A UNION BOUND OF THE BIT ERROR RATIO FOR DATA TRANSMISSION OVER CORRELATED WIRELESS MIMO CHANNELS

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ABSTRACT

In this contribution we derive a union bound of the Bit Error Ratio (BER) in wireless Multiple Input Multiple Output (MIMO) data transmission systems using Maximum Likelihood (ML) receivers. A novel channel model is used to take into account channel correlation. Relying on the Euclidean distance between different transmit signals, we show that the error performance is governed by a few so-called Error Types (ETs). Simulation results are shown for a 4×4 MIMO system with BPSK modulation and for a 2×2 MIMO system with 16QAM modulation for several channel correlations. Our union bound becomes tight for low BERs for all MIMO systems under investigation.

1. INTRODUCTION

Recent research has shown that wireless Multiple Input Multiple Output (MIMO) systems offer very high capacity [1]. To exploit this increased capacity several approaches are under investigation in these days. One possibility to achieve high data rate is uncoded data transmission using high order modulation formats (MQAM). To keep the Bit Error Ratio (BER) within reasonably low values, Maximum Likelihood (ML) receivers, which are the optimal receivers, are used. Performance evaluation of such systems is mostly done by simulation, although the computational effort is extremely high. For this reason it is very important to find performance bounds. One possibility of approximating the BER performance of such systems is to use a union bound. Such a bound has already been derived by M. Kiessling in [2]. In this paper the so-called Kronecker channel model [3] is used to model a correlated MIMO channel. However, the Kronecker model has some deficiencies [4] and therefore we use a novel channel model recently presented in [5]. Moreover, our calculations are based on so-called ETs that govern the overall error performance. In this way, the BER calculations can be based on a few ETs and thus the derivation becomes more obvious.

The rest of the paper is organized as follows. In Sec. 2 the novel channel model is explained. The union bounds of the BER based on the above mentioned ETs are calculated in Sec. 3 for uncorrelated and correlated MIMO channels. In Sec. 4 the union bounds and simulation results for a 4×4 MIMO system with BPSK modulation and for a 2×2 MIMO system with 16QAM modulation for several channel correlations are presented. A summary and conclusions close the paper.

2. CHANNEL MODEL

In the following sections, two different approaches are used to model the MIMO channel: the well known i.i.d. channel model and a novel correlation model[1]. Our transmission system can be modeled as:

$$\mathbf{y} = \mathbf{H}\mathbf{c} + \mathbf{n} \,, \tag{1}$$

where $\mathbf{y}$ is the receive signal vector, $\mathbf{H}$ is the channel matrix, $\mathbf{c}$ is the transmit signal vector and $\mathbf{n}$ is the additive noise vector.

2.1. Uncorrelated Channels

Uncorrelated MIMO channels are modeled by a channel matrix $\mathbf{H}$ with $n_R \times n_T$ independent complex Gaussian distributed random variables with zero mean and unit variance:

$$\mathbf{H} = \mathcal{N}_C^{n_R \times n_T}(0,1) \,, \tag{2}$$

where n_T and n_R denote the number of transmit and receive antennas.

2.2. Correlated Channels

Correlated channels are modeled in this paper by the W-model, which has been presented in [5]. This model is a generalization of the well known Kronecker model. Note that the Kronecker model is a special case of the W-model [5]. In the Kronecker model it is assumed, that the transmit correlation and the receive correlation are independent from each other. This assumption is quite often not valid. For the W-model it is only assumed that the Eigenbasis $\mathbf{U}_{TX}$ and $\mathbf{U}_{RX}$ of the correlation matrices $\mathbf{R}_{TX}$ and $\mathbf{R}_{RX}$ are fixed. The Eigenbasis can be interpreted as the influence of the scatterers around the transmitter and the receiver. The power coupling from the transmitter Eigenbasis $\mathbf{U}_{TX}$ to the receiver Eigenbasis $\mathbf{U}_{RX}$ is modeled by an additional matrix $\mathbf{\Omega}$. Due to this coupling, transmit and receive correlation are not independent, as it is assumed in the Kronecker model. More details on this model are provided in [5]. The channel model can be written as:

$$\mathbf{H} = \mathbf{U}_{RX}\left(\tilde{\mathbf{\Omega}} \odot \mathbf{G}\right)\mathbf{U}_{TX}^T \,, \tag{3}$$

where $\tilde{\mathbf{\Omega}}$ is the element-wise square root of the power coupling matrix $\mathbf{\Omega}$ and $\mathbf{G}$ is an independent complex Gaussian distributed random matrix, where all entries have zero mean and unit variance. $\odot$ stands for element-wise multiplication. In [5] it is shown how the model parameters ($\mathbf{\Omega}$, $\mathbf{U}_{TX}$, $\mathbf{U}_{RX}$) are obtained. For our purpose, two correlation types (moderate and strong correlation) are investigated. The used model

[1] In the following, this novel model is called W-model due to its inventor Werner Weichselberger [5].

parameters are extracted from measurement data obtained from MIMO channel measurements which were performed at our Institute. A detailed description of the measurement scenarios can be found in [6][2]. In order to get a quantitative value of the amount of correlation, we have calculated the condition numbers c_N (strongest Eigenvalue / weakest Eigenvalue) of the correlation matrix $\mathbf{R} = E_{\mathbf{H}}\left\{\mathbf{H}^H\mathbf{H}\right\}$ for uncorrelated, moderately and strongly correlated 4×4 and 2×2 MIMO channels:

$$c_N^{4\times 4} = 1;\ 8.14;\ 22.92 \quad \text{and} \quad c_N^{2\times 2} = 1;\ 2.74;\ 6.76\ . \tag{4}$$

3. UNION BOUND OF THE BER

In this paper we focus on ML receiver. The Pairwise Error Probability (PEP) P_e between two signals with Euclidean distance d and the noise variance σ_n^2 of the additive Gaussian noise is given by:

$$P_e = Q\left(\sqrt{\frac{d^2}{2\,\sigma_n^2}}\right)\ , \tag{5}$$

where $Q(.)$ denotes the well known Marcum Q-function. From this equation it is obvious, that the Euclidean distance d governs the error performance of the ML receiver. Therefore, we are interested in calculating the mutual Euclidean distances between all received signals. The Euclidean distance between the transmit signal vector $\mathbf{c}_i$ and $\mathbf{c}_j$ at the receiver can be calculated as:

$$\begin{aligned} d_{i,j}^2 &= \|\mathbf{y}_i - \mathbf{y}_j\|^2 = \|\mathbf{H}\mathbf{c}_i + \mathbf{n} - (\mathbf{H}\mathbf{c}_j + \mathbf{n})\,\|^2 \\ &= \|\mathbf{H}\underbrace{(\mathbf{c}_j - \mathbf{c}_i)}_{\mathbf{b}_{i,j}}\|^2 = \mathbf{b}_{i,j}{}^H\mathbf{H}^H\underbrace{\mathbf{H}\mathbf{b}_{i,j}}_{\mathbf{z}_{i,j}}\ . \end{aligned} \tag{6}$$

Here, $\mathbf{b}_{i,j} = \mathbf{c}_i - \mathbf{c}_j$ is called difference vector and $\mathbf{z}_{i,j} = \mathbf{H}\mathbf{b}_{i,j}$. For our purpose, it is sufficient to investigate the statistics of the Euclidean distance $d_{i,j}$. A linear transformation of a complex Gaussian random vector / matrix results again in a complex Gaussian random vector / matrix and therefore the random vector $\mathbf{z}_{i,j}$ is complex Gaussian distributed with the following first and second order moment:

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{z}_{i,j}} &= \mathrm{E}_{\mathbf{H}}\left\{\mathbf{H}\mathbf{b}_{i,j}\right\} = \mathbf{0}\ , \\ \mathbf{R}_{\mathbf{z}_{i,j}} &= \mathrm{E}_{\mathbf{H}}\left\{\mathbf{H}\mathbf{b}_{i,j}\mathbf{b}_{i,j}^H\mathbf{H}^H\right\}\ . \end{aligned} \tag{7}$$

Therefore, $\mathbf{z}_{i,j}$ can be modeled as:

$$\mathbf{z}_{i,j} = \sqrt{\mathbf{R}_{z_{i,j}}}\,\mathbf{g}\ , \tag{8}$$

where $\mathbf{g}$ is an independent complex Gaussian distributed vector with zero mean and unit variance. Then $d_{i,j}^2$ can be written as:

$$d_{i,j}^2 = \mathbf{z}_{i,j}^H\mathbf{z}_{i,j} = \mathbf{g}^H\mathbf{R}_{z_{i,j}}\,\mathbf{g}\ . \tag{9}$$

3.1. Uncorrelated Channels

The main goal of this section is to show our specific way of calculating the union bound, although a quite similar bound already exists for uncorrelated MIMO channels [10]. For uncorrelated channels $\mathbf{R}_{z_{i,j}}$ degenerates to a scaled unity matrix:

$$\mathbf{R}_{z_{i,j}} = \underbrace{\mathbf{b}_{i,j}{}^H\mathbf{b}_{i,j}}_{d^2_{i,j\,(\mathrm{TX})}}\mathbf{I}\ , \tag{10}$$

[2] Moderately correlated channels and strongly correlated channels are modeled by the model parameters extracted from the measurements indicated in [6] by 1D3 and 14D3, respectively.

where $d^2_{i,j\,(\mathrm{TX})}$ is the squared Euclidean distance at the transmitter and $\mathbf{I}$ is the unity matrix. Hence, Eqn. (9) simplifies to:

$$d_{i,j}^2 = d^2_{i,j\,(\mathrm{TX})}\left(|g_1|^2 + |g_2|^2 + \ldots + |g_{n_R}|^2\right)\ . \tag{11}$$

The sum over n_R squared magnitudes of independent complex Gaussian random variables with the same variance $d^2_{i,j\,(\mathrm{TX})}$ is a χ^2 distributed random variable with $2n_R$ degrees of freedom. Thus, the Probability Density Function (PDF) of $d_{i,j}^2$ is:

$$p_{d^2_{i,j}}(\xi) = \frac{\xi^{n_R-1}}{\left(d^2_{i,j\,(\mathrm{TX})}\right)^{n_R}\Gamma(n_R)}e^{-\frac{\xi}{d^2_{i,j\,(\mathrm{TX})}}}\ . \tag{12}$$

Now, we know the PDF of the squared Euclidean distance $d_{i,j}^2$ at the receiver, but actually we are interested in the mean error performance. The mean PEP is calculated as:

$$\begin{aligned} \mathrm{PEP}_{i,j}^{\mathrm{i.i.d.}} &= \mathrm{E}_{\mathbf{H}}\left\{Q\left(\sqrt{\frac{d_{i,j}^2}{2\,\sigma_n^2}}\right)\right\} \\ &= \int_0^\infty Q\left(\sqrt{\frac{\xi}{2\sigma_n^2}}\right)\frac{\xi^{n_R-1}}{\left(d^2_{i,j\,(\mathrm{TX})}\right)^{n_R}\Gamma(n_R)}e^{-\frac{\xi}{d^2_{i,j\,(\mathrm{TX})}}}\,d\xi \\ &= \left(\frac{1-\mu_{i,j}}{2}\right)^{n_R}\sum_{k=0}^{n_R-1}\binom{n_R-1+k}{k}\left(\frac{1+\mu_{i,j}}{2}\right)^k\ , \end{aligned}$$

with

$$\mu_{i,j} = \sqrt{\frac{d^2_{i,j\,(\mathrm{TX})}}{4\sigma_n^2 + d^2_{i,j\,(\mathrm{TX})}}}\ . \tag{13}$$

The superscript i.i.d. is used to distinguish the PEPs for uncorrelated and correlated channels. The integral is taken from [8]. As it can be seen in Eqn. (13) the only essential parameter, which determines the PEP is $d^2_{i,j\,(\mathrm{TX})}$. The main idea of this paper is to simplify the evaluation of a union bound of the overall BER via the introduction of ETs. Our method can be explained best by an illustrative example. As illustrative example No.1 we discuss the case of a $n_R = n_T = 4$ MIMO system with BPSK modulation. Using BPSK modulation ($s_i \in \pm 1$), there are $2^{n_T} = 2^4 = 16$ different transmit signal vectors $\mathbf{c}_i$. Therefore, there are $16\cdot 16 = 256$ possible crossover events, where the signal vector $\mathbf{c}_i$ is transmitted and the receiver decides in favor of $\mathbf{c}_j$. Thus, we have to consider 256 difference vectors $\mathbf{b}_{i,j} = \mathbf{c}_i - \mathbf{c}_j$. For each of these 256 difference vectors $\mathbf{b}_{i,j}$, the Euclidean distance at the transmitter $d^2_{i,j\,(\mathrm{TX})} = \mathbf{b}_{i,j}{}^H\mathbf{b}_{i,j}$ and the number of corresponding bit errors $n_{BE\,i,j}$ can be calculated.

Definition: A so-called Error Type (ET) is defined as the set of all crossover events which have the same $d^2_{i,j\,(\mathrm{TX})}$ and $n_{BE\,i,j}$.

The number of crossover events with the same parameter pair ($d^2_{i,j\,(\mathrm{TX})}$, $n_{BE\,i,j}$) is denoted by f_k. The surprising result is, that in our example there are only four different ETs. In general, the number of different ETs is denoted by n_{ET}. The concept of ETs was first presented in [7]. It is important to note that in the following the index (i,j) is replaced by (k), because now there are only four different values for the parameter pair $d^2_{i,j\,(\mathrm{TX})}$ and $n_{BE\,i,j}$. Numerical values for these parameters are listed in Table 1. With this, the union bound of the BER, which is the sum over all 256

ET	$d^2_{k\,(\mathrm{TX})}$	f_k	n_{BE_k}
	0	16	0
1	4	64	1
2	8	96	2
3	12	64	3
4	16	16	4

Table 1: The list of the characteristic parameters for all four ETs, namely $d^2_{k\,(\mathrm{TX})}$, f_k and n_{BE_k} (Example No.1).

crossover events, can be written as the weighted sum over all ETs:

$$\mathrm{BER} \leq \sum_{k}^{n_{ET}} w_k \, \mathrm{PEP}_k^{\mathrm{i.i.d.}} \qquad w_k = \frac{f_k}{|\mathcal{A}|^{n_T}} \frac{n_{BE_k}}{\mathrm{ld}(|\mathcal{A}|)\, n_T} \,. \quad (14)$$

Here, $|\mathcal{A}|$ denotes the size of the symbol alphabet of the modulation format and ld denotes the base 2 logarithm.

3.2. Correlated Channels

For correlated channels (correlation incorporated by the W-model) the equations

$$\begin{aligned} \mathbf{R}_{z_{i,j}} &= \mathbf{U}_{RX} \mathrm{diag}(\lambda_1^{(i,j)}, \lambda_2^{(i,j)}, \ldots, \lambda_{n_R}^{(i,j)}) \mathbf{U}_{RX}{}^H \,, \\ \lambda_m^{(i,j)} &= \left(\left(\mathbf{b}_{i,j}{}^T \mathbf{U}_{TX} \right) \odot \mathbf{\Omega}_m \right) \mathbf{U}_{TX}{}^H \mathbf{b}_{i,j}{}^* \end{aligned} \quad (15)$$

hold, where $\mathbf{\Omega}_m$ denotes the m-th row of the power coupling matrix $\mathbf{\Omega}$. The number of non zero Eigenvalues of $\mathbf{R}_{z_{i,j}}$ is denoted by n_{NZ}. Hence, in the correlated case Eqn. (9) changes to:

$$d^2_{i,j} = |g_1|^2 \lambda_1^{(i,j)} + |g_2|^2 \lambda_2^{(i,j)} + \ldots + |g_{n_{NZ}}|^2 \lambda_{n_{NZ}}^{(i,j)} \,. \quad (16)$$

The sum over independent random variables corresponds to the product of the characteristic functions [9]. Since the random variables $|g_i|^2$ follow a χ^2-distribution with two degrees of freedom, the characteristic function of $d^2_{i,j}$ is

$$\Psi_{d^2_{i,j}}(j\omega) = \prod_{m=1}^{n_{NZ}} \frac{1}{1 - j\omega \lambda_m^{(i,j)}} \,. \quad (17)$$

Applying a partial fraction expansion to Eqn. (17), the PDF of $d^2_{i,j}$ is easily obtained by the inverse Fourier transform of $\Psi_{d^2_{i,j}}(-j\omega)$, resulting in:

$$p_{d^2_{i,j}}(\xi) = \sum_{m=1}^{n_{NZ}} \frac{\left(\lambda_m^{(i,j)}\right)^{n_R - 2}}{\prod\limits_{\substack{n=1 \\ n \neq m}}^{n_{NZ}} \left(\lambda_m^{(i,j)} - \lambda_n^{(i,j)}\right)} e^{-\frac{\xi}{\lambda_m^{(i,j)}}} \,. \quad (18)$$

Then, the mean PEP results in:

$$\begin{aligned} \mathrm{PEP}_{i,j}^{\mathrm{W}} &= \mathrm{E}_{\mathbf{H}} \left\{ Q\left(\sqrt{\frac{d^2_{i,j}}{2\,\sigma_n^2}} \right) \right\} \quad (19) \\ &= \sum_{m=1}^{n_{NZ}} \frac{\left(\lambda_m^{(i,j)}\right)^{n_{NZ}-2}}{\prod\limits_{\substack{n=1 \\ n \neq m}}^{n_{NZ}} \left(\lambda_m^{(i,j)} - \lambda_n^{(i,j)}\right)} \int_0^\infty Q\left(\sqrt{\frac{\xi}{2\sigma_n^2}} \right) e^{-\frac{\xi}{\lambda_m^{(i,j)}}} d\xi \\ &= \frac{1}{2} \sum_{m=1}^{n_{NZ}} \frac{\left(\lambda_m^{(i,j)}\right)^{n_{NZ}-1}}{\prod\limits_{\substack{n=1 \\ n \neq m}}^{n_{NZ}} \left(\lambda_m^{(i,j)} - \lambda_n^{(i,j)}\right)} \left(1 - \sqrt{\frac{\lambda_m^{(i,j)}}{4\sigma_n^2 + \lambda_m^{(i,j)}}} \right) \,. \end{aligned}$$

The integral has the same structure as in Eqn. (13). As it can be seen in Eqn. (18) the relevant parameters, which influence the PEP are the Eigenvalues $\lambda_m^{(i,j)}$ (m=1, 2, ..., n_{NZ}). For this reason in principle we approach in the same way as in the case of an uncorrelated channel: Again, we list all ETs with their parameters in a new table quite similar to Tab. (1). The only difference is that there are now n_{NZ} parameters $\lambda_m^{(i,j)}$ (m=1, 2, ..., n_{NZ}) instead of the single parameter $d^2_{k\,(\mathrm{TX})}$. In the correlated case of our example No.1, there are 40 ETs ($n_{ET} = 40$) and therefore the table is not shown here in detail. Whereby these ETs were found by an exhaustive search.

The union bound of the BER, which is the sum over all 256 crossover events, again can be written as the weighted sum over all ETs:

$$\mathrm{BER} \leq \sum_{k}^{n_{ET}} w_k \, \mathrm{PEP}_k^{\mathrm{W}} \qquad w_k = \frac{f_k}{|\mathcal{A}|^{n_T}} \frac{n_{BE_k}}{\mathrm{ld}(|\mathcal{A}|)\, n_T} \,. \quad (20)$$

4. RESULTS

The simulations are based on MIMO channel realizations which are generated by the W-model. The parameters of the W-model are extracted from measurement data as mentioned in Sec. 2. Fig.1.a shows the union bounds (dashed curves) and the simulation results (solid curves) of the BER vs. SNR for a 4×4 system with BPSK modulation. Fig.1.b shows the results for a 2×2 system with 16QAM modulation. The red curve labeled by the ∘-markers shows the performance for uncorrelated channels, the green curve labeled by the △-markers shows the performance for moderately correlated channels and the blue curve labeled by the ×-markers shows the performance for strongly correlated channels. As it can be seen, the union bounds for the 4×4 system with BPSK modulation are tight for BERs lower than 10^{-2} and for the 2×2 system with 16QAM modulation the union bound is almost tight for BERs lower than 10^{-3}.

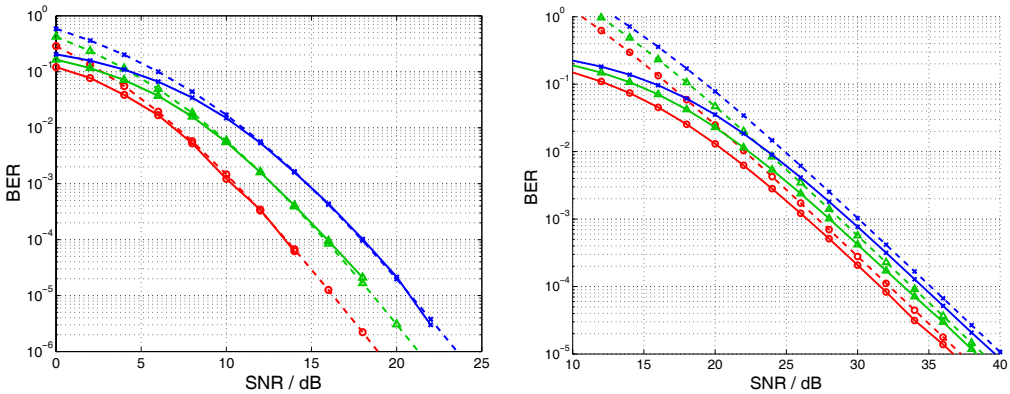
At this point we want to point out an essential difference to the calculations of the union bound in [2] [10]. In these contributions the authors make the assumption the one symbol error results in exactly one bit error. This assumption is not justified in general, especially if high order modulation formats are used. Therefore the "union bounds" in [2] [10] are in fact rather performance approximations than union bounds.

Comparing the results in Fig.1 of [10] with our results in Fig.1.b, we observe that the "union bound" for the 2×2 system with 16QAM modulation in [10] is very tight for high SNR values. We suppose that this effect is due to the wrong assumption, of one bit error for each symbol error, made in [10].

Secondly, we would like to point out that the curves in Fig.1.a have all identical slopes of four decades per 10dB SNR and in Fig.1.b identical slopes of two decades per 10dB SNR . This is because all PEPs in Eqn. (18) for correlated channels have the slope:

$$\lim_{\mathrm{SNR} \to \infty} \frac{d\left(\log_{10} \mathrm{PEP}_k^{\mathrm{W}}\right)}{d\left(\log_{10} \mathrm{SNR}\right)} = -n_{NZ} \,. \quad (21)$$

Consequently, the total BER has also this slope of $-n_{NZ}$ and therefore it can be concluded that full diversity ($n_{NZ} = n_R$)



a. b.

Figure 1: Figure a. shows the union bounds (dashed curves) and the simulation results (solid curves) for a 4×4 MIMO system with BPSK modulation and different channel correlations (uncorrelated - red, ∘-marker | moderately correlated - green, △-marker | strongly correlated - blue ×-marker). Figure b. shows corresponding results for a 2×2 MIMO system with 16QAM modulation.

is achieved, if all Eigenvalues $\lambda_m^{(i,j)}$ are non zero. This is true if the sum of elements over each row of the matrix $\mathbf{\Omega}$ in Eqn.(15) is non zero. Note that the Eigenvalues of the correlation matrices of the Kronecker model are related to the rows and the columns of the matrix $\mathbf{\Omega}$ of the W-model. More details can be found in [5]. The matrices $\mathbf{\Omega}$ used in our simulations indeed have the above mentioned property and therefore full diversity is achieved in all cases. Hence, it is the matrix $\mathbf{\Omega}$ that determines the amount of diversity. I.e., a 4×4 matrix $\mathbf{\Omega}$ with two rows filled with zeros, leads to a diversity of two instead of four and thus a diversity loss of two is observed due to a severe deficiency of $\mathbf{\Omega}$. This loss of diversity is not observed in [2], because there always full rank correlation matrices are assumed and therefore no diversity loss due to fading correlation can occur. However, channel correlation causes an SNR penalty that depends on the amount of correlation as can be seen in Fig.1.

5. CONCLUSION

In this paper an efficient and clear analytical calculation of a tight union bound of the BER of uncoded MIMO systems including uncorrelated and correlated MIMO channels using ML receivers is presented. Spatial channel correlation is incorporated by a novel correlation model, called W-model, introduced in [5]. The calculated union bounds are compared to simulation results in order to verify the tightness of these union bounds. As it can be seen from the simulation results in most cases, fading correlation only causes an SNR penalty but no diversity loss (equal slope of the BER curves for different amount of correlation). Full diversity is achieved, if the power coupling matrix $\mathbf{\Omega}$ does not have rows containing zeros only. An l-fold diversity loss only occurs if all entries of l rows of the matrix $\mathbf{\Omega}$ are zero.

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