

On Extended Alamouti Schemes for Space-Time Coding

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Abstract

The Alamouti space-time code for transmission diversity is extended recursively to $2^n \times 2^n$ antenna elements as well as to unsymmetrical arrays of the size $m \times n$. The proposed schemes exhibit two significant advantages: they allow for higher diversity (of degree 2^n for the symmetrical case) and at the same time can be implemented as zero-forcing or MMSE type at low complexity avoiding any matrix inversion due to the structure in the transmission matrices as well as low complexity ML.

Keywords

Transmit Diversity, MIMO transmission, STC, Low Complexity Receivers

INTRODUCTION

Alamouti [1] introduced a very simple scheme allowing to transmit at two antennas with the same data rate as on a single antenna but increasing the diversity at the receiver from one to two in a flat-fading channel. While the scheme works for BPSK even with four and eight antennas, it was proven for QPSK that only the two antenna scheme offers the full diversity gain [6, 2].

In this contribution, the Alamouti space-time code for transmission diversity is extended recursively to $M = 2^m$ antenna elements at the transmitter applying QPSK, the receiver need only use a single antenna, but may use more. While it is true that the resulting transmission matrix for flat-fading loses its orthogonality for $m \geq 2$, it is shown that the loss in orthogonality is not severe. Starting with a four antenna scheme in Section it will be demonstrated that linear receivers perform close to the theoretical bound for four-path diversity offering significant gain over the two-antenna case proposed by Alamouti. Even more interestingly, linear interference suppression can be implemented with low complexity because the channel matrix exhibits a high degree of structure enabling a factorization in closed-form. Transmission schemes with more than one receive antenna will be considered and it will be shown that even in cases with $N \neq 2^n$ receive antennas, preservation of orthogonality is possible. Variable bit-rate services and bursty packet arrivals are handled flexibly in UMTS by dynamically changing the spreading factor in conjunction with the transmit power, thus preserving an average E_b/N_o , but without changing the diversity order and outage probability.

REVIEW OF ALAMOUTI SCHEME

A very simple and effective scheme for two transmit antenna-elements and a single receive antenna achieving diversity order two was introduced by Alamouti [1, 6]. It sends the sequence $\{s_1, s_2^*\}$ on the first antenna and $\{s_2, -s_1^*\}$ on the other. Assuming a flat-fading channel with coefficients h_1, h_2 , the received vector \mathbf{r} is formed by stacking two consecutive data samples $[r_1, r_2]^T$ in time $\mathbf{r} = \mathbf{S}\mathbf{h} + \bar{\mathbf{v}}$. Here, the symbol block \mathbf{S} and the channel vector \mathbf{h} are introduced,

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

This can be reformulated as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2^* \end{bmatrix} \quad (1)$$

or in short notation $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}$, where the vector $\mathbf{y} = [r_1, r_2^*]^T$ was introduced. The resulting channel matrix \mathbf{H} is orthogonal, i.e. $\mathbf{H}^H\mathbf{H} = \mathbf{H}\mathbf{H}^H = h^2\mathbf{I}_2$, where \mathbf{I}_2 is the 2×2 identity matrix and the gain of the channel is $h^2 = |h_1|^2 + |h_2|^2$. The transmitted symbols can be computed by the ZF approach, i.e.

$$\hat{\mathbf{s}} = \frac{1}{h^2} \mathbf{H}^H \mathbf{y} = \mathbf{s} + [\mathbf{H}^H \mathbf{H}]^{-1} \mathbf{H}^H \mathbf{v}. \quad (2)$$

The relevant noise variance is given by $\sigma_v^2 \text{tr}([\mathbf{H}^H \mathbf{H}]^{-1}) = 2\sigma_v^2/h^2$. The nominator h^2 indicates diversity order two for the reception of both symbols. Using unspecified complex-valued modulation, such an improvement is possible only for the two antenna scheme. Higher schemes with 4, and 8 antennas exist only in the case of binary transmission [2]. In UMTS, QPSK is utilized on CDMA preventing perfectly orthogonal schemes with code-rate 1. An orthogonal scheme exists for complex-valued modulation with code-rate $\frac{3}{4}$ and it is known that a higher code-rate cannot be achieved without sacrificing orthogonality [3].

FOUR ANTENNA SCHEME

In the following, a four times diversity scheme for UMTS is proposed¹. It comes with a small caveat which will be explained in the following.

Proposition 1 Starting with the 2×2 -Alamouti scheme the following recursive construction rule (similar to the

¹The scheme was already proposed in [4], however only for the interest of outage capacity and was not considered for further investigation.

construction of a Walsh-Hadamard code) is applied

$$\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \rightarrow \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}. \quad (3)$$

i.e. the complex scalars h_1 and h_2 appearing to the left of the arrow “ \rightarrow ” are replaced by the 2×2 matrices

$$\mathbf{H}_1 = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \quad \text{and} \quad \mathbf{H}_2 = \begin{bmatrix} h_3 & h_4 \\ -h_4^* & h_3^* \end{bmatrix}$$

and then re-inserted into the Alamouti Space-Time channel matrix

$$\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \end{bmatrix}$$

where $*$ denotes complex conjugation without transposition.

This scheme is implementable via the following symbol block \mathbf{S} for transmitting the four symbols $\mathbf{s} = [s_1, \dots, s_4]^T$:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}, \quad (4)$$

and the element $[\mathbf{S}]_{ik}$ is transmitted over the k -th antenna in the i -th time step. Assuming a flat-fading channel with coefficient vector $\mathbf{h} = [h_1, \dots, h_4]^T$, the received vector \mathbf{r} is formed by stacking four consecutive data samples $[r_1, \dots, r_4]^T$ in time, $\mathbf{r} = \mathbf{S}\mathbf{h} + \bar{\mathbf{v}}$. Converting the received vector \mathbf{r} by complex conjugation

$$\begin{aligned} y_i &= r_i, & v_i &= \bar{v}_i, & \text{for } i &= 1, 4 \\ y_i &= r_i^*, & v_i &= \bar{v}_i^*, & \text{for } i &= 2, 3 \end{aligned} \quad (5)$$

results in the equivalent transmission scheme $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}$, in which now \mathbf{H} appears as the channel transmission matrix. If $\bar{\mathbf{v}}$ is a complex Gaussian vector with i.i.d. elements then so is \mathbf{v} . The resulting matrix \mathbf{H} is not unitary anymore, instead

$$\mathbf{G} = \mathbf{H}^H \mathbf{H} = \mathbf{H} \mathbf{H}^H = h^2 \begin{bmatrix} \mathbf{I}_2 & X \mathbf{J}_2 \\ -X \mathbf{J}_2 & \mathbf{I}_2 \end{bmatrix} \quad (6)$$

where $\mathbf{J}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. The gain of the channel is $h^2 = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$ and the channel dependent real-valued random variable X is defined by

$$X = 2 \operatorname{Re}(h_1 h_4^* - h_2 h_3^*) / h^2. \quad (7)$$

Applying the matched filter \mathbf{H}^H , results in the reception of the following vector:

$$\mathbf{z} = \mathbf{H}^H \mathbf{y} = h^2 \begin{bmatrix} s_1 + X s_4 \\ s_2 - X s_3 \\ s_3 - X s_2 \\ s_4 + X s_1 \end{bmatrix} + \mathbf{H}^H \mathbf{v} \quad (8)$$

in which the pair $\{s_1, s_4\}$ is decoupled from $\{s_2, s_3\}$ allowing for a low complexity solution based on the newly formed received vector \mathbf{z} .

Performance of linear receivers

Linear receivers suffer of noise enhancement. In this section, the increased noise caused by the ZF and Minimum Mean Squared Error (MMSE) detectors is investigated. Both receivers can be described by the following detection principle

$$\hat{\mathbf{s}} = (\mathbf{G} + \nu \mathbf{I}_4)^{-1} \mathbf{z}, \quad (9)$$

where \mathbf{I}_4 is the 4×4 identity matrix. We set $\nu = 0$ for ZF and $\nu = \sigma_v^2$ for MMSE. It turns out that both detection principles have essentially the same receiver complexity.

Lemma 1 Given the 4×4 Alamouti scheme as described in (3), the eigenvalues λ_i of \mathbf{G} in Eq.(6) are given by

$$\lambda_1 = \lambda_2 = h^2(1 + X), \quad \lambda_3 = \lambda_4 = h^2(1 - X). \quad (10)$$

Proof: The Grammian \mathbf{G} is diagonalized by $\mathbf{V} \mathbf{G} \mathbf{V}^T$ with the orthogonal matrix

$$\mathbf{V} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{J}_2 & -\mathbf{J}_2 \\ \mathbf{I}_2 & \mathbf{I}_2 \end{bmatrix} \quad (11)$$

□.

Some favorable properties are worth mentioning. The eigenvectors of \mathbf{G} which are stacked in the columns of \mathbf{V} do not depend on the channel; they are constant. The scaled matrix $\sqrt{2}\mathbf{V}$ is sparse, i.e., half of its elements vanish and the non-zero entries are ± 1 .

Lemma 2 If the channel coefficients h_i are i.i.d. complex Gaussian variates then the scaled eigenvalue $u_i = \lambda_i/2$ is Beta(2,2)-distributed for all $i = 1, \dots, 4$, i.e. the probability density of the eigenvalues is given by $f_4(\lambda) = \frac{3}{4}\lambda(2 - \lambda)$ for $0 < \lambda < 2$ and zero elsewhere.

Proof: see the Appendix. □

Let $\gamma \geq 1$ be the following random variable which depends on the channel gain if $\nu > 0$,

$$\gamma = \frac{h^2 + \nu}{h^2} = \begin{cases} 1, & \text{for ZF,} \\ 1 + \frac{\sigma_v^2}{h^2}, & \text{for MMSE,} \end{cases} \quad (12)$$

For evaluating the BER of the linear receiver for general $\nu \neq 0$,

$$\begin{aligned} \operatorname{tr} [(\mathbf{G} + \nu \mathbf{I}_4)^{-1} \mathbf{G} (\mathbf{G} + \nu \mathbf{I}_4)^{-1}] &= \\ \left(\frac{4}{h^2}\right) \left(\frac{1}{\gamma^2 - X^2} - 2(\gamma - 1) \frac{\gamma^2 + X^2}{(\gamma^2 - X^2)^2}\right) & \quad (13) \end{aligned}$$

needs to be evaluated which is obtained via

$$[\mathbf{G} + \nu \mathbf{I}_4]^{-1} = \frac{1}{h^2(\gamma^2 - X^2)} \begin{bmatrix} \gamma \mathbf{I}_2 & -X \mathbf{J}_2 \\ X \mathbf{J}_2 & \gamma \mathbf{I}_2 \end{bmatrix}. \quad (14)$$

When replacing the arguments of the complementary error function with (13), two interpretations can be discussed. In h^2 the required form for four times diversity is recognized together with an additional term starting with $1/(\gamma^2 - X^2)$. This term can be interpreted as an increase in noise. Alternatively, one can interpret the whole expression as defining the true diversity order without noise increase. Both interpretations can be used to describe the scheme's performance. The

first interpretation is favoured in this deliverable and leads to the following result.

Lemma 3 Given the 4×4 Alamouti scheme as described in (3), a four times diversity is obtained at the expense of a noise enhancement of

$$E \left[\frac{1}{\gamma^2 - X^2} - 2(\gamma - 1) \frac{\gamma^2 + X^2}{(\gamma^2 - X^2)^2} \right] = \frac{3}{2} - 2\nu^2 + 2\nu e^{2\nu} \text{Ei}(1, 2\nu) (2\nu^2 + \nu - 2) \quad (15)$$

where $\text{Ei}(n, x)$ denotes the exponential integral defined for $\text{Re}(x) > 0$ by $\text{Ei}(n, x) \triangleq \int_1^\infty \frac{e^{-xt}}{t^n} dt$.

Proof: In the case of i.i.d. complex-valued Gaussian distributed variables h_i , the noise enhancement can be evaluated in closed form. As shown in the Appendix, the noise is increased by (15). In case of a ZF receiver, the value is $3/2$ which corresponds to 1.76 dB, a value for which the four times diversity scheme gives much better results as long as E_b/N_o is larger than about 3 dB. \square

Therefore, the noise enhancement is maximum for $\nu = 0$ and does not exceed 1.76 dB. Further ahead noise enhancement will be discussed in a more general form. The behavior of (15) vs. $1/\nu$ (which equals \bar{E}_b/N_o for the MMSE) is shown in Figure 1 indicated by little crosses labelled “ \times ”.

Maximum-Likelihood Receiver

The Maximum-Likelihood (ML) receiver selects the $\hat{\mathbf{s}}$ which minimizes

$$\Lambda_1(\mathbf{s}) = \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \mathbf{s}^H \mathbf{G}\mathbf{s} - 2 \text{Re}(\mathbf{y}^H \mathbf{H}\mathbf{s}) + \|\mathbf{y}\|^2 \quad (16)$$

for all permissible symbol vectors \mathbf{s} from the transmitter alphabet. We have assumed spatially white interference plus noise. In the following we specialize to QPSK which is relevant for UMTS. The transmitter alphabet of the Four Antenna Scheme using QPSK consists of $4^4 = 256$ symbol vectors \mathbf{s} .

Alternatively, we can apply the matched filter \mathbf{H}^H to \mathbf{y} and implement the ML estimator on its output \mathbf{z} given in Eq.(8). However, we need to take into account that the additive noise plus interference is spatially correlated after filtering: Assuming \mathbf{v} to be zero-mean and spatially white with 4×4 covariance matrix $\sigma^2 \mathbf{I}_4$, gives a $\mathbf{w} = \mathbf{H}^H \mathbf{v}$ with covariance matrix $\sigma^2 \mathbf{G}$. The advantage of this approach is that this partly decouples the symbols. The pair $\{s_1, s_4\}$ is decoupled from $\{s_2, s_3\}$ allowing for an ML receiver which needs to search over $4^2 = 16$ vector symbols only twice,

$$\Lambda_2(\mathbf{s}) = (\mathbf{z} - \mathbf{G}\mathbf{s})^H \mathbf{G}^{-1} (\mathbf{z} - \mathbf{G}\mathbf{s}) \quad (17)$$

The metric Λ_2 can be re-written as the sum of two independent terms $\Lambda_{2a} + \Lambda_{2b}$ which depend on mutually exclusive sets of variables each.

$$\Lambda_{2a} = \Lambda_{2a}(s_1, s_4) = |z_1 - h^2(s_1 + X s_4)|^2 + |z_4 - h^2(s_4 + X s_1)|^2$$

$$\begin{aligned} & -2X \text{Re} \{ [z_1 - h^2(s_1 + X s_4)] [z_4^* - h^2(s_4^* + X s_1^*)] \}, \\ \Lambda_{2b} & = \Lambda_{2b}(s_2, s_3) = \\ & |z_2 - h^2(s_2 - X s_3)|^2 + |z_3 - h^2(s_3 - X s_2)|^2 \\ & + 2X \text{Re} \{ [z_2 - h^2(s_2 - X s_3)] [z_3^* - h^2(s_3^* - X s_2^*)] \}. \end{aligned}$$

Note that the two metrics $\Lambda_{2a}, \Lambda_{2b}$ are positive definite quadratics when $|X| < 1$. Therefore, s_1, s_4 can be detected regardless of s_2, s_3 and vice versa.

Simulation Results

Figure 1 displays the simulated behavior of the uncoded bit error rate of the linear MMSE receiver and vanishing fading correlation between the four transmit paths. The BER results were averaged over 16000 symbols and 3200 selections of channel matrices \mathbf{H} for each simulated E_b/N_o . For comparison, the BER from the ZF receiver and the cases of ideal two- and four-path diversity are also shown. The values marked by circles (‘o’) labeled “Theory including n.e. of 1.76 dB” are the same as for four-path diversity, but shifted by the noise enhancement (n.e.) of 1.76 dB. Compared to the ZF receiver performance, there is only little improvement for MMSE.

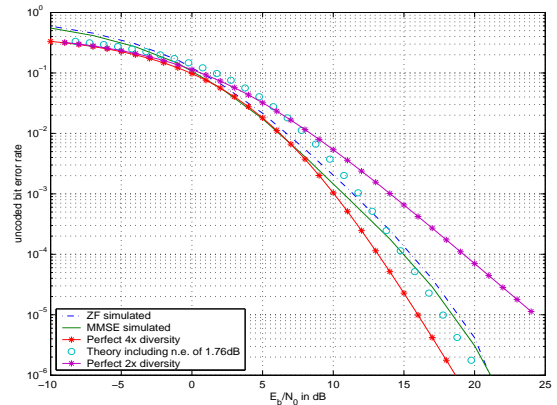


Figure 1. BER for Four Antenna Scheme with linear MMSE and ZF receiver and no correlation between antennas.

For practical considerations, it is of interest to investigate the performance when the L paths are correlated, as can be expected in a typical transmission environment. Figure 2 displays the situation when the antenna elements are correlated by a factor of $\rho \in \{0.5, 0.75, 0.95\}$. The figure shows that a fading correlation above 0.5 results in some losses. Only with very strong correlation (0.95) a degradation towards the two-times diversity performance was noticed.

The results for the BER of the ML receiver are given in Figure 3. The BER results were averaged over 12800 QPSK symbols for each channel matrix \mathbf{H} and 8000 selections of channel matrices \mathbf{H} for each simulated E_b/N_o . For comparison, the BER from the ZF receiver and the cases of ideal two- and four-path diversity are also shown. The magnified portion of this plot shows the vicinity around a BER of 10% which is where typical packet data services in UMTS FDD

will operate. In this range of E_b/N_0 , we see that the ZF receiver performs rather weakly, i.e. worse than the two-antenna Alamouti scheme by approximately 0.5 dB. The ML receiver for the Four Antenna Scheme, however, outperforms the two-antenna Alamouti scheme by 0.1 to 0.5 dB depending on the chosen point of operation.

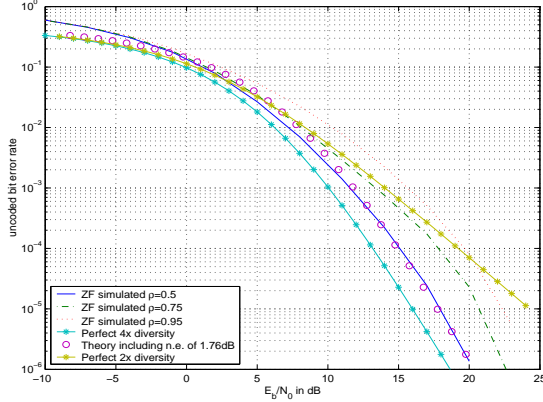


Figure 2. BER for Four Antenna Scheme with Zero-Forcing receiver, no correlation between adjacent antenna elements is $\rho \in \{0.5, 0.75, 0.95\}$.

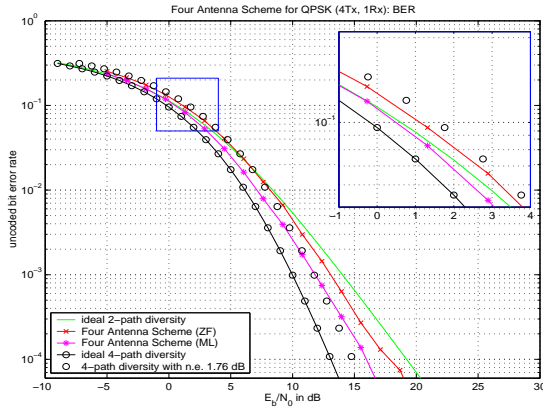


Figure 3. BER for Four Antenna Scheme with ML receiver for QPSK, No correlation between antenna elements.

Let us consider the ML receiver for the Four Antenna Scheme proposed in Sec. for two symbol alphabets in further detail. For the QPSK symbol alphabet $\{1, j, -1, -j\}$, there exist $4^4(4^4 - 1) = 65280$ error events $\mathbf{S} \rightarrow \tilde{\mathbf{S}}$ where we have counted only those pairs $\mathbf{S}, \tilde{\mathbf{S}}$ with $\mathbf{S} \neq \tilde{\mathbf{S}}$. It turns out that exactly 2080 error events are characterized by a rank-deficient matrix $\mathbf{A}(\mathbf{S}, \tilde{\mathbf{S}})$ of rank $r = 2$ instead of 4. The corresponding simulation result is shown in Figure 3. For a slightly modified QPSK symbol alphabet where the constellation is rotated by $e^{j\pi/4}$ for the symbols s_2 and s_4 , i.e.

$$s_k \in \{e^{j(k-3)\pi/4}, e^{j(k-1)\pi/4}, e^{j(k+1)\pi/4}, e^{j(k+3)\pi/4}\},$$

it turns out that all error events are characterized by a full-

rank matrix $\mathbf{A}(\mathbf{S}, \tilde{\mathbf{S}})$. This results in a space-time code with diversity order 4 in flat Rayleigh fading. The corresponding simulation result is shown in Figure 4.

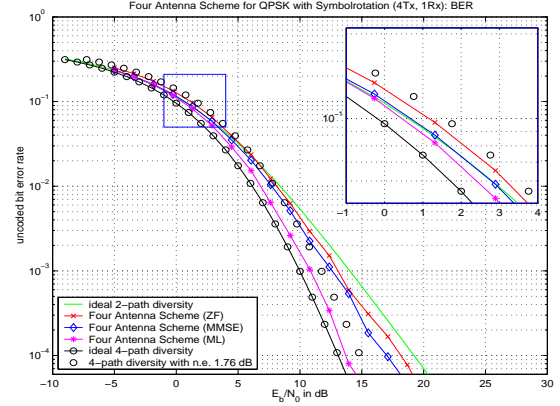


Figure 4. BER for Four Antenna Scheme with ML receiver, rotated QPSK constellation, no correlation between antenna elements.

THE UNSYMMETRICAL CASE

An interesting approach to increase the diversity when the number of receiver antennas is more than one, is given by the following. Assume that a block matrix form of the channel matrix \mathbf{H} is given by

$$\mathbf{H} = [\mathbf{H}_1 \mathbf{H}_2]. \quad (18)$$

Then, the scheme can be Alamouti by performing the following operation:

$$\mathbf{F} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\mathbf{H}_2^* & \mathbf{H}_1^* \\ \mathbf{H}_2^* & \mathbf{H}_1^* \\ \mathbf{H}_1 & -\mathbf{H}_2 \end{bmatrix} \quad (19)$$

The corresponding term $\mathbf{F}^H \mathbf{F} =$

$$2 \begin{bmatrix} \mathbf{H}_1^* \mathbf{H}_1 + \mathbf{H}_2^* \mathbf{H}_2 & 0 \\ 0 & \mathbf{H}_1^* \mathbf{H}_1 + \mathbf{H}_2^* \mathbf{H}_2 \end{bmatrix}. \quad (20)$$

Example: a two transmit-two receive antenna system is considered. We set

$$\mathbf{H}_1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} h_3 \\ h_4 \end{bmatrix}$$

The matrix $\mathbf{F}^H \mathbf{F}$ becomes

$$\mathbf{F}^H \mathbf{F} = 2(|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2) \mathbf{I}_2$$

Thus, the full 4 times diversity can be explored, without a matrix inverse to compute! Note that in this case, the sequence to transmit at the two antennas reads

$$\left\{ \begin{array}{cccccccc} s_1 & s_2 & -s_3^* & -s_4^* & s_3^* & s_4^* & s_1 & s_2 \\ s_3 & s_4 & s_1^* & s_2^* & s_1^* & s_2^* & -s_3 & -s_4 \end{array} \right\}$$

Note that during 8 time periods only four symbols are transmitted, i.e., this general scheme has the drawback that it can

only offer half the symbol rate! Many other examples will be provided in the full paper.

Consider now a 4 by 2 transmission scheme. The matrices are identified to

$$\mathbf{H}_1 = \begin{bmatrix} h_1 & h_2 \\ h_5 & h_6 \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} h_3 & h_4 \\ h_7 & h_8 \end{bmatrix}$$

We now obtain a matrix $\mathbf{F}^H \mathbf{F}$ consisting of two block matrices of size 2×2 on the diagonal. Thus, the scheme is still rather simple, since only a 2×2 -matrix is to be inverted although a four-path diversity is achieved. Note that now the data rate is at full speed!

The previously discussed four antenna system can be obtained when setting

$$\mathbf{H}_1 = [h_1 \quad h_2], \quad \mathbf{H}_2 = [h_3 \quad h_4]$$

A comparison of the trace term shows that for the 4×2 antenna system 3 dB is gained compared to the 4×1 antenna system.

APPENDIX

Starting from the definition of X in (7) it is observed that the squares in the denominator can be appended,

$$X + 1 = \frac{|h_1 + h_4|^2 + |h_2 - h_3|^2}{|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2}.$$

In the case of i.i.d. complex-valued Gaussian distributed variables h_i , the two variates $h_1 + h_4$ and $h_2 + h_3$ become complex Gaussian distributed and independent of each other. They depend, however, on the variates in the nominator. Now, a linear orthogonal coordinate transformation

$$\begin{aligned} u &= (h_1 + h_4)/\sqrt{2}, & v &= (h_2 - h_3)/\sqrt{2}, \\ u' &= (h_1 - h_4)/\sqrt{2}, & v' &= (h_2 + h_3)/\sqrt{2}. \end{aligned}$$

is defined and in the new variables,

$$\frac{X + 1}{2} = \frac{|u|^2 + |v|^2}{|u|^2 + |v|^2 + |u'|^2 + |v'|^2}. \quad (21)$$

Generally, if X_1^2 and X_2^2 are independent random variables following chi-square distributions with ν_1 and ν_2 degrees of freedom respectively, then $\frac{X_1^2}{X_1^2 + X_2^2}$ is said to follow a Beta(p, q) distribution with $\nu_1 = 2p$ and $\nu_2 = 2q$ degrees of freedom and the probability density is given by $\frac{1}{B(p, q)} \xi^{p-1} (1 - \xi)^{q-1}$, with $p = \frac{\nu_1}{2}$, $q = \frac{\nu_2}{2}$. This matches our case (21) for $\nu_1 = \nu_2 = 4$ and the probability density specializes to $6\xi(1 - \xi)$. Transforming this back to X gives the probability density

$$f_X(x) = \begin{cases} \frac{3}{4}(1 - x^2), & \text{for } |x| < 1, \\ 0, & \text{elsewhere.} \end{cases} \quad (22)$$

For the ZF receiver, it follows that the noise is increased by a factor of

$$\mathbb{E} \left[\frac{1}{1 - X^2} \right] = \int_0^2 \lambda^{-1} f_X(\lambda - 1) d\lambda = \frac{3}{2}.$$

For the linear receiver with $\nu > 0$,

$$\frac{1}{2} \mathbb{E} \left[\frac{1 + X}{\left(1 + \frac{\nu}{h^2} + X\right)^2} + \frac{1 - X}{\left(1 + \frac{\nu}{h^2} - X\right)^2} \right]$$

we substitute $\eta = h^2$ and evaluate the double integral

$$\int_{\eta=0}^{\infty} \int_{x=-1}^1 \frac{1+x}{\left(1 + \frac{\nu}{\eta} + x\right)^2} f_X(x) f_{\eta}(\eta) dx d\eta,$$

where $f_{\eta}(\eta) = \frac{128}{3} \eta^3 e^{-4\eta}$ is the χ_8^2 -density re-scaled to $\mathbb{E}[\eta] = 1$.

CONCLUSION

In this article we provided a deep insight into the possibility of extending the Alamouti scheme for more than two antennas. With some small modifications, the scheme allows high diversity even for UMTS like scenarios. Extending to higher than four antennas is straight forward. By structuring the transmitted data, it is possible to utilize much of the advantages of low complexity receivers without sacrificing too much in diversity gain. More details of this will be shown in[5].

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