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# Internal Structure and Ice Content of Reichenkar Rock Glacier (Stubai Alps, Austria) Assessed by Geophysical Investigations

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#### **ABSTRACT**

The Reichenkar rock glacier (Tyrol, Austria) is a typical tongue-shaped, 1400 m long, ice-cored active rock glacier, which connects to a debris-free cirque glacier. Aerial photographs from 1954 and 1990 indicate its mean surface velocity to be 0.6 m/a while a photograph from 2003 and annual global positioning system (GPS) measurements since 1998 show that velocities in the past decade have increased to 3 m/a. Integration of ground-penetrating radar (GPR), seismic and gravimetric data reveals that the Reichenkar rock glacier consists of four layers. The uppermost debris layer has an average thickness of about 5 m and is underlain by ice-rich permafrost with an average thickness of about 25 m. A prominent reflector detected by GPR is identified as the top of an unfrozen till layer located a few metres above the bedrock. Seismic refraction data clearly indicate the boundary between till and bedrock. The geophysical interpretation shows that the ice-rich permafrost of the rock glacier has an ice content of 45–60%, depending on assumptions concerning the air content of the ice. Creep velocities calculated from the geophysical model, ice contents and an extension of Glen's flow law are in good accordance with observed surface velocities. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: rock glacier; gravimetry; GPR; seismic refraction; creep; mountain permafrost

# INTRODUCTION

Internal structure and ice content are key factors controlling the dynamics and evolution of active rock glaciers. This information can be derived either from direct observations (e.g. borehole logs, outcrops and tunnels) or from surface-based geophysical methods such as seismic refraction, geoelectrics, ground-penetrating radar (GPR) and gravimetry. Until the early 1990s seismic refraction and direct current (DC) resistivity soundings were common methods to assess

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the thickness of permafrost and to deduce the origin of the permafrost ice (Barsch, 1973; King, 1976; King et al., 1987). During the project Permafrost and Climate in Europe (PACE) a broad range of geophysical methods was applied to determine the internal structure as well as the vertical and lateral distribution of permafrost in high mountain environments (Vonder Mühll et al., 2001, 2002; Hauck, 2001; Hauck and Vonder Mühll, 2003a, 2003b; Hauck et al., 2004). Vonder Mühll et al. (2000) suggest that useful combinations of geophysical techniques in Alpine permafrost environments include seismic surveys with DC resistivity and gravimetry with GPR. Nowadays even DC resistivity data are usually processed with algorithms that allow two-dimensional imaging of the subsurface (Hauck et al., 2003).

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Few boreholes are available from rock glaciers (e.g. Haeberli, 1990; Vonder Mühll and Holub, 1992; Hoelzle et al., 1998; Arenson et al., 2002; Vonder Mühll et al., 2003; Arenson and Springman, 2005b). Haeberli et al. (2006) summarised these observations in a threefold classification based on ice distribution: (1) massive ice with dispersed debris and intercalated debris-laden ice, overlying a basal layer of rock; (2) a few metres to tens of metres of debris-laden ice overlying massive ice; and (3) ice/rock mélange throughout. Besides solid rock and ice, ice-rich core samples of rock glaciers contain air voids with a volumetric air content up to 25% (Wagner, 1990; Arenson, 2002; Arenson et al., 2004). Thus, zones of circulating water or air can exist within or below the permafrost body of a rock glacier (Haeberli et al., 2006).

Previous publications on the Reichenkar rock glacier focused on geology, geomorphology, hydrology and surface velocities (Krainer and Mostler, 2000a, 2000b, 2002, 2006; Krainer et al., 2002; Chesi et al., 2003). In this study, we present an integrated geophysical approach to investigate its internal structure and provide information on the ice content of the permafrost body using a three-phase model (rock, ice and air). We calculate the creep velocity from the structural model, the ice content and an extension of Glen's flow law, in order to test whether the geophysical model can explain recently observed high surface velocities.

## STUDY SITE

The Reichenkar rock glacier is located in a small, northeast-facing side-valley ('Inneres Reichenkar') of the Sulztal in the western Stubai Alps, Tyrol, Austria (Figure 1). The landfrom is a tongue-shaped, 1400 m long, ice-cored rock glacier, up to 260 m wide in its upper part, 170–190 m wide in its lower part and with an area of 0.27 km<sup>2</sup> (Krainer and Mostler, 2000b) (Figure 2a). The rock glacier extends from an elevation of 2750 m a.s.l. down to 2310 m a.s.l. The active front is 33 m high and has a steep slope (41°).

The surface of the Reichenkar rock glacier is characterised by well-developed, longitudinal furrows and ridges in the upper part and transverse furrows and ridges in the lower part. The debris layer is very coarse near the surface, containing blocks up to several metres in diameter, but is much finer grained beneath. This debris is about 70% amphibolite and 30% eclogite by volume (Figure 2a). Bedrock in the lower part of the rock glacier (the investigated area) is biotite-plagioclase gneiss (Krainer and Mostler, 2000a).

Analysis of aerial photographs from 1954, 1990 and 2003 (Figure 2b and 2c) shows that the creep velocity of the rock glacier increased from 0.6 to 3 m/a over this period. Annual GPS measurements between 1997 and 2004 give a mean horizontal velocity of 2.2 m/a and a mean vertical change of 0.5 m/a, and also show a steady increase in horizontal velocities (Figure 3)

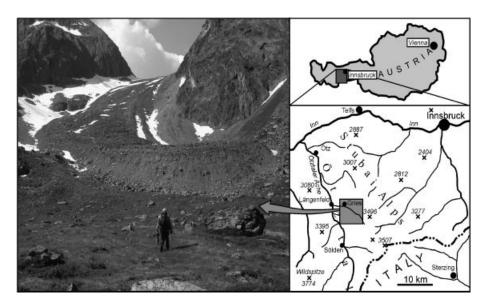


Figure 1 Location map and view of the Reichenkar rock glacier towards the south.

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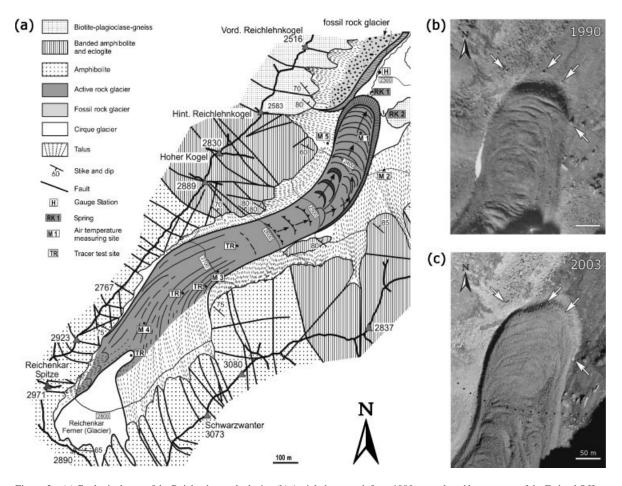


Figure 2 (a) Geological map of the Reichenkar rock glacier. (b) Aerial photograph from 1990, reproduced by courtesy of the Federal Office of Metrology and Surveying in Vienna (EB 2006/00073). (c) Aerial photograph from 2003, reproduced by permission of the Administration of Tyrol/tiris. The white arrows (in (b) and (c)) are reference points to display the large displacements between 1990 and 2003. Notable is the disappearance of two blocks in the front of the rock glacier. Geophysical investigations were carried out in the area between the front wall and the dotted line (c). In the northeastern part of (b) and (c) discharge from the rock glacier emerges from above a silty till layer.

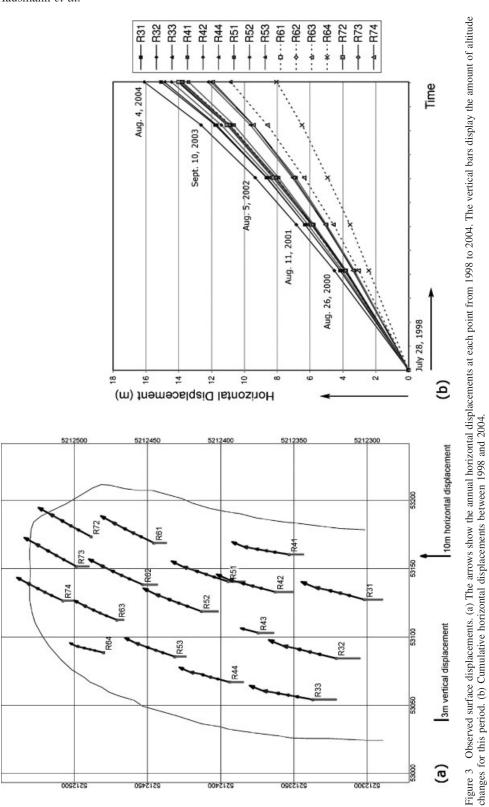
(Chesi et al., 2003; Krainer and Mostler, 2006). The maximum measured surface velocity of almost 3 m occurred in 2003-04 along the longitudinal axis of the lower part of the rock glacier. The rock glacier front advanced  $\sim 30 \,\mathrm{m}$  from 1990 to 2003, which is in agreement with the observed average displacement determined from GPS. Continuous GPS measurements over 46 days during summer 2002 showed fairly constant creep rates averaging 6 mm/d, independent of precipitation/discharge and ground surface temperature.

Meltwater from the rock glacier is released from a spring above a fine-grained (silty) till layer at the foot of the steep front (location RK1 in Figure 2a). Dye tracer tests showed high velocities

(3.5–5.5 m/min) of the meltwater flow through the rock glacier, indicating channelised flow along a network of conduits (Krainer and Mostler, 2002). Strong seasonal and diurnal variations characterise the discharge.

# GEOPHYSICAL INVESTIGATIONS

GPR, seismic refraction and gravimetry were used to interpret the internal structure and ice content of the Reichenkar rock glacier. First, GPR was used to derive the top of an unfrozen sediment layer (Figure 4a). Second, the depth to ice-rich permafrost was determined using seismic refraction (Figure 4b).



9212300

9212350

2212500

9212450

9212400

(a)

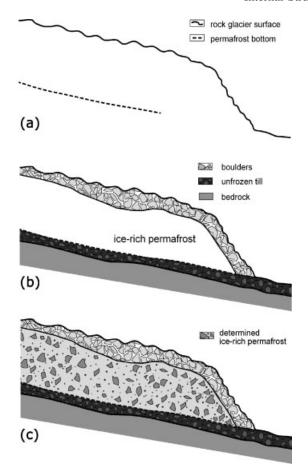


Figure 4 Integration of geophysical methods: (a) Information from GPR; (b) combination of (a) with information from seismic refraction; (c) combination of (a) and (b) with information from gravimetry.

The combination of these results also provided the depth to bedrock. Finally, gravimetric information was used to calculate the ice content of the permafrost body (Figure 4c).

#### **GPR**

The internal structure of rock glaciers has been studied by GPR since 1982 using centre frequencies between 50 and 300 MHz (Haeberli et al., 1982; Haeberli, 1985; King et al., 1987; Vonder Mühll, 1993). Penetration depths up to 40 m were obtained recently at rock glaciers using centre frequencies between 15 and 50 MHz (e.g. Berthling et al., 2000, 2003; Isaksen et al., 2000; Degenhardt, 2002; Degenhardt et al., 2003; Maurer and Hauck, 2007). Arcone et al. (1998) highlighted the potential of GPR for permafrost investigations in Alaska. Lehmann and Green (2000) emphasised the importance of topographic migration for steep shallow reflectors below rugged surfaces like rock glaciers. Typical values for the mean wave velocity of the ice-rich permafrost of rock glaciers range between 0.14 and 0.15 m/ns (Schmöller and Fruhwirth, 1996; Wale, 1999; Isaksen *et al.*, 2000; Lehmann and Green, 2000; Berthling et al., 2003). Degenhardt and Giardino (2003) produced a different value (0.12 m/ns).

We used the GSSI SIR System 2000 in combination with a multiple low frequency (MLF) antenna. Profiles were measured using a centre frequency of 35 MHz and constant antenna spacing in point mode (constant-offset profiling). The distance between transmitter and receiver was 4 m and the sampling interval was 1 m. The antennas were oriented perpendicular to the profile direction. The main recording parameters were 1000 ns time range, 1024 samples/scan, 16 bits/ sample and 32-fold vertical stacking. One longitudinal profile (147) and two transverse profiles (146, 148) were measured (Figure 5a). Best results were achieved in winter when the rock glacier was covered with a thick layer of snow and when meltwater was absent (Krainer et al., 2002). Figure 6a shows an unprocessed section of profile 147. A continuous reflector (R1) is apparent and can be correlated in all other sections after data processing (Figure 6). No transparent zone (Berthling et al., 2003) was detected but a few vague reflections and scattering can be observed above reflector R1.

The lack of availability of common midpoint (CMP)-profiles complicates determination of the mean wave velocity. Therefore, deconvolution was applied to identify parts of diffraction hyperbolae in the lower parts of the sections. The corresponding values for the mean velocity range between 0.14 and 0.15 m/ns. To verify these values we used different velocities for the migration  $(0.11 < v_{till,gpr} < 0.19 \text{ m/ns})$  (for a list of symbols see Appendix A) and evaluated the shape of the migrated diffraction hyperbolae. We arrived at a value of  $v_{till,gpr} = 0.145 \text{ m/ns}$  for the conversion of two-way reflection times  $(t_{gpr})$  to depths  $(h_{till})$  which agrees with previously reported CMP measurements. The penetration depth obtained (30 m in case of R1) is typical for rock glaciers.

Amplitude and waveform modelling was used to investigate the origin of the reflector R1. The different media in the model (debris, ice-rich permafrost and bedrock) are described by dielectric permittivity and conductivity (Arcone et al., 1995; Berthling et al., 2003; Hauck et al., 2003; Musil et al., 2006). For the till layer a range of values was used for dielectric

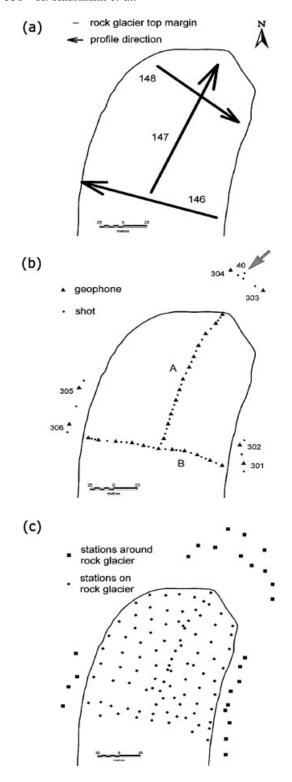


Figure 5 Layout of geophysical measurements; (a) GPR; (b) seismic refraction; (c) gravimetry.

permittivity (10–30) and conductivity (0.00002–0.2 S/m). The reflection coefficient of radar waves at normal incidence is +0.27 for an ice-air boundary, -0.67 for an ice-water boundary and -0.8 for an air-water boundary (Arcone *et al.*, 1995; Stuart *et al.*, 2003). The modelled radar data show that a high contrast in permittivity and conductivity is necessary to produce the observed high amplitudes and the waveform shape of the reflector R1. Thus, the reflector R1 is interpreted as the top of a wet till layer. This layer can be extrapolated to the terrain surface at the front of the rock glacier, where a fine-grained (silty) till layer is present.

## **Seismic Refraction**

Seismic refraction has been successfully used on rock glaciers since about 1970 (e.g. Barsch, 1971; Potter, 1972; King, 1976; Vonder Mühll, 1993; Musil *et al.*, 2002) and more recently seismic refraction tomography has been employed for permafrost studies (Hauck, 2001; Vonder Mühll *et al.*, 2002; Hauck *et al.*, 2004). Seismic P-wave velocities of rock glaciers range between 250 and 1100 m/s for the debris layer and between 2700 and 4050 m/s for ice-rich permafrost (e.g. Vonder Mühll, 1993; Musil *et al.*, 2002).

Seismic refraction measurements were carried out using 40 shots and 30 single channel miniature recorders (Reftek 125) placed along profiles A and B and at locations outside the rock glacier (Figure 5b). Seismic energy was generated by the use of detonating cord blasted at the surface and all shots were recorded simultaneously by all receivers. The average spacing of the geophones (4.5 Hz, vertical) along the two profiles on the rock glacier was 15 m. Since one or two shot points were arranged between two consecutive geophones, the average sampling distance was 6.5 m along profiles A and B. The temporal sample interval was 1 ms. A sample record is shown in Figure 7a. All first arrival travel times are plotted in Figure 7b. Figure 7c shows all traces with geophones and shots on the rock glacier stacked to offset bins of 8 m width. Both the stack and the travel times resolve three layers: debris, ice-rich permafrost and bedrock. Average velocities estimated by correlation of the different phases are:  $v_{debris} = 950 \text{ m/s}, \ v_{icePf} = 3300 \text{ m/s}$  and  $v_{bedrock} = 4100 \text{ m/s}$ . The P-wave velocity of the ice-rich permafrost is significantly lower than the velocity of pure ice at  $0^{\circ}$ C ( $\sim$ 3750 m/s). This may indicate (i) a reduction of the frozen contact between ice and debris within the ice-rich permafrost and subsequent formation of a water film (cf. Vonder Mühll, 1993; Barsch, 1996; Musil, 2002) and/or (ii) numerous air voids within the ice (Arenson, 2002). The velocity of the

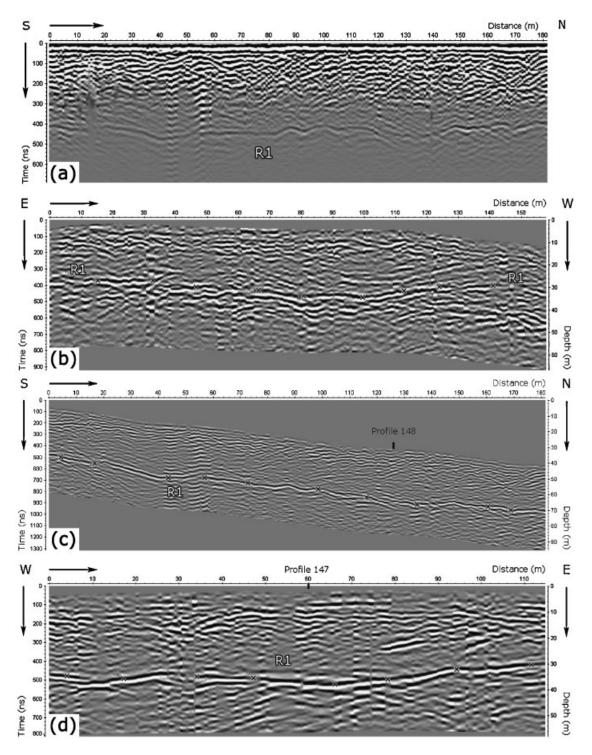


Figure 6 (a) GPR profile 147 without data processing (constant gain). (b-d) GPR profiles processed with bandpass filter, background removal, gain adjustment, depth conversion and elevation correction. The black crosses indicate the interpreted horizon R1: (b) profile 146; (c) profile 147; (d) profile 148.

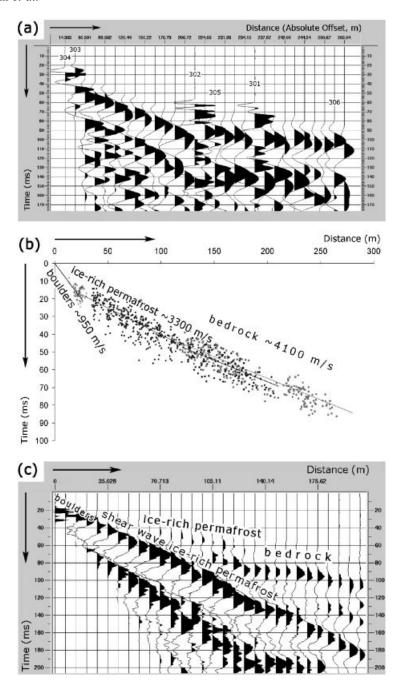


Figure 7 Representation of seismic data: (a) Record section (seismogram arranged by the absolute offset) of shot 40 (highlighted in Figure 5b). Noticeable are the shorter travel times to geophones (301–304) placed outside the rock glacier. (b) First arrival travel times versus absolute offset (distance). (c) Offset bin stack (from geophones and shots placed on the rock glacier).

bedrock is compatible with the velocity of slightly jointed gneiss.

The depths to ice-rich permafrost along profiles A and B were calculated by the Plus-Minus method (Hagedorn, 1959). Depths range between 1 m and

10 m, with an average of 4.6 m. For the ice head wave the mean value is 3430 m/s for profile A ( $\sigma$  = ±66 m/s) and 3260 m/s for profile B ( $\sigma$  = ±70 m/s). The calculation of the depth to the bedrock takes advantage of the three dimensional (3D) aspect of the shot and

Wave type	Shot	Geophone	Offset range (m)	v <sub>p</sub> (m/s)	Used for the determination of:
Direct wave	RG	RG	5–30	950	Depth to ice-rich permafrost
Direct wave	Out	Out	8–55	1000–2000	Unfrozen till and depth to the bedrock outside the rock glacier
Ice head wave Profile A Profile B	RG	RG	35–120	3363–3527 3143–3376	Depth to ice-rich permafrost
Rock head wave	RG	RG	120-205	4100	Bedrock surface
Rock head wave	Out/RG	RG/Out	120-265	4100	Bedrock surface

Table 1 Types of waves used in seismic refraction analysis. Geophone and shots are placed either on the rock glacier (RG) or outside the rock glacier (Out).

receiver layout and uses a 3D inversion (e.g. Kirchheimer, 1988a, 1988b; Iwasaki, 2002) in which delay times are proxies for refractor depths (Appendix B; Gardner, 1939; Telford et al., 1990). Prior to the inversion, we reduced the travel times of P-waves refracted from the bedrock statically to the top of the ice-rich permafrost by an amount accounting for the debris layer. The delay times were transformed to refractor depths using the velocities v<sub>icePf</sub> and v<sub>bedrock</sub>.

The wave types employed for calculation of velocities and depths are summarised in Table 1. In this three-layer model, the bedrock depth calculated by seismic refraction data exceeds the depth of the GPR reflector R1 by 17 m on average. Because of this significant disagreement, we introduced a seismic low velocity zone (i.e. an unfrozen layer) as a fourth layer between the ice-rich permafrost and the bedrock surface. Figure 8 presents two possible four-layer models, which are in agreement with both the GPR and the seismic refraction data. In the first model, the fourth layer is arranged below the GPR reflector R1 and might represent unfrozen sediments. In the second model, the fourth layer is arranged above the GPR reflector R1 and might represent unfrozen debris. We favour the first model because seismic data acquired in front of the rock glacier indicate the presence of an unfrozen layer with an average thickness of 5 m. This layer is composed of till and its upper surface could well represent the prominent reflector R1. The velocity of the till ( $v_{till} \sim 1600 \,\mathrm{m/s}$ ) was deduced from seismic data where the layer is outcropping. We derived thicknesses between 3 and 12 m for the till laver.

## Gravimetry

Gravimetric measurements have been carried out in a number of rock glacier studies (e.g. Klingele and Vonder Mühll, 1993; Vonder Mühll, 1993; Vonder Mühll and Klingele, 1994) and are summarised in Barsch (1996). Gravimetry yields additional information on density and ice content. In order to constrain the gravity inversion we used the four-layer model of the rock glacier obtained from the GPR and seismic refraction analyses.

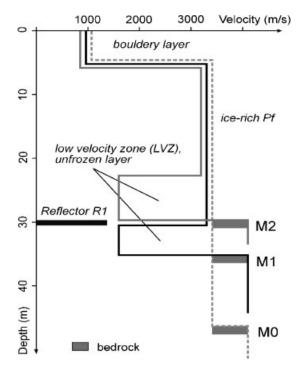


Figure 8 The initial seismic three-layer model (M0) and two possible four-layer models (M1, M2) that are consistent with the observed GPR reflector R1. In model M1, the introduced layer is arranged below the GPR reflector (e.g. unfrozen sediments). In contrast, the low velocity zone of model M2 is arranged above R1 (e.g. unfrozen debris) and the bedrock surface corresponds to R1.

A Scintrex CG-3 gravity meter (accuracy of  $\pm 10 \,\mu\text{Gal}$ ) was deployed to make 128 measurements on the lower part of the rock glacier (Figure 9a) and at 25 adjoining locations (Figure 5c) in order to estimate the regional trend. The regional density of 2670 kg/m<sup>3</sup>, used for the calculation of topographic correction and Bouguer anomaly, corresponds to a regional density map (Granser et al., 1989). A linear regional trend is subtracted from the Bouguer anomaly. The residual values up to -1 mGal show two minima (Figure 9b). Densities for the debris  $(d_{debris} = 1920 \text{ kg/m}^3)$  and till  $(d_{till} = 1950 \text{ kg/m}^3)$  layers were estimated by a conversion of seismic velocities to density using the relation of Watkins et al. (1972). Finally, modelling of the residual values yielded densities between 1620 and 2120 kg/m<sup>3</sup> for the ice-rich permafrost layer along profiles A and B (Figure 10).

#### **Ice Content**

Based on the gravimetrically derived densities we calculated the relative ice content *i* using a three-phase

model (rock, ice and air) for the ice-rich permafrost within the rock glacier:

$$d_{icePf} = i \cdot d_{ice0} + (1 - i - a) \cdot d_{rockPf} + a \cdot d_{air} \quad (1)$$

In Equation (1)  $d_{icePf}$  is the density of the ice-rich permafrost calculated in the previous section;  $d_{ice0}$  (917 kg/m³; Andersland and Ladanyi, 2004) and  $d_{air}$  (1.3 kg/m³ at 0°C) are the densities of ice and air, respectively. The rock density  $d_{rockPf}$  (2990 kg/m³) was derived from buoyancy measurements on 11 samples of the debris layer. The relative ice content (Figure 10) calculated from Equation (1) is ~60% for a=0 (two-phase model; rock and ice) and ~45% for a=0.1 (three-phase model; rock, ice and air) along profiles A and B. A one dimensional (1D) model representing the average values (Figure 11) was used to estimate the total ice content (250 000 m³) in the lower central part of the rock glacier (0.017 km²) where the surface displacements are uniform.

A sensitivity analysis and evaluation of the accuracy (Appendix C) use this representative 1D model and the

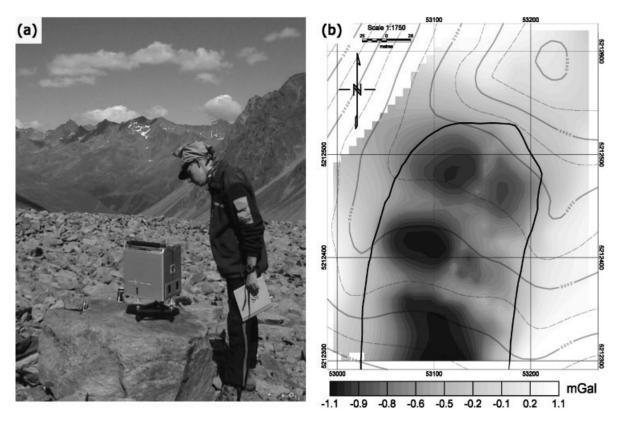


Figure 9 (a) Gravimetric data acquisition and (b) residual Bouguer anomaly.

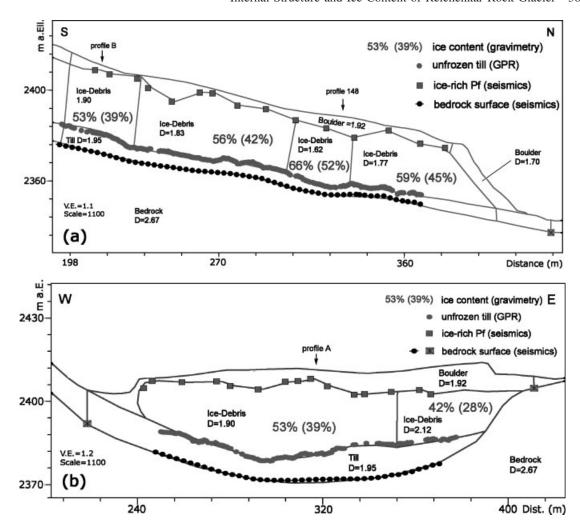


Figure 10 The combined result of GPR, seismic refraction and gravimetry: The density model and ice content on (a) longitudinal profile and (b) transverse profile. Ice content values in parenthesis refer to the three-phase model with 10% relative air content.

two-phase model of the ice-rich permafrost. Table 2 shows the computed accuracy of all the calculated parameters. The most important parameter, the total ice content of the ice-rich permafrost, has a standard deviation of about  $\pm 15\%$ .

# Creep Model

By comparing observed surface velocities and calculated creep velocities, we tested the plausibility of our geophysical interpretation. Creep of mountain permafrost depends mainly on surface slope and ice content, but temperature and air content of the ice are also important parameters. Some studies have used a pure ice body for a tentative description of rock glacier creep (e.g. Kääb et al., 2006) while others determined

the parameters for the standard creep expression (e.g. Wagner, 1992). Because of the heterogeneous composition (mixture of debris, silt, water, air and ice) (Hoelzle et al., 1998; Arenson et al., 2002; Musil, 2002) and because the temperature is close to the melting point, the observed parameters for the standard creep expression show large variations (Ikeda et al., 2003; Arenson and Springman, 2005a). Ladanyi (2003) described how the creep of ice/rock mixtures can be calculated using known principles of soil and rock mechanics.

To tentatively describe the creep process we created a rheological model using a refinement of Glen's flow law. Our model uses simplified geometry (representative 1D model, Figure 11) and assumptions (e.g. homogeneous ice/rock concentration). We apply

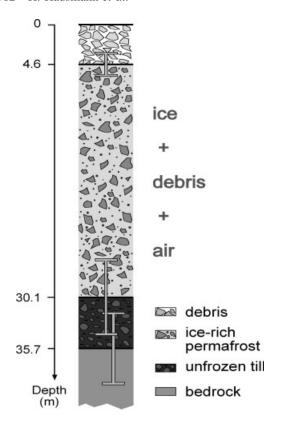


Figure 11 Representative vertical profile with error bars. It is calculated by average values from the profiles (a) and (b) (Figure 10). For calculation of the total ice content as well as the sensitivity and error analysis the lateral extension of the 1D model is assumed to be 1 in both directions.

Glen's law (Glen, 1955) to describe the rheology of pure ice in the case of simple shear:

$$\dot{\varepsilon}_{xy} = A \cdot \tau_{xy}^n \tag{2}$$

In Equation (2)  $\dot{\varepsilon}_{xy}$  is the shear strain rate, A is a temperature-dependent parameter,  $\tau_{xy}$  is the shear

stress and n = 3 is the exponent in the non-linear flow law. An expression of the surface velocity can be derived by integration of Equation (2) (Paterson, 1994; Whalley and Martin, 1992):

$$v_{x} = (i+a) \cdot 2 \cdot A \cdot \int_{0}^{dh_{icePf}} \left[ \left( \tau_{0} + \frac{g \cdot d_{icePf} \cdot y \cdot \sin(\alpha)}{1000} \right) \cdot \left( \frac{i+a}{i} \right) \right]^{n} dy$$
(3)

$$\tau_0 = \frac{g \cdot d_{debris} \cdot dh_{debris} \cdot \sin(\alpha)}{1000} \tag{4}$$

In Equation (3)  $v_x$  is the surface velocity, g is the gravity acceleration and  $\alpha$  is the (surface) slope. The quantities i and a describe the relative ice and air content of the rock glacier. Introducing the sum (i + a)in Equation (3) has the effect that flow is only allowed where ice or air exist. The presence of air voids within the ice media causes stress concentrations at the rate of  $\lfloor (i+a)/i \rfloor$ . Thickness and density of the ice-rich permafrost are  $dh_{icePf}$  and  $d_{icePf}$ . The mean surface slope  $\alpha = 11.2^{\circ}$  was computed by a leastsquares fit through 128 surface points. The shear stress  $\tau_0$  (Equation (4)) considers the additional loading by the debris layer. The thickness of the debris layer and its density are  $dh_{debris}$  and  $d_{debris}$ . We used an exponential expression given by Kääb et al. (2006) to calculate the temperature-dependent parameter A (Equation (5)).

$$A = 6.6 \cdot e^{(0.4 \cdot T)} \cdot 10^{-15} \,\mathrm{s}^{-1} (\mathrm{kPa})^{-3} \quad \text{for T} \le 0 \quad (5)$$

We calculated A for ice temperatures of 0, -1, -2 and  $-3^{\circ}$ C and varied the relative air content between 0 and 20% to derive creep velocities using Equation (3) (Figure 12). The magnitudes of the calculated creep

Table 2 Accuracy of the determined parameters as result of an analytical covariance error propagation using the two-phase model and the representative 1D model ( $\sigma$  = standard deviation).

Parameter	$h_{till}$	$h_{icePf}$	$h_{rock}$	$d_{debris} \\$	$d_{till}$	$d_{icePf}$	i	V <sub>ice</sub>
	Depth (m) to the:			Density (g/cm <sup>3</sup> ) of the:			Relative ice	Total ice
	Till layer	Ice-rich permafrost	Bedrock	Debris layer	Till layer	Ice-rich permafrost	content	content (m <sup>3</sup> )
Abs. value σ σ (%)	30.1 4.2 14	4.6 1.4 30	35.2 4.2 12	1.92 0.033 2	1.95 0.087 4	1.79 0.115 6	0.577 0.057 10	14.7 1.8 14

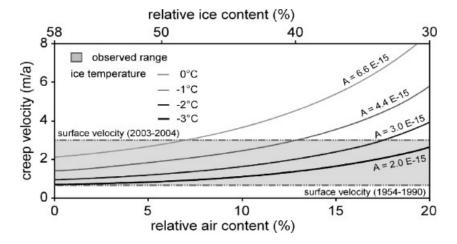


Figure 12 Effect of increasing relative air content on the creep velocity when the ice temperature varies between 0 and  $-3^{\circ}$ C. The grey area displays the range of observed surface velocities for the lower part of the rock glacier.

velocities agree with the observed surface velocities, which point towards the validity of the geophysical model. Our model was also tested with various values for the parameters A, n, i and a (Wagner, 1992; Paterson, 1994; Ikeda et al., 2003; Arenson and Springman, 2005a). In particular, a 55% ice content (Arenson and Springman, 2005a) gives a creep velocity of 2.96 m/a, very similar to that observed in the field.

The geophysical model was also tested to see if it can qualitatively explain the increase in surface velocities observed during the last decade. A surface velocity of  $\sim$ 0.6 m/a between 1954 and 1990 can be explained by an ice temperature of  $\sim -3^{\circ}$ C and almost zero air content. Surface velocities of  $\sim$ 3 m/a between 2003-04 can be produced by increasing internal deformation caused by a combination of higher ice temperature  $(0 > T^{\circ}C > -1)$  and air content (5 < a < 15%). However, because of the lack of information on the relative air content, ice temperature and their recent variations, the possible onset of sliding processes to explain the increase in surface velocity cannot be excluded. Internal sliding can occur between ice and debris, or in a distinct shear zone ('soft layer') within the permafrost body (Arenson et al., 2002; Kääb et al., 2006). At the Reichenkar rock glacier, basal sliding could take place along the water-saturated, fine-grained till layer which lies between the ice-rich permafrost and the bedrock.

It should be noted that a homogenous ice/debris concentration in vertical direction was used in the creep model. There are other possible ice/debris concentrations. For example, if most of the ice is

concentrated in the upper part of the permafrost layer, the creep model (at temperatures of  $0^{\circ}$ C) produces creep velocities that are too low.

## **CONCLUSIONS**

A combination of GPR, seismic refraction and gravimetry was successful in revealing the internal structure and the ice content of the Reichenkar rock glacier, and resulted in a four-layer model. The debris layer has an average thickness of 4.6 m and is underlain by ice-rich permafrost. At the base of the ice-rich permafrost, a prominent reflector was detected by GPR and interpreted as an unfrozen till layer. This reflector is located a few metres above the surface of the bedrock, which was clearly detected by seismic refraction. The average thickness of the ice-rich permafrost is about 25 m. The geophysical data indicate that the ice-rich permafrost is composed of about 60% ice in the case of a two-phase model (rock and ice) or by about 45% ice, if a relative air content of 10% is assumed (three-phase model).

The strengths of the geophysical investigations are:

- The combination of GPR and seismic refraction data allows the complex structure of the rock glacier (e.g. seismic low velocity zone on top of the bedrock) to be resolved.
- The cost-effective 3D refraction seismic layout can be applied to determine the bedrock surface.
- The density of the ice-rich permafrost is determined as a unique parameter (there is only one free parameter in the gravimetric model).

 The rheological model can be used as benchmark for the geophysical model.

A sensitivity analysis of the total ice content shows its dependency on the following parameters:  $d_{rockPf}$ ,  $del_{icePf}$ ,  $v_{till,gpr}$ ,  $t_{gpr}$  (Appendix C). The estimated parameter  $v_{till}$  does not greatly affect the computed ice content.

Based on the structural model and the ice content we calculated the creep velocity of the rock glacier by an extension of Glen's flow law to a homogenous three-phase model (rock, ice and air) for the ice-rich permafrost. An additional shear stress caused by the load of the debris layer was also taken into account. Calculated creep velocities were in agreement with observed values within the range of uncertainty of parameters of the flow law. Plausible explanations for the recent acceleration of surface velocities are compatible with the geophysical model.

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# APPENDIX A: LIST OF SYMBOLS

a	relative	air	content	of	the	ice-rich
	permafro	ost				

d<sub>air0</sub> density of air

d<sub>debris</sub> density of the debris layer

dice0 density of ice fractionally below zero

celsius

 $d_{icePf}$  density of the ice-rich permafrost (density

model)

d<sub>till</sub> density of the till layer

d<sub>rockPf</sub> density of rock samples from the debris

layer

del<sub>bedrock</sub> computed delay time of the bedrock

surface

del<sub>icePf</sub> computed delay time of the ice-rich

permafrost

dh<sub>debris</sub> thickness of the debris layer

dh<sub>icePf</sub> thickness of the ice-rich permafrost

 $h_{icePf}$  depth to the ice-rich permafrost

$h_{till}$	depth to unfrozen sediments (top of the					
	till layer)					
$h_{rock}$	depth to the bedrock surface					
i	relative ice content of the ice-rich					
	permafrost					
$t_{\rm gpr}$	two way travel time of the till layer					
V <sub>bedrock</sub>	P-wave velocity of the ice-rich perma-					
	frost					
V <sub>debris</sub>	P-wave velocity of the debris layer					
$V_{icePf}$	P-wave velocity of the ice-rich perma-					
	frost					
$v_{till}$	estimated P-wave velocity for the till layer					
V <sub>till,gpr</sub>	mean wave velocity of the till layer					
Vice	calculated total ice volume for the					
	representative 1D model $\approx$					
	f (t <sub>gpr</sub> , V <sub>till,gpr</sub> , V <sub>till</sub> , V <sub>icePf</sub> , V <sub>debris</sub> , V <sub>bedrock</sub> ,					
	del <sub>bedrock</sub> , del <sub>icePf</sub> , d <sub>icePf</sub> , d <sub>rockPf</sub> )					

# APPENDIX B: THE PRINCIPLE OF DELAY TIME MODELLING

A travel time t of a wave passing a refractor can be decomposed according to:

$$t(aoff) = td_{SRC} + \frac{aoff}{v_p} + td_{RCV}$$
 (B1)

with travel time of a refracted wave refractor (here: bedrock) delay times  $td_{SRC}$ ,  $td_{RCV}$ at source and receiver locations absolute offset (distance between aoff source and receiver) refractor velocity at the CMP-location

 $v_P$ 

2D and 3D refraction surveys lead to equation systems based on (B1) which can be solved for laterally variable delay times and refractor velocities. In case of a 1D velocity distribution  $v_P(z)$  above and a constant velocity  $v_P$  below the refractor (bedrock), the delay time td and the depth D of the refractor at an arbitrary location are related by:

$$td = \int_{z=0}^{z=D} \sqrt{\frac{1}{v_P(z)^2} - \frac{1}{v_P^2}} \cdot dz$$
 (B2)

In case of a dipping refractor or lateral variations of  $v_P$  the delay time depends on the azimuth of the ray.

# APPENDIX C: SENSITIVITY ANALYSIS AND ACCURACY

In order to define how accurate an observation must be, the analytical covariance error propagation was applied. The standard deviation of the total ice content of the ice-rich permafrost as well as the sensitivity of all unknown parameters is important. Therefore, we have to declare a functional relation between observations (observables: velocities, delay times, densities) and unknown parameters (depths to each layer, densities, relative and total ice content). We used a two-phase model (rock and ice) and a representative 1D model of the rock glacier (Figure 11). The covariance propagation was developed by C.F. Gauß and is mainly used in geodesy. It is only valid for random and not systematic errors (e.g. model errors). The covariance propagation law (e.g. Mikhail, 1976; Reissmann, 1976) is:

$$\sum_{yy} = A \cdot \sum_{xx} \cdot A^T \tag{C1}$$

It uses the functional matrix A (Jacobi matrix) and the covariance matrix  $\Sigma_{xx}$ . The elements of matrix A contain the first derivative of each observation with respect to each equation and the matrix  $\Sigma_{xx}$  contains all variances ( $\sigma^2$ ) for the observations. As a result, all standard deviations from the unknown parameters are obtained  $(\Sigma_{yy})$ . For the total ice content, a standard deviation of  $\pm 15\%$  can be quoted (Table 2).

To test the sensitivity of the geophysical model we applied (for each test run and observation) the value of the half-standard deviation to the corresponding absolute value. Relative values of 5% and 10% were also applied to each observation or their corresponding standard deviation. Analysis showed that the seismic velocity of the till layer  $(v_{till})$  has a relatively small effect on the final ice volume. However, the parameters  $d_{rockPf}$ ,  $del_{icePf}$ ,  $v_{till,gpr}$ , and  $t_{gpr}$  have significant effects.

Figure 11 shows the simplified internal structure as result of the determined quantities and associated error bars (Table 2). For covariance error propagation, the following standard deviations were used: for  $t_{gpr}$ ,  $v_{till,gpr}$ ,  $v_{till}$   $\pm 10\%$  of their total value. The standard deviations from the parameters del<sub>bedrock</sub>,  $del_{icePf}$ ,  $d_{icePf}$  and  $d_{rockPf}$  could be calculated as 1.56 ms, 1.3 ms, 0.03 mGal and 0.07 g/cm<sup>3</sup> respectively. For the parameters  $v_{icePf}$ ,  $v_{debris}$ , and  $v_{bedrock}$  a standard deviation of  $\pm 70 \,\text{m/s}$  was used. The computation of the bedrock surface relies heavily on the GPR data.

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