Non-unique self-similar turbulent boundary layers in the limit of large Reynolds numbers

Abstract

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As pointed out in the pioneering experimental work of Clauser [1], self-similar or, equivalently, equilibrium turbulent boundary layer flows may be non-unique in a certain range of the controlling parameters: Specifically, Clauser observed that if the imposed (adverse) pressure gradient varies as a power of the streamwise distance in order to maintain equilibrium and if the associated exponent m falls below a certain limit, the boundary layer forming initially rapidly approaches a different state (most probably due to instabilities). The initial flow is seen to resemble a classical turbulent boundary layer, characterized by an essentially inviscid outer part having a small velocity defect and the logarithmic law of the wall. In contrast, the second state exhibits a much larger defect and momentum thickness reflecting a wake-type velocity profile and possibly approaches separation.

Clauser's observation regarding the non-uniqueness of turbulent boundary layer flow formed the basis of theoretical considerations carried out by a number of authors. The essence of these studies is discussed and summarized by the analysis presented in [2]. Basically, all of them exploit the idea of quasi-equilibrium flows including weak variations of m in the streamwise direction, in combination with conclusions gained from a (rather ad-hoc) simplified integral momentum balance. However, by adopting the classical asymptotic concept of small-defect turbulent boundary layers in the limit of large (globally defined) Reynolds numbers *Re*, it is readily shown that self-preserving flows exist for $\mu = m + 1/3 > 0$ only (see [2], for example). Since the approaches available at present are essentially relying on the classical picture of a small-defect wall-bounded turbulent flow, they are inevitably not capable of incorporating the effect of nonlinearity arising from the inertia terms in the Reynolds equations in a rational manner. Thus they predict an unbounded growth of displacement and momentum thickness as $\mu \to 0_+$.

The theory presented here is solely based on a strictly self-consistent treatment of the equations of motion and resolves those shortcomings by considering the distinguished limit $Re \to \infty$, $\mu^{3/2} \log Re = O(1)$.

Starting with the classical two-tiered small-defect form of the flow holding for $\mu = O(1)$, the asymptotically correct treatment of the inertia terms is shown to point to the existence of a turning point in the function $G(\mu, Re)$ as $\mu \to 0_+$ where G denotes the well-known shape factor, [2]. To leading order, for strict equilibrium the associated non-unique flow structure is captured by a simple canonical algebraic relationship between suitably rescaled corresponding quantities $\hat{G}$, $\hat{\mu}$ (solid curve in Fig. 1). Let $\hat{G}^*$, $\hat{\mu}^*$ represent the turning point, (steady) flow is found to exist for $\hat{\mu} > \hat{\mu}^*$ only. Interestingly, a quasi-equilibrium-type change from a state A to a different one B due to a pressure rise is always characterized by an increase of G.

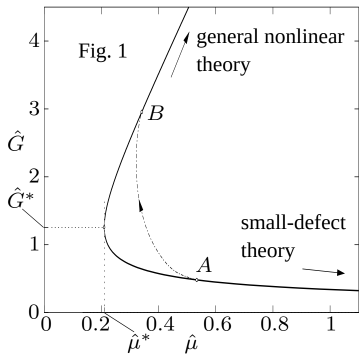

Fig. 1

Finally, an outlook devoted to the aspect of extending the present theory by tracking the upper branch into the fully nonlinear regime with a large velocity deficit will be given. The results obtained so far suggest the possibility for a rational description of flows even on the verge of separation, which emerge in this regime in agreement with Clauser's original observations.

References

[1] CLAUSER, F. H. 1954 Turbulent Boundary Layers in Adverse Pressure Gradients. *J. Aeron. Sci.* **21**, 91–108.
[2] SCHLICHTING, H. & GERSTEN, K. 2000 *Boundary-layer theory*. Springer.