# AN ANALOG-DIGITAL ADAPTIVE ECHO CANCELLER FOR HYBRIDS

Markus Rupp
Institut für Netzwerk- und Signaltheorie, TH Darmstadt, 6100 Darmstadt, FRG

*Abstract*—**Adaptive hybrids are one way of canceling the echoes in telephone systems. Since the ratio of the far-end and the local speaker's signal level can be very high the quantization effects of the A/D converters can diminish the speech signal's quality additional to the echo. A simple automatic gain control (AGC) cannot solve the problem. In this paper a new analog-digital solution is proposed to overcome the problem. Since the error path transfer function can mainly be constructed as a pure delay a least mean-square algorithm with delayed update (DLMS) is used to find the optimal echo estimator.**

## I. INTRODUCTION

One disturbing phenomenon of hybrids in telephone networks is the occurence of unavoidable electric echoes. Echo cancellation is necessitated due to imperfect conditions at the hybrid and to ensure a natural speaking environment. However, when transmitting speech signals the signal level of the far-end speaker can be very low compared to the actual echo of the hybrid. Since digital solutions for echo compensation need A/D converters to calculate with sampled values the speech quality is not only diminished by the echo of the hybrid but also by quantization effects. Fig. 1 pictures the signal-to-noise ratio as a function of the input signal variance when using a 12bit A/D converter with a range of ±2.5V. A $K_0$-density function has been used to closely resemble speech signals and a Gaussian density to compare. As can be seen, with $K_0$ only 55dB can be achieved maximally, but an increase of the variance by a factor 5 already results in a 30dB decrease of the signal-to-noise ratio. On the contrary the Gaussian density allows a wide range of almost constant signal-to-noise ratio.

## II. AN ANALOG-DIGITAL SOLUTION

The undesired quantization effect can be diminished by using an automatic gain control (AGC) before quantizing. However, the ratio of the local speaker echo and the subscriber's signal remains unchanged by an AGC. Therefore, it seems better to subtract the estimated echo first, before quantizing. To improve the situation a combined analog-digital solution is proposed as illustrated in Fig. 2. The

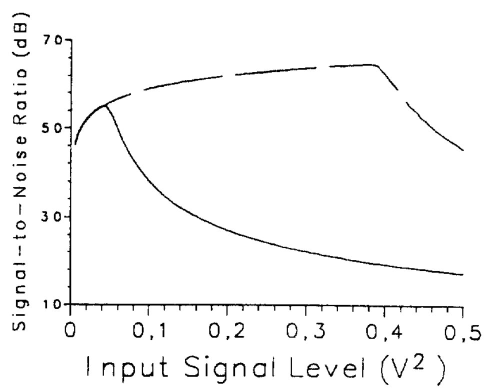


Figure 1: Signal-to-noise ratio on the output of a 12bit A/D converter (±2.5V) as a function of the variance for $K_0$ process (solid line) and Gaussian process (dashed line)

bandpasses $F_1$ and $F_2$ ensure the correct use of the sampling theorem. The subtraction node to cancel the echo is now implemented in the analog part by an operational amplifier (OpAmp). This allows locating the AGC direct in the hybrid path before the echo cancellation node, so that the AGC's transfer function is added to this path and not to the error path from $\hat{y}(n)$ to $e_D(n)$. The advantage of this movement is to obtain a error path transfer function close to a simple delay. To make the adaptive filter insensitive to the gain variations of the AGC the signals $\hat{y}(n)$ and $e_D(n)$ are scaled corresponding to the actual AGC gain. The adaptive filter itself is a 32tap transversal filter. Subtracting the echo by an OpAmp a filter function is added in the error path. Fig. 3 illustrates the measured impulse response of the error path setting $d(n) = 0$ (Sampled signals are written with discreete argument $n$, whereas the corresponding continuous signals are written with argument $t$ and tilde.):

$$e_D(n) = H[\hat{y}(n)]|_{d(n)=0} \quad (1)$$

$$= \sum_{i=0}^{P} h_i \, \hat{y}(n-i) \, . \quad (2)$$

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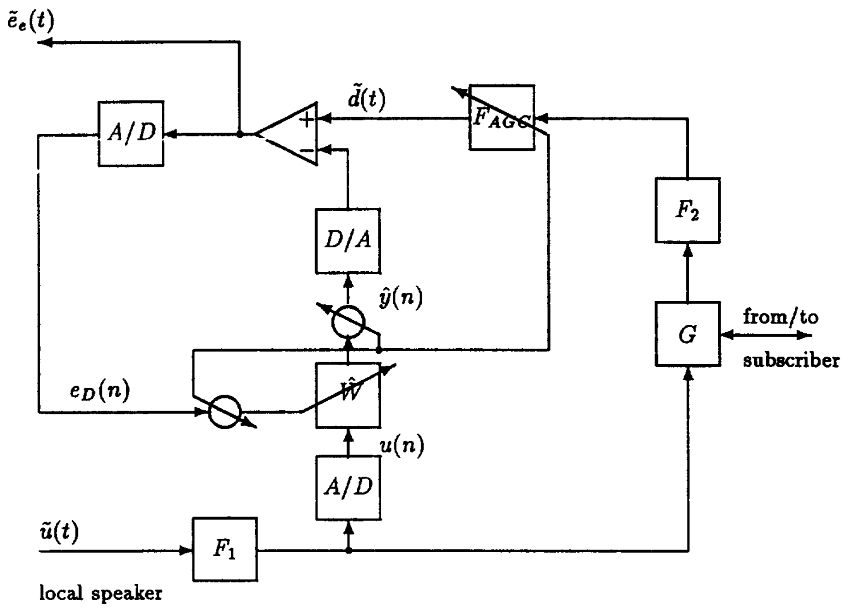


Figure 2: Adaptive transversal filter with AGC and analog subtraction node

The most interesting fact in Fig. 3 is the shape of the

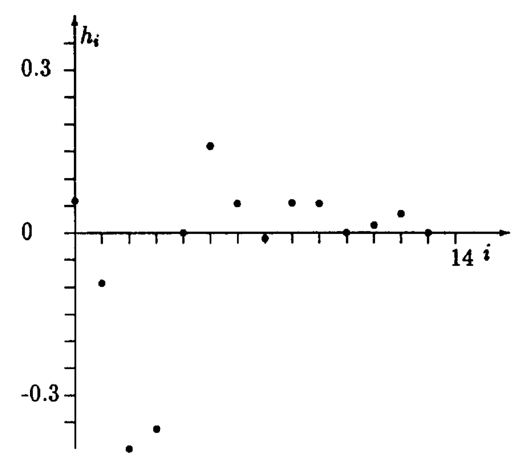


Figure 3: Measured error path impulse response $h_i$

impulse response. Only two coefficients are dominant, whereas the remaining coefficients are smaller. The measured function consists of one unit delay by A/D and D/A converter and of the OpAmp's transfer function. The special shape of the transfer function has important consequences for the choice of the algorithm.

## III. THE CHOSEN ALGORITHM

To adaptate the filter $\hat{W}$ in Fig. 2, two algorithms are possible. The more complex solution uses the FXLMS [6] algorithm which needs the measurement of the error path transfer function. A simpler solution is the usage of the LMS algorithm with delayed coefficient update (DLMS) [5]. To deal with this algorithm a pure delay as transfer function for the error path is necessary. Although the measured transfer function $H$ does not show this desired behavior, the concentration around a delay $D$ between two and three is obvious. The observed error signal can be modelled as

$$e_D(n) = h_D\left(d(n-D) - \underline{\hat{w}}^T(n-D)\underline{u}(n-D)\right) \qquad (3)$$

using the abbreviations:

$$\begin{aligned} \underline{\hat{w}}^T(n) &= (\hat{w}_0, \hat{w}_1 ..., \hat{w}_{M-1}) \\ \underline{u}^T(n) &= (u(n), u(n-1), ..., u(n-M+1)) \,. \end{aligned}$$

Assuming this model the DLMS algorithm can be applied and is supposed to have good results. To make the algorithm independent of the input signal level, a normalized version of the DLMS algorithm (NDLMS) has been used. The update equations are as follows:

$$\underline{\hat{w}}(n+1) = \underline{\hat{w}}(n) + \mu(n)e_D(n)\underline{u}(n-D) \,. \qquad (4)$$

To analyze the properties of the algorithm two ways are possible. On one hand the input sequences can be looked upon as random signals. An analysis of the unnormalized algorithm can be found in [5], when using spherically invariant random processes [7] which closely resemble speech signals in [8]. On the other hand the input signals can be handled as deterministic sequences. An analysis of this kind will be presented in the following section.

## IV. ANALYSIS OF THE ALGORITHM

For the analysis the system to identify $W(z) = F_{AGC}(z)F_2(z)G(z)$ is supposed to be time-invariant with an impulse response series $w_i$ $(i = 0..M-1)$, defining a vector $\underline{w}$. With this definition the weight-error vector $\underline{\epsilon}(n) = \hat{\underline{w}}(n) - \underline{w}$ is introduced. The requirement for a convergent algorithm is a vector $\underline{\epsilon}(n)$ whose length decreases with growing time $n$. The error from (3) can be rewritten as

$$e_D(n) = h_D(e_m(n) - \underline{\epsilon}^T(n-D)\underline{u}(n-D)) , \qquad (5)$$

where $e_m(n)$ is the remaining signal after echo compensation (originated mainly from the subscriber side when the far-end speaker is active) and $h_D$ is the gain constant of the error path. The adaptation (4) can now be written in terms of the weight-error vector

$$\begin{aligned} \underline{\epsilon}(n+1) &= \underline{\epsilon}(n) - \mu(n)h_D\underline{u}(n-D)\underline{u}^T(n-D)\underline{\epsilon}(n-D) \\ &\quad + \mu(n)h_D e_m(n)\underline{u}(n-D) . \qquad (6) \end{aligned}$$

This is a vector difference equation of order $D+1$. It can be written in a compact form using

$$\begin{aligned} \underline{\epsilon}_v^T(n) &= (\underline{\epsilon}^T(n-1), ..., \underline{\epsilon}^T(n-D)) \\ \underline{u}_v^T(n) &= (\underline{u}^T(n-D), \underline{0}^T, ..., \underline{0}^T) \\ R(n) &= \underline{u}(n-D)\underline{u}^T(n-D) , \\ B(n) &= \begin{pmatrix} I & O & \ldots & O & -\mu(n)h_D R(n) \\ I & O & \ldots & O & O \\ O & I & \ldots & O & O \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & \ldots & I & O \end{pmatrix} \end{aligned}$$

$$\underline{\epsilon}_v(n+1) = B(n)\,\underline{\epsilon}_v(n) + \mu(n)h_D e_m(n)\underline{u}_v(n) . \quad (7)$$

Here, $I$ is the $M \times M$ identity matrix and $O$ is a $M \times M$ matrix with zero elements. Calculating the eigenvalues $\lambda_i(i = 1..M(D+1))$ of the matrix $B(n)$, only $D+1$ eigenvalues influence the convergence behavior. For these the following characteristic equation is obtained:

$$z^D(z-1) + \mu(n)h_D\underline{u}^T(n-D)\underline{u}(n-D) . \qquad (8)$$

Equation (8) can be rewritten using the normalized stepsize $\alpha$:

$$\mu(n) = \frac{\alpha}{h_D\ \underline{u}^T(n-D)\underline{u}(n-D)} \qquad (9)$$

($h_D$ has to be measured before using the algorithm) to:

$$z^D(z-1) + \alpha = 0 . \qquad (10)$$

Obviously, the solutions of (10), i.e. the eigenvalues, are not depending on the input sequence any further and are constant with time. Fig. 4 depicts the root loci for $D = 3$ corresponding to the measured error path transfer function when $\alpha$ varies from zero to one. Each root once passing the unit circle due to an increasing $\alpha$ never returns. The root loci behavior is approximately determined using classical rules [3] for evaluating. It can be seen that $D$ roots start in the origin with $\alpha = 0$, then radiate like beams towards the outside of the circle. At the origin adjacent radials have an angle equal to $\frac{360°}{D+1}$, while asymptotical radials have an angle equal to $\frac{360°}{D+2}$. Only one initial root lies at the point $(1,0)$. This root runs first in an inwards direction before it changes and leaves the circle as well. This root is decisive for both the speed of convergence and the stability bound of the weight-error vector. Similar to the considerations in [4] a bound for convergence and stability can be found:

$$0 < \alpha < 2\cos\left(\frac{D}{2D+1}\pi\right) . \qquad (11)$$

For $D = 3$ the bound $\alpha_l = 0.445$ is obtained. Simulations

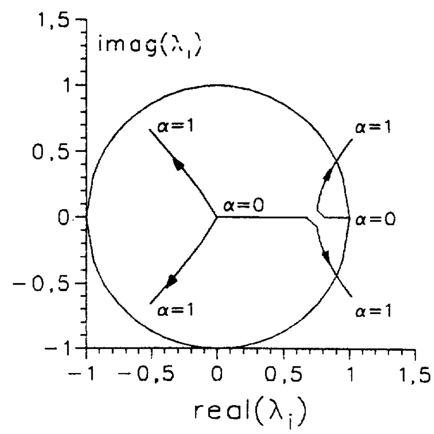


Figure 4: Root loci of the characteristic polynomial for $D = 3$ as a function of $\alpha$

proved the correctness of this bound. The more correlated the input process is, the smaller is the difference of the simulated and the calculated bound. For white processes however, the bound could be almost twice as much. The calculation of optimal step-sizes depending on the correlation of the input process can be found in [5,8].

## V. MEASUREMENT RESULTS

Fig. 5 illustrates a comparison of the achievable gain depending on the input signal level using a 32tap transversal filter $\hat{W}$. The solid line demonstrates the improvement of using an AGC while the dashed line indicates the classical normalized LMS (NLMS) solution. As can be seen in this application the NLMS algorithm depends strongly upon the input level. Especially for small input levels the behavior is improved using the AGC. For all measurements a white Gaussian process has been used. According to Fig. 1 the improvement for $K_0$ processes is expected to be much bigger. Fig. 6 illustrates the measured Echo return loss

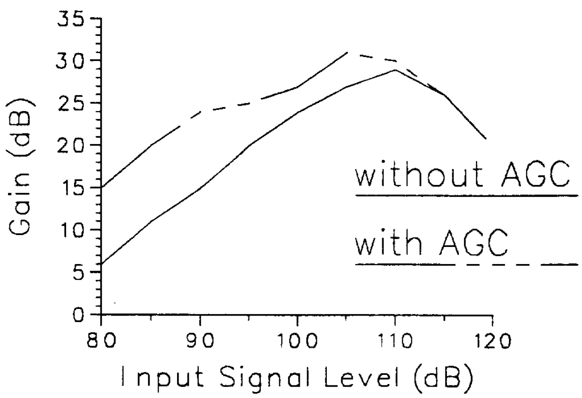


Figure 5: Classical NLMS (dashed line) in comparison to NDLMS algorithm (solid line) with AGC

enhancement (ERLE) using the analog-digital structure of Fig. 2. The convergence rate is fast and the obtained echo cancellation is as good as in pure digital solutions. However, the quality of a low level subscriber signal is not lowered any further. For the delays $D = 2$ and $D = 3$ the measured ERLE in Fig. 6 was the same. Even for an FXLMS algorithm with 14 taps for the error path the curve did not change. But a simple delay unequal to two or three made the algorithm unstable.

To control the AGC an estimate for the level of the error signal $e_D(n)$ is used. This works perfectly to improve the ERLE when the error signal consists mainly of uncompensated echoes, originated from $u(n)$. However, if the far-end signal has a higher level the AGC control adapts to it. A switching between the two states –far-end active/inactive– is currently being investigated.

Since adaptive IIR filters showed good results for telephone hybrids [2] instead of an NLMS algorithm for transversal filters the SHARF algorithm has also been implemented. Here, three coefficients for the recursive and only nine coefficients for the transversal part have been used, achieving the same steady-state error. With the normalization rule from [2] the algorithm possessed its fastest behavior with a step-size of $\alpha = 0.05$, which is much smaller than the step-size of the NLMS algorithm ($\alpha = 0.25$). A smaller step-size, however, causes a smaller steady-state error. This seems to be a slight advantage when using the SHARF algorithm.

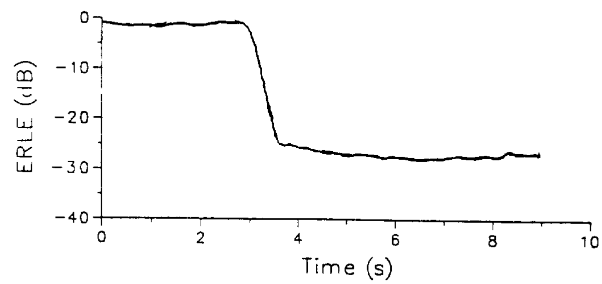


Figure 6: Measured ERLE for NDLMS algorithm


ACKNOWLEDGMENT

The author would like to thank Udo Petri from Telenorma for his helpful comments and his company for supplying this work.


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