ON THE INFLUENCE OF UNCERTAINTIES IN MIMO DECODING ALGORITHMS

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ABSTRACT

With the introduction of multiple antennas at transmitter and receiver new challenging problems have occurred. Many MIMO decoding algorithms have been proposed in the literature. However, many assumptions are too simplifying and obtained results based on oversimplified simulations showed too optimistic receiver performances. This paper treats uncertainties a receiver designer has to deal with: unknown noise level, channel estimation errors, and different channel qualities, and presents receiver performances with respect to such uncertainties. Furthermore, few details in the ordering process of iterative MIMO decoding schemes are discussed.

1 INTODUCTION

Since the discovery of Telatar[1] and Foschini and Gans[2], MIMO transmission proposals are coming out at high rates. In the past years, the new dimensions of space and time have inspired many researchers. However, exploration of ideas is typically performed in idealistic environments, in which channels are known perfectly at the receiver; clocks at receiver and transmitter are ticking synchronously, and channels have perfect (Rayleigh-) statistics. Once transmission systems are designed, the impacts of such idealizations should be taken into account.

The following section will take a closer look at the radio channels, in particular the flat Rayleigh fading assumption and compare values obtained via Rayleigh distributions with measured data. Section 3 will present the typical MIMO decoding algorithms and study their performance with respect to the radio channels. In [3] and [4] an ordering scheme for iterative decoding has been proposed and claimed to be optimal. In Section 4 it will be shown that this is not correct and that depending on the choice of algorithm very different behavior can be obtained. MMSE techniques require perfect knowledge of the received noise level. The impact of noise level estimates is discussed in Section 5. Finally, Section 6 presents how the algorithmic behavior depends on the channel estimation precision.

2 CHANNEL DISTRIBUTION

Throughout the paper the simple transmission model

$$\mathbf{r}(k) = \mathbf{H}\mathbf{a}(k) + \mathbf{v}(k) \tag{1}$$

will be considered. Here, $\mathbf{a}(k)$ is an $M \times 1$ symbol vector containing transmitted binary symbols $\{-1, 1\}$. The additive $N \times 1$ noise vector $\mathbf{v}(k)$ is zero mean, i.i.d. complex Gaussian with covariance matrix $\sigma_v^2\mathbf{I}$. The $N \times M$ channel matrix $\mathbf{H}$ has complex entries and is normalized so that

$$\text{trace}(\mathbf{H}^H\mathbf{H}) = \|\mathbf{H}\|_F^2 = N. \tag{2}$$

The upper H denotes conjugate transpose. Throughout the paper $M = N = 4$ will be considered, selecting numbers that are very likely to be realized in near future systems. Note that (1) can be written in terms of singular values ($\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^H$), arriving at

$$\begin{aligned} \mathbf{U}^H\mathbf{r}(k) &= \mathbf{S}\mathbf{V}^H\mathbf{a}(k) + \mathbf{U}^H\mathbf{v}(k) & (3)\\ \tilde{\mathbf{r}}(k) &= \mathbf{S}\tilde{\mathbf{a}}(k) + \tilde{\mathbf{v}}(k), & (4) \end{aligned}$$

representing transformed receiver vectors $\tilde{\mathbf{r}}(k)$, transmitted symbols $\tilde{\mathbf{a}}(k)$ and additive noise $\tilde{\mathbf{v}}(k)$. Since the matrices $\mathbf{V}$ and $\mathbf{U}$ do not change the energy of the vectors, (4) can now be interpreted as a set of independent transmission streams, each weighted with the singular values in the diagonal of $\mathbf{S}$. Note that with the previous constraint (2) the singular values cannot be chosen arbitrarily. The ratio of largest to smallest is certainly a good measure to express the fact that at least one data stream is impacted by noise more than others. Since this ratio is known as the condition number of the matrix, it is of interest what condition numbers are expected.

Assuming a flat Rayleigh fading channel, all entries of $\mathbf{H}$ are independent complex Gaussian. The condition number of such matrices can be evaluated and ordered by size, obtaining the cumulative distribution function of their occurrence. This is presented in Figure 1. Also shown in Figure 1 are results from outdoor narrowband

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measurements at 2.44GHz with a measurement system according to[5], however, with 4in spacing of a 4×4 antenna system. As the Figure indicates, the measurements exhibit higher condition numbers than the simple Rayleigh model. Measurements with larger antenna spacing show even higher condition numbers. This may or may not be surprising. Note that the Rayleigh fading assumption is applied to an ensemble averaging, while measurement data are obtained by driving a mobile along a street, thus having scattering characteristics from one measurement point to another that are not entirely independent. A good fitting with measurement (labeled as mixed random matrix) was obtained when amplitudes remained Rayleigh distributed but the phases were given a different distribution. Since driving along a street may not change the major scattering characteristics for a longer time period, it was assumed that the phases are constant with some small random component. Recent indoor measurements at 5.2GHz[6] showed even higher condition numbers (typically by a magnitude) than shown in the figure. The reason for this can be contributed mainly to a strong correlation of the random variables. Such measurements show that the condition number of the channel matrix H is of crucial importance if the decoding algorithms are dependent on it.

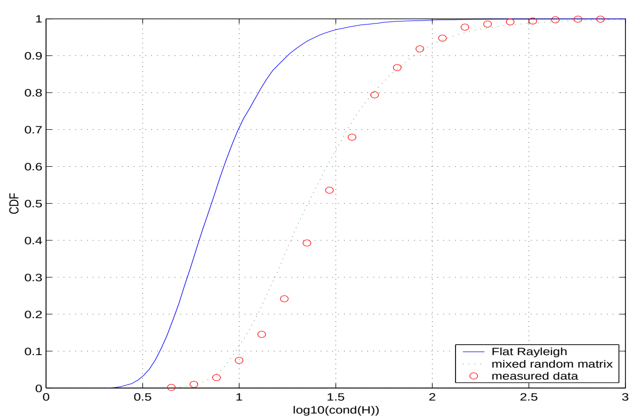

Figure 1: *CDF of condition number of* $\mathbf{H}$*; comparison of measured and synthetic data.*

3 IMPACT ON CHANNEL CONDITION NUMBER

As discussed in the previous section, varying the channel condition number can be expected to have quite some impact on the receiver. Such impact is different for each algorithm and is thus investigated. The following algorithms are considered:

- Maximum Likelihood: the receiver optimizes $\min_{\hat{\mathbf{a}}(k)} \|\mathbf{r}(k) - \mathbf{H}\hat{\mathbf{a}}(k)\|_2$ by a full search. In a small system (N, M small) the complexity of ML can be comparable to alternatives[7, 8].

- Zero-Forcing: the receiver computes the expression $\mathbf{G}\mathbf{r}(k) = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{r}(k)$ and maps the obtained result onto the nearest symbol in the alphabet.

- ZF-VBLAST: uses Zero-Forcing, however in an iterative scheme. This is also known as downdating (see, e.g., [9]). The complete algorithm is given in [3, 4]. Note that although originally named ZF-VBLAST, the algorithm is often called VBLAST, confusing it with the transmitting scheme and with alternatives like the MMSE version.

- MMSE: the receiver computes $(\mathbf{H}^H\mathbf{H} + \sigma_v^2\mathbf{I})^{-1}\mathbf{H}^H\mathbf{r}(k)$ and maps the obtained result onto the closest symbol in the alphabet. Note that this requires further knowledge of σ_v^2.

- MMSE-VBLAST: similar to ZF-VBLAST, the receiver computes iteratively estimated symbols, however, now with the matrix $\mathbf{G} = (\mathbf{H}^H\mathbf{H} + \sigma_v^2\mathbf{I})^{-1}\mathbf{H}^H$. Unlike in ZF-BLAST, the ordering decision is performed on $\max_i \|(\mathbf{G}\mathbf{H})_i\|/\|(\mathbf{G})_i\|$.

Figure 2 depicts the algorithmic behavior for these algorithms when setting the channel condition number to $\{6, 40, 121, 416\}$. While ML and MMSE-VBLAST only face small losses, the other (simpler) approaches loose substantial performance for high condition numbers.

4 IMPACT ON ORDERING

In the previous section, the ordering scheme as proposed in the original papers[3, 4] were applied for ZF-VBLAST and in some modified form for MMSE-VBLAST. The idea is very intuitive: select the smallest row norm of $\mathbf{G}$ since it will give the smallest noise power in the decision. Since this is the best decision at this point, continue with the reduced system until you are done. In [3] it is shown that this ordering scheme is optimal. However, selecting the row with lowest noise power will also change the structure of $\mathbf{H}$. The resulting $\mathbf{G}$-rows of the reduced matrix $\mathbf{H}$, can show very different behavior dependent from the history of reducing $\mathbf{H}$. In the following experiment, all 24 possible orders were investigated for both VBLAST-decoder variants. Figures 3 and 4 show the results for condition numbers 40 and 416, respectively. Depicted are for each algorithm the best and the worst case, compared to the automatic selected case. While for small condition numbers the automatic selection is at least close to optimal, the scheme looses for ZF-VBLAST in high condition numbers but keeps up the performance for MMSE VBLAST-decoding. Note that the complexity of an iterative decoding scheme like VBLAST is quite high due to the required multiple matrix inverses. In [10] an interesting alternative was proposed using QR decomposition only once. Also of interest is the impact of error propagation. An error made in the first decision influences all following decisions. This has been analyzed in [11].

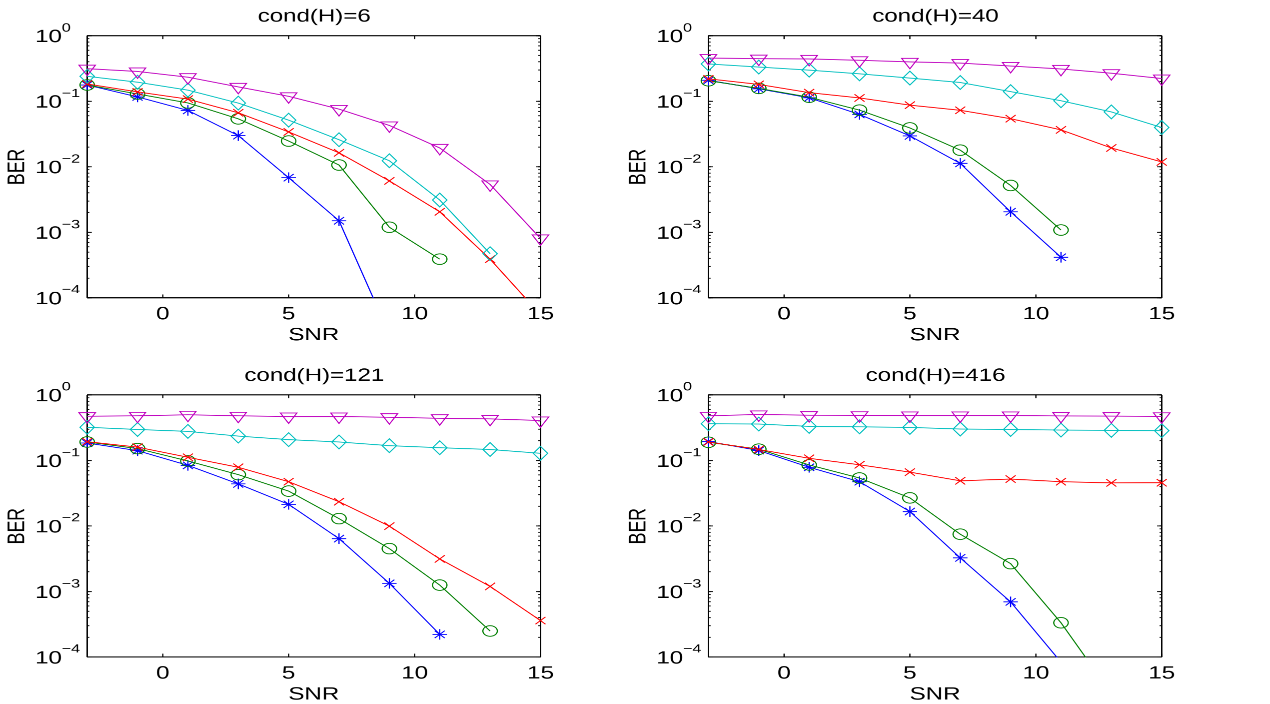

Figure 2: *Algorithmic performance for different channel condition numbers (*=ML, o=MMSE-VBLAST, ×=MMSE, ⋄=ZF-VBLAST, ∇=ZF).*

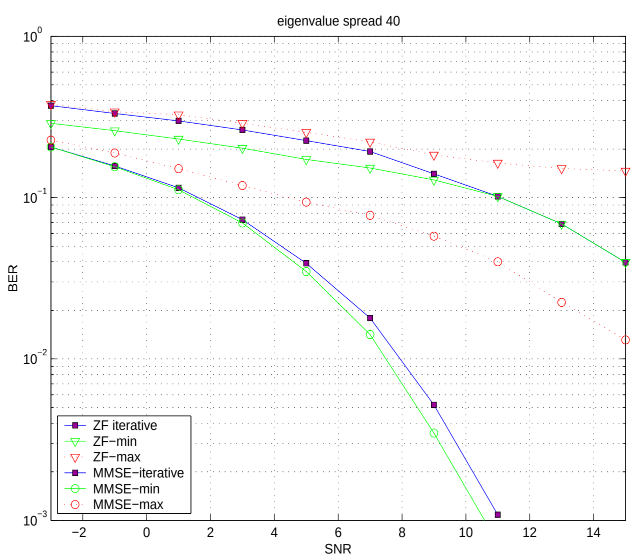

Figure 3: *BLAST performance for cond*$(\mathbf{H}) = 40$.

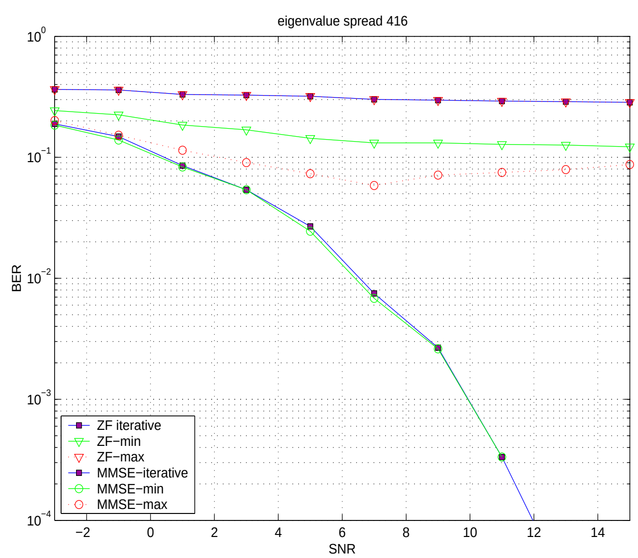

Figure 4: *BLAST performance for cond*$(\mathbf{H}) = 416$.

5 IMPACT ON NOISE LEVEL ESTIMATION

Both MMSE schemes require perfect knowledge of the noise level σ_v^2. Practically, this is not a simple task. Simple estimation schemes typically deliver only rough estimates. In the following experiment the noise level σ_v^2 in the matrix $\mathbf{G}$ was set to different values relative to the true noise level. The results for the MMSE-BLAST schemes are shown in Figures 5 and 6. While low SNR scenarios are quite insensitive to the choice of σ_v^2, for higher SNR the losses become visible. Once the estimation is off by 10dB, there is hardly any further performance loss. Too high values for σ_v^2 are typically harming less than too low values. This is consistent with the poor ZF behavior. Similar behavior, however with higher BER level was obtained for MMSE decoding schemes.

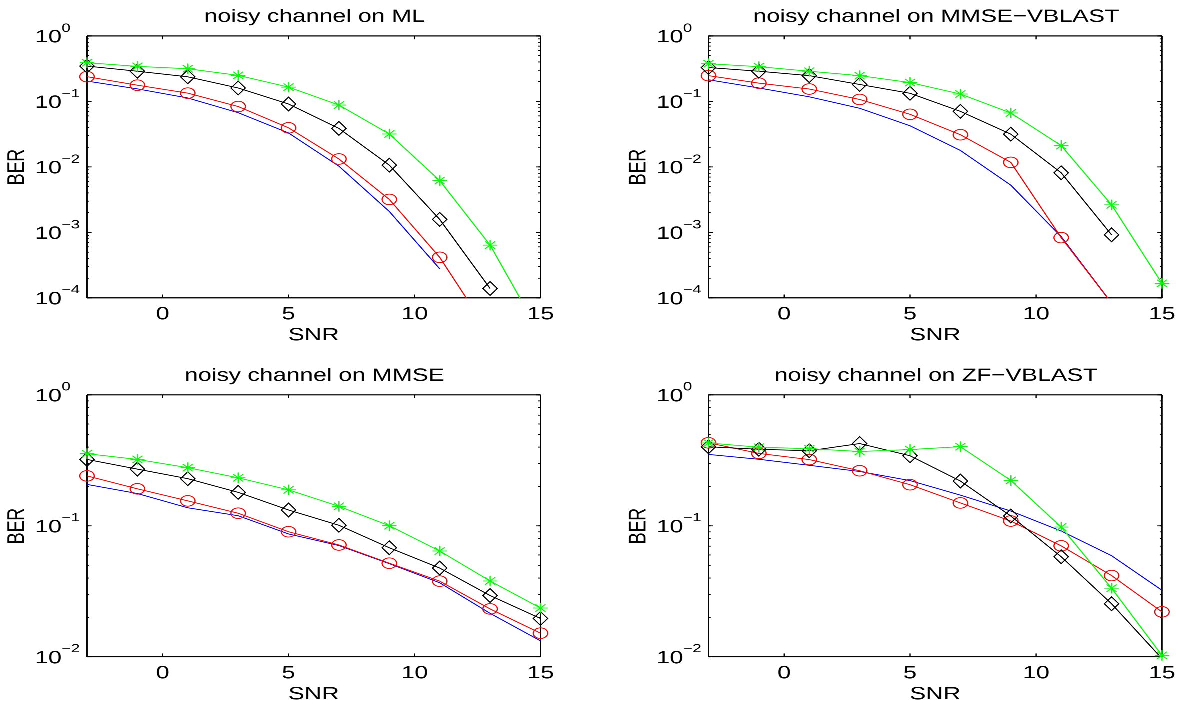

Figure 7: *Impact of channel estimation (cond*$(\mathbf{H}) = 40$*,*:-20dB,◇:-10dB,o:-3dB, and –:0dB).*

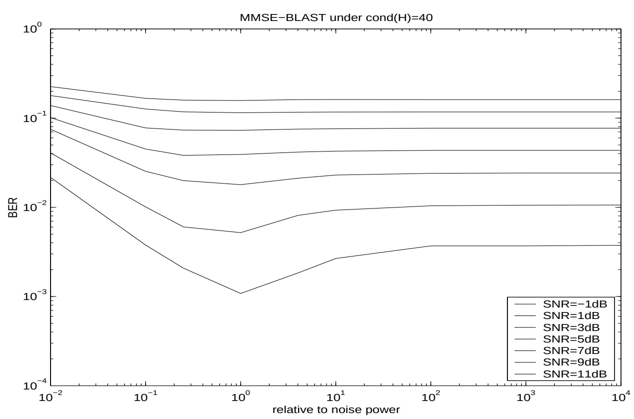

Figure 5: *Impact on noise estimation for cond*$(\mathbf{H}) = 40$.

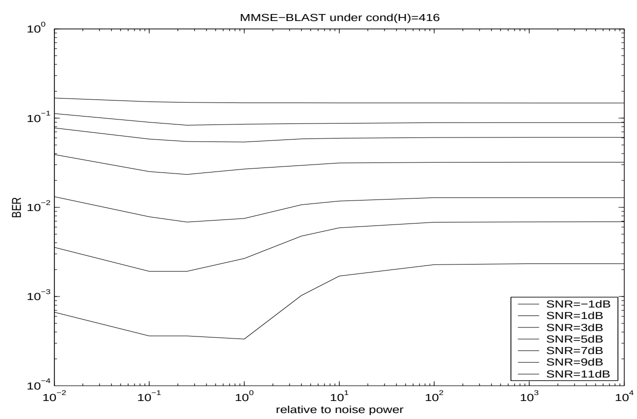

Figure 6: *Impact on noise estimation for cond*$(\mathbf{H}) = 416$.

6 IMPACT ON CHANNEL ESTIMATION

Until here, the channel matrix $\mathbf{H}$ was assumed to be known perfectly at the receiver. In order to investigate the algorithmic sensitivity on the channel estimation precision, noise was added to the true channel matrix $\mathbf{H}$. The level of this additive noise was set relative to the SNR so that with higher SNR the channel estimates are corrupted less. Figures 7 and 8 depict the results for ML,MMSE-VBLAST,MMSE and ZF-VBLAST decoding applied to a channel with condition number 40 and 416, respectively. Channel estimation noise was added at *:-20dB,◇:-10dB,o:-3dB, and −:0dB relative to the noise level. While ML and the MMSE methods exhibit a somewhat expected behavior, ZF-VBLAST shows much unexpected performance. For some medium values of SNR the behavior suddenly improves. To explain this, the channel estimate can be decomposed in its true ($\mathbf{H}$) and its noise component $\mathbf{W}$. The ZF-VBLAST algorithm thus has to compute $[(\mathbf{H}^H + \mathbf{W}^H)(\mathbf{H} + \mathbf{W})]^{-1}$. This gives in the mean: $[\mathbf{H}^H\mathbf{H}+\sigma_w^2\mathbf{I}]^{-1}$, nothing else but an additive noise term just like in the MMSE case. As we have seen from Section 3, adding a noise component in ZF improves considerably the decoding performance. It is thus not surprising that the algorithm behaves in some cases better for poorly estimated channels.

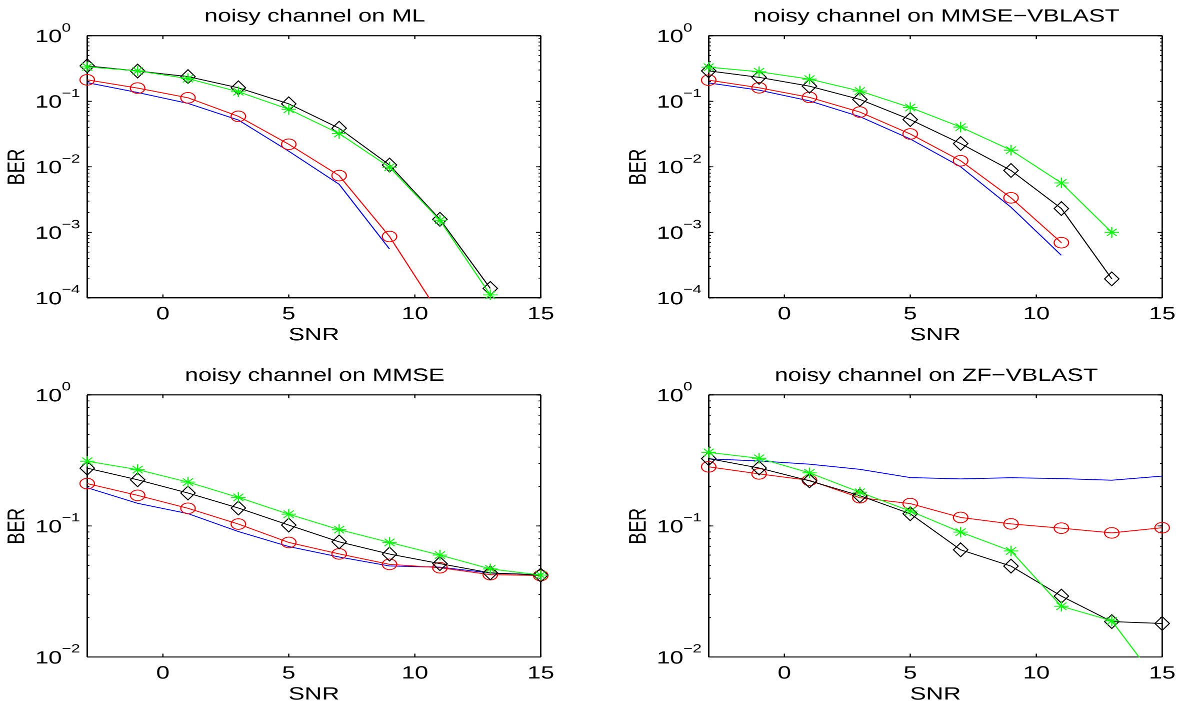

Figure 8: *Impact of channel estimation* ($cond(\mathbf{H}) = 416$, **:-20dB,◇:-10dB,o:-3dB, and –:0dB).*

7 CONCLUSION

Uncertainties in MIMO decoding algorithms have attracted little attention in literature so far. The paper closes this gap by investigating the algorithmic sensitivity on noise level estimation in MMSE schemes, ordering sensitivity in iterative (VBLAST) schemes and channel impact on all kind of MIMO decoding algorithms.

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