AN ALGORITHM FOR ESTIMATING THE DOPPLER SPEED

Markus Rupp

Bell-Labs, Lucent Technologies,Wireless Research Laboratory
791 Holmdel-Keyport Rd.,Holmdel, NJ 07733-0400, USA
Fax: (732) 888 7074, Tel: (732) 888 7104
e-mail:rupp@lucent.com

1. ABSTRACT

Adaptive algorithms are obligatory for tracking the channel or its inverse in mobile communication systems. Typical are either the Least-Mean-Squares (LMS) algorithm or the Recursive-Least-Squares (RLS) algorithm. Their performance considerably improves when the step-size for LMS (or forgetting factor for RLS) is chosen according to the Doppler speed, that is the speed of a vehicle in which the receiver is located relative to the base station. A higher Doppler speed usually requires a higher step-size. However, it is not straightforward to derive Doppler speed estimates from the received data. Theoretical analyses have been done in the context of zero-crossings [1, 2], a method that typically requires several thousand samples in order to deliver satisfying estimates. In IS-136 based TDMA phones an estimate is required every slot (162 symbols). This paper describes a very simple scheme for Doppler speed estimation that is well suited for real-time implementations in fixed point and derives its properties.

2. TRANSMISSION MODEL

In the following it is assumed that the received data $y(k)$ refer to a single path flat Rayleigh fading with additive noise, i.e.,

$$y(k) = c(k)s(k) + v(k)\,. \tag{1}$$

The channel's fading properties are all concentrated in one random process $c(k)$

$$c(k) = fc(k-1) + \sqrt{1-f^2}\, w(k)\,. \tag{2}$$

The driving process $w(k)$ is assumed to be complex-valued and statistically white Gaussian with unit variance. The only parameter f defines the dynamic of the process and thus is related to the Doppler speed. The autocorrelation (ACF) function of this process is given by

$$r_c(l) = E[c(k)c^*(k+l)] = f^{|l|}\,.$$

Although the true acf is known to be a Bessel-function $r_c(l) = J_0(2\pi f_D T_s l)$ it will be continued with this simpler model. In order to come up with practical values the data model will be needed to simplified even more. A comparison with a more realistic model following the Bessel-functions is discussed at the end of the paper. The simple model has the advantage that the influence of all parameters can be studied easily since closed-form expressions can be derived. Although less accurate the simple model provides essential information about the basic properties of the estimation scheme. The final evaluation with Bessel-functions can only be done numerically and although accurate, lacks the possible insight in the various terms that are involved.

3. THE METHOD

IS-136 delivers 176=162+14 (one slot plus adjacent sync word) data symbols each data slot which will be cut in N smaller blocks of $L = 176/N$ elements. Based on the variance of these blocks the relation to the unknown Doppler speed is derived.

This variance is averaged over the N blocks so that finally an estimate for the expectation value is obtained. Intuitively speaking, if the Doppler speed is high, the variance will become high and thus a high step-size can be derived. However, as the following derivations will show, the variance is far from being a monotone function in the Doppler speed.

In order to understand the properties of the estimate, some expressions have to be computed. Throughout the method only energy values will be used since they are easy to compute in particular with a fixed point DSP. The expression

$$E[|y(k)|^2|y(k+l)|^2] \approx E[|c(k)|^2|c(k+l)|^2]$$

is approximated by the simpler one containing only the channel coefficient. It is assumed that the transmitted symbols in (1) are statistically white and the noise is relatively small compared to the transmitted symbol energy. Simulation results showed reasonable agreement with this assumption.

With the given channel model the expression can be computed to (see Appendix A)

$$E[|c(k)|^2|c(k+l)|^2] = 1 + f^{2|l|} ,$$

thus giving a value between one and two depending on the correlation.

For the proposed method small blocks of averaged energy have to be computed:

$$a(l) = \frac{1}{L}\sum_{i=1}^{L} |y(i+(l-1)L)|^2 , l = 1..N .$$

These blocks are used to compute the mean and variance:

$$\overline{a} = \frac{1}{N}\sum_{i=1}^{N} a(i), \tag{3}$$

$$\sigma_a^2 = \frac{1}{N}\sum_{i=1}^{N} (a(i)-\overline{a})^2 . \tag{4}$$

In order to compute $\mathrm{E}[\sigma_a^2]$, the autocorrelation $r_a(m)$ for two blocks

$$r_a(m) = E[a(l)a(l+m)]$$

has to be computed. After a lengthy derivation, given in Appendix B,

$$r_a(m) = 1 - \frac{f^{2(mL+1)}}{L^2(1-f^2)^2}\left(2 - f^{2L} - f^{-2L}\right)$$

is obtained. Now,

$$E[a^2(l)] = r_a(0) = 1 - \frac{f^2}{L^2(1-f^2)^2}\left(2 - f^{2L} - f^{-2L}\right) .$$

Finally,

$$E[\sigma_a^2] = \beta\left[1 - \frac{1}{N^2}\frac{1}{1-f^{2L}}\left(N(1+f^{2L}) - 2f^{2L}\frac{1-(f^{2L})^N}{1-f^{2L}}\right)\right] \tag{5}$$

is obtained with

$$\beta = \frac{1}{(1-f^2)L^2}\left[L(1+f^2) + 2f^2\frac{f^{2L}-1}{1-f^2}\right] .$$

In the following $L = 8, N = 22$ was chosen. Figure 1 plots the estimator as a function of the channel dynamic f. The function is monotonically increasing in f until the maximum is reached at about $f = 0.97$. It then decreases monotonically. The picture also shows some markers at which the function has been checked via simulation. Since the values of interest are on the down slope (f between 0.99 and one), Figure 2 depicts the descending slope in terms of $1-f$. Even for values very small to one the simulation results show that the estimator is still accurate. Notice that a double logarithmic scaling has been used for this figure.

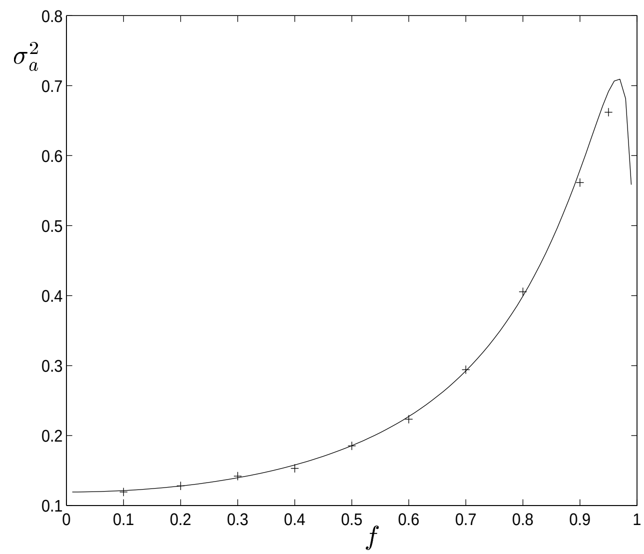

Figure 1: Estimator σ_a^2 as function of channel dynamic f.

Thus, given the optimal step-size for a certain Doppler speed and the relation from f to Doppler speed, this method can be used to map the estimate σ_a^2 directly to the optimal step-size. Figure 3 depicts the estimation of Doppler speed by estimating the variance σ_a^2 in a "real" scenario. The given Doppler speed is based on a 800MHz carrier frequency.

4. ROBUSTNESS AND ACCURACY

As Figure 3 shows, the curve for two path Rayleigh fading becomes lower with higher delay spread. Fortunately, the variation is not too big and since the optimal step-size values for the LMS algorithm are from rather flat curves around the optimum, the method can be used independent of the delay spread. Simulations also show that the method is independent of offsets in the sample timing as well as frequency offsets. The later property can easily be proven. If the received sequence $y(k)$ is corrupted by a frequency offset, the sequence $y(k)e^{j\Omega k}$ will be received. Since only the magnitudes are used $|y(k)e^{j\Omega k}|^2 = |y(k)|^2$, the influence of the frequency offset is completely removed.

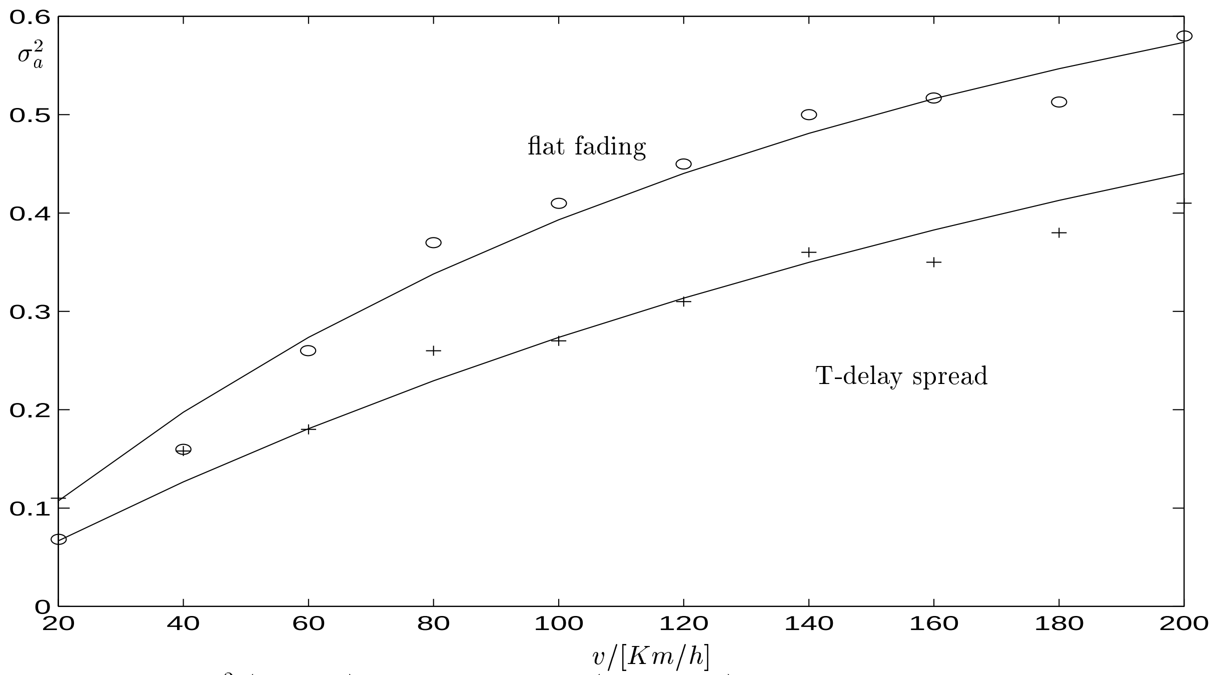

Figure 3: Estimator σ_a^2 (markers) vs. theoretical line (continuous) as function of Doppler speed.

Simulations also confirmed little sensitivity to the additive noise level. For SNR< 20dB only the estimates for small speeds are corrupted. The noise increases the estimates for 10Km/h by about 15% (SNR=20dB) to 80% (SNR=12dB). The values for high speeds remain untouched.

5. MORE ACCURATE COMPUTATION

The only drawback of the theory so far is that the model of the channel dynamics is relatively simple compared to the true dynamics. Since the coefficient $c(k)$ is a Gaussian process, the following can be deduced (see Appendix A for derivation)

$$E[|c(k)|^2|c(k+l)|^2] = r_c^2(0) + r_c^2(l)$$

with $r_c(l) = E[c(k)c^*(k+l)]$. Since the acf is known for this process

$$E[|c(k)|^2|c(k+l)|^2] = 1 + J_0^2(\gamma l)$$

with $\gamma = 2\pi T_2 f_d$. Unfortunately, there are no close expressions for finite sums of Bessel-functions. Thus, a program has been written to numerically compute all the required terms. For fixed block-length L=8, and N=44 blocks the results shown in Figure 4 were obtained. The measured data are from a set of T/2 sampled data, thus giving 352 data elements for every slot. The comparison for 630MHz and 880MHz carrier frequency is depicted. The simulation data was generated for 800MHz carrier frequency. Although still a slight mismatch, the data fit well into the predicted curve.

Table 1 lists optimal step-sizes for the normalized LMS algorithm found by simulations for various channel conditions. The optimal step-size ($\mu(k) = \alpha(k)/M$, M being the length of the adaptive filter) choices are roughly independent of the delay spread. Since the corresponding BER are not very sensitive around the optimum point, a simple mapping can be chosen. Good results were obtained for

$$\alpha(k) = \begin{cases} 0.05 & \sigma_a^2 < 0.05 \\ 0.6\sigma_a^2 & 0.05 < \sigma_a^2 < 0.25 \\ 0.25 & 0.25 < \sigma_a^2 \end{cases} .$$

Channel	100Km/h	50Km/h	8Km/h
flat	0.24	0.18	0.12
$T/4$	0.27	0.18	0.09
$T/2$	0.24	0.18	0.09
T	0.3	0.21	0.09

Table 1: Optimal step-sizes $\alpha(k)$ for PCS band based on $M = 3$.

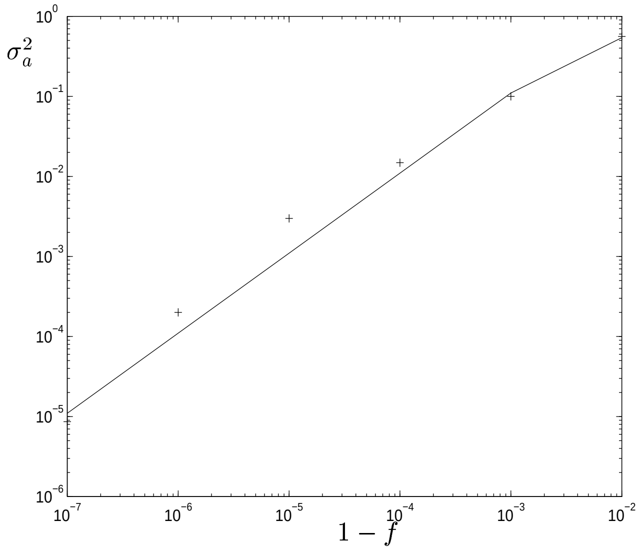

Figure 2: Estimator σ_a^2 as function of channel dynamic $1-f$.

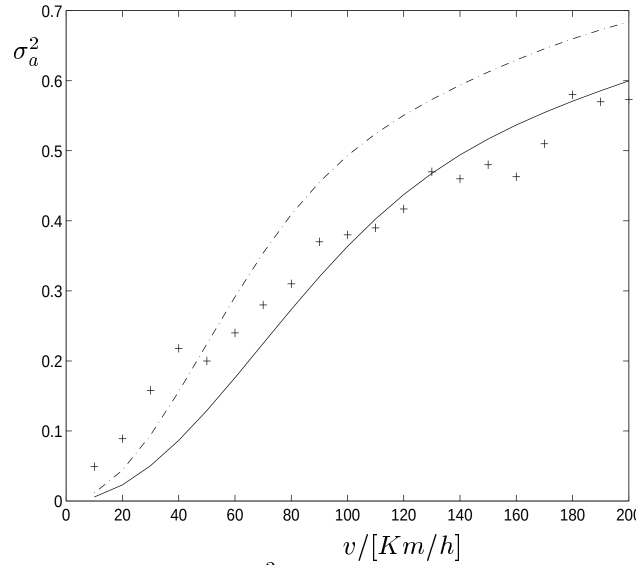

Figure 4: Estimator σ_a^2 as function of Doppler speed with accurate computation. continuous line (630MHz), dashed line (880MHz).

APPENDIX A.

An important expression is

$$E[|c(k)|^2|c(k+l)|^2] = r_c^2(0) + r_c^2(l)\ ,$$

which can be reformulated by $c(k) = x + jy$ and $c(k+l) = u + jv$. Now $\mathrm{E}[(x^2+y^2)(u^2+v^2)]$ has to be computed. Because of the dynamic equation 2 x is only statistically dependent on u, and y only on v. With the well known identity for Gaussian processes

$$E[xyuv] = E[xy]E[uv] + E[xu]E[yv] + E[xv]E[yu]$$

the following is obtained

$$E[(x^2+y^2)(u^2+v^2)] = 4(E[x^2] + E[xu]^2)\ .$$

Since symmetric correlation is assumed, i.e, $r_{xu} = r_{yv}$ and $r_{xx} = r_{yy} = r_{uu} = r_{vv}$, it can eventually be concluded that

$$E[|c(k)|^2|c(k+l)|^2] = 4(r_{xx}^2(0)+r_{xx}^2(l)) = r_c^2(0)+r_c^2(l)\,.$$

APPENDIX B.

Without giving the complete derivation line by line, a basic expression that appears several times in the derivations is explained more in detail. Thus, consider the following expression

$$\sum_{l=1}^{N}\sum_{k=1}^{N} g^{|k-l|} =$$

$$\begin{aligned}
&= \sum_{l=1}^{N}\left[\sum_{k=1}^{l} g^{k-l} + \sum_{k=l+1}^{N} g^{l-k}\right] \\
&= \frac{1}{1-g}\sum_{l=1}^{N}\left[1 - g^l + g - g^{N+1-l}\right] \\
&= \frac{1}{1-g}\left[N(1+g) - 2\frac{g-g^{N+1}}{1-g}\right] \\
&= \frac{2g^{N+1} - Ng^2 - 2g + N}{(1-g)^2}\ .
\end{aligned}$$

6. CONCLUSION

This paper presents a simple and efficient method to estimate the Doppler speed out of received sampled data. The method is well suited for DSP or chip implementation. Simulations with realistic data samples show that the proposed method is very robust with respect to noise and model misassumptions as well as insensitive towards frequency offsets.

7. REFERENCES

[1] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw Hill, 1987.

[2] William C. Jakes, Microwave Mobile Communication, IEEE Press, 1974.