On the Separation of Channel and Frequency Offset Estimation

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ABSTRACT

Accurate channel estimation is crucial to many digital receiver schemes. The LMS algorithm run on a training sequence often does not show the expected behavior since additional frequency offset corrupts the estimation. This paper presents a new channel estimation procedure, called constant modulus channel estimator (CMCE). It works similar to a Constant Modulus Algorithm (CMA) but uses a training sequence. It has in common with the CMA that the estimation process is robust against frequency offsets.

1 Introduction

The Constant Modulus Algorithm (CMA) is by now well established in literature (see for example [1] and the references therein) as well as in practical experiments and even products (see for example products by Applied Signal Technology, Inc. [2]). Although theoretically not completely understood the algorithm seems to work satisfactory in practical situations. One big advantage (among other disadvantages) of this algorithm is its insensitivity to frequency offsets. This paper, however, does not deal with the CMA but a similar algorithm for estimating the channel rather than its inverse. It is assumed that the transmitted symbols $u(k)$ are known to the receiver, i.e., during the training phase. They are arranged in a vector

$$\mathbf{u}_k = [u(k), u(k-1), ..., u(k-M)] .$$

These symbols $u(k)$ are corrupted in three directions: the channel $\mathbf{c}$ (column vector) adds intersymbol interference (ISI), additive noise $v(k)$ and a frequency offset Ω. The received symbol is thus given by:

$$d(k) = (\mathbf{u}_k\mathbf{c} + v(k)) \times e^{j\Omega k} .$$

The problem is to estimate the channel $\mathbf{c}$ and the frequency offset at the same time while the observations are corrupted by noise. If the LMS algorithm is used for this estimation, the channel estimates are only accurate for relatively small frequency offsets. The LMS algorithm is not capable of tracking fast changes caused by frequency offsets and thus looses also its capability to estimate the channel [3]. A slightly better algorithm to estimate frequency offsets was proposed in [4], however, it turned out that this algorithm is very sensitive on timing offset errors while the LMS algorithm is more robust against it.

The algorithm proposed here is similar to the CMA but operates on the symbols rather than on the received signal. In the following a theoretical evaluation of the algorithm is presented and on typical examples the usefulness of the new algorithm is shown and compared to LMS performance.

2 The Algorithm

The following algorithm called the constant modulus channel estimator (CMCE) is largely insensitive to frequency offsets:

$$\begin{aligned}\mathbf{w}_{k+1} &= \mathbf{w}_k + \mu(|d(k)|^2 - |y(k)|^2)y(k)\mathbf{u}_k^* \\ y(k) &= \mathbf{u}_k\mathbf{w}_k\end{aligned}$$

where the column vector $\mathbf{w}_k$ is an estimate of the channel coefficients $\mathbf{c}$ at time instant k. The symbol '*' denotes conjugate transpose. The idea is that $|d(k)|$ is not dependent on the frequency offset. Therefore, it is of advantage to use $|d(k)|$ rather than $d(k)$. This, of course, leads to a loss of phase information when estimating the channel. On the other hand the absolute phase of the channel is not important for many digital modulation schemes and can, if necessary, be corrected.

Since the step-size μ is a free parameter, it is of interest to know bounds on μ so that the algorithm converges and to know the relation of the algorithms accuracy and convergence speed in relation to the step-size μ. It has been shown that rewriting the algorithm in

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an a-posteriori form is usually of advantage (see [5]), and restricts the free parameter considerably. The a-posteriori form of this algorithm is similar to the one for the CMA (normalized CMA) and can be found to be

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \alpha(|d(k)| - |y(k)|)\frac{y(k)}{|y(k)|}\frac{\mathbf{u}_k^*}{\|\mathbf{u}_k\|^2} \quad (1)$$

where a proper normalization has been introduced. The update equation also leads to the relation of $y(k)$ to the a-posteriori $y_p(k)$:

$$|y_p(k)|^2 = |y(k)|^2\left(1 + \alpha\frac{|d(k)| - |y(k)|}{|y(k)|}\right)^2$$

when multiplied the update (1) by $\mathbf{u}_k$ from the left. Replacing the later relation in the update and computing the l_2-norm leads to

$$\|\mathbf{w}_{k+1}\|^2 + \frac{|y_p(k)|^2}{\|\mathbf{u}_k\|^2} = \|\mathbf{w}_k\|^2 + \frac{|y(k)|^2}{\|\mathbf{u}_k\|^2}$$

This can be rewritten into

$$\|\mathbf{w}_{k+1}\|^2 + \frac{1-\gamma(k)}{\|\mathbf{u}_k\|^2}|y(k)|^2 = \|\mathbf{w}_k\|^2 + \frac{1-\gamma(k)}{\|\mathbf{u}_k\|^2}|d(k)|^2$$

with the abbreviation

$$\gamma(k) = \frac{|d(k)|(1-\alpha^2) + |y(k)|(1-\alpha)^2}{|d(k)| + |y(k)|}$$

a value that lies between zero and one for $\alpha \in [0, 1]$. Now, summing up over a number N of elements leads to:

$$\sum_{k=0}^{N}\frac{1-\gamma(k)}{\|\mathbf{u}_k\|^2}|y(k)|^2 \leq \|\mathbf{w}_0\|^2 + \sum_{k=0}^{N}\frac{1-\gamma(k)}{\|\mathbf{u}_k\|^2}|d(k)|^2$$

Since $\|\mathbf{w}_0\|$ can be neglected when N tends to infinity, both sums approach each other and therefore $|y(k)|$ will approach $|d(k)|$ asymptotically.

Problem:
Note that the error running toward zero does not mean the channel is identified. In fact for constant modulus signals it can be shown that the channel -even with arbitrary phase- is not always a unique information, i.e., there can exist several channels with the same output amplitude. For example consider the channel $\mathbf{c} = [a, b]$. The squared amplitude of the outcome is given by:

$$\begin{aligned} & a^2|u(k)|^2 + b^2|u(k-1)|^2 + 2ab \times \\ \times\ & [\mathrm{Re}\{u(k)\}\mathrm{Re}\{u(k-1)\} \\ & + \mathrm{Im}\{u(k)\}\mathrm{Im}\{u(k-1)\}]\,. \end{aligned}$$

Note that the product ab is already ambiguous. Now, if $|u(k)| = c$ is a constant, the first two terms are also ambiguous and so is the whole expression. In other words, the channel $[b, a]$ is a solution as well. As soon as the signal is not constant modulus any more, the solution becomes unique. It might seem that this channel is just one exception. However, every channel with just two coefficients is as well, for example $\mathbf{c} = [a, 0, 0, b]$ behaves like $\mathbf{c} = [b, 0, 0, a]$. In practice it is not necessary that the remaining channel coefficients are perfectly zero. If they are small compared to the two major coefficients the same effect can occur. Thus, the algorithm is to be used carefully in the context of constant modulus sequences. For higher constellation QAM this effect was not found.

3 Experiments

3.1 Learning Curve

In the first experiment a 16QAM is used with a three tap channel $\mathbf{c} = [j, 2, 3]$ and an SNR of 30 dB. We run the CMCE algorithm with a normalized step-size of $\alpha = 0.5$ and compare its behavior with the LMS algorithm. Plotted in Figure 3 is the relative system match over number of iterations. In order to have a fair comparison the relative system mismatch was now chosen to be

$$S_{rel}(k) = \frac{\|\mathbf{c}\|^2 + \|\mathbf{w}_k\|^2 - 2|\mathbf{c}^*\mathbf{w}_k|}{\|\mathbf{c}\|^2} \quad (2)$$

which is different to a conventional system mismatch. However, because of the frequency offset and the chosen algorithm the channel cannot be estimated uniquely. Every value $\mathbf{c}e^{j\phi}$ is a valid solution. Thus, in order to minimize the estimate with respect to ϕ, the definition (2) has been selected.
The experiment was run for a frequency offset of 8000/24300, thus a relatively large amount. In this case the LMS algorithm does not exhibit any learning behavior while the CMCE algorithm converges rapidly.

3.2 Frequency Offset Estimation for 16QAM

In the next experiment the algorithms capability of estimating the frequency offset is investigated. For this reason the frequency offset was estimated based on the channel estimate $\mathbf{w}_k$, obtained after 150 iterations of training. The amount of 150 symbols for training might appear very high for practical reasons. However, a given smaller set (for example only 15 symbols in IS-136) can be reused in order to obtain this accuracy. Figure 4 depicts the relative system mismatch after 150 iterations for LMS and CMCE algorithm respectively over various frequency offsets.

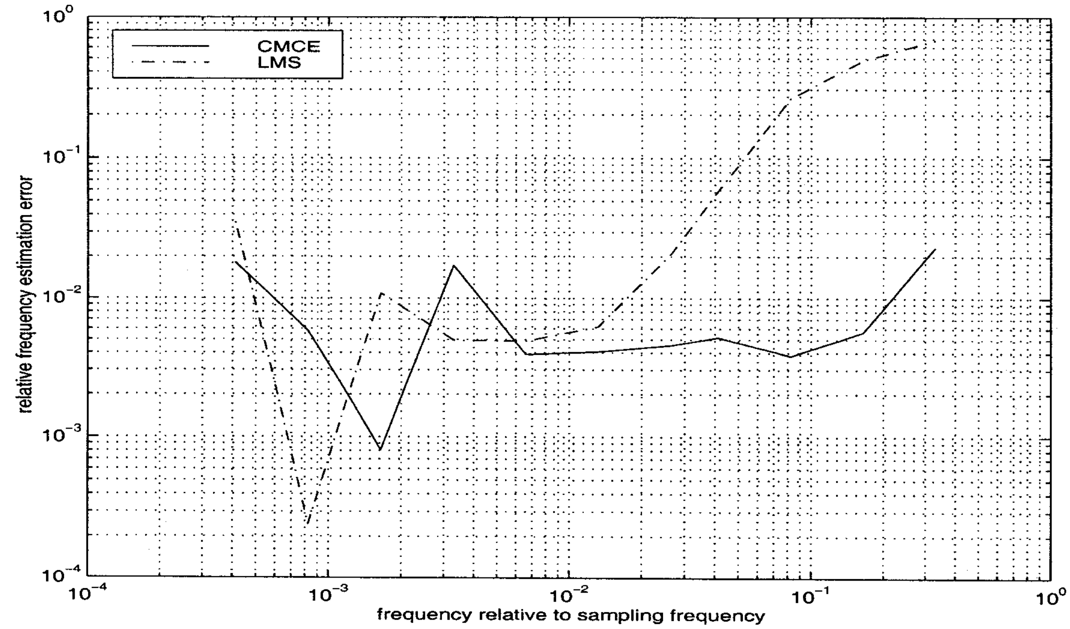

Figure 1: *Relative frequency estimation error for SNR=10dB and 16QAM.*

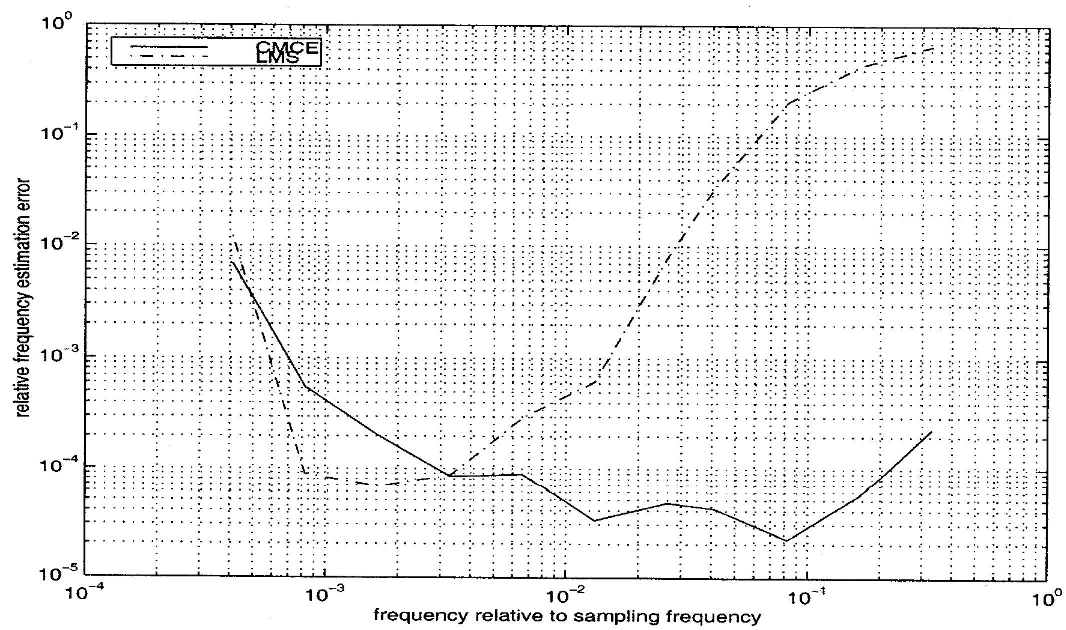

Figure 2: *Relative frequency estimation error for SNR=20dB and 16QAM.*

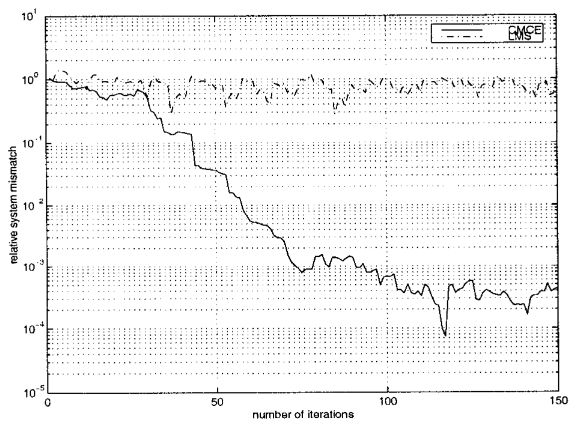

Figure 3: *Learning curve comparison of LMS and CMCE algorithm.*

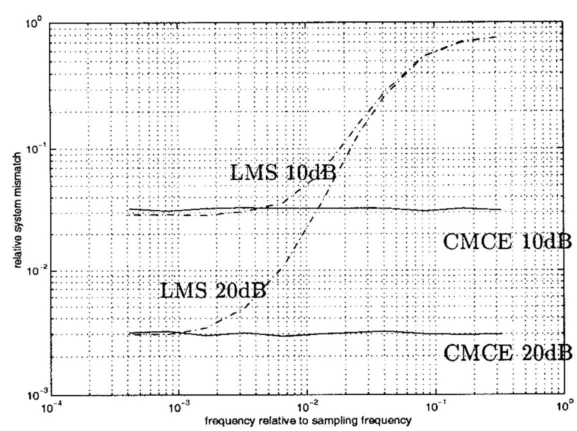

Figure 4: *Relative system mismatch for 16QAM at an SNR of 10dB and 20dB, respectively.*

The angle estimate is obtained by

$$\hat{\Omega} = 1/N \sum_N \angle(d(k)y^*(k)d^*(k+1)y(k+1))$$

The squared differences of

$$\delta f = (1 - \hat{\Omega}/(2\pi T f_o))^2$$

are depicted in Figures 1 and 2 for SNR of 10dB and 20dB, respectively. The results are averaged over 1000 runs. In this experiment a three tap channel with three independent Rayleigh fading weights is assumed. During the training the channel is assumed to be constant and the random variables are changed for every run. Every tap is given a variance of one. The transmitted sequence is chosen to be 16QAM and the additive noise is white complex Gaussian. The channel estimator is thus of $M = 3$ coefficients. Offset frequencies are used at [10,20,40,80,160,320,640,1000,2000,4000,8000]Hz for an IS-136 system (see [6]) with 24300Hz sampling rate. The step-size was chosen to $\alpha = 0.5$, a compromise between fastest possible convergence ($\alpha = 1$) and smallest noise influence $\alpha \approx 0$. The initial channel estimate was set to [0.0001,0,0] in all cases.

Both algorithm show high sensitivity towards additive noise. The CMCE algorithm typically depicts strong improvement compared to the LMS algorithm once the frequency offset is larger than 30Hz at 20dB SNR and 150Hz at 10dB SNR. Thus, a possibility is to run CMCE first, estimate the frequency offset, and than run LMS on the de-rotated signal to eventually obtain a precise channel estimate with correct phase information.

3.3 Frequency Offset Estimation for QPSK

In a third experiment the algorithm will be employed in a QPSK modulated scheme. A channel length of $M = 5$ was used in order to avoid (or at least make it very unlikely) to have a situation with only two strong coefficients. since a longer channel is employed a training time of $N = 300$ symbols was chosen. Running QPSK showed slower convergence than 16QAM. In order to compensate for that the step-size was increased to $\alpha = 0.9$. Apart from that all conditions from the previous experiment were repeated. The results are shown in Figure 5 for the relative system mismatch and in Figure 6 for the frequency estimation at SNR of 10dB. If compared to the previous experiment the precision of the frequency estimate is even better. However, if compared CMCE results with those of LMS, the improvement for QAM was much larger than it is now for the QPSK case. The relative system mismatch reveals that the channel estimate is even less accurate for QPSK than for QAM.

A possible application can be to combine the LMS and CMCE algorithm. If the estimated frequency offset is small (below a certain threshold), LMS is used otherwise CMCE. Figure 7 depicts the relative frequency error over the iterated data blocks. If only the LMS algorithm is used, it takes about ten blocks until the frequency offset of 8kHz (sampling rate 24.3kHz) drops to a small value, while for the CMCE algorithm only two blocks are required to achieve an estimate closer than one percent of the true value. Note that if a pure CMCE is used the fine estimate is somehow rough

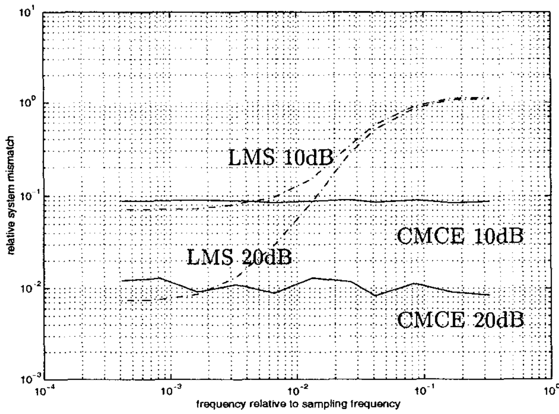

Figure 5: *Relative system mismatch for QPSK at an SNR of 10dB and 20dB, respectively.*

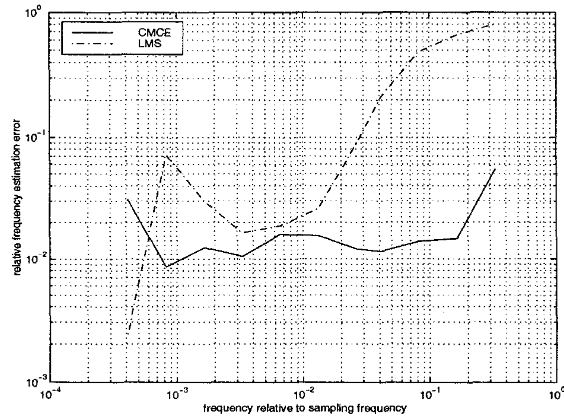

Figure 6: *Relative frequency estimation error for SNR=10dB and QPSK.*

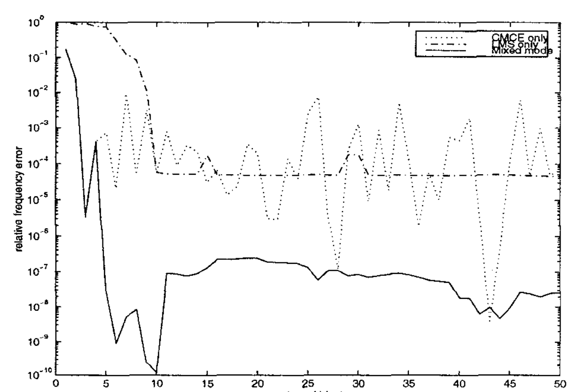

Figure 7: *Relative frequency estimation error for SNR=20dB .*

while if switched to LMS it can be further improved. Therefore, a logic was introduced that monitors the additional frequency offset estimates and switches to pure LMS once the error is small enough. The SNR in this experiment was 20dB.

4 Conclusion

A new algorithm that is as simple to implement as the LMS algorithm has been proposed with the intent to decompose channel estimation from frequency offset estimation. On typical examples for digital data transmission it has been shown that the new algorithm can achieve considerable improvement compared to LMS when the frequency offset becomes relatively large.

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