat{x}(n)}{1 + \hat{\theta}(n)\,|\hat{x}(n)|^2} \quad , \tag{11}$$

matching the nonlinear function in (7). The vector $\hat{\underline{\theta}}(n)$ reduces to a complex scalar $\hat{\theta}(n)$. Subsuming the actual input signal $u(n)$ and the last $M-1$ input samples into the input-vector $\underline{u}(n)$ and the estimated weights $\hat{h}_m(n)$ of the linear filter into the vector $\hat{\underline{h}}(n)$

$$\begin{aligned} \underline{u}(n) &= [u(n), u(n-1), \ldots, u(n-M+1)]^T \\ \hat{\underline{h}}(n) &= \left[\hat{h}_0(n), \hat{h}_1(n), \ldots, \hat{h}_{M-1}(n)\right]^T \quad , \end{aligned} \tag{12}$$

the output of the linear filter is $\hat{x}(n) = \underline{u}^T(n)\hat{\underline{h}}(n)$. After some simple calculations, the gradients of the a-priori error with respect to the parameter-vectors $\hat{\underline{\theta}}(n)$ and $\hat{\underline{h}}(n)$ are obtained as

$$\begin{aligned} \nabla_{\hat{\underline{h}}}\tilde{e}_a(n) &= -\nabla_{\hat{\underline{h}}}\hat{y}(n) = -\underline{u}^T(n)\frac{1}{\left(1 + \hat{\theta}\,|\hat{x}(n)|^2\right)^2} \quad , \\ \nabla_{\hat{\underline{\theta}}}\tilde{e}_a(n) &= -\nabla_{\hat{\underline{\theta}}}\hat{y}(n) = \hat{x}^*(n)\hat{y}^2(n) \end{aligned} \tag{13}$$

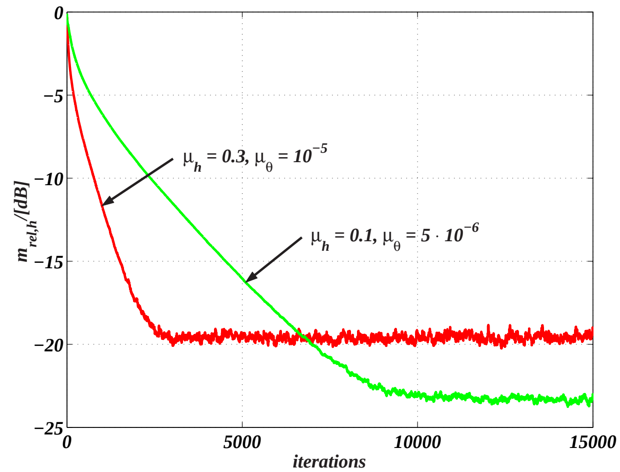

Fig. 5. Relative misadjustment of the linear filter part parameter vector $\hat{\underline{h}}$ for two different step-sizes.

where $(\cdot)^*$ stands for complex conjugation. The LMS-algorithm can therefore be formulated for the parameter-vectors

$$\begin{aligned} \hat{\underline{h}}(n+1) &= \hat{\underline{h}}(n) + \mu_h\tilde{e}_a(n)\left(-\nabla_{\hat{\underline{h}}}\tilde{e}_a(n)\right)^H \\ \hat{\underline{\theta}}(n+1) &= \hat{\underline{\theta}}(n) + \mu_\theta\tilde{e}_a(n)\left(-\nabla_{\hat{\underline{\theta}}}\tilde{e}_a(n)\right)^H \end{aligned} \tag{14}$$

using two different step-size μ_h and μ_θ. Finally, the LMS-algorithm for updating the parameters reads

$$\begin{aligned} \hat{\underline{h}}(n+1) &= \hat{\underline{h}}(n) + \mu_h\tilde{e}_a(n)\underline{u}^*(n)\left[\frac{1}{\left(1 + \hat{\theta}\,|\hat{x}\,(n)|^2\right)^2}\right]^* \\ \hat{\underline{\theta}}(n+1) &= \hat{\underline{\theta}}(n) - \mu_\theta\tilde{e}_a(n)\hat{x}\,(n)\left(\hat{y}^2\,(n)\right)^* \quad . \end{aligned} \tag{15}$$

Since the model output depends nonlinearly on the parameters $\hat{\underline{h}}(n)$ and $\hat{\underline{\theta}}(n)$ the adaptation process ending in a global minimum of the cost function $J_c\,(n) = E\left[|\tilde{e}_a(n)|^2\right]$ can not be guaranteed. The performance depends as well on the starting point of the optimization, i.e., the initial guess for the parameters.

In a real-time system the optimization of the parameters would be achieved by a LMS algorithm rather than a complex Matlab function. It is thus of interest to study the behavior of the LMS algorithm when adapting to the coefficients of a Wiener-model. The learning behavior of the derived LMS-algorithm using the parameters of Table 3 was simulated. The relative misadjustments of the parameter

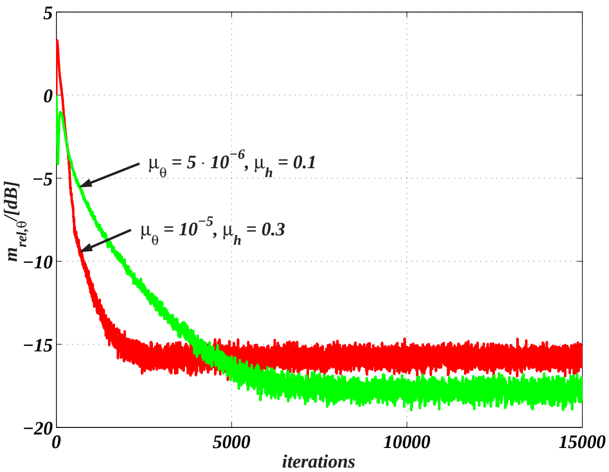

Fig. 6. Relative misadjustment of the nonlinear filter part parameter vector $\hat{\underline{\theta}}$ for two different step-sizes.

vectors

$$\begin{aligned} m_{rel,\hat{h}}(n) &= \frac{\left\| \underline{h}(n) - \hat{\underline{h}}(n) \right\|^2}{\left\| \underline{h}(n) \right\|^2} \\ m_{rel,\hat{\theta}}(n) &= \frac{\left\| \underline{\theta}(n) - \hat{\underline{\theta}}(n) \right\|^2}{\left\| \underline{\theta}(n) \right\|^2} \end{aligned} \tag{16}$$

are shown in Fig. 5 and Fig. 6, respectively. The mean value of the misadjustment is plotted, averaging over 50 runs. A white gaussian input signal with unit variance was applied. The desired signal was additively disturbed by statistically independent white gaussian noise with variance equal to 10^{-4}. The initial guess for the parameter-vectors were zero vectors, respectively. The algorithm showed good convergence for this particular choice. The step sizes where chosen heuristically. Note that the convergence time using the larger step-sizes of approx. 2000 iterations corresponds to $195\mu s$.

Using small step-sizes the influence on the squared a-priori error can be neglected. For small step-sizes ($\mu_h = 0.01, \mu_\theta = 10^{-7}$) simulations showed a remaining squared a-priori error energy of $\approx 4 \cdot 10^{-4}$, after more than $8 \cdot 10^4$ iterations, applying a noise variance of 10^{-4} and unit variance for the input signal. If the model error and the unknown measurement noise in section 4 are assumed to be equivalent to additional white gaussian noise, the simulations show that the derived adaptation scheme can identify the unknown power amplifier, at least for a white gaussian input signal.

Calculating optimal values and convergence bounds for the step-sizes remains an open problem.

6. CONCLUSION

The popular memoryless nonlinear Saleh-model describing power amplifiers was compared with a nonlinear Wiener-model. With a moderate signal bandwidth of 2MHz, the Wiener-model showed a better performance. Since in a predistortion setting it is necessary to estimate continuously the model parameters, the adaptive LMS algorithm for the Wiener-model was derived. The convergence properties in the mean square sense of the algorithm where investigated using simulations. Theoretical results concerning the convergence properties and the choice of the step-sizes remain open problems.

7. ACKNOWLEDGEMENT

We would like to thank Prof. Gottfried Magerl and Holger Arthaber for their support performing the measurements.

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