Adaptive TDMA-DFE Algorithms Under IS-136

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Abstract

The IS-136 standards define conditions under which wireless TDMA systems are required to work. Adaptive equalizers are unavoidable in order to satisfy these requirements. We extend common results in tracking theory for system identification to the equalization case. Unlike in system identification where a steady-state error energy is minimized, the optimization criterion here is the minimum of the BER. However, we show that both are related by a monotone function and therefore minimizing the BER is equivalent to minimizing the steady-state-error energy. Optimum parameters for LMS as well as RLS algorithms are derived and simulation results indicate that under the conditions defined in the IS-136 standards, the performance of the two methods is comparable.

1 Introduction

The IS-136 standards[1] define transmission protocol (offset 4DPSK) and conditions under which a wireless system is supposed to work. For a Rayleigh fading channel, for example the standard requires that the Bit-Error-Rate (BER) for a Signal-to-Noise Ratio of about 20dB does not exceed 3%. Although it is known how a communication system with this modulation performs for an ideal equalizer, the tracking effects of the equalizer itself have not been investigated. Usually, slow fading channels are assumed so that the effect of the equalizer is much smaller than the BER caused from the Rayleigh channel. The IS-136 standards however, require the 3% BER also for Doppler speeds up to 100Km/h. In this case the tracking noise of the equalizer becomes much larger than the error caused by the Rayleigh channel. Hence, any analysis of the performance of such a communication system must take the equalizer tracking noise into account. Note that tracking analyzes for LMS and RLS algorithms in the context of system identification can be found in literature (see[2,3]). In this paper we show

1. How to extend the tracking theory from [2,3] to the equalization case.

2. How to compute the steady-state-error energy and

3. minimize this energy with respect to the free parameters.

4. How to map the steady-state-energy to the BER.

2 The DFE Reference Model

In order to treat the equalization problem like an identification problem, we assume that there exists an optimal equalizer (model reference) with the structure depicted in Figure 1. The reference model consists of a linear filter that performs an equalization of the channel. Since a general channel can be perfectly equalized only by a filter of (double) infinite length, a finite (and general small) filter order can only achieve a rough equalization. The nonlinear decision device (slicer) following the equalizer guarantees that the outcome of the reference model equals the transmitted signal $y(k) = s(k - D)$, where we allow for a delay of D samples. The difference of the reference model output $s(k - D)$ and the estimate $\hat{z}(k)$ leads to the error $e(k)$ that is used for updating the estimates $\hat{\mathbf{w}}_k$. A final decision device delivers estimates $\hat{s}(k - D)$ of the transmitted sequence. In the following the delay D will be dropped for convenience. The outcome of the linear part is denoted $z(k) = \mathbf{u}_k \mathbf{w}_{k-1}$ where the sampled receiver values have been combined in a row vector $\mathbf{u}_k$ and the filter taps in a column vector $\mathbf{w}_{k-1}$. In order
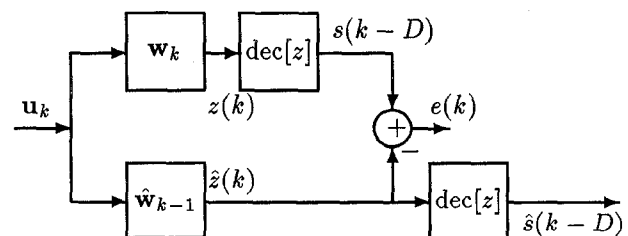

Figure 1: *Model reference structure for DFE equalizer.*

to describe an adaptive DFE structure in terms of the reference model and investigate the desired tracking behavior we have to make some assumptions.

Assumption 1 (Input Statistics)
The white symbol sequence $s(k)$ is linearly filtered

by the channel C and thus the received sequence $u(k) = C[s(k)] + v(k)$ is expected to be correlated. We assume that the received sequence has nearly Gaussian statistics. This assumption has a strong impact on the tracking behavior of the (N)LMS algorithm, where the expectation of $\bar{\mu}(k) = 1/\|\mathbf{u}_k\|_2^2$ plays an important role. If we assume $u(k)$ to be a complex valued white Gaussian process of variance one, the expectation of $\bar{\mu}(k) = 1/\|\mathbf{u}_k\|_2^2$ is given by

$$E[\bar{\mu}(k)] = \frac{1}{M-1} , \tag{1}$$

M being the linear equalizer length. Although this assumption is not precisely satisfied, more accurate terms are not easily available. For larger filter order M, the expression becomes more precise. We also assume the additive noise $v(k)$ to be a Gaussian random process.

Assumption 2 (Approximation)
We obtain for the update error

$$e(k) = \hat{s}(k) - \hat{z}(k) . \tag{2}$$

By adding and subtracting the symbol $s(k)$ this error can be rewritten as

$$
\begin{aligned}
e(k) &= \hat{s}(k) - s(k) + s(k) - \hat{z}(k) \\
&= \hat{s}(k) - s(k) + g[z(k)] + e_a(k)
\end{aligned}
\tag{3}
$$

where the *a-priori* error

$$e_a(k) \triangleq z(k) - \hat{z}(k) = \mathbf{u}_k[\mathbf{w}_{k-1} - \hat{\mathbf{w}}_{k-1}] = \mathbf{u}_k\tilde{\mathbf{w}}_{k-1} , \tag{4}$$

and the error function describing the equalizer imperfection is defined as

$$g[z(k)] \triangleq \mathrm{dec}[z(k)] - z(k) . \tag{5}$$

The definition (3) for the error $e(k)$ in terms of the a-priori error $e_a(k)$ and the equalizer imperfection $g[z(k)]$ requires modification when a DFE structure is used. The elements of the vector $\mathbf{u}_k$ are then

$$\mathbf{u}_k = [u(k), ..., u(k-M+1), s(k-1), ..., s(k-N)] ,$$

while the adaptive branch uses a different vector

$$\bar{\mathbf{u}}_k = [u(k), ..., u(k-M+1), \hat{s}(k-1), ..., \hat{s}(k-N)] .$$

The linear combiner $\mathbf{w}_k$ then consists of weights for the received sequence $u(k)$ and the feedback symbols $\{s(k-1), ..., s(k-N)\}$. Correspondingly, $\hat{\mathbf{w}}_k$ consists

of weights for the received sequence $u(k)$ and the *estimated* feedback symbols $\{\hat{s}(k-1), ..., \hat{s}(k-N)\}$. The a-priori error can now be redefined in terms of $\bar{\mathbf{u}}_k$:

$$e_a(k) \triangleq \bar{\mathbf{u}}_k[\mathbf{w}_{k-1} - \hat{\mathbf{w}}_{k-1}] \tag{6}$$

The relation between $z(k) - \hat{z}(k)$ and the a-priori error is given by

$$z(k) - \hat{z}(k) = e_a(k) + A[s(k) - \hat{s}(k)] , \tag{7}$$

with the linear operator A containing the feedback coefficients of the reference equalizer. Thus, for the DFE structure we obtain instead of (3)

$$e(k) = (1 - A)[\hat{s}(k) - s(k)] + g[z(k)] + e_a(k) \tag{8}$$

As long as the symbols are estimated correctly, $s(k) = \hat{s}(k)$, and we assume that

$$e(k) \approx e_a(k) + g[z(k)] . \tag{9}$$

Assumption 3 (Equalizer Imperfection)
When (9) is compared to a system identification scheme, the term $g[z]$ plays the role of additive noise. Note that $g[z]$ can also be expressed as

$$g[z] = (1 - WC - A)[s(k)] - W[v(k)] , \tag{10}$$

where W is an operator that describes the feedforward part of the equalizer coefficients. The first term in (10) describes the equalization effect. If the equalization is perfect, it is close to zero. This is rarely the case, however, and we therefore have to deal with equalizer imperfection. Thus, we assume for the symbol error-free case that $e_a(k)$ in (6) is disturbed by the compound noise $g[z(k)]$. Since we assume $s(k)$ and $v(k)$ to be independent, the variance of $g[z(k)]$ is given by

$$\sigma_g^2 = \|W\|_2^2\sigma_v^2 + \sigma_e^2 \tag{11}$$

with $\sigma_e^2 = \|1 - WC - A\|^2$. With this approximation we can treat the reference model completely like a linear identification scheme. The additive noise is described by the compound noise with variance given by (11) and the unknown system by the linear combiner $\mathbf{w}_k$. Its dynamics are described in the following section.

3 Channel and Reference Model Dynamics

In IS-136 either a one or a two path Rayleigh model is assumed for modeling the channel. Thus, we can describe the channel coefficients $\mathbf{c}_k$ as

$$\mathbf{c}_k = c_1(k)\mathbf{h}_1 + c_2(k)\mathbf{h}_2 , \tag{12}$$

the vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ being the transfer functions of the actual paths and $c_1(k), c_2(k)$ the random coefficients with Rayleigh distribution. We assume the following linear AR model for $c_i(k)$:

$$c_i(k) = \sum_{l=1}^{L} f_l\, c_i(k-l) + \zeta_i(k)\ , i = 1,2, \qquad (13)$$

The driving processes $\zeta_i(k)$ are assumed to be white complex Gaussian and independent of each other. The filter coefficients $\{f_l\}$ define the correlation of the process and are determined by the Doppler speed. Combining (12) and (13) yields

$$\mathbf{c}_k = \sum_{l=1}^{L} f_l \mathbf{c}_{k-l} + \mathbf{h}_1 \zeta_1(k) + \mathbf{h}_2 \zeta_2(k)\ . \qquad (14)$$

Based on (14) we make the following assumptions on the channel and reference model:

Assumption 4 (Channel Model)
In order to simplify matters we assume $L = 1$ and combine the driving terms into one new white noise term.

$$\mathbf{c}_k = f\, \mathbf{c}_{k-1} + \sqrt{1-f^2}\, \mathbf{q}_k \qquad (15)$$
$$u(k) = \mathbf{s}_k \mathbf{c}_k + v(k) \qquad (16)$$

where $\mathbf{q}_k$ is a white complex-valued Gaussian vector random process of unit variance, i.e., $E[\|\mathbf{q}_k\|_2^2] = 1$ for a one path model or twice that amount for a two path model. The row vector s_k consists of the transmitted symbol sequence $s(k)$.

Assumption 5 (Reference Model Fluctuations)
We assume that the dynamic of the reference model equalizer behaves like the channel, i.e., it follows the same model (15):

$$\mathbf{w}_k = f\, \mathbf{w}_{k-1} + \sqrt{1-f^2}\, \mathbf{q}_k\ . \qquad (17)$$

This is true in particular when only one coefficient f is used with values close to one (as it is the case here). The variance of the fluctuations becomes

$$\Delta_w = E[\|\mathbf{w}_k - \mathbf{w}_{k-1}\|^2] = 2(1-f)E[\|\mathbf{q}_k\|^2]\ . \quad (18)$$

4 Tracking Theory

We first write the LMS update equation

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu(k)\mathbf{u}_k^* e(k) \qquad (19)$$

in terms of $\tilde{\mathbf{w}}_k = \mathbf{w}_k - \hat{\mathbf{w}}_k$ as

$$\tilde{\mathbf{w}}_k = \Delta\mathbf{w}_k + \tilde{\mathbf{w}}_{k-1} - \mu(k)\mathbf{u}_k^* e(k)\ , \qquad (20)$$

with $\Delta\mathbf{w}_k = \mathbf{w}_k - \mathbf{w}_{k-1}$. As long as no error occurs we simply have $e(k) = \mathbf{u}_k \tilde{\mathbf{w}}_{k-1} + g[z(k)]$. We replace the step-size $\mu(k)$ by the projection step-size $\alpha\bar{\mu}(k)$ with

$$\bar{\mu}(k) = 1/\|\mathbf{u}_k\|_2^2$$

and calculate the quadratic l_2- norm of the parameter error vector

$$\begin{aligned}
\|\tilde{\mathbf{w}}_k\|_2^2 &= \|\Delta\mathbf{w}_k\|_2^2 + \|\tilde{\mathbf{w}}_{k-1}\|_2^2 + \alpha^2\bar{\mu}(k)|e(k)|^2 \\
&+ 2\ \mathrm{Re}\{\tilde{\mathbf{w}}_{k-1}^* \Delta\mathbf{w}_k - \alpha e^*(k)\bar{\mu}(k)\mathbf{u}_k\Delta\mathbf{w}_k \\
&\quad -\alpha e^*(k)\bar{\mu}(k)\mathbf{u}_k\tilde{\mathbf{w}}_{k-1}\}\ . \qquad (21)
\end{aligned}$$

We take the expectation of both sides and use the following terms:

$$E[\bar{\mu}(k)|\mathbf{u}_k\tilde{\mathbf{w}}_{k-1}|^2] = \gamma\|\tilde{\mathbf{w}}_{k-1}\|_2^2 \qquad (22)$$
$$E[\bar{\mu}(k)e^*(k)\mathbf{u}_k\Delta\mathbf{w}_k^*] = \gamma E[\tilde{\mathbf{w}}_{k-1}\Delta\mathbf{w}_k^*] \qquad (23)$$
$$E[\mathrm{Re}\{\tilde{\mathbf{w}}_{k-1}\Delta\mathbf{w}_k\}] = \frac{-(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2]\ ,$$

with γ between zero and one depending on the correlation of the received sequence. The derivation for the last line is somewhat lengthy and therefore not given here.

Assumption 6 (Correlations)
In order to continue we assume that all processes $u(k), g[z(k)]$, and $\mathbf{q}_k$ are mutually independent. Note that if the equalizer imperfection is caused mainly by the linear part $(1 - WC - A)$, it will be correlated with the received sequence $u(k)$, and this assumption does not hold. We thus consider two cases: one for which the linear part is small but uncorrelated to the signal, the other where it is part of the signal (see (33) ahead). We will later give the results in form of bounds between these two cases.

With these assumptions, (21) now reads

$$\begin{aligned}
E[\|\tilde{\mathbf{w}}_k\|_2^2] &= \Delta_w + (2\alpha\gamma)E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2] \\
&+ \alpha^2\left[\gamma E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2] + E[\bar{\mu}(k)]\sigma_g^2\right] \\
&- 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2] \\
&= \Delta_w + E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2]\left[1 + \gamma\alpha(\alpha-2)\right] \\
&- 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2] \\
&+ \alpha^2\sigma_g^2 E[\bar{\mu}(k)]\ . \qquad (24)
\end{aligned}$$

In steady-state, $E[\|\tilde{\mathbf{w}}_k\|_2^2] = E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2]$ and (24) becomes

$$E[\|\tilde{\mathbf{w}}_k\|_2^2] = \frac{\Delta_w + \alpha^2\sigma_g^2 E[\bar{\mu}(k)] - 2\frac{(1-\alpha\gamma)(1-f)^2}{1-f(1-\alpha\gamma)}E[\|\mathbf{q}_k\|^2]}{\gamma\alpha(2-\alpha)}\ . \tag{25}$$

Substituting for Δ_w from (18), we finally get

$$E[\|\tilde{\mathbf{w}}_k\|_2^2] = \frac{\Delta_w \frac{\alpha\gamma}{1-f(1-\alpha\gamma)} + \alpha^2\sigma_g^2 E[\bar{\mu}(k)]}{\gamma\alpha(2-\alpha)} . \quad (26)$$

For the update error $e(k)$ we use the relation

$$E[|e(k)|^2] = \gamma M \sigma_u^2 E[\|\tilde{\mathbf{w}}_{k-1}\|_2^2] + \sigma_g^2 ,$$

and obtain for $\sigma_u^2 = 1$

$$\lim_{k\to\infty} E[|e(k)|^2] = \frac{\gamma M \Delta_w}{(1-f(1-\alpha\gamma))(2-\alpha)} \quad (27)$$
$$+\frac{2-\alpha(1-ME[\bar{\mu}(k)])}{2-\alpha}\sigma_g^2 .$$

If we compare this result with the one from [1] we note that $1/\alpha$ is replaced by $1/[1-f+f\alpha\gamma]$. In fact, as $\alpha \to 0$, the steady-state error energy will not grow beyond all limits but will remain bounded by the equalizer fluctuations Δ_w. Thus, (27) is a more accurate description.

Let us consider the flat Rayleigh fading channel for which the receiver sequence $u(k)$ can be assumed to be a white random process with $\gamma = 1/M$. If we further recall that the compound noise $\sigma_g^2 = E[|W[v(k)] + e_e(k)|^2] = E[|\mathbf{w}_k(0)|^2]\sigma_v^2 + \sigma_e^2$, and that $\Delta_w = 2(1-f)E[|\mathbf{w}_k(0)|^2]$, we find the final expression for flat Rayleigh fading

$$\lim_{k\to\infty} E[|e(k)|^2] = \frac{\Delta_w}{(1-f+\frac{\alpha f}{M})(2-\alpha)} \quad (28)$$
$$+\frac{2-\alpha(1-ME[\bar{\mu}])}{2-\alpha}\left[\sigma_v^2 E[|\mathbf{w}_k(0)|^2] + \sigma_e^2\right] .$$

Thus, it is possible to compute the optimal step-size for minimal steady-state error. We obtain a quadratic equation in the step-size α

$$\alpha^2 + 2\alpha A + B = 0 , \quad (29)$$

with the terms

$$A = \left[\frac{M(1-f)}{f}\right](1+C)$$

$$B = \left[\frac{M(1-f)}{f}\right](A-2C)$$

$$C = \frac{|\mathbf{w}_k(0)|^2}{ME[\bar{\mu}(k)][|\mathbf{w}_k(0)|^2\sigma_v^2 + \sigma_e^2]} .$$

The optimal solution is then given by

$$\alpha_{opt} = -A + \sqrt{A^2 - B} . \quad (30)$$

The result for the RLS algorithm is not shown here explicitly but can be obtained following similar arguments.

5 Computing the BER

Although the steady-state-error energy is a good measure for the tracking performance of an equalizer, one is more interested in the final BER for such systems. We therefore need to relate the steady-state error energy to the BER. To this end, we first note that the outcome of the linear equalizer filter $\hat{z}(k)$ is an estimate of $s(k)$, i.e.,

$$\hat{z}(k) = s(k) - e(k) . \quad (31)$$

In the steady-state situation the error term $e(k)$ plays the role of additive noise. Using (9) and (31) we have

$$\hat{s}(k) = \text{dec}[\hat{z}(k)] = \text{dec}[s(k) - e(k)] \quad (32)$$
$$\approx \text{dec}[s(k) - g[z(k)] - e_a(k)] . \quad (33)$$

Since we can consider the channel and equalizer combination as an equivalent additive noise channel with two noise sources $g[z(k)]$ and $e_a(k)$. If the noise sources are assumed to be independent, their energy is given by $\sigma_g^2 + E[|e_a(k)|^2]$ which leads to an upper bound on the BER. However, the compound noise $g[z(k)]$ also consists of signal components. If they are large, they cannot be considered independent of the actual symbol $s(k)$ any more. In this case we consider

$$\hat{s}(k) = \text{dec}[(WC+A)[s(k)] - W[v(k)] - e_a(k)] , \quad (34)$$

which leads to a lower bound.

For a Rayleigh fading channel and additive Gaussian noise the BER for the differentially encoded case is given by [4,5]

$$\text{BER} = \frac{1}{2+\text{SNR}} . \quad (35)$$

For the LMS algorithm the SNR is given by

$$\text{SNR}_{LMS}^{-1} = \frac{2(1-f)}{(1-f+\frac{\alpha f}{M})(2-\alpha)} - \frac{\sigma_v^2}{E[|\mathbf{w}_k(0)|^2]} \quad (36)$$
$$+ \frac{2-\alpha(1-ME[\bar{\mu}])}{2-\alpha}\left[\sigma_v^2 + \frac{\sigma_e^2}{E[|\mathbf{w}_k(0)|^2]}\right] .$$

The optimal step-size for minimum BER can be obtained by differentiating (35) with respect to α. The result for RLS is similar and can be obtained by substituting $2-\alpha(1-ME[\bar{\mu}])$ by 2 and $\lambda = 1 - \alpha/M$ in the above expression.

6 Simulation Results

We provide three simulation runs for 8, 100 and 237 Km/h with the optimal step-sizes $\alpha_{opt} = 0.02, 0.21$ and

0.42 computed for the LMS algorithm and optimal forgetting factors $\lambda_{opt} = 0.99, 0.95$ and 0.9 for the RLS algorithm. Both sets are obtained for a channel SNR of 19.2dB. The modulating frequency was assumed to 900MHz. The speed of 237Km/h is not included in the IS-136 specifications but corresponds to 100Km/h when using a modulating frequency of 1900MHz which is the condition for the PN 3386 PCS standards. We averaged the simulations over 1000 runs.

The BER for the LMS case is shown in Figure 2 and for RLS in Figure 3. For LMS we obtained the BER values (theoretical lower and upper bounds in parenthesis) 0.0106 (0.0082-0.012), 0.0145 (0.0109- 0.0151) and 0.024 (0.017-0.022) for the three speeds respectively. Clearly the theoretical values are very close to the simulation results. In particular, the upper bounds that incorporate the correlation in the equalizer imperfection are very precise. At the iteration number 100 we switched from training to data mode. For the low speed nothing noticeable changes while there is a slight increase in BER (14% increase for 100Km/h and 40% for 237Km/h) for higher speeds.

The behavior for RLS is similar, with BERs of 0.0097 (0.0082-0.012), 0.0138 (0.00109-0.0151) and 0.022 (0.017-0.022) for the three speeds. These error rates are comparable to those from the LMS algorithm. When there is a delay spread the differences between the two algorithms are more pronounced, but both equalization methods satisfy the standards when using a DFE structure.
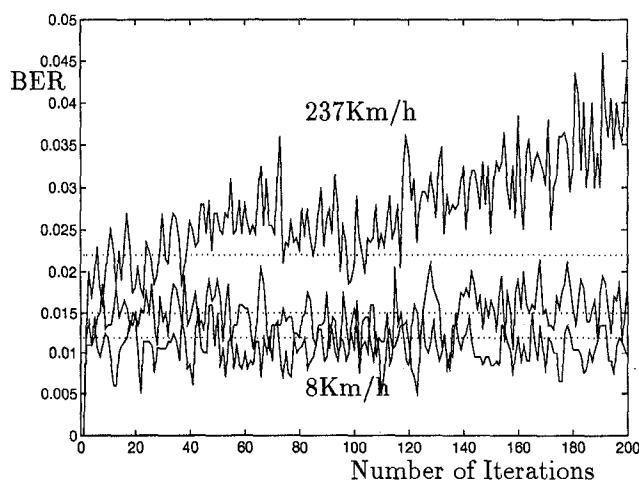

Figure 2: *BER curves for DFE-LMS algorithms for the speeds 8,100 and 237 Km/h when using differentially encoded 4QPSK.*
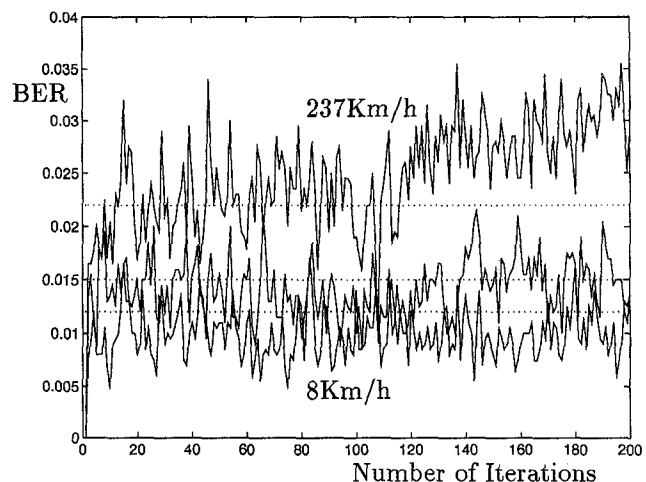

Figure 3: *BER curves for DFE-RLS algorithms for the speeds 8,100 and 237 Km/h when using differentially encoded 4QPSK.*

7   Conclusions

Our theoretical investigations as well as our simulations indicate that the LMS and the RLS algorithms exhibit similar performance in their tracking behavior when used under IS-136 conditions. Both meet the 3% BER bound as required by IS-136 for Doppler speeds up to 100Km/h. Hence the LMS algorithm with much lower complexity can be applied for IS-136. Finally, neither of these schemes meet the BER requirements for the PCS band.

References:

[1] TIA/EIA/Interim Standard IS 136 Telecommunication Association, Dec. 1994.

[2] S. Marcos, O. Macchi, "Tracking capability of the least mean square algorithm: application to an asynchronous echo canceller," *IEEE Trans. Acoustics, Speech and Signal Proc.*, vol. ASSP-35, no. 11, pp. 1570-1578, Nov. 1987.

[3] E. Eleftheriou, D.D. Falconer, "Tracking properties and steady-state performance of RLS adaptive filter algorithms," *IEEE Trans. Acoustics, Speech and Signal Proc.*, vol. ASSP-34, no. 5, pp. 1097-1110, Oct. 1986.

[4] S. Stein, "Fading channel issues in system engineering," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 68-89, Feb. 1987.

[5] J.A. Roberts, J.M.Bargallo, "DPSK performance for indoor wireless Rician fading channels," *IEEE Trans. Commun.*, vol. 42, no. 2-4, pp. 595-596, Feb-Apr. 1994.