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Comparison of Non-Orthogonal Space-Time Block Codes using partial Feedback in Correlated Channels

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Abstract

In this paper we analyze and compare transmission schemes that apply non-orthogonal space-time block codes in uncorrelated and correlated MIMO channels using four transmit antennas and a zero-forcing receiver. The ABBA code and a slightly different code, the so-called Extended Alamouti Code (EAC) are compared. For both codes, partial channel state information is sent back from the receiver to the transmitter quantized to one bit per code block. In this way, we improve the diversity of signal transmission and decorrelate the system using the ABBA code as well as the EAC.

I Introduction

The idea of space-time block coding (STBC) has been proposed by Alamouti [1] for transmitting signals in a complex modulation alphabet over two independently fading channels. This code achieves diversity two and has rate one, as it transmits two symbols in two time intervals. The theory of space-time block codes has been further developed by Tarokh, Jafarkhani and Calderbank [2]. They defined orthogonal space-time block codes (O-STBC) that lead to full diversity and low decoding complexity at the receiver, at the price of some loss in data rate. Full data rate is achievable in connection with full diversity only in the case of two transmit antennas. In the case of four antennas the maximum rate is 1/2 or 3/4. The basic technique to overcome the code rate limitation of orthogonal STBCs for more than two transmit antennas is to introduce a small amount of non-orthogonality in the STBC matrices. As a result, several full rate quasi-orthogonal STBC (Q-OSTBC) schemes have been introduced [3], [4], [5], [6]. However, some of these codes, e.g. the ABBA code [3] are very sensitive against spatially correlated channels. To overcome this problem, a randomization strategy is proposed in [7].

Most STBC are designed under the assumption that the transmitter has no knowledge about the channel. On the other hand, it has been shown that exploiting perfect channel state information (CSI) at the transmitter and at the receiver, outage performance is improved compared

to the case when only the receiver has perfect channel knowledge. In some schemes, low decoding complexity, high diversity and a higher code rate can be obtained even if only partial channel information is sent back to the transmitter [8]. In the scheme presented in [9] [10], with partial channel information at the transmitter, diversity is improved up to nearly the optimum value. Research on adapting the STBC to partial feedback has been an intensive area of research [11–13]. However, mostly channel models with independent and identical by distributed (i.i.d.) coefficients have been considered. While this is far from practical setups, the advantage of such simplification is that much of the performance can be predicted in closed form mathematical expressions. In this paper, Q-OSTBCs are investigated over spatially correlated channels. In our simulations we utilize the so called Kronecker channel model [14] to analyze the performance of Alamouti-like space-time block codes, designed for four transmit antennas. We present a transmission system where only one channel information bit b per code block is returned to the transmitter. Depending on the value of b the transmitter switches between two predefined Q-OSTBCs and chooses that code matrix which achieves better performance.

The paper is organized as follows. First, we give a briefly overview of an orthogonal design in Section II. Section III describes quasi-orthogonal space-time block codes, namely ABBA and the EAC. In Section IV we explain our simple transmission scheme using one feedback control bit per code block. The simulation results are given in Section V and conclusions are summarized in Section VI.

II Alamouti-Code and Orthogonal Code Design

The Alamouti code [1] is an orthogonal code with rate one, allowing to transmit two complex symbols s_1 and s_2 from two transmit antennas in two symbol intervals. The code matrix is defined as:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}. \quad (1)$$

The received symbol vector is given as: $\mathbf{r} = \mathbf{S}\mathbf{h} + \mathbf{v}$, where $\mathbf{h}$ is channel vector and $\mathbf{v}$ is noise vector. The received vector can be written alternatively as $\mathbf{y} = \mathbf{H}_v \mathbf{s} + \bar{\mathbf{v}}$, where $\mathbf{y} = [r_1, r_2]^T$, $\mathbf{s} = [s_1, s_2]^T$ and the virtual 2×2 channel matrix $\mathbf{H}_v$ results in

$$\mathbf{H}_v = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}. \quad (2)$$

The data s_1 and s_2 can be decoupled by the matched filter channel matrix $\mathbf{H}_v^H$, since $\mathbf{H}_v \mathbf{H}_v^H = \mathbf{H}_v^H \mathbf{H}_v = h^2 \mathbf{I}_2$, where $\mathbf{I}_2$ is the 2×2 identity matrix and $h^2 = |h_1|^2 + |h_2|^2$ is the channel gain. In this way, the Alamouti scheme decouples the MISO-channel into two virtually independent channels with channel gain h^2 and diversity $d = 2$.

For more than two transmit antennas, a full diversity space-time block code has been proposed in [17]. This code has rate 3/4 and it is problematic due to its severe power-imbalance. The power transmitted from a given antenna varies in time. Another code with the same properties has been proposed in [16] where the peak to average power ratio is slightly smaller. Again, the code rate there is 3/4..

III Quasi-Orthogonal Space-Time Block Codes

Non-orthogonal code design sacrifices code-orthogonality for a higher code rate. As an example for a rate one design we discuss the codes defined in [3], [4] and [6].

ABBA Code

Two Alamouti codes for two transmit antennas are used as building blocks of the ABBA [3] code for 4 transmit antennas:

$$\mathbf{S}_{ABBA} = \begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_2 & \mathbf{S}_1 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix}. \quad (3)$$

The equivalent virtual channel matrix $\mathbf{H}_{ABBA}$ results in:

$$\mathbf{H}_{ABBA} = \begin{bmatrix} \mathbf{H}_{v1} & \mathbf{H}_{v2} \\ \mathbf{H}_{v2} & \mathbf{H}_{v1} \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix}. \quad (4)$$

Applying a matched filter $\mathbf{H}^H$ at the receiver, the non-orthogonality of the ABBA code shows up in the Gram-mian matrix $\mathbf{G}_{ABBA}$:

$$\mathbf{G}_{ABBA} = \mathbf{H}_{ABBA}^H \mathbf{H}_{ABBA} = h^2 \begin{bmatrix} 1 & 0 & X & 0 \\ 0 & 1 & 0 & X \\ X & 0 & 1 & 0 \\ 0 & X & 0 & 1 \end{bmatrix}, \quad (5)$$

where $h^2 = h_1^2 + h_2^2 + h_3^2 + h_4^2$ is the channel gain and X is a channel dependent interference parameter:

$$X_{ABBA} = \frac{2\text{Re}(h_1 h_3^* + h_2 h_4^*)}{h^2}. \quad (6)$$

One major limitation of the ABBA code is its loss of robustness in highly correlated channels, especially in the case when $h_1 = h_2 = h_3 = h_4$, which leads to the collapse of all detection/decoding algorithms [15].

EAC

The extended Alamouti code for four transmit antennas was first presented in [2]. Later on it has been studied in [4] and [6]. The EAC for four transmit antennas is given as:

$$\mathbf{S}_{EA} = \begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_2^* & -\mathbf{S}_1^* \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & s_4 & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}. \quad (7)$$

The corresponding virtual channel matrix results in:

$$\mathbf{H}_{EA} = \begin{bmatrix} \mathbf{H}_{v1} & \mathbf{H}_{v2} \\ -\mathbf{H}_{v2}^* & \mathbf{H}_{v1}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3 & -h_4 & h_1 & h_2 \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}. \quad (8)$$

Applying a matched filter at the receiver as in the case of the ABBA code we obtain the Grammian matrix $\mathbf{G}_{EA}$:

$$\mathbf{G}_{EA} = \mathbf{H}_{EA}^H \mathbf{H}_{EA} = \mathbf{H}_{EA} \mathbf{H}_{EA}^H = h^2 \begin{bmatrix} \mathbf{I}_2 & X_{EA} \mathbf{J}_2 \\ -X_{EA} \mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix},$$

$$\text{with } \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{J}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad (9)$$

$$h^2 = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$$

and X_{EA} as

$$X_{EA} = \frac{2\text{Re}(h_1 h_4^* - h_2 h_3^*)}{h^2}. \quad (10)$$

It is well known that $\mathbf{G}_{EA}$ should approximate a scaled identity-matrix as far as possible to get full diversity and optimum BER performance. This means, $|X|$ should be as small as possible. In the case of $X = 0$, full orthogonality is obtained.

IV System Model and Feedback Approach

The transmission scheme under investigation is shown in Fig. 1 [9, 10]. The transmitter selects **one of two** very similar Q-OSTBCs to transmit the signal vector $\mathbf{s}$ over the actual channel $\mathbf{h}$. The selection algorithm is controlled by the receiver, that tells the transmitter which one of the two Q-OSTBCs will achieve a more reliable transmission. We assume that the receiver can estimate the channel and has perfect channel knowledge. The channel transfer vector $\mathbf{h} = [h_1, h_2, h_3, h_4]^T$ may fade in any arbitrary way but it is assumed that $\mathbf{h}$ remains constant during the code block of length $T = 4$. The signal transmission can be described by

$$\mathbf{r}_i = \mathbf{S}_i \mathbf{h} + \mathbf{v} \quad i = 1, 2, \quad (11)$$

where $\mathbf{r}_i$ is the (4×1) vector of received signals resulting from the code-block $\mathbf{S}_i$ of length $T = 4$. The code matrix $\mathbf{S}_i$ is either $\mathbf{S}_1$ or $\mathbf{S}_2$ depending on the channel parameter b , defined below. b is fed back from the receiver to the transmitter, and $\mathbf{v}$ is the (4×1) noise vector at the receiver input with independent complex Gaussian components with zero mean and variance σ_v^2 .

Figure 1: Switching Strategy

We simulated the scheme for both space-time block codes, namely the ABBA code and the EAC.

In case of the ABBA code we use an alternative second signal matrix

$$\mathbf{S}_{ABBA_2} = \begin{bmatrix} -s_1 & s_2 & s_3 & s_4 \\ s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3 & s_4 & s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \quad (12)$$

at the transmitter.

Then, the corresponding channel matrix $\mathbf{H}$ is:

$$\mathbf{H}_{ABBA_2} = \begin{bmatrix} -h_1 & h_2 & h_3 & h_4 \\ h_2^* & h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & -h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix} \quad (13)$$

and the interference parameter of the corresponding Gram-mian matrix is

$$X_2 = \frac{2\text{Re}(-h_1 h_3^* + h_2 h_4^*)}{h^2}. \quad (14)$$

In the case of EAC, we use the alternative second code matrix, already defined in [9]

$$\mathbf{S}_{EA_2} = \begin{bmatrix} -s_1 & s_2 & s_3 & s_4 \\ -s_2^* & -s_1^* & s_4^* & -s_3^* \\ -s_3^* & s_4^* & -s_1^* & -s_2^* \\ -s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}. \quad (15)$$

The corresponding virtual MIMO channel matrix is:

$$\mathbf{H}_{EA_2} = \begin{bmatrix} -h_1 & h_2 & h_3 & h_4 \\ -h_2^* & -h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & -h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & -h_1 \end{bmatrix}. \quad (16)$$

The interference parameter of the corresponding Gram-mian matrix is

$$X_{EA_2} = \frac{2\text{Re}(-h_1 h_4^* - h_2 h_3^*)}{h^2}. \quad (17)$$

The principle of our code selection scheme is very simple: The transmitter chooses that code $\mathbf{S}_1$ or $\mathbf{S}_2$ that minimizes $|X|$. As it is assumed that the receiver has full information

of the channel state, knowing h_1 to h_4 , the receiver can compute X_1 and X_2 . With this information the receiver returns the feedback bit b informing the transmitter to select that code block $\mathbf{S}_i$ ($i = 1, 2$) which leads to the smaller absolute value of X_i . With this information the transmitter switches between the Q-OSTBCs $\mathbf{S}_1$ and $\mathbf{S}_2$ such that the resulting $|X|$ will be $\min(|X_1|, |X_2|)$. Obviously the control information sent back to the transmitter only needs one feedback bit per code block. During simulation it is assumed that the channel varies slowly such that the delay of the feedback information can be neglected.

V Simulation Results

In our simulations, we have used a QPSK signal constellation. The Rayleigh fading channel has been kept constant during the transmission of each code block but has faded independently between successive blocks. At the receiver side, we have implemented a zero forcing (ZF) receiver. In general, we model fading correlation by the following channel matrix

$$\mathbf{H} = \frac{1}{\sqrt{\text{tr}(\mathbf{R}_T)}} \mathbf{R}_R^{\frac{1}{2}} \mathbf{V} \mathbf{R}_T^{\frac{1}{2}} \quad (18)$$

where $\mathbf{R}_R = \mathbf{E}\{\mathbf{H}\mathbf{H}^H\}$ is the $n_R \times n_R$ receive correlation matrix, and $\mathbf{R}_T = \mathbf{E}\{\mathbf{H}^H \mathbf{H}\}$ is the $n_T \times n_T$ transmit correlation matrix. The term $\mathbf{V}$ describes a random $n_R \times n_T$ matrix with independent complex-valued elements with zero mean and unit variance. The elements of $\mathbf{V}$ are Gaussian distributed. The normalization coefficient

$$\text{tr}(\mathbf{R}_T) = \text{tr}(\mathbf{R}_R) = \mathbf{E} \left\{ \sum_{i=1}^{n_R} \sum_{j=1}^{n_T} |h_{ij}|^2 \right\} \quad (19)$$

can be interpreted as the channel's total power gain. We simulated the (4×1) MIMO system with $\mathbf{R}_R = \mathbf{I}$ and

$$\mathbf{R}_T = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \quad (20)$$

assuming that the transmit antenna elements are spatially correlated. Four values of the correlation parameter have been used in our simulations: $\rho = \{0, 0.5, 0.75, 0.95\}$

Uncorrelated channels

First, a channel without spatial correlation ($\rho = 0$) has been investigated. As the simulation results in Figure 2 show, the ABBA code and EAC achieve practically identical performance results. With feedback the optimal performance of an OSTBC is closely achieved.

Correlated channels

Figures 3 to 5 show simulation results assuming that antenna elements are correlated with $\rho = \{0.5, 0.75, 0.95\}$. It turns out that for small correlation values up to $\rho = 0.5$ (Figure 3), the ABBA and the EAC are robust against spatial correlation and the feedback information still provides a substantial improvement of the bit error ratio (BER) performance.

In case of higher correlation the ABBA code performance deteriorates dramatically with increasing ρ and collapses in the case of heavily correlated channels (Figure 5). In contrast, the EAC is more robust in spatially correlated channels and shows better BER performance than the ABBA code. As simulation results show, our simple feedback scheme helps in decorrelating the channel using the ABBA code as well as the EA code.

The BER-curve of the ABBA code and the EAC agree completely for all values of ρ in case of feedback of channel characteristics. That means, applying our transmission scheme the choice of a specific quasi-orthogonal code is of no importance - due to minimization of the channel dependent interference parameter X in both cases.

To show that our simple scheme minimizes the channel dependent interference parameter X , we simulated the probability density function of X for correlated channels for both space-time block codes, the ABBA code and the EAC.

Without feedback X_{EA} is already small in correlated channels, since due to Eqn.(10) two approximately equal valued terms are subtracted (Figure 6) which at least partially compensate each other. Note that in average $E[X_{EA}] \approx \frac{1}{2}\rho(1 - \rho^2)$, keeping X_{EA} values with small and with large correlation small. Therefore, feedback does not decrease X_{EA} any further. However, in highly correlated channels a small value of X_{EA} is not sufficient to obtain a good BER performance (Figure 5) since now the performance is essentially controlled by h^2 , a value appearing in all O-STBCs as well as Q-OSTBCs.

In contrast, in strongly correlated channels without feedback, X_{ABBA} is rather high (Figure 7). This is due to Eqn.(6) where two approximately equal terms are summed up resulting in rather high values of X_{ABBA} . Therefore, feedback and code switching reduces X_{ABBA} substantially and thus improves the BER-performance. We conjecture that applying our feedback scheme and switching between channel dependent code versions improves any quasi-orthogonal code due to the minimization of the interference parameter X .

Figure 2: BER results for uncorrelated channels

Figure 3: BER results in case of correlated channels with $\rho = 0.5$

Figure 4: BER results in case of correlated channels with $\rho = 0.75$

VI Conclusion

In this work two sets of STBCs, namely EACs and ABBA codes for four transmit antennas are combined with very low-rate channel feedback information. The BER-performances of all systems are compared in case of spatially correlated and uncorrelated channels. It turns out that this simple transmit scheme, selecting the best transmission code for a given transmit channel, improves diversity and BER-performance over the whole SNR range in spatially correlated and uncorrelated transmission environment. Even a small amount of channel state information helps to decorrelate the transmission system using the ABBA code and the EAC. However, in strongly correlated channels, the transmission performance deteriorates equally in both cases compared to the performance in uncorrelated MIMO-channels.

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Figure 5: BER results in case of correlated channels with $\rho = 0.95$

Figure 6: One sided PDF of X_{EA} for correlated channels

Figure 7: One sided PDF of X_{ABBA} for correlated channels

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