Optimization of Coded MIMO-Transmission with Antenna Selection

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***Abstract*— The performance of a wireless system can be significantly improved by using multiple antenna elements at the transmitter and/or the receiver side. The major problems of such systems are high cost and complexity due to more than one radio frequency (RF) chains at both end links. An effective technique to reduce this overhead is to allow the transmitter and/or the receiver to select an optimum antenna subset according to some optimization criterion. In this paper we analyze coded multiple-input/multiple output (MIMO) systems with antenna selection at the transmitter side. We propose an optimum selection criterion to minimize the Bit Error Ratio (BER) when a channel adapted quasi-orthogonal space time code (QSTBC) and a low complexity zero forcing receiver (ZF) are used. We show that antenna selection is a promising low-cost technique that provides additional diversity and coding gain and therefore improves the system performance even if the simple linear receiver is used.**

I. Introduction

The performance of a wireless transmission system can be significantly improved by using multiple antenna elements at the transmitter and/or at the receiver side. This technique promises a significant enhancement of system performance without requiring the allocation of extra spectrum. It has been shown that MIMO systems can achieve high capacity and performance improvement compared to a single antenna system (see [1] and references therein).
However, in practical situations, the major problems of MIMO systems are high cost and complexity due to more than one RF chains at both link ends. Therefore methods have been developed that reduce the number of active antennas at the transmitter and/or the receiver side.
A promising technique is channel adaptive antenna selection. It allows the transmitter and/or the receiver to use an optimum antenna subset according to some antenna selection criterion [2], [3], [4]. Systems equipped with this capability choose a subset of the available transmit and/or receive antennas and only process the signal transmitted across them. This preserves most of the benefit from the multiple available antennas but with fixed RF complexity and cost constraints.

Space-time codes (STBCs) have been introduced to improve the reliability of MIMO systems [5]. Since the pioneering work of Alamouti [6] several orthogonal and quasi-orthogonal STBCs have been developed [7], [8], [9]. Most of the STBCs are designed assuming that the transmitter has no knowledge about the channel. It has been shown that outage performance with perfect channel state information (CSI) at the transmitter and at the receiver is considerably improved compared to the case when only the receiver has perfect knowledge of the channel [10], [11]. In [3], [4] orthogonal STBCs with transmit antenna selection have been investigated.

In this paper we study antenna selection at the transmitter and joint transmit antenna/code selection combined with QSTBCs for 4 transmit antennas. The main goal is to improve the BER performance of the QSTBC using a simple linear decoding algorithm by using an additional channel dependent antenna selection scheme. We investigate three different selection criteria and propose an optimum selection criterion for a ZF receiver. We show that transmitting over a time varying transmit antenna subset selected according to some channel dependent selection criterion, we can achieve a significantly improved diversity advantage compared to a transmission over a fixed set of antennas.

II. Transmission Scheme

We consider space-time coded transmission over $n_t = 4$ out of N_t available transmit antennas and $n_r = 1$ receive antenna. The transmission system is described by

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is the (4×1) vector of signal samples received at the single receive antenna within 4 successive time slots, $\mathbf{S}$ is the QSTBC code matrix [8], [9] given as:

$$\mathbf{S} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{pmatrix}, \tag{2}$$

consisting of the four information symbols s_1 to s_4 and complex conjugated versions of them. $\mathbf{h}$ is the complex valued (4×1) channel gain vector and $\mathbf{n}$ is (4×1) complex Gaussian noise vector. By complex conjugation of the second and third element of $\mathbf{y}$, as already described in [9], the input/output relation (1) can be reformulated as

$$\tilde{\mathbf{y}} = \mathbf{H}_v \mathbf{s} + \mathbf{v}, \tag{3}$$

where $\mathbf{H}_v$ is an equivalent virtual (4×4) channel matrix consisting of elements taken from the original (4×1) channel vector $\mathbf{h}$ (including some complex conjugate versions of the original channel coefficients) [9]. $\mathbf{s}$ is the (4×1) vector consisting of the signal samples s_1 to s_4.

The non-orthogonality of this QSTBCs becomes obvious when maximum ratio combining (MRC) of $\tilde{\mathbf{y}}$ with $\mathbf{H}_v^H$ is applied at the receiver resulting only in a partial decoupling of the data streams, described by

$$\mathbf{r} = \mathbf{H}_v^H \mathbf{H} \mathbf{s} + \mathbf{H}_v^H \mathbf{v} = \begin{bmatrix} s_1 & + & X s_4 \\ s_2 & - & X s_3 \\ s_3 & - & X s_2 \\ s_4 & + & X s_1 \end{bmatrix} + \mathbf{H}_\mathrm{v}^H \mathbf{v} \ . \quad (4)$$

Obviously, due to the interference parameter X, there remains a coupling between the symbols s_1, s_4 and s_2, s_3. The resulting equivalent (4×4) Grammian channel matrix $\mathbf{G}$ results in

$$\mathbf{G} = \mathbf{H}_v^H \mathbf{H}_v = \mathbf{H}_v \mathbf{H}_v^\mathrm{H} = \mathrm{h}^2 \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix} . \quad (5)$$

$h^2 = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$ characterizes the total channel gain and indicates a fourfold transmit diversity. X can be interpreted as a channel dependent interference parameter, given by

$$X = \frac{2\mathrm{Re}(h_1 h_4^* - h_2 h_3^*)}{h^2}. \quad (6)$$

It is well known that $\mathbf{G}$ should approximate a scaled identity-matrix as far as possible to achieve ideal data stream decoupling. Hence, $|X|$ should be as small as possible [10].

A. Zero-Forcing (ZF) receiver

To simplify the transmission on the receiver side we apply a low-complexity ZF receiver. Due to its quasi orthogonality the results for this transmission system using a linear receiver differ from the optimum maximum-likelihood (ML) receiver performance. Nevertheless, the simple ZF detection approach exhibits very low computational complexity and also benefits from the fact that complex matrix inversion is not necessary in the case of QSTBCs. The ZF receiver algorithm leads to

$$\hat{\mathbf{y}} = (\mathbf{H}_v^H \mathbf{H}_v)^{-1} \mathbf{r} = s + (\mathbf{H}_v^H \mathbf{H}_v)^{-1} \mathbf{H}_v^H \tilde{\mathbf{v}}. \quad (7)$$

The correlation of the resulting modified noise samples is given by:

$$\mathrm{E}\left\{\tilde{\mathbf{v}}\tilde{\mathbf{v}}^H\right\} = \frac{N_0}{h^2(1-X^2)} \begin{bmatrix} 1 & 0 & 0 & -X \\ 0 & 1 & X & 0 \\ 0 & X & 1 & 0 \\ -X & 0 & 0 & 1 \end{bmatrix},$$

which leads to two important conclusions:

The power of the modified noise increases with the interference parameter X:

$$\sigma_{\tilde{v}_i}^2 = \frac{N_0}{h^2(1-X^2)}, \quad 1 \le i \le 4 \ . \quad (8)$$

Secondly, the correlation between noise samples $\tilde{v}_1, \tilde{v}_4$ and $\tilde{v}_2, \tilde{v}_3$ grows even faster with increasing X what is the reason for the resulting performance gap compared to the ML receiver.

With (7) and (8) the expression for the instantaneous bit error ratio can be obtained [9]:

$$\mathrm{BER}_{ZF} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\left(\frac{h^2(1-X^2)}{4\sigma_v^2}\right)}. \quad (9)$$

The factor 4 in the denominator of the square root originates from the fact that the transmit symbols are scaled by $\frac{1}{2}$ since the total transmit power is equally split between four transmit antennas. In fact, the scaling-factor h^2 as well as the interference parameter X determine the BER performance as already shown in [9], [10].

III. Antenna Selection Criteria

We assume that the receiver has perfect channel knowledge and it returns some channel information to the transmitter in form of some control bits. At the transmitter, four out of N_t transmit antennas are selected for transmitting the QSTBC. In total, there are

$$q_\mathrm{eff} = \binom{N_t}{4}$$

possibilities to select a subset of $n_t = 4$ transmit antennas out of a set of N_t available antennas and therefore

$$b_\mathrm{feedback} = \log_2(q_\mathrm{eff}) = \log_2\left[\binom{N_t}{4}\right] \text{ bits}$$

have to be returned to inform the transmitter which transmit antenna subset should be used. We assume that the receiver knows all N_t complex channel gains h_1 to h_{N_t} and calculates the best subset of n_t transmit antennas to be used, according to one of the three optimization criteria discussed below. Then, the receiver provides the transmitter with this information via a low-rate feedback channel (we assume that the channel varies slowly and is constant during each QSTBC block).

From the previous section, it is obvious that the interference parameter X and the channel gain h^2 need to be jointly optimized when the best antenna subset is determined. In fact, our optimization criterion is based on the analytic expression of the BER performance (9) for the QSTBC. Obviously, we have to maximize the term $h^2(1-X^2)$ in order to minimize the BER. That means, those four transmit antennas will be selected which maximize the term $h^2(1-X^2)$. This criterion trades off a maximization of the channel gain h^2 and a minimization of the channel dependent interference parameter X.

We have compared this criterion to several other selection rules like maximizing of the channel gain h^2 or minimizing the channel dependent interference parameter X and we have found that this joint optimization criterion leads to the best BER performance over the entire SNR range. In

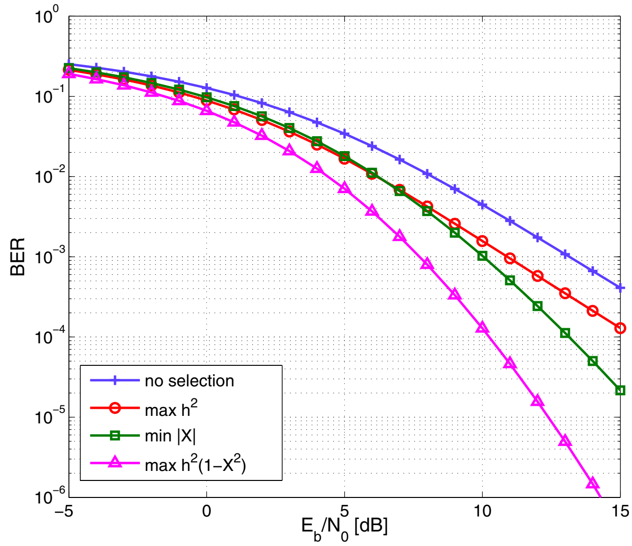

Fig. 1. Comparison of antenna selection criteria for $n_t = 4$ out of $N_t = 6$.

Fig. 1. we show the simulated BER over E_b/N_0 curves for an i.i.d MIMO channel with $N_t = 6, n_t = 4$ and $n_r = 1$. In all our simulations we have used 4QAM signal symbols. Maximization of h^2 only and minimization of $|X|$ lead to a coding gain of about 3 dB at BER = 10^{-3} and some additional diversity gain compared to the case of no transmit antenna selection with $N_t = n_t = 4$. Applying the best optimization (minimizing $h^2(1 - X^2)$) criterion for transmit antenna selection yields by far the best overall performance with the highest coding gain and highest diversity.

IV. Results of Antenna Selection with fixed QSTBC

First we will show the improvement of coded data transmission by additional antenna selection, where the QSTBC is fixed to $\mathbf{S}_1 = \mathbf{S}$ (and not adapted to the channel parameters). We have simulated the BER as a function of E_b/N_0 using 4QAM symbols leading to an information rate of 2 bits/channel use. The channel coefficients are modelled as zero mean i.i.d complex Gaussian random variables with unit variance and are assumed to be invariant during a frame length of 2048 4QAM data symbols

In Fig. 2 we present the simulation results for $4 \leq N_t \leq 7$ available transmit antennas and $n_r = 1$, where 4 transmit antennas are selected according to the best optimization rule, that is maximizing $h^2(1 - X^2)$. For a BER = 10^{-3} and $N_t = 5$, the coding gain is about 3 dB compared to $N_t = 4$, the case without antenna selection. Increasing the number N_t of available transmit antennas by one more, the coding gain increases again by about 1 dB. Most important, Fig. 2 shows that the system diversity increases substantially with the number N_t of the available transmit antennas.

Fig. 3 shows the simulation results for transmit antenna selection in case of spatially strongly correlated channels ($E[h_i h_{i+1}^*] = \rho = 0.95$). QSTBC alone improves the transmission very similar to the case of an uncorrelated system. However, due to the strong spatial correlation, additional antenna selection leads only to a small additional BER improvement.

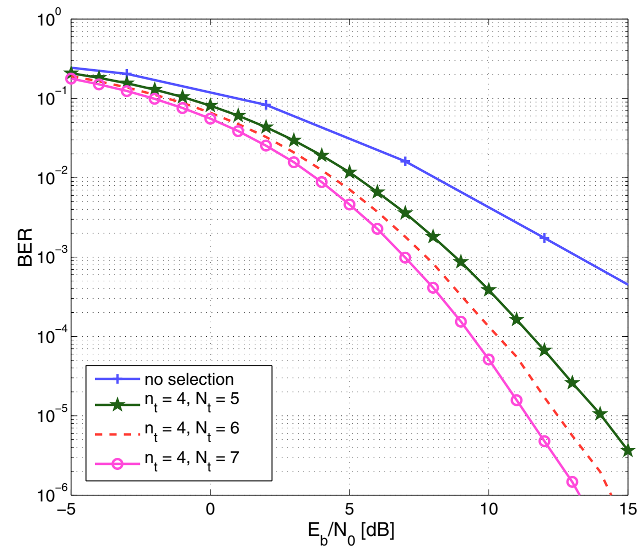

Fig. 2. Transmit antenna selection, iid MISO channels, $n_t = 4$ out of $5 \leq N_t \leq 7$.

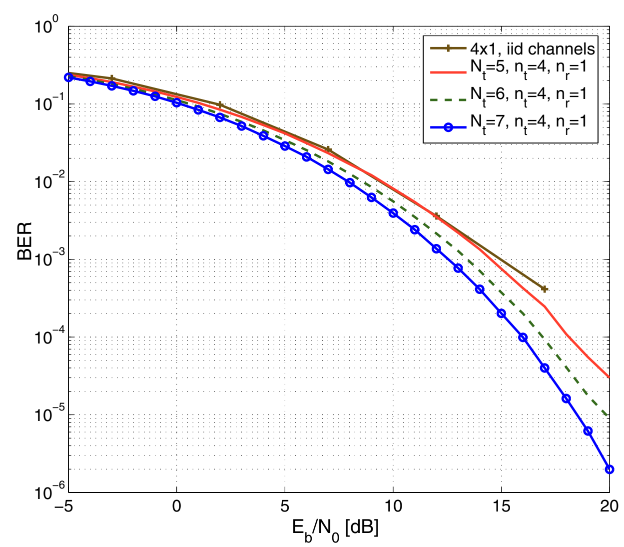

Fig. 3. Transmit antenna selection, spatially correlated MISO channels, ($\rho = 0.95$).

V. Joint Antenna/Code Selection

An alternative way of improving the statistics of X and thereby enhancing the performance of the resulting closed loop system is to adapt the QSTBC to the instantaneous channel as proposed in [10]. In this case, the feedback link is utilized to select one out of a small set of available QSTBCs which is then used for transmission. These QSTBCs are all similarly structured and have the instantaneous bit error ratio as it has been given in (9) but with a different interference parameter X. We use two very similar QSTBCs at the transmitter. Depending on the feedback information the transmitter selects the best antenna subset and one of the two QSTBCs, namely, either $\mathbf{S}_1$ as given in (2) or $\mathbf{S}_2$ given as

$$\mathbf{S}_2 = \begin{pmatrix} -s_1 & s_2 & s_3 & s_4 \\ -s_2^* & -s_1^* & s_4^* & -s_3^* \\ -s_3^* & s_4^* & -s_1^* & -s_2^* \\ -s_4 & -s_3 & -s_2 & s_1 \end{pmatrix}, \qquad (10)$$

such that the term $h^2(1 - X^2)$ is maximized. Transmitting $\mathbf{S}_2$ the corresponding channel dependent interference parameter X_2 is given as:

$$X_2 = \frac{2\mathrm{Re}(h_1 h_4^* + h_2 h_3^*)}{h^2} .$$

As already shown in [10] by this simple code selection the channel interference parameter X can indeed be reduced. In this way full diversity and better quasi orthogonality can be achieved even using a simple ZF receiver.

In case of joint antenna/code selection the amount of required feedback information increases to:

$$b_{\text{feedback}} = \log_2 \left[\binom{N_t}{4} \right] + b_{\text{code}} .$$

b_{code} is number of feedback bits necessary for the code selection. In case if the transmitter switches between two predefined QSTBCs $b_{\text{code}} = 1$, in case of 4 available QSTBCs $b_{\text{code}} = 2$ and so on. Unfortunately, analyzing the results shown in Fig. 4 it turns out that the achievable additional performance gains obtained by code selection are negligible. In fact, only a very small BER performance improvement is achieved due to the additional channel dependent code selection, since the channel dependent interference parameter X is already decreased by the antenna selection algorithm.

An important point of interest are the statistical properties of $|X|$ [9], [10]. Fig. 5 illustrates the probability density function (pdf) of $|X|$ without antenna selection, applying antenna selection and using additional code selection. The mean absolut values of the resulting interference parameters in case of antenna selection ($\mu_{|X|} = 0.158$) and joint antenna/code selection ($\mu_{|X|} = 0.082$) are substantially smaller compared to the mean value in case of no antenna selection ($\mu_{|X|} = 0.375$). Surprisingly, in contrast to the remarkable difference between these pdfs of $|X|$ only a very small gain in BER performance (0.1dB) shown in Fig. 4 results from the additional code selection. This result can be explained by the fact, that both, channel gain h^2 and interference parameter X effect the code performance.

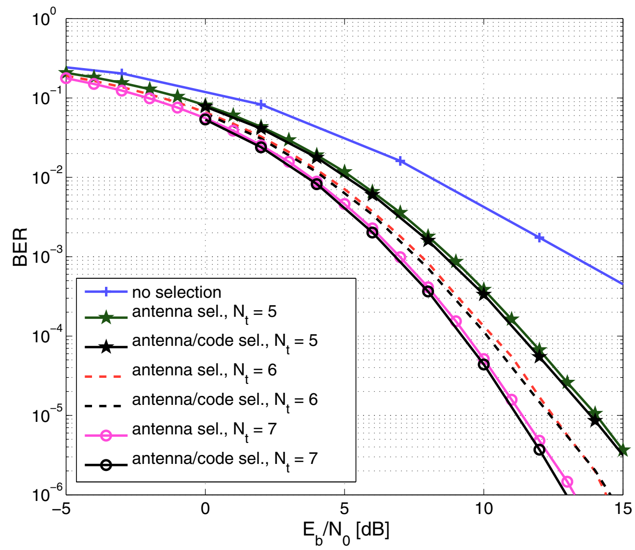

Fig. 4. Joint antenna/code selection, $n_t = 4$ out of $5 \leq N_t \leq 7$.

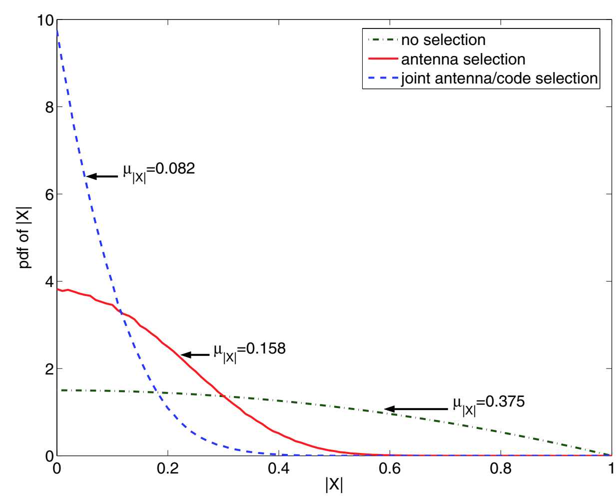

Fig. 5. Statistical properties of $X, N_t = 6$.

VI. Concluding Remarks

In this last Section we discuss some additional interesting aspects. One important question is: Does it really offer a considerable benefit if space-time coding is combined with antenna selection, or is it more advisable to apply a simple selection principle that only utilizes the simple "best" transmit antenna without applying any STBC?

Fig. 6 shows that under the assumption that the number of available transmit antennas is kept fixed at $N_t = 6$, the QSTBC as well as the standard Alamouti transmission system with two transmit antennas [6] are outperformed by the single antenna selection scheme without any STBC.

This can be explained as follows: Basically, the closed loop approach allows to assign the transmit power onto the antenna with the lowest path attenuation (i.e. highest path

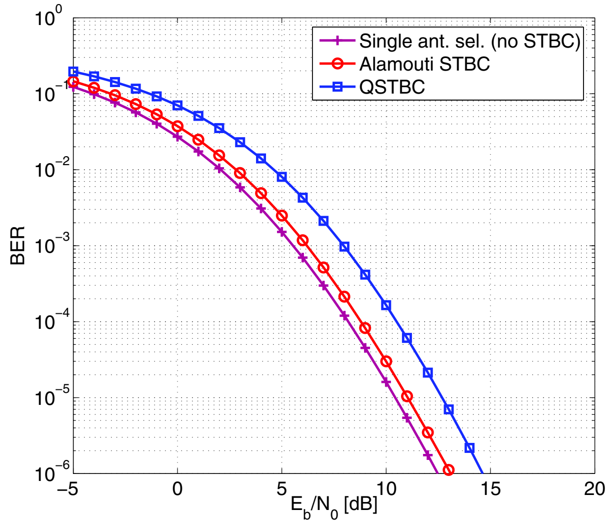

Fig. 6. Transmit antenna selection performance for the three considered transmission schemes, $N_t = 6$.

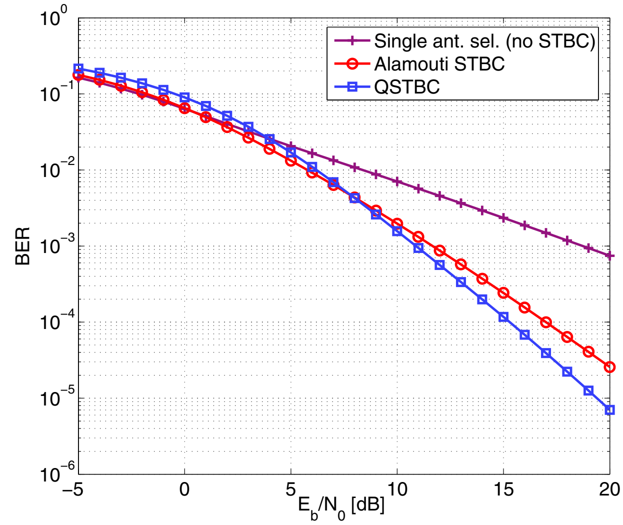

Fig. 7. Transmit antenna selection performance when a bit error ratio of $\mathrm{BER}_{\mathrm{feedback}} = 10^{-2}$ at the feedback link is assumed, ($N_t = 6$).

gain) to the receive antenna. If the number of simultaneously used transmit antennas is increased to two and the Alamouti scheme is applied, then the total transmit power is equally split between the best and the second best transmit antenna and using QSTBC for 4 transmit antennas the total transmit power is equally distributed among the 4 best antennas and the best transmitt path is not fully used.
Note that the results shown in Fig. 6 are based on the following assumptions:

- Quasi-static channel,
- Perfect channel estimation,
- Ideal feedback link (error free, zero-latency).

As a final simulation result we show the BER performance in case of non perfect feedback transmission in Fig. 7. In this figure it is shown that the performance of the simple antenna selection scheme heavily deteriorates in case of feedback errors, whereas both STBC based schemes still achieve reasonable transmit diversities.

Therefore, QSTBCs with antenna selection are more important when the idealistic model assumptions are replaced by more realistic ones.

VII. Conclusion

In this paper we investigated optimization criteria of coded MIMO transmission combined with antenna selection and with joint antenna/code selection using a low cost zero-forcing receiver. Simulation results showed, that just only one additional transmit antenna leads to a substantial improvement in diversity and coding gain. An additional channel adaptive code selection does not improve BER performance essentially.

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