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Combined Heat and Power Technologies
Applied Studies of Options including Micro turbines

Diploma Thesis for Mechanical Engineering

First Research Project Made in Cooperation with the Department of Mechanical Engineering from Universidad Nacional de Colombia

Scholarship of Austrian Exchange Service (Österreichische Austauschdienst, ÖAD)

Under the supervision of
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Chapter 1

Introduction

The project Combined Heat and Power - Application Studies of Options including Microturbines, consist on the development of several models for some cogeneration technologies, as well for trigeneration, whose purpose is the optimisation of the design and the operation of a plant. This optimisation is carried out though the solving of a thermoeconomical model, using a software for nonlinear programming.

For the realisation of the project, it was defined the concept of energy systems and all that is related with it, like the components, inputs and outputs. A survey of cogeneration technologies was made, describing the basis of thermodynamics, the range of size, applications, and some others relevant characteristics. An analysis of the demand has been also included. The determination of the user requirements and the aspects involved with it, are topics particularly studied. Six different cases of demand have been defined, with different energy demand, climate conditions and geographical position.

Thermodynamics of the technologies selected –gas turbine, backpressure steam turbine, micro gas turbine, heat pumps, compression and absorption chillers– was described in detail. On the other hand the economics of these technologies was presented, based on several studies done by scientists and academics specialists in this field. It was also investigated the currently prices and forecasts for fuels used by the technologies and electricity in one specified place; economical engineering concepts were applied. Combining all this, optimisation models were built, known as thermoeconomical models.

Finally, an analysis of the results of the modelling was made, and conclusions and recommendations are presented. With this project it is tried to have an useful guide for the selecting of a cogeneration technology for a specific case of demand. It is also tried to establish a comparison between the energy supply for a specific user producing electricity and heat separately, and with cogeneration technologies. This comparison is raised specially from the economical viewpoint, although the environmental aspect is also taken into account. This work pretends to show the convenience of implementing cogeneration plants in some cases, which has an important impulse currently in many countries, not only because of the economical benefits, but also because of the excellent efficient use of the energy got with these

Chapter 2

Energy Systems

Energy exists in a variety of forms. All human activities involve conversion of energy from one form to another. In fact, life itself depends on energy conversion processes. Some of the processes for transformation between important forms of energy and the names of some associated energy converters are summarized in the Table 1.

From / To	Thermal Energy	Mechanical Energy	Electrical Energy
Chemical Energy	Furnace	Diesel engine	Fuel cell
Thermal Energy	Heat exchanger	Steam turbine	Thermocouple
Mechanical Energy	Refrigerator, Heat pump	Gearbox	Electrical Generator
Nuclear Energy	Fission reactor	Nuclear steam turbine	Nuclear power plant

Table 1. Energy transformation matrix [3]

In this work energy system is defined as a schematic representation of the set of equipments, installations and the respective interconnections, required to supply different energy forms (power, heat and cooling) demanded by specific users, using primary energy sources (fuel and other sources). The energy system has all the time the same set of equipments installations and interconnections, that means bounded by a closed and ideal surface; all other matter that can interact with the system is called the surroundings. Energy conversion is concerned with changes in the energy system and in its interactions with the surroundings. All the forms of energy, such as power, heat and cooling (extraction of heat) are transported across the energy system boundaries to or from the environment. The Figure 1 shows the energy system defined previously.

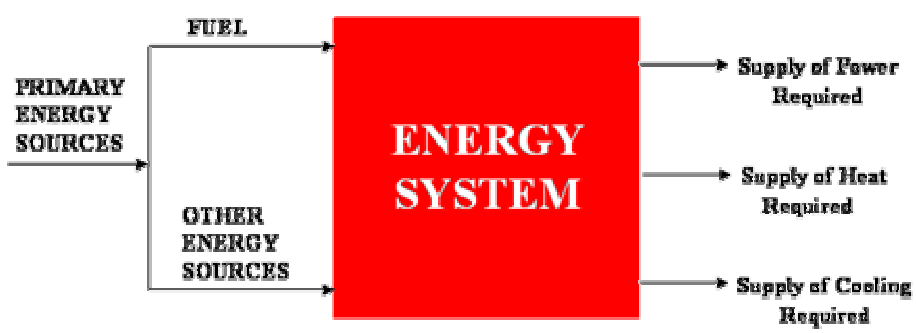


Figure 1. Energy System

2.1 Classification of Energy Systems

The energy systems can be classified in two categories in accord with several topics; the Figure 2 shows the topics that define the classification of energy systems.

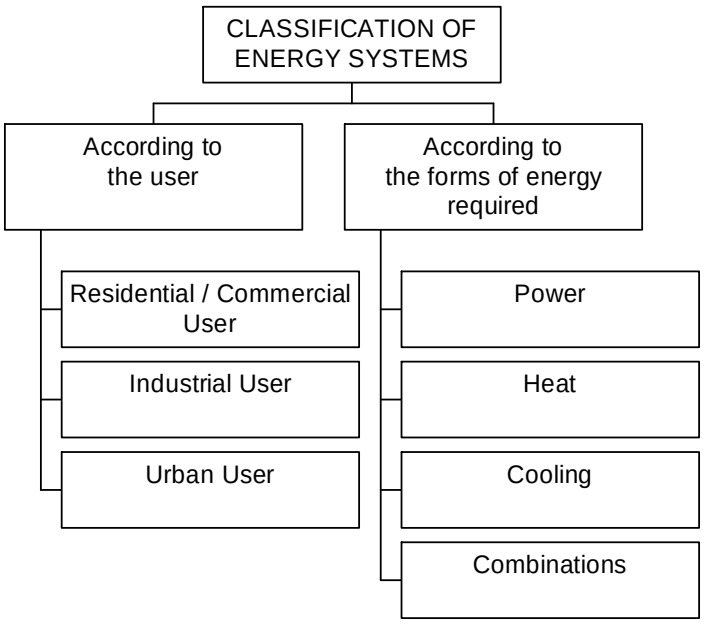


Figure 2. Classification of energy systems

2.1.1 According to the user

Residential / Commercial users are specifically houses, households, buildings, offices, schools, hotels, shopping-centres, hospitals, universities, sport centres, etc. In these cases the heat demand varies significantly within a year, being high in winter and low in summer; the power demand varies in an interval of ±15% of a base load. Industrial users require different forms of energy for technological purposes (production of chemicals, food, paper, etc). These users are characterized by a constant head demand and a variation of the power demand less of ±15% of the base load. Urban users are industrialized community areas consisting of private settlements, buildings for administration and related services as well as production locations. The power and heat demands are the addition of the respective demand of each component.

2.1.2 According to the forms of energy required

An energy system can be designed for only a form of energy, as such power, heat or cooling or as combinations of those. Heat and power is the combination most used worldwide, the most commonly applications are district heating, space heating and hot water for domestic, commercial or industrial use. Heat, power and cooling is a combination used in the industrial sector, when both heat and cooling are required for special processes, as such food industry and meat industry.

2.2 Efficient Energy Systems

In this work, efficient energy systems are based on a synthesis of cogeneration plants, refrigeration systems and auxiliary equipments. The Figure 3 shows the schematic representation of an efficient energy system.

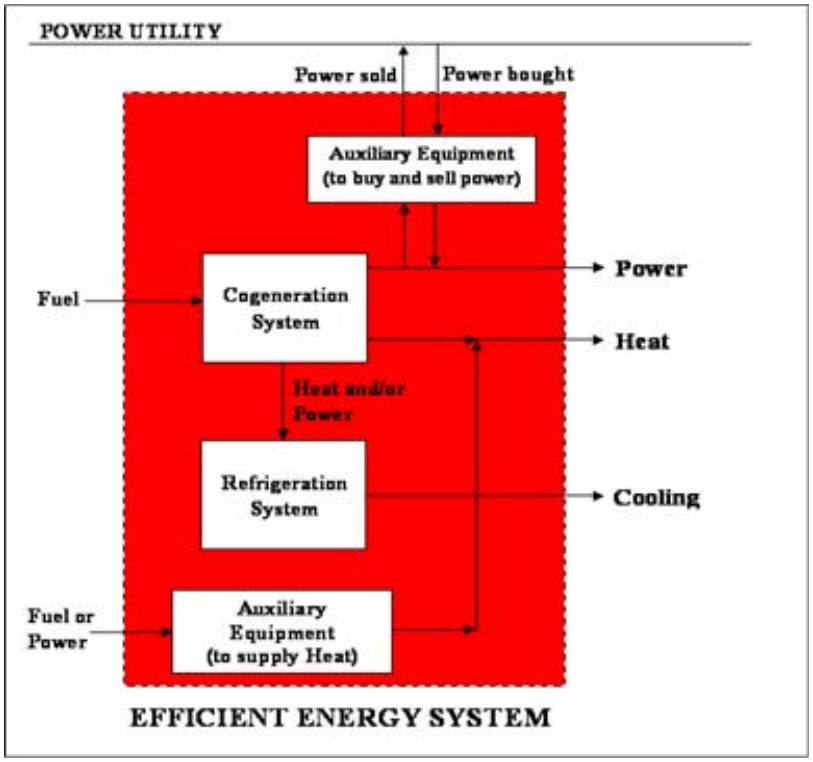


Figure 3. Efficient energy system

The cogeneration system produces two different forms of energy from a primary energy source; normally these two forms of energy are heat and power. Is the principal component of the efficient energy system, it produces the energy demanded for the user and for other equipments, as the refrigeration systems. The refrigeration systems produce cooling and can be classified in two main groups, the electrical refrigeration systems and the absorption refrigeration system; both are explained with details in this chapter.

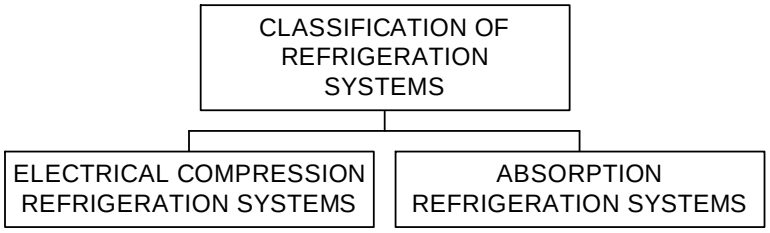


Figure 4. Classification of refrigeration systems

When the refrigeration systems are directly provided by the cogeneration system, the whole cogeneration-refrigeration arrangement is called trigeneration, that means, the generation of three different forms of energy from the primary energy source, namely, heating, cooling and power generation.

The auxiliary equipment of an energy system is basically used to supply an excess of any kind of energy to the user and also to give the option to buy energy when is needed.The principal auxiliary equipments employed in energy systems are defined by the Figure 5.

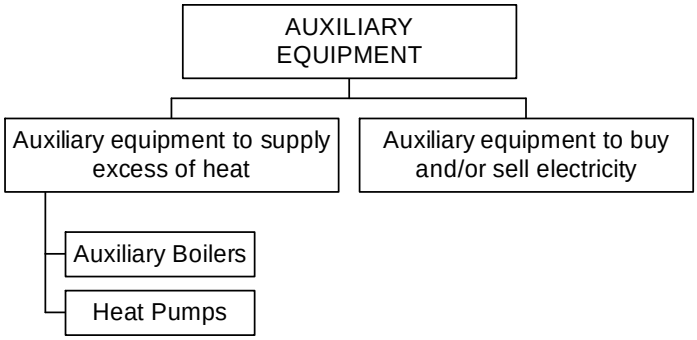


Figure 5. Auxiliary equipment

2.3 Cogeneration

Cogeneration or Combined Heat and Power (CHP) is defined as the thermodynamically sequential production of two or more useful forms of energy from a single primary energy source [1].

The most usual forms of energy are mechanical and thermal energy. Mechanical energy is usually used to drive an electric generator to produce electricity, and the thermal energy produced can be used for heating or for cooling.

2.3.1 Comparison between cogeneration energy systems and conventional energy systems

The conventional power plant, shown in the Figure 6 , receives fuel energy, producing work and rejecting heat to a sink at low temperature. The efficiency of a conventional power plant, that is, the work produced divided by the supplied fuel energy, is commonly between 25%-35%.

During the operation of the conventional power plant, large quantities of heat are rejected in the atmosphere either through the cooling units (steam condensers, cooling towers, water coolers in Diesel or Otto engines, etc) or with the exhaust gases.

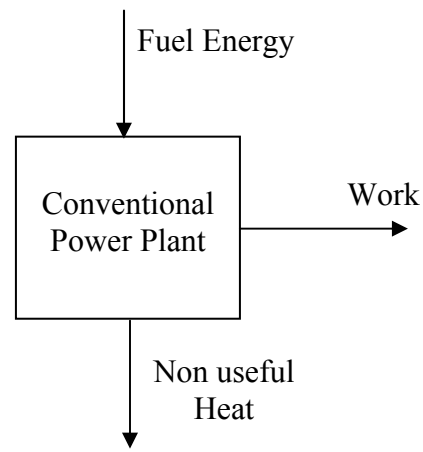
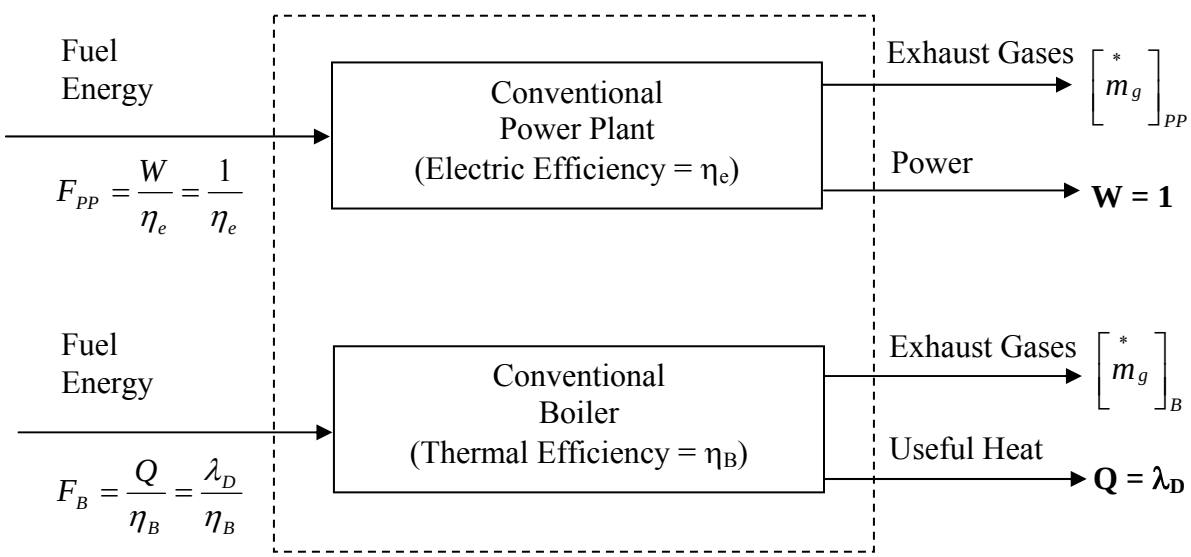


Figure 6. Conventional Power Plant [2]

The conventional energy systems produce power with a conventional power plant and produces useful heat with auxiliary equipments as boilers, that is, separate production of heat and power. Conventional boilers receive also fuel energy and produce heat at the required temperature. The efficiency of a conventional boiler, that is, the heat produced divided by the supplied fuel energy, is commonly between 75%-85%.

A separate production of power and heat by a conventional energy system (conventional power plant and a conventional boiler) is shown in the Figure 7.



Conventional Energy System

Figure 7. Separate production of power and heat by a conventional energy system

In order to illustrate better the performance, the electrical load W is unity. The total fuel energy required in the conventional energy system is therefore:

Fuel Energy = $F_C = F_{PP} + F_B = \frac{1}{\eta_e} + \frac{\lambda_D}{\eta_B} = \frac{\eta_B + \lambda_D \eta_e}{\eta_e \eta_B}$ (1.)

The fuel mass flow required in the conventional energy system is:

Fuel mass flow (Power Plant) = $\left[\overset{*}{m}_F \right]_{PP} = \frac{F_{PP}}{H_U}$ (2.)

Fuel mass flow (Boiler) = $\left[\overset{*}{m}_F \right]_B = \frac{F_B}{H_U}$ (3.)

Total fuel mass flow = $\overset{*}{m}_{FC} = \frac{F_{PP} + F_B}{H_U} = \frac{F_C}{H_U}$ (4.)

The total efficiency of the conventional energy system (usually named energy utilisation factor) is:

Total Efficiency = $\eta_{TC} = \frac{W + Q}{F_C} = \frac{1 + \lambda_D}{F_C} = \frac{(1 + \lambda_D)(\eta_e \eta_B)}{\eta_B + \lambda_D \eta_e}$ (5.)

The carbon dioxide emissions of the conventional energy system are:

Carbon dioxide emissions (Power Plant) = $\mu_{CO_2} \left[\overset{*}{m}_f \right]_{PP} = \frac{\mu_{CO_2} F_{PP}}{H_U}$ (6.)

Carbon dioxide emissions (Boiler) = $\mu_{CO_2} \left[\overset{*}{m}_f \right]_B = \frac{\mu_{CO_2} F_B}{H_U}$ (7.)

Total carbon dioxide emissions = $\mu_{CO_2} \overset{*}{m}_{fC} = \frac{\mu_{CO_2} (F_B + F_{PP})}{H_U} = \frac{\mu_{CO_2} F_C}{H_U}$ (8.)

In conventional energy systems (separate production of heat and power) more than 30% of the fuel energy is being discharged as waste heat. Some of the waste heat is unrecoverable, such as radiation and stack loss, but much of the waste heat can be used for industrial applications of for district heating. [1] [2]

The principal objective of cogeneration is the determination and maximisation of thermal efficiency, that is, the most efficient production of energy (power and thermal energy) from a supply of fuel with chemical energy.

A common cogeneration system has an electric efficiency of 20%-40%, and a total efficiency of 75%-95%. The heat produced for the cogeneration system, depends on an important parameter, the heat to power ratio that is special for each cogeneration technology, normally between 1.5 and 8 [1] [2]. The Figure 8 shows an Energy system based in cogeneration, producing heat and power simultaneously.

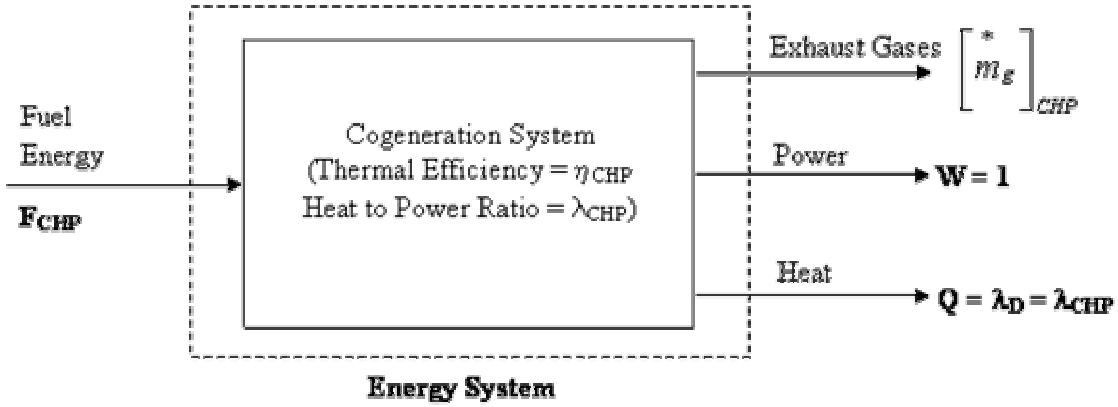


Figure 8. Simultaneous production of heat and power by a cogeneration energy system

The total fuel energy required in the cogeneration energy system is therefore:

$$\text{Fuel Energy} = F_{CHP} = \frac{W}{\eta_{CHP}} = \frac{1}{\eta_{CHP}} \qquad (9.)$$

The fuel mass flow required in the cogeneration energy system is:

$$\text{Total fuel mass flow} = \dot{m}_{f_{CHP}} = \frac{F_{CHP}}{H_U} = \frac{1}{\eta_{CHP} H_U} \qquad (10.)$$

The total efficiency of the conventional energy system is determined by the next equation, where λ_D (ratio heat to electrical demands) is equal to λ_{CHP} (heat to power ratio in the cogeneration system):

$$\text{Total Efficiency} = \eta_{TCHP} = \frac{W+Q}{F_{CHP}} = \frac{1+\lambda_D}{F_{CHP}} = (1+\lambda_D)\eta_{CHP} \qquad (11.)$$

The carbon dioxide emissions of the conventional energy system are:

$$\text{Total carbon dioxide emissions} = \mu_{CO_2} \dot{m}_{fCHP} = \frac{\mu_{CO_2} F_{CHP}}{H_U} = \frac{\mu_{CO_2}}{\eta_{CHP} H_U} \qquad (12.)$$

The summary of the comparison between conventional energy system and cogeneration energy system is presented in the Appendix B, Table 37.

2.3.2 Classification of CHP (combined heat and power) Systems

Cogeneration systems are normally classified according to the sequence of energy use, and according to the technology. Below is shown the Figure 9, which shows the classification of cogeneration systems:

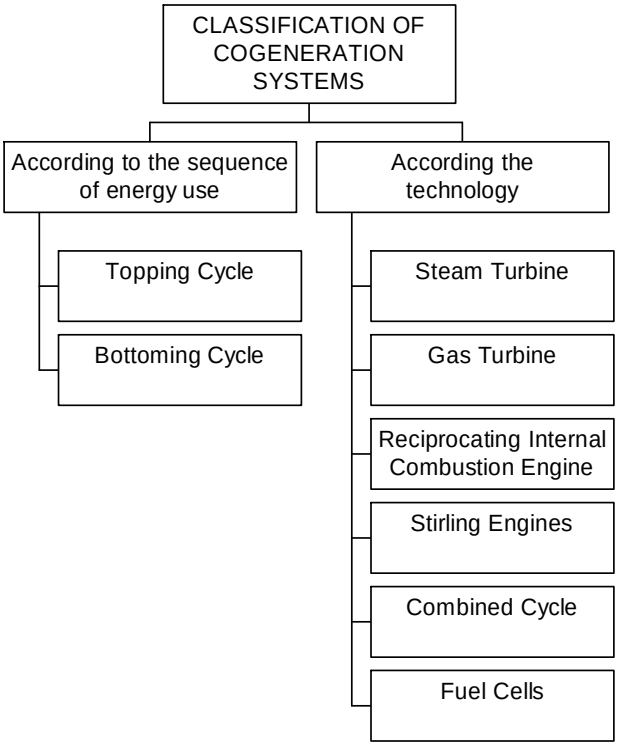


Figure 9. Classification of Cogeneration Systems

2.3.2.1 Classification according to the sequence of energy use

In the topping cycle, the prime mover is used to generate electric power and the waste heat or the by-product steam from it is used for thermal processes or space heating (or cooling). Topping cycles are significantly more fuel-efficient than conventional systems that generate electric and thermal energy separately. [5]

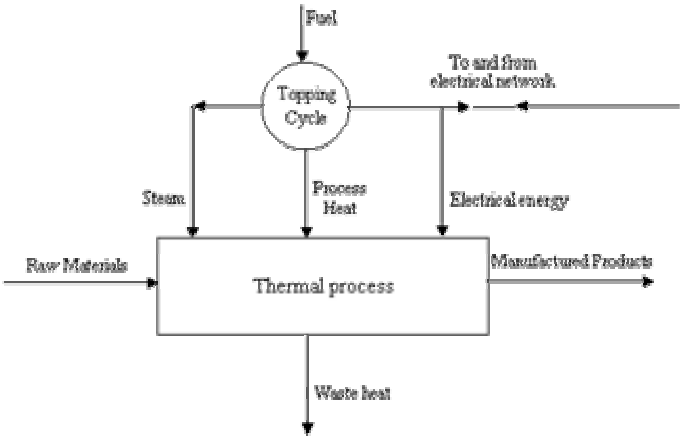


Figure 10. Topping cycle [5]

In bottoming systems, high temperature heat is first produced for a process (e.g. in a furnace of a steel mill or of glass-works, in a cement kiln) and after the process hot gases are used either directly to drive a gas-turbine generator, if their pressure is adequate, or indirectly to produce steam in a heat recovery boiler, which drives a steam-turbine generator. Bottoming cycles also save fuel when compared with conventional systems. [5]

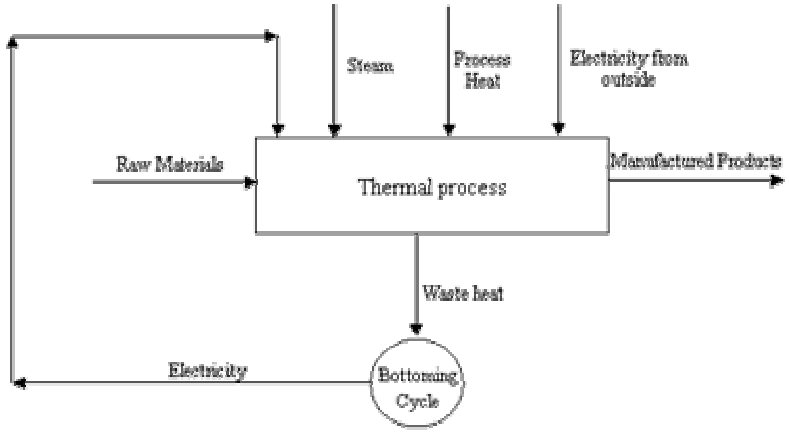


Figure 11. Bottoming cycle [5]

2.3.2.2 Classification in accord with the technology

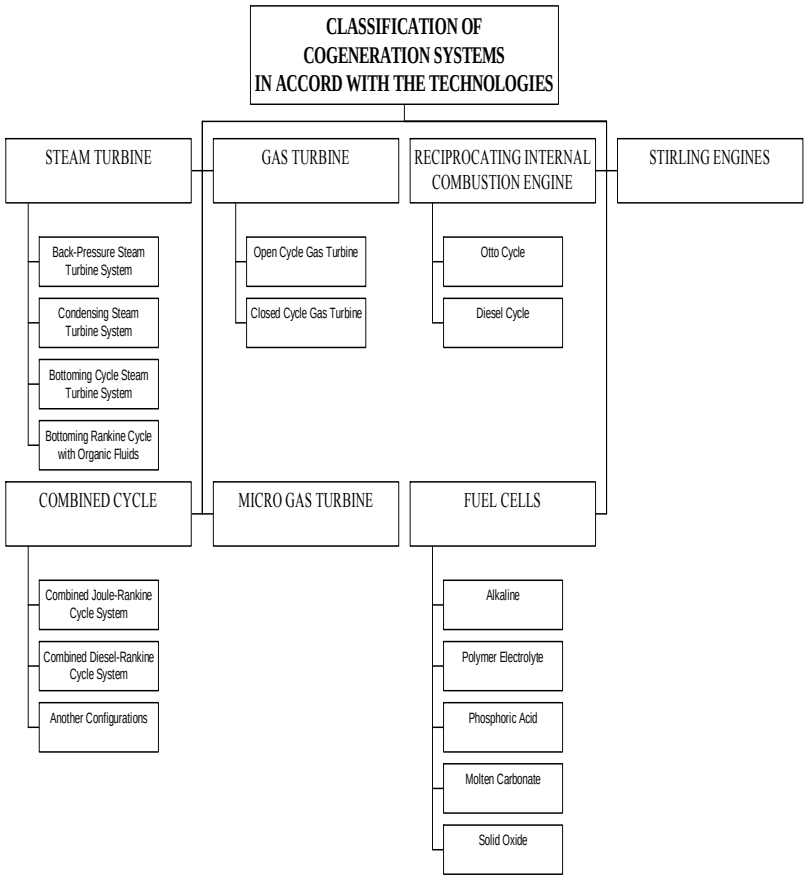


Figure 12. Classification of the cogeneration systems in accord with the technologies [1]

2.3.2.2.1 Steam turbines cogeneration systems

A system based on steam turbine consists of three major components: a heat source, a steam turbine and a heat sink. The system operates on the Rankine cycle, either in its basic form or in its improved versions with steam reheating and regenerative water preheating.

The most common heat source is a boiler, which can burn any type of fuel or certain combinations of fuels, and produces superheated steam. In place of a boiler, nuclear reactors can be used. On the other hand, the system can use renewable energy such as biomass or concentrated solar radiation. The operating conditions can vary in a wide range. For cogeneration applications, steam pressure can range from a few bars to about 100 bars; in the utility sector, higher pressures can also be used. Steam temperature can range from a few degrees of superheat to about 450°C, and, in the utility sector, to about 540°C. The power output is usually in the range of 0.2-100 MW, even though higher power is also possible. [1] [2]

Configurations of Steam Turbine Cogeneration Systems:

- Back pressure steam turbine systems
- Condensing steam turbine systems
- Bottoming cycle steam turbine systems
- Bottoming Rankine cycle systems with organic fluids

Backpressure steam turbine systems

It is the simplest configuration. Steam exits the turbine at a pressure higher or at least equal to the atmospheric pressure, which depends on the needs of the thermal load. This is why the thermal backpressure is used. It is also possible to extract steam from intermediate stages of the steam turbine, at a pressure and temperature appropriate for the thermal load. After the exit from the turbine, the steam is fed to the load, where it releases heat and is condensed. The condensate returns to the system with a flow rate which can be lower than the steam flow rate, if steam mass is used in the process or if there are losses along the piping.

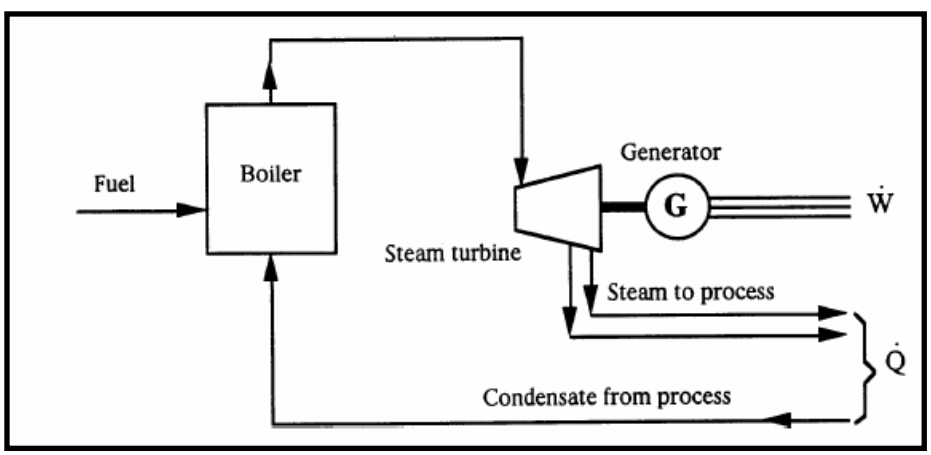


Figure 13. Back pressure steam turbine system [1]

Condensing steam turbine systems

In such a system, steam for the thermal load is obtained by extraction from one or more intermediate stages at the appropriate pressure and temperature. The remaining steam is exhausted to the pressure of the condenser, which can be as low as 0.05 bar with corresponding condensing temperature of about 33°C. It is rather improbable for such a low temperature heat to find a useful application and consequently it is rejected to the environment.

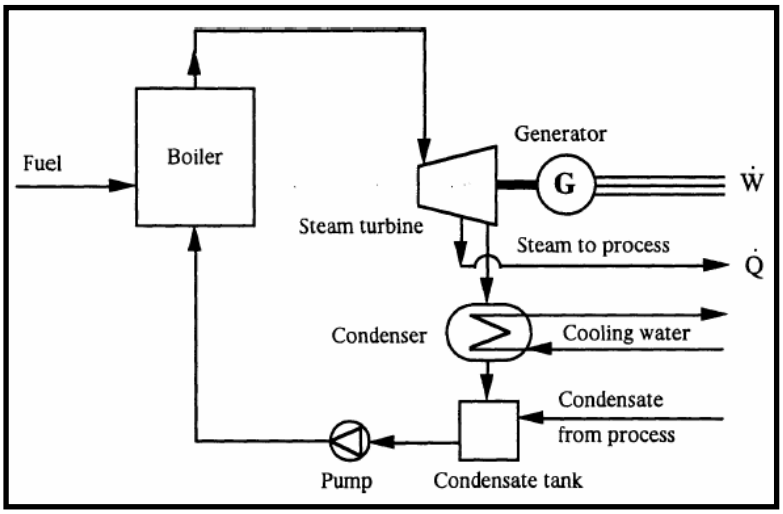


Figure 14. Condensing steam turbine system [1]

Bottoming cycle steam turbine systems

Many industrial processes (steel mills, glass-works, ceramic factories, cement mills, oil refineries) operate with high temperature exhaust gases (1000-1200°C). After the process, the gases are still at high temperature (500-600°C). Instead of releasing them directly into the atmosphere, they can pass through a Heat Recovery Steam Generator (HRSG) producing steam, which then drives a steam turbine.

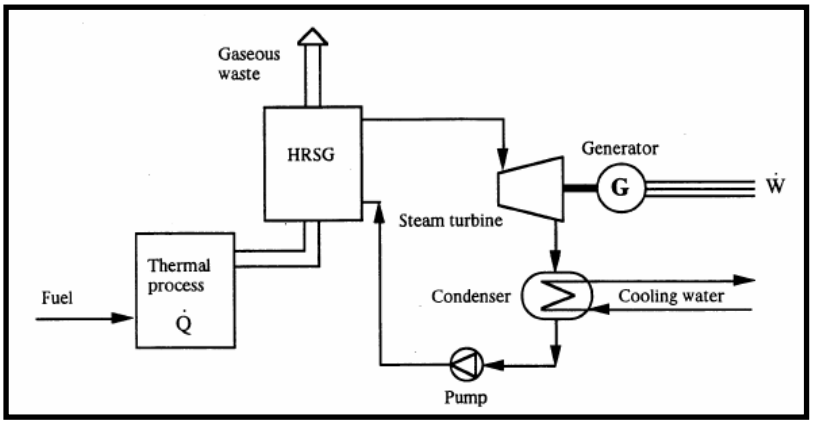


Figure 15. Bottoming cycle steam turbine systems [1]

Bottoming Rankine cycle with organic fluids

These systems are usually referred to as organic Rankine cycles (ORC). In the bottoming cycle, water is the working fluid, which evaporates by heat recovery at high temperature (500°C or higher). However, when heat is available at relatively low temperatures (80-300°C), organic fluids with low evaporation temperatures can be used, such as toluene, improving the heat recovery and the performance of the system. ORC's can be effective also in geothermal applications, where only low temperature heat is available. In certain cases, the working fluid can be a mixture of two different fluids such as water and ammonia, which increases the cycle efficiency.

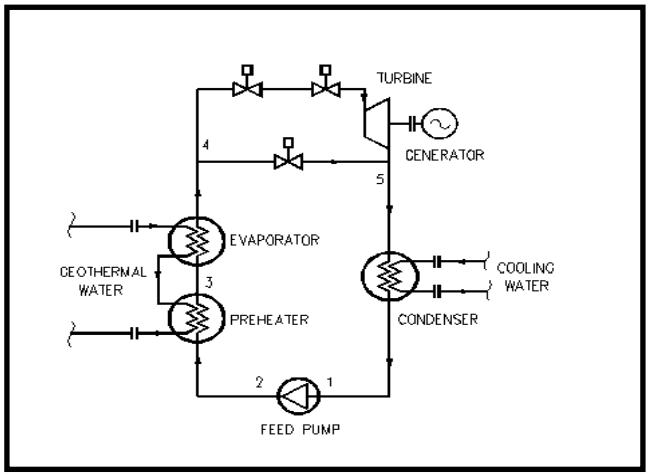


Figure 16. Bottoming Rankine cycle with organic fluids [1]

2.3.2.2.2 Gas turbines cogeneration systems

The thermodynamic cycle associated with gas turbine systems is the Brayton cycle; the thermodynamic steps include compression of atmospheric air, introduction and ignition of fuel, and expansion of the heated combustion gases through the gas producing and power turbines. The developed power is used to drive the compressor and the electric generator. There are two kinds of gas turbines (accord with the design) Aero derivative gas turbines and Industrial gas turbines.

Aero derivative gas turbines for stationary power are adapted from their jet engine counterpart. These turbines are lightweight and thermally efficient, however, are limited in capacity. The largest aero derivatives are approximately 40 MW, with compression ratios up to 30:1 and simple cycle efficiencies of 45% (approx). Industrial gas turbines are more rugged (1 MW to 250 MW). They can operate longer between overhauls, and are more suited for continuous base load operation. However, they are less efficient and much heavier than the aero derivative. The compression ratios are up to 16:1 and often not require an external compressor. The efficiency is 40% (approx.). [1] [2]

Configurations of Gas Turbine Cogeneration Systems:

- Open Cycle Gas Turbine Cogeneration Systems
- Closed Cycle Gas Turbine Cogeneration Systems

Open cycle gas turbine cogeneration systems

A compressor takes in air from the atmosphere and derives it at increased pressure to the combustor. The air temperature is also increased due to compression. The compressed air is delivered through a diffuser to a constant-pressure combustion chamber, where fuel is injected and burned. The diffuser reduces the air velocity to values acceptable in the combustor. Combustion takes place with high excess air. The highest temperature of the cycle appears at this point; the higher this temperature is, the higher the cycle efficiency is. The high pressure and temperature exhaust gases enter the gas turbine producing mechanical work to drive the compressor and the load.

The exhaust gases leave the turbine at a considerable temperature (450-600°C), which makes high-temperature heat recovery ideal. This is effectuated by a heat recovery boiler of single-pressure or double-pressure, for more efficient recovery of heat. The steam produced can have high quality, which makes it appropriate not only for thermal processes but also for driving a steam turbine thus producing additional power.

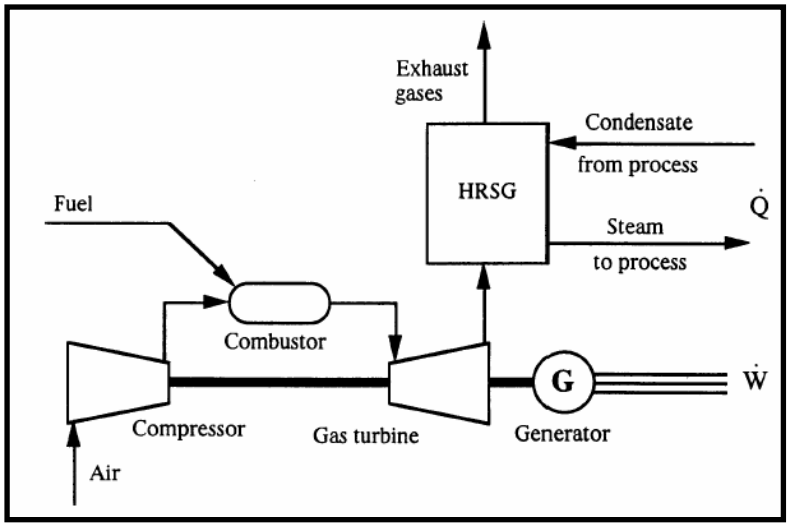


Figure 17. Open cycle gas turbine cogeneration system [1]

Closed cycle gas turbine cogeneration systems

In the closed-cycle system, the working fluid (usually helium or air) circulates in a closed circuit. It is heated in a heat exchanger before entering the turbine, and it is cooled down after the exit of the turbine releasing useful heat. Thus, the working fluid remains clean and it does not cause corrosion or erosion. Source of heat can be the external combustion of any fuel, even city or industrial wastes. Also, nuclear energy or solar energy can be used.
After accumulation of experience, the reliability of close-cycle systems is expected to be at least equal to that of open-cycle systems, while the availability is expected to be higher thanks to the clean working fluid.

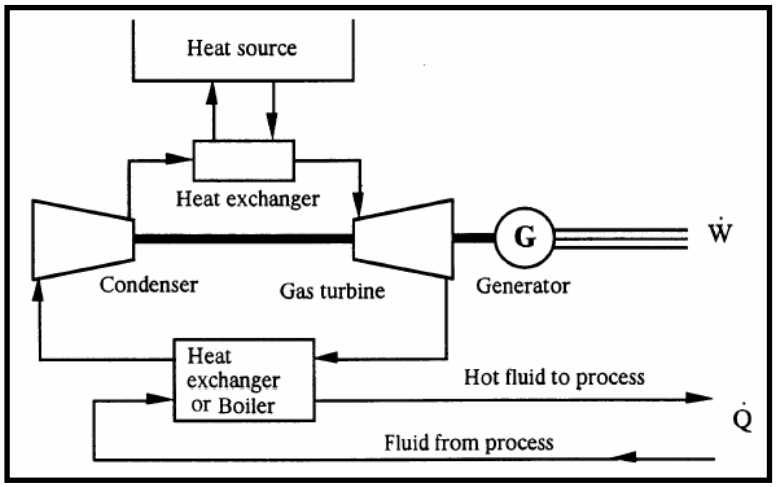


Figure 18. Closed cycle gas turbine cogeneration system [1]

Micro gas turbines cogeneration systems

Micro Gas Turbines are being successfully used in industry for on-site power generation and as mechanical drivers. Turbine sizes are typically between 0.025-0.5 MW for these applications. Small gas turbines drive compressors along natural gas pipelines for cross-country transport. In the petroleum industry they drive gas compressors to maintain well pressures. In the steel industry they drive air compressors used for blast furnaces. [1]

The operating theory of the micro turbine is similar to the gas turbine, except that most designs incorporate a recuperator to recover part of the exhaust heat for preheating the combustion air. Air is drawn through a compressor section, mixed with fuel and ignited to power the turbine section and the generator. The high frequency power that is generated is converted to grid compatible 60HZ through power conditioning electronics. For single shaft machines, a standard induction or synchronous generator can be used without any power conditioning electronics.

2.3.2.2.3 Reciprocating internal combustion engines cogeneration systems

Reciprocating internal combustion engines have high efficiencies, even in small sizes. They are the first choice, up to now, for cogeneration applications in the institutional, commercial and residential sector, as well as in the industrial sector when low to medium voltage is required.

The classification of these systems is based on the internal combustion engine cycle: Otto cycle and Diesel cycle. In an Otto engine, a mixture of air and fuel is compressed in each cylinder and an externally supplied spark causes the ignition. In a Diesel engine, only air is compressed in the cylinder and the fuel, which is injected in the cylinder towards the end of the compression stroke, ignites spontaneously due to the high temperature of the compressed air. Otto engines can operate on a broad range of fuels such as gasoline, natural gas, propane and landfill methane (They are often called gas engines if they use gaseous fuel). Diesel engines operate on higher pressure and temperature levels, and for this reason heavier fuels are used: Diesel oil, fuel oil and, in large two-stroke engines, residual fuel oil.

The principal characteristics of the both engine types in cogeneration, Diesel systems and gas engines are shown in the Table 38 (Appendix B). [1]

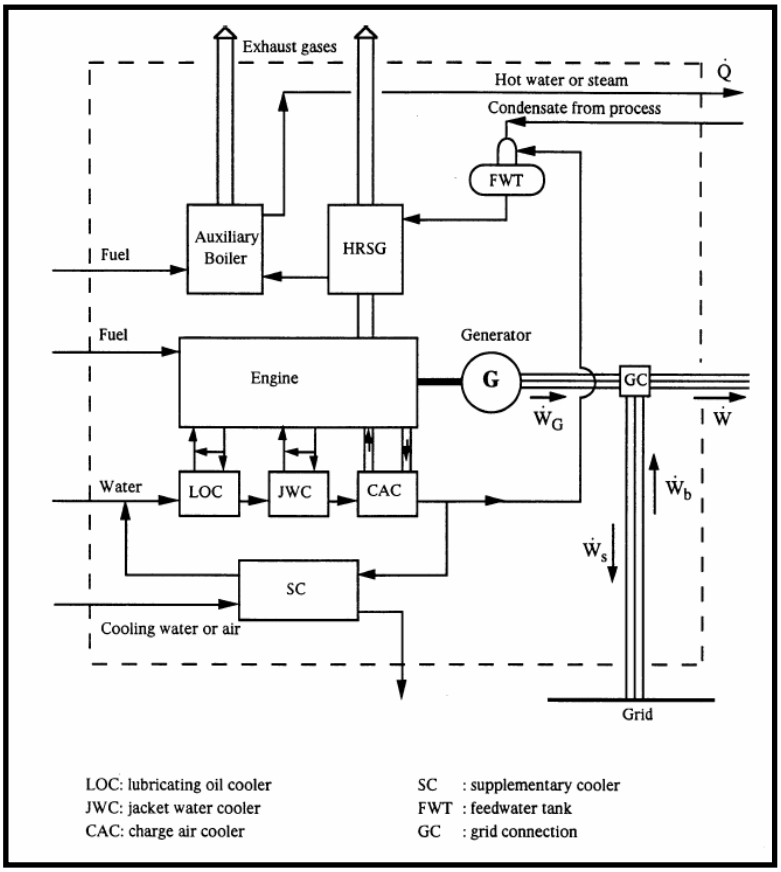


Figure 19. Reciprocating internal combustion engines cogeneration systems [1]

2.3.2.2.4 Stirling engines cogeneration systems

This technology is not fully developed yet and there is no wide-spread application, but there is an increasing interest because of certain advantages: prospect of high efficiency, good performance at partial load, fuel flexibility, low emission level, low vibration and noise level.

In the ideal Stirling cycle (which is reversible) the positions of the pistons are shown at the four extreme state points of the cycle as seen in the pressure-volume and temperature-entropy diagrams. Process 1-2 is an isothermal compression process; during which the heat is removed from the engine at the cold sink temperature. Process 3-4 is an isothermal expansion process, during which heat is added to the engine at the hot source temperature. Processes 2-3 and 4-1 are constant-volume displacement processes, in which the working gas (usually air, helium or air) is passed through the regenerator. During the process 4-1 the gas gives its heat up to the regenerator matrix, to be recovered subsequently during the process 2-3. The regenerator substantially improves the efficiency of the cycle. It comprises a matrix of fine wires, porous metal, or sometimes simply the metal wall surfaces enclosing an annular gap.

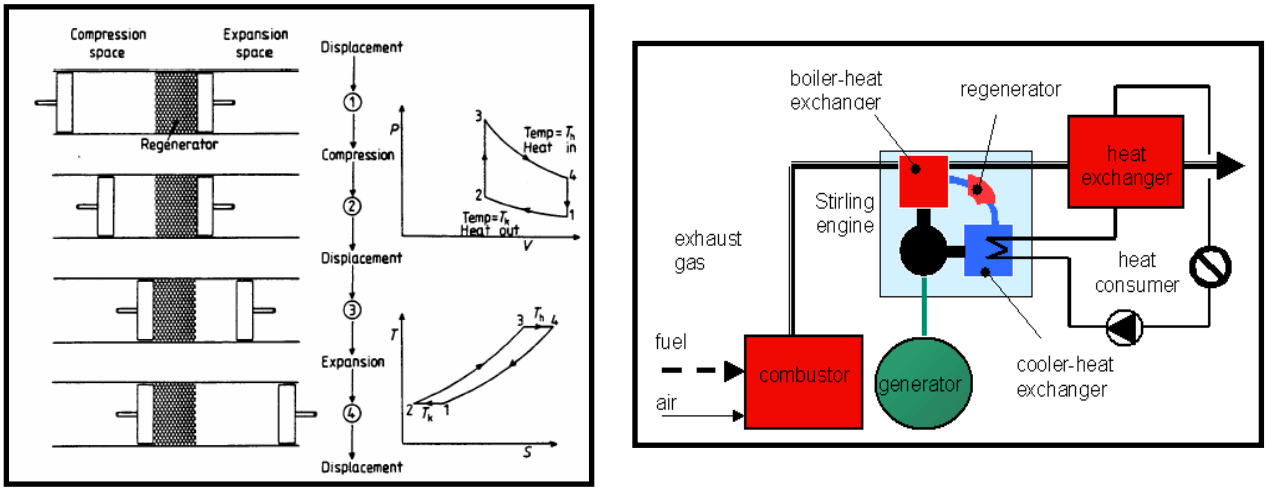


Figure 20. Stirling engine cycle [1]

2.3.2.2.5 Combined cycle cogeneration systems

The term 'combined cycle' is used for systems consisting of two thermodynamic cycles, which are connected with a working fluid and operate at different temperature levels. The high temperature cycle (topping cycle) rejects heat, which is recovered and used by the low temperature cycle (bottoming cycle) to produce additional electrical (or mechanical) energy, thus increasing the electrical efficiency. The most used configurations of combined cycle cogeneration systems are presented next.

Combined Joule–Rankine cycle systems

The most widely used combined cycle systems are those of gas turbine-steam turbine (combined Joule-Rankine cycle) as is shown in Figure 21. The steam turbine is a backpressure one but condensing turbine is also possible, while extraction can also be used with either the backpressure or the condensing turbine. The power concentration (i.e. power per unit volume) of the combined cycle systems is higher than the one of the simple gas turbine (Joule) or steam turbine (Rankine) cycle. The maximum possible steam temperature with no supplementary firing is by 25-40°C lower than the exhaust gas temperature at the exit of the gas turbine, while the steam pressure can reach 80 bar. If higher temperature and pressure is required, then an exhaust gas boiler with burner(s) is used for firing supplementary fuel.

Usually there is no need of supplementary air, because the exhaust gases contain oxygen at a concentration of 15-16%. With supplementary firing, steam temperature can approach 540°C and pressure can exceed 100 bar. Supplementary firing not only increases the capacity of the system but also improves its partial load efficiency. Initially combined cycle systems were constructed with medium and high power output (20-400 MW). During the last years, also smaller systems (4-15 MW) have started being constructed, while there is a tendency to further decrease the power limit.

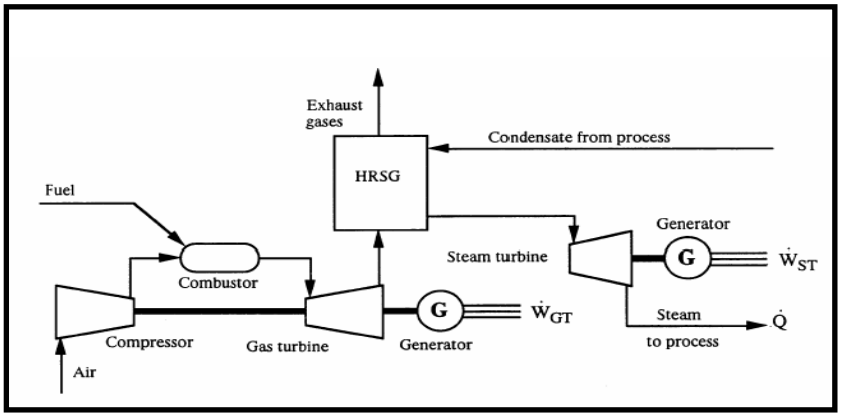


Figure 21. Combined Joule-Rankine cycle cogeneration system [1]

Combined Diesel-Rankine cycle systems

It is also possible to combine Diesel cycle with Rankine cycle. In this configuration is close to the Combined Diesel-Rankine Cycle Systems, but the gas turbine unit (compressor-combustor-gas turbine) is replaced by a Diesel engine. Medium to high power engines may make the addition of the Rankine cycle economically feasible. Supplementary firing in the exhaust gas boiler is also possible. Since the oxygen content in the exhaust gases of a Diesel engine is low, supply of additional air for the combustor in the boiler is necessary.

2.3.2.2.6 Fuel cells cogeneration systems

A fuel cell is an electrochemical device, which converts the chemical energy of fuel into electricity directly, without intermediate stages of combustion and production of mechanical work. Certain types of fuel cells are available, although at high cost. Fuel cells are still considered as an emerging technology and very promising both for electricity generation and for cogeneration. In its basic form a fuel cell operates as follows: hydrogen reacts with oxygen in the presence of an electrolyte and produces water, while at the same time an electrochemical potential is developed, which causes the flow of an electric current in the external circuit (load). At the anode, ions and free electrons are produced. Ions move towards the cathode through the electrolyte. Electrons move towards the cathode through the external circuit, which includes the load (external resistance). The reaction is exothermic. The released heat can be used in thermal processes.

2.4 Trigeneration

Trigeneration is the concept of deriving three different forms of energy from the primary energy source, namely, heating, cooling and power generation. Also referred to as CHCP (combined heating, cooling and power generation), this option allows having greater operational flexibility at sites with demand for energy in the form of heating as well as cooling.

This is particularly relevant in tropical countries where buildings need to be air-conditioned and many industries require process cooling. A typical trigeneration facility consists of a cogeneration plant, and a vapour absorption chiller, which produces cooling by making use of some of the heat recovered from the cogeneration system.

In food industry cogeneration plants are widely introduced. Many industries use cogeneration plants with either gas engines or turbines to cover their steam, hot water and electrical demands. The combination of absorption refrigeration with a cogeneration plant allows using all generated heat for the production of cooling. In the field of air condition and ventilation, the production of cooling out of heat is getting more and more important since the beginning of the 1990's. The reason for this is the boom of block-type thermal power stations and the connection to district heating systems.

The most important advantages of trigeneration are:

- Reduction of primary energy requirements for cooling
- Use of ecologic refrigerants like water and ammonia instead of chlorofluorocarbons (CFC)
- Trigeneration units operate with much less noise and vibration than the conventional alternatives, this has definite benefits in some types of building and other sensitive areas (hospitals, universities, colleges, etc)
- Applications of trigeneration with absorption chillers can be very attractive, because the absorption cooling represents an additional consumer of heat, which improves the global efficiency of an existing cogeneration plant or can improve the viability of a proposed cogeneration plant
- Low maintenance costs and high availability due to very few moving components

2.4.1 Classification of Trigeneration Energy Systems

The Figure 22 shows the components of a trigeneration system, that is, a basic cogeneration system, which supply the heat and the power demand, and a refrigeration system.

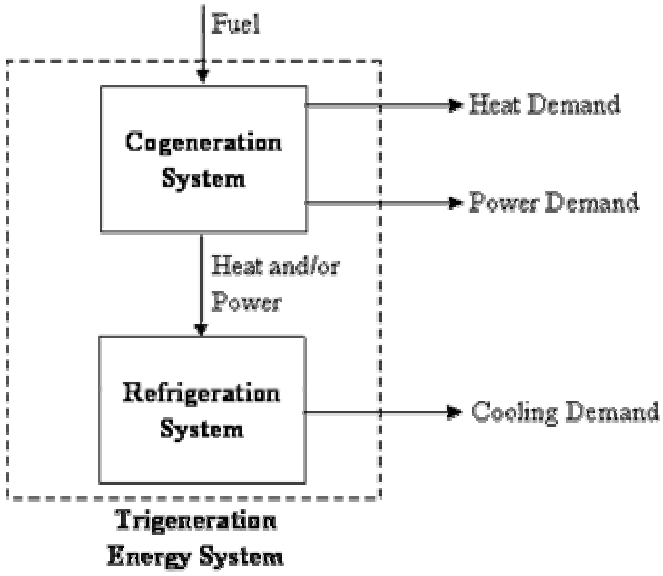


Figure 22. Components of a trigeneration energy system

The trigeneration systems can be classified in accord with the kind of technology used to refrigerate. Principally trigeneration systems are based in a cogeneration system, which use refrigeration equipments to reach the cooling demand.

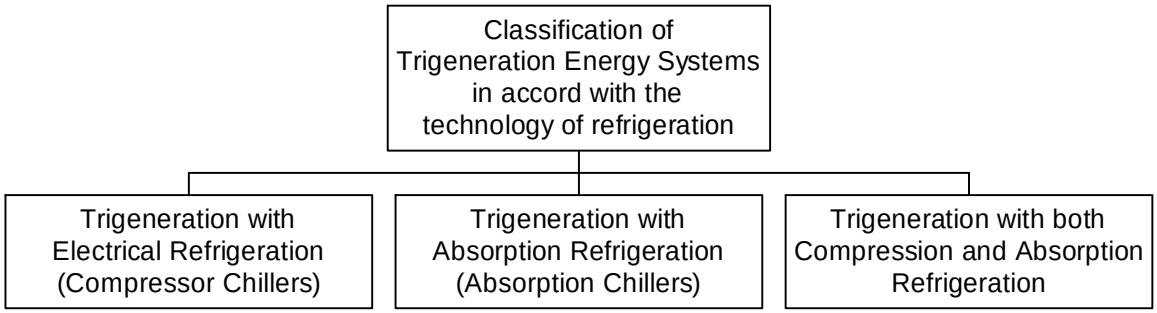


Figure 23. Classification of trigeneration energy systems

2.4.1.1 Trigeneration with electrical refrigeration

These trigeneration systems use typical machines for refrigeration using electric energy, that is, equipments with conventional mechanical vapour compression cycle. In a compression cycle refrigeration system, the refrigerant evaporates at low pressure, producing cooling. It is compressed in a mechanical compressor to a higher pressure, where it condenses. The compressors are either positive displacement or centrifugal.

A positive displacement compressor increases the vapours pressure reducing the refrigerant volume; these compressors can be reciprocating, screw or scroll machines. The centrifugal compressors increase the vapours pressure by conversion of kinetic energy into static energy. The refrigerants used are chlorofluorocarbons (CFC), bromoflourocarbons (BFC), hydrochloroflourocarbons (HCFC), hydroflourocarbons (HFC), but all these chemicals are undesirable for the environment. Other refrigerants as ammonia and azeotropes, like R-500 and R-400 present better environmentally performance.

The electrical refrigeration system can be described with the Coefficient of performance (COP), defined by the equation:

$$(COP)_{Elec.} = \frac{Q_{REFRIGERATION}}{W_{SUPPLIED}} = \frac{Q_{\text{Refri}}}{W_{\text{Refri}}} \qquad (13.)$$

The COP for an electrical refrigeration system is normally between 2.5 and 7, which depends on the size of the equipment.

2.4.1.2 Trigeneration with absorption refrigeration

The absorption refrigeration systems use heat to produce coldness. This heat can be supplied as steam, hot water of by direct firing. An absorption refrigeration cycle works with a binary mixture of two components. The liquid with the lower boiling point operates as refrigerant, the one with the higher boiling point as absorbent solvent. In absorption refrigeration systems occurs a thermal compression. This is possible because the boiling point of the solution is higher than the boiling point of the refrigerant.

The increase of pressure between evaporator and condenser is realized with two heat exchangers, the absorber and the generator. The vaporized refrigerant is desorbed in a liquid or solid absorbent in the absorber, than is pumped into the generator, and finally is rectified from the solution, at a higher temperature level with aid of hot water of steam. The vaporized refrigerant is condensed and passed back to the evaporator through a throttle valve. The separated absorbent returns from the generator into the absorber. The absorption cycle is presented in more detail in the chapter of thermodynamics of energy systems.

The most used fluids in trigeneration with absorption rerigeration are:

- Water (as refrigerant) and lithium bromide (as absorbent)
- Ammonia (as refrigerant) and water (as absorbent)
- Water (as refrigerant) and silica-gel (as absorbent)

Commonly the COP for absorption refrigeration is between 0.68 and 0.74 (Single Effect) and 1 and 1.3 (Double Effect) [12]. The Table 39 shows a summary with the principal characteristics of each absorption system.

2.4.2 Comparison between trigeneration energy systems (with electrical refrigeration) and conventional energy systems

A conventional energy system with refrigeration, shown in the Figure 24, consists of a conventional power plant, an auxiliary boiler and an electrical refrigeration system. The conventional power plant generates electricity to satisfy the power demand and also supply electricity to the electrical refrigeration system; the auxiliary boiler produces the useful heat.
As was mentioned before, the efficiency of a conventional power plant is between 25%-35%, the efficiency of the conventional boiler is between 75%-85%.

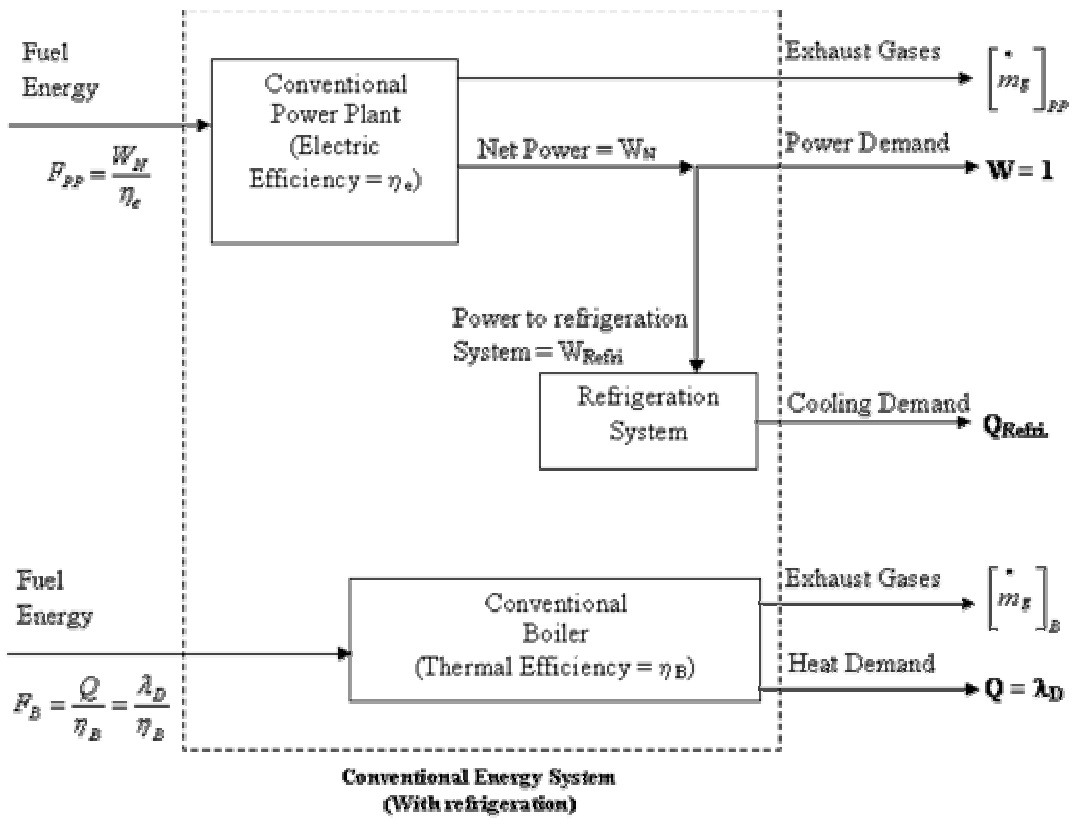


Figure 24. Separate production of power, heating and cooling by a conventional energy system

As for the comparison of cogeneration systems, the electrical demand W is unity. The total fuel energy required in the conventional energy system with refrigeration is:

$$\text{Fuel Energy} = F_C = F_{PP} + F_B = \frac{W_N}{\eta_e} + \frac{\lambda_D}{\eta_B} = \frac{\eta_B W_N + \lambda_D \eta_e}{\eta_e \eta_B} \qquad (14.)$$

The work required for the electrical refrigeration system is:

$$W_{Refri} = \frac{Q_{Refri}}{(COP)_{Elec.}} \qquad (15.)$$

Where $(COP)_{Elec.}$ is the coefficient of performance for a electrical refrigeration system

The net power produced for the conventional power plant is:

$$W_N = W + W_{Refri} = 1 + \frac{Q_{Refri}}{(COP)_{Elec.}} \quad (16.)$$

The fuel mass flow required in this energy system is:

$$\text{Fuel mass flow (Power Plant)} = \left[\overset{*}{m}_f\right]_{PP} = \frac{F_{PP}}{H_U} \quad (17.)$$

$$\text{Fuel mass flow (Boiler)} = \left[\overset{*}{m}_f\right]_{B} = \frac{F_B}{H_U} \quad (18.)$$

$$\text{Total fuel mass flow} = \overset{*}{m}_{fC} = \frac{F_{PP} + F_B}{H_U} = \frac{F_C}{H_U} \quad (19.)$$

The Energy Utilisation Factor (EUF) of the conventional energy system is:

$$(EUF)_c = \frac{W + Q + Q_{Refri}}{F_C} = \frac{1 + \lambda_D + Q_{Refri}}{F_C} \quad (20.)$$

The carbon dioxide emissions of the conventional energy system are:

$$\text{Carbon dioxide emissions (Power Plant)} = \mu_{CO_2}\left[\overset{*}{m}_F\right]_{PP} = \frac{\mu_{CO_2} F_{PP}}{H_U} \quad (21.)$$

$$\text{Carbon dioxide emissions (Boiler)} = \mu_{CO_2}\left[\overset{*}{m}_F\right]_{B} = \frac{\mu_{CO_2} F_B}{H_U} \quad (22.)$$

$$\text{Total carbon dioxide emissions} = \mu_{CO_2}\overset{*}{m}_{FC} = \frac{\mu_{CO_2}(F_B + F_{PP})}{H_U} = \frac{\mu_{CO_2} F_C}{H_U} \quad (23.)$$

The next figure shows an energy system based in trigeneration, producing heating cooling and power simultaneously. This system is basically a cogeneration system using an electric refrigeration unit. As was mentioned, a common cogeneration system has an electric efficiency of 20%-40%, and a total efficiency of 75%-95%, the heat to power ratio is between 1.5 and 8. The COP for an electrical refrigeration system is normally between 2.5 and 7, which depends on the size of the equipment.

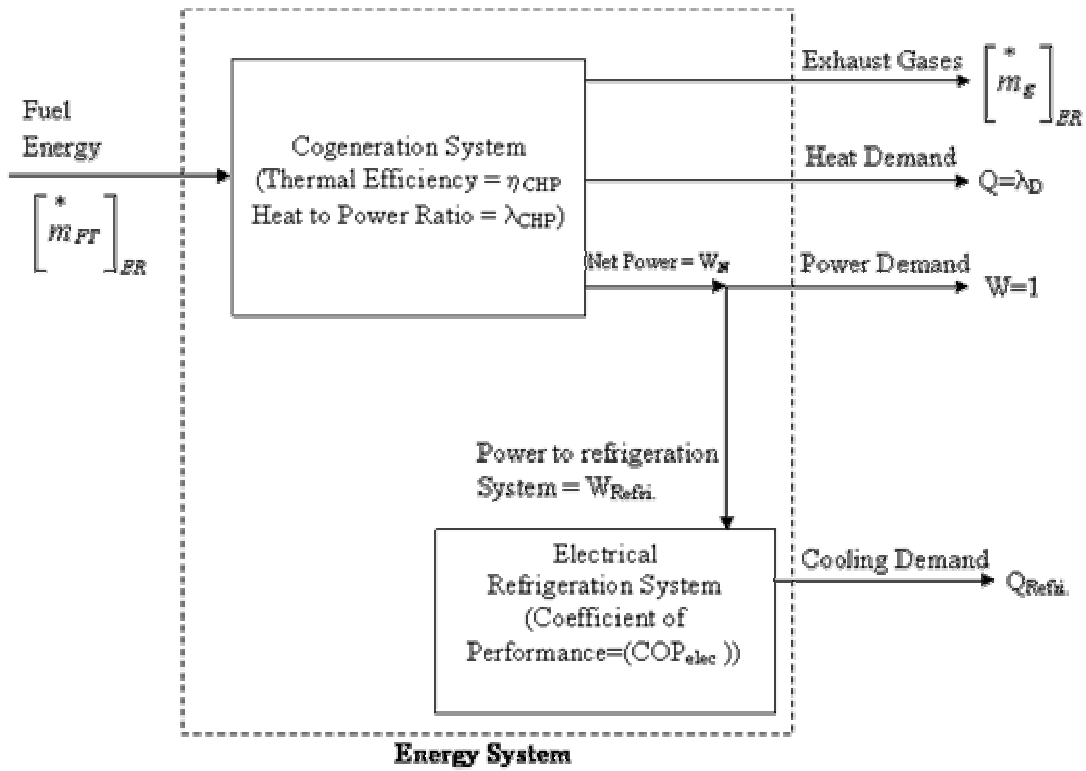


Figure 25. Simultaneous production of heating, cooling and power by a trigeneration energy system (with electrical refrigeration)

The total fuel energy required in the trigeneration energy system with electrical refrigeration is:

$$\text{Fuel Energy} = [F_T]_{ER} = \frac{W_N}{\eta_{CHP}} = \frac{1 + \dfrac{Q_{\text{Refri}}}{(COP)_{Elec.}}}{\eta_{CHP}} \qquad (24.)$$

The work required for the electrical refrigeration system is:

$$W_{\text{Refri.}} = \frac{Q_{\text{Refri}}}{(COP)_{Elec.}} \qquad (25.)$$

The net power produced for the cogeneration system is:

$$W_N = W + W_{Refri.} = 1 + \frac{Q_{Refri.}}{(COP)_{Elec.}} \quad (26.)$$

The heat demand to power demand ratio is defined as follows:

$$\lambda_D = \frac{Q}{W} = Q \quad (27.)$$

The heat to power ratio of the cogeneration system is:

$$[\lambda_{CHP}]_{ER} = \frac{Q}{W_N} = \frac{\lambda_D}{W_N} \quad (28.)$$

The fuel mass flow required in this energy system is:

$$\text{Total mass flow} = \left[\overset{*}{m}_{f_T}\right]_{ER} = \frac{[F_T]_{ER}}{H_U} = \frac{W_N}{\eta_{CHP} H_U} = \frac{1 + \frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP} H_U} \quad (29.)$$

The Energy Utilisation Factor of the trigeneration energy system (with electric refrigeration) is:

$$(EUF)_{ER} = \frac{W + Q + Q_{Refri.}}{[F_T]_{ER}} = \frac{1 + \lambda_D + Q_{Refri.}}{[F_T]_{ER}} = [1 + \lambda_D + Q_{Refri.}] \Bigg/ \left[\frac{1 + \frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP}}\right] \quad (30.)$$

The carbon dioxide emissions of the trigeneration energy system are:

$$\text{Total carbon dioxide emissions} = \mu_{CO_2}\left[\overset{*}{m}_{FT}\right]_{ER} = \frac{\mu_{CO_2}[F_T]_{ER}}{H_U} = \mu_{CO_2}\left[\frac{1 + \frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP} H_U}\right] \quad (31.)$$

2.4.3 Comparison between trigeneration energy systems (with absorption refrigeration) and conventional energy systems

The conventional energy system with electrical refrigeration is shown in the Figure 24. Trigeneration energy systems reach the cooling demand using absorption equipments, like absorption chillers, which do not need electricity to produce cooling. A trigeneration energy system with absorption refrigeration is shown in the Figure 26.

This system consists of a cogeneration system, an auxiliary boiler and an absorption refrigeration unit. The cogeneration unit produces all the power demand and generates useful heat to satisfy the heat demand and also supply heat to the absorption refrigeration system. The absorption refrigeration system is described with the Coefficient of performance (COP), defined by the equation:

$$(COP)_{AC.} = \frac{Q_{REFRIGERATION}}{Q_{SUPPLIED}} = \frac{Q_{Refri}}{Q_{AC}} \qquad (32.)$$

The COP for an absorption refrigeration system is normally between 0.68-0.74 (Single Effect) or 1-1.3 (Double Effect):

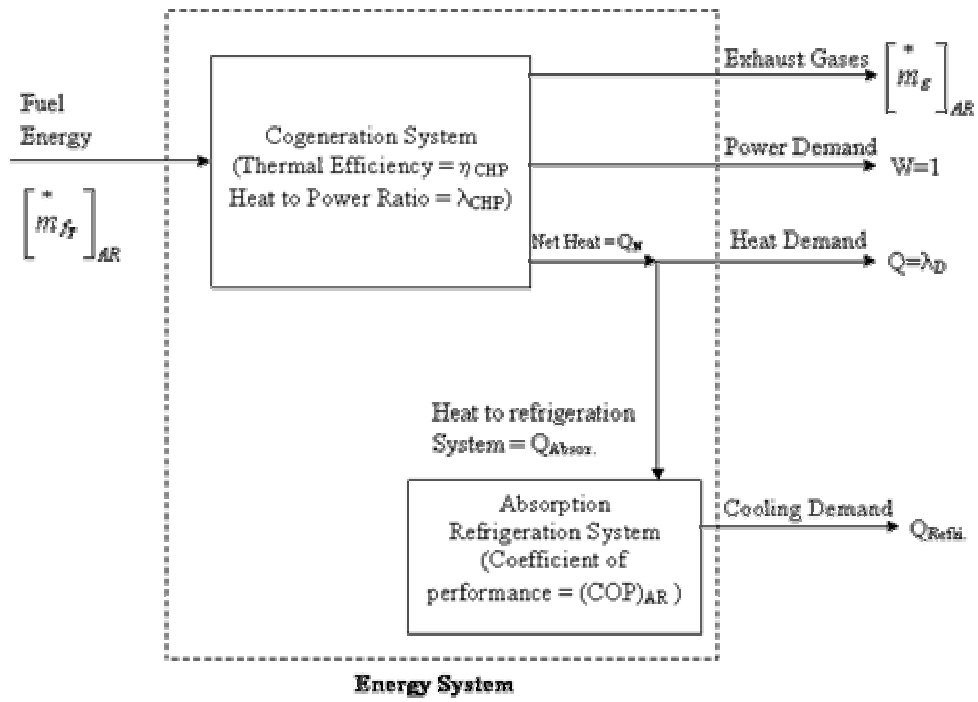


Figure 26. Simultaneous production of heating, cooling and power by a trigeneration energy system (with absorption refrigeration)

The total fuel energy required in the trigeneration energy system with absorption refrigeration is:

$$\text{Fuel Energy} = [F_T]_{AR} = \frac{W}{\eta_{CHP}} = \frac{1}{\eta_{CHP}} \qquad (33.)$$

The heat required for the electrical refrigeration system is:

$$Q_{AC} = \frac{Q_{Refri.}}{(COP)_{AR}} \qquad (34.)$$

The net heat produced for the cogeneration system is:

$$Q_N = Q + Q_{AC.} = \lambda_D + \frac{Q_{Refri.}}{(COP)_{AR}} \quad (35.)$$

The ratio heat demand to power demand is defined so:

$$\lambda_D = \frac{Q}{W} = Q \quad (36.)$$

The heat to power ratio of the cogeneration system is:

$$[\lambda_{CHP}]_{AR} = \frac{Q_N}{W} = \frac{Q + Q_{AC.}}{W} = \frac{\lambda_D + \frac{Q_{Refri.}}{(COP)_{AR}}}{W} \quad (37.)$$

The fuel mass flow required in this energy system is:

$$\text{Total mass flow} = \left[\overset{*}{m}_{f_T}\right]_{AR} = \frac{[F_T]_{AR}}{H_U} = \frac{W}{\eta_{CHP} H_U} = \frac{1}{\eta_{CHP} H_U} \quad (38.)$$

The Energy Utilisation Factor of the trigeneration energy system (with absorption refrigeration) is:

$$(EUF)_{AR} = \frac{W + Q + Q_{Refri.}}{[F_T]_{AR}} = \frac{1 + \lambda_D + Q_{Refri.}}{[F_T]_{AR}} = \eta_{CHP}(1 + \lambda_D + Q_{Refri.}) \quad (39.)$$

The carbon dioxide emissions of the trigeneration energy system are:

$$\text{Total carbon dioxide emissions} = \mu_{CO_2}\left[\overset{*}{m}_{f_T}\right]_{AR} = \frac{\mu_{CO_2}[F_T]_{AR}}{H_U} = \frac{\mu_{CO_2}}{\eta_{CHP} H_U} \quad (40.)$$

The Table 40 (Appendix B) summarizes the comparison of three different systems to produce heating, cooling and power (conventional systems, trigeneration systems).

2.5 Auxiliary Equipment

The auxiliary equipment of an energy system is basically used for supplying excess of any kind of energy to the user and also for giving the option to buy energy when it is required. The most used auxiliary equipments in energy systems are defined by the Figure 27.

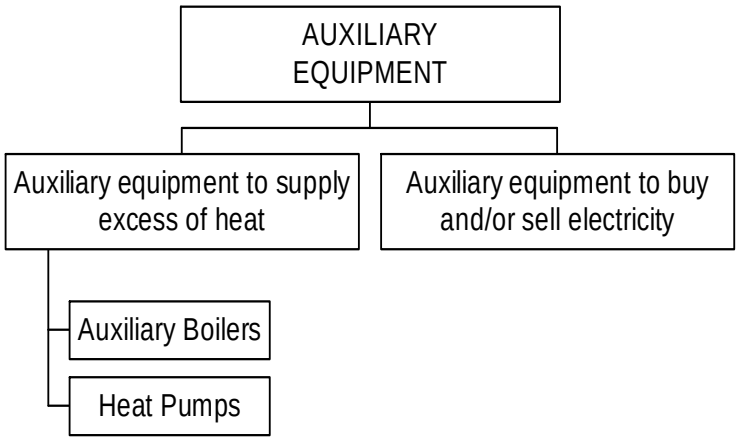


Figure 27. Auxiliary equipment used in energy systems

2.5.1 Electrical equipment

In this section is reviewed the electrical equipment required in an energy system to connect the plant to the electric utility network. This equipment gives to the energy system the option of buy or sells electricity from/to the electric utility network. The main planning factors of this electrical equipment are:

- Generator
- Transformer connections
- Protection systems
- Regulation systems
- Metering arrangements

2.5.1.1 Generator

Electric alternating current (AC) generating sets can be of two types, synchronous generators and induction (asynchronous) generators. The use of any of the above machines poses advantages and disadvantages depending on their operating conditions and attached load along with its corresponding supply system.

There is great disparity between these two types of generators both during normal (steady state) and during abnormal (transient) conditions. These should be carefully considered in choosing either machine. [9]

2.5.1.2 Transformer connections

A transformer is a device that transfers energy from one AC system to another. A transformer can accept energy at one voltage and deliver it at another voltage. This permits electrical energy to be generated at relatively low voltages and transmitted at high voltages and low currents, thus reducing line losses, and to be used at safe voltages.

2.5.1.3 Protection systems

The design of electrical equipments for energy systems is influenced by knowledge of fault conditions; the major use of fault analysis is in the specification of switchgear and protective gear. There are many varieties of automatic protective systems, ranging from simple over-current electromechanical relays to sophisticated electronic systems transmitting high-frequency signals along the power lines. The simplest but extremely effective form of protection is the electromechanical relay, which closes contacts and hence energizes the circuit-breaker opening mechanisms when currents larger than specified pass through the equipment.

2.5.1.4 Regulation systems

Depending on the existing operating conditions the embedded generator may produce reactive power (power factor lagging), absorb reactive (power factor leading) and neither of the above (unity power factor)

The introduction of a energy system with production of power potentially has the capacity to support voltage control. That is, absorb or produce reactive power when the network requires voltage compensation. The above, will be dependent on the generating technology adopted.
In spite of this, the energy system also has the capacity to disrupt voltage control if not adequately operated. For instance, because rural loads are usually scattered the networks are design to make maximum use of voltage variations (long lines with small conductors). Voltage must be maintained within fixed statutory limits, e.g. +10% and .5% for 11/0.45 kV in the UK. This is normally done through tap-changing actions of transformers. However, tap changing at low voltage level is usually carried out by off-load tap changing transformers. In which case, given the situation when the energy system is connected at the remote end of a radial network, depending on the plant size it may be needed to alter the tap position of one or more of the transformers along the feeders. If the energy system trips or is shut down the voltage at the remote end can fall below the statutory limits. [9] [10]

2.5.1.5 Metering arrangements

Two basic metering arrangements are used when excess in-plant power is sold to the utility. These arrangements allow either the sale of excess power only or the sale of all generated power to the utility with simultaneous purchase of all the plant's power needs. The last one is generally known as a *buy all, sell-all arrangement*, see Figure 28 [9]. An excess power sale requires only one tie line connecting the costumer to the utility. Two watt-hour meters are installed in the line by the utility, each ratcheted to operate in opposite directions of power flow (Figure 28).

During normal operation, power generated in-house flows directly to plant processes and is supplemented by utility power, which is recorded on the in-meter. When the plant is generating more power than it needs, usually in the evening and on weekends, power flows into the utility network and is recorded on the out-meter. Both meters are usually time-of-use devices.

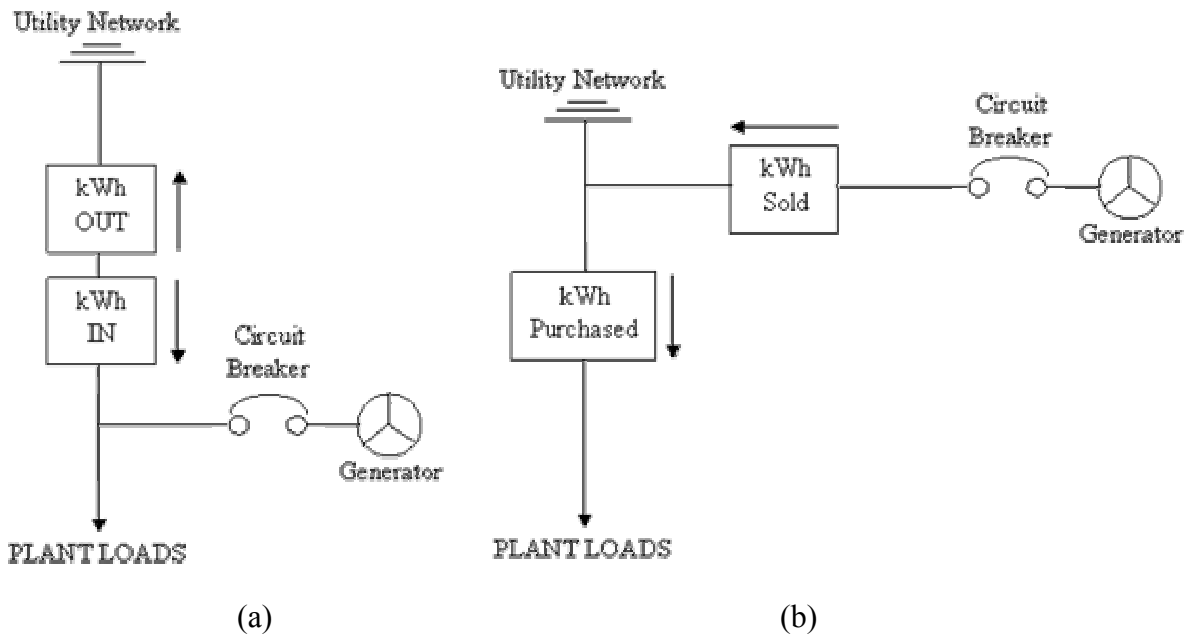


Figure 28. (a) Excess sale metering arrangement with cogeneration (b) Buy-all, sell-all metering arrangement with cogeneration [9]

A buy-all, sell-all metering arrangement is designed to take advantage of state or federal regulations that require the utility to pay at its avoided cost for purchased power. For many utilities, the avoided cost is higher than the standard industrial rates at which they sell power. Thus, it becomes advantageous to connect the in-plant generator directly to the utility network through one meter and to sell all the generated power at the avoided cost. Than plant power is purchased at standard industrial rates through another meter. [9]

2.5.2 Equipment to supply excess of heat

2.5.2.1 Auxiliary Boilers

The boiler is a steam generating system, which transfers the heat from the products of combustion to water and produces hot water or steam. The combustion is accomplished in a furnace. Heat is transferred in the furnace mainly by radiation to water-cooled walls, which constitutes the evaporation section of the steam generation system. After leaving the furnace, the gases pass through a superheater in which steam receives heat from the gases and has its temperature raised above the saturation temperature. Since the temperature or the gases leaving the superheater section is still high, modern steam generators often employ additional heat transfer surfaces to utilize the thermal energy of the gases. These include the surfaces of reheaters, economizers and air-preheaters.

The boilers used in power plants can produce steam at the rate up to several million pounds per hour. The steam pressure may be either supercritical or subcritical and the temperature is frequently around 450° C. Boilers are operated by firing various fuels. These fuels include bituminous coal, lignite, biomass, natural gas and oil. [5]

Boiler efficiency can be expressed in the next form:

$$\eta_B = \frac{\text{Heat Produced}}{\text{Fuel Energy Required}} = \frac{Q}{H_U} \qquad (41.)$$

2.5.2.2 Heat Pumps

A heat pump is a device, which is able to force a heat flow from a cold reservoir, extracting an amount of heat Q_C, to a hot reservoir, delivering a heat amount Q_H, applying a relatively small external work. Since its ability to move heat from one space to another, this device is denominated a "pump". Heat pumps are able to transfer heat from a source -natural or as the result of some domestic or industrial processes- at lower temperatures (called cold reservoir), to enclosed areas, which would be heated, or to be used for processes that require heat for an specific application. The following table summarizes some typical sources of heat and their range of temperatures.

Heat source	Range of temperatures (^oC)
Ambient air	-10/15
Extraction air	15/25
Underground water	4/10
Water from lakes and rivers	0/10
Water from oceans	3/8
Ground	0/5
Subsoil	0/10
Sewage or water from industrial processes	>10

Table 2. Heat sources (cold reservoir) for heat pumps and their range of temperatures [13]

The cycle of the work fluid in a heat pump is subjected to four processes (see Figure 29).

Compression, where the pressure and the temperature are increased; condensation, where the fluid transfers its heat to the environment which is to be heated; expansion, where the pressure of the work fluid (as a liquid) is reduced to the same value that it had before the compression stage; and evaporation, where the heat exchange occurs between the fluid and the heat source.

Heat pumps are characterized for by the coefficient of performance (COP), which is the ratio between the amount of heat delivered to the ambient for heating (Q_H), and the work (W) that is necessary to apply for the process.

$$COP = \frac{Q_H}{W} \qquad (42.)$$

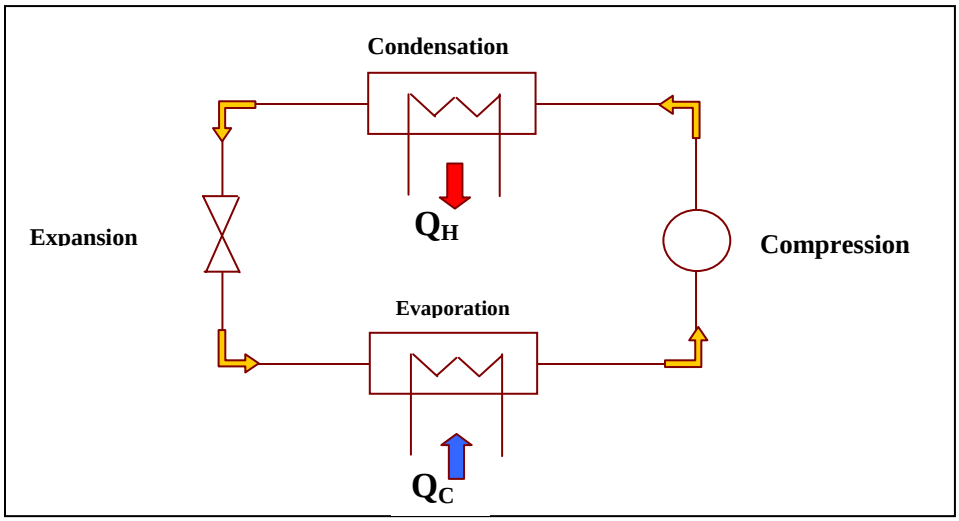


Figure 29. Scheme of the operation of a heat pump

This coefficient has a maximum theoretical value, which correspond to the Carnot cycle. The COP is then expressed as a function of the absolute temperatures of the cold and the hot reservoir, T_C and T_H respectively:

$$COP_{theoretical} = \frac{T_H}{T_H - T_C} \qquad (43.)$$

The practical COP is equal to the theoretical COP increased by a factor α, since the real cycle is not isentropic, that is, the processes are not reversible:

$$\mathrm{COP}_{\mathrm{practical}} = \alpha \frac{\mathrm{T_H}}{\mathrm{T_H} - T_C} \qquad (44.)$$

References

[1] EDUCOGEN. *The European Educational Tool on Cogeneration*. 2001. (http://www.cogen.org/projects/educogen.htm)

[2] Horlock J.H. *Cogeneration: combined heat and power*. Pergamon Press. New York. 1987

[3] Brown C. *World Energy Resources*, International Geohydroscience and Energy Research Institute. Springer. Berlin. 2002

[4] Ristinen R., Kraushaar J. *Energy and the Environment*. John Wiley & Sons, Inc. New York. 1999

[5] Li Kam W., Priddy A. Paul. *Power Plant System Design.* John Wiley & Sons, Inc. New York.1985

[6] Energytech. Austria. http://www.energytech.at

[7] Protermo. Manual for calculating CHP electricity and heat. Suomen Kaukolämpö. January 2000

[8] An introduction to absorption cooling. Good Practice Guide 256. Energy Efficiency Enquiries Bureau. 1999

[9] Elliot Thomas C. *Standard Handbook of Powerplant Engineering.* Mc Graw Hill Publishing Company. 1989

[10] Brithish Electricity International. *Modern Power Station Practice.* Electrical Systems and Equipment. Volume D. Pergamon Press.1992

[11] Aschner F. S. *Planning Fundamentals of Thermal Power Plants.* John Wiley & Sons, Inc. New York.1978

[12] Transformer Book. http://leeh.ee.tut.fi/transformer/

[13] H. L. von Cube, F. Steimle. *Wärmepumpen*. VDI-Verlag GmbH. Düsseldorf.1984

[14] F.A. Holland, F.A. Watson, S. Devotta. Thermodynamic Design Data for Heat Pump Systems. Pergamon Press. 1982

[15] International Energy Agency (IEA). IEA Heat Pump Conference. Graz, Austria. 1984

[16] International Energy Agency (IEA). Selected Issues on CO_2 as Working Fluid in Compression Systems. Trondheim, Norway. 2000

Chapter 3

Economics of energy systems

Economics is a decisive factor –and many times the most important- on the designing of a power plant. The system must not only be adequate from the technical, but also from the economical viewpoint.

The objective is then that the plant be able to satisfy the energy demand, with an optimal combination of investment and operational costs, which gives the lowest total cost. For this reason, it is important to clear some aspects related to the economics, which will be taken into account in next chapters.

3.1 Economic concepts

3.1.1 Costs

Defined in a general way, cost is the price paid or required for acquiring, producing, or maintaining something, usually measured in money, time, or energy [9]. Nevertheless, the costs can be classified by many different forms, for example, in fixed or variable relevant or irrelevant, direct or indirect, historical or predefined, etc. But in this part, it will be specified some kind of costs involved on the thermo-economical modelling of the cogeneration technologies.

3.1.1.1 Fixed costs

Fixed costs are those unaffected by changes in activity level over a feasible range of operations for the capacity or capability available of the plant [1]. Examples of these costs are the insurance costs, taxes, administrative salaries, etc. The fixed costs are also subjected to change because of some external or internal factors, but they tend to remain constant or between a specific range of operation.

3.1.1.2 Variable cost

Variable costs are those that vary with the value adopted by other measures of activity level. That is the case of the fuel costs per period, which depend on the power output of the turbine, both gas and steam. For the modelling of the cogeneration systems, these variable costs are represented by the *operational costs*, because these are associated to the energy demand (heat, power and cooling), which varies along the year.

3.1.1.3 Capital cost

Capital costs are costs for acquiring, developing, or installing capital assets. A capital asset is a tangible asset that has a useful life of greater than one year and that is intended for continuing use over time [10]. For the power system, in these assets are included all the equipments, which make it up: turbines, compressors, pumps, pipelines, etc.

For this reason, it will be adopted the convention, that all the investment costs for the energy system, will be called as capital costs. Here are also included the construction costs (planning and design, land and site preparation, building and machinery foundation, plant equipment, erection and start-up, administrative work, etc). In order to make the modelling, the simulation and the economical comparison of the technologies, it is necessary to annualise these capital costs, and it will be explain next.

3.1.1.4 Unit cost

For the energy systems, unit cost has been defined as the factor that specifies the cost of a unit of energy, expressed in dollars per kilowatt-hour ($/kWh). These costs are specified for the fuel and for the electricity, purchased or sold, which vary according to the period of time of the year.

3.1.2 Inflation

Inflation is a general rise in prices across the economy. This is distinct from a rise in the price of a particular good or service. Individual prices rise and fall all the time in a market economy, reflecting consumer choices and preferences, and changing costs. The inflation is expressed in percentage, and it means a measure of the average change in prices across the economy over a specified period, most commonly 12 months, also called annual rate of inflation. [7].

3.1.3 Interest

Interest is a charge for borrowed money, generally a percentage of the amount borrowed. It can be also the profit in goods or money that is made on invested capital [11]. The ratio between the interest and the borrowed money is called interest rate, and it can be expressed as a percentage, but for the equations, it is written as a decimal. When the interest is calculated, not only on the principal, or the amount originally borrowed, but also on the interest that has accrued, or built up, at the time of the calculation, it is called compound interest.

The interest rate (i) used commercially and designed henceforth simply as interest rate, has three main components: the inflation (i_f), the risk (i_s), and the real interest rate (i_r) [8]. This last one is the money paid for the use of capital, without the adjustment for the inflation rate, that is, it represents the time value change in future real money cash flows based only on the potential real earning power or money [1]. The interest rate is then defined by the following equation:

$$i = (1 + i_f) * (1 + i_s) * (1 + i_r) - 1 \qquad (45.)$$

Nevertheless, making the assumption that the interest rate is free of risk, the equation 45, changes:

$$i = (1 + i_f) * (1 + i_r) - 1 \qquad (46.)$$

3.1.4 Financial Math

The methodical expansion of the interest formulas or conversion factors is known in the literature as financial math. The objective is to try to determinate one between five variables: the interest rate (i), the number of analysed periods (n), Present Value (PV), Future Value (FV) and annuity (A).

3.1.4.1 Conversion from Present to Future Value

The Future value of PV dollars at the year number n, with an interest rate i, is equal to:

$$FV = PV * (1 + i)^n \qquad (47.)$$

Graphically, it can be expressed as follows:

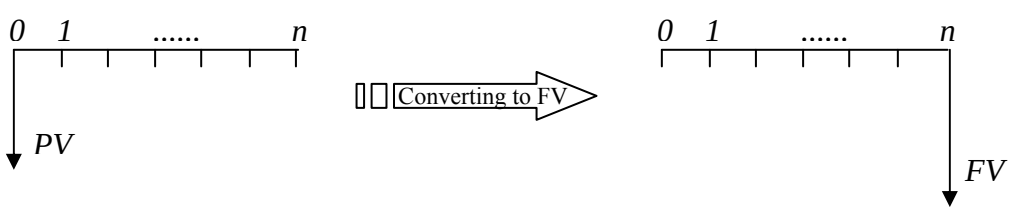


Figure 30. Conversion from present to future value

3.1.4.2 Conversion from Present Value to Annuity

The annual equivalency of PV dollars distributed in n years, with an interest rate i, is,

$$A = PV * a, \qquad (48.)$$

Where a is the annualised factor:

$$a = \left(\frac{i * (1 + i)^n}{(1 + i)^n - 1} \right) \qquad (49.)$$

Graphically, it means:

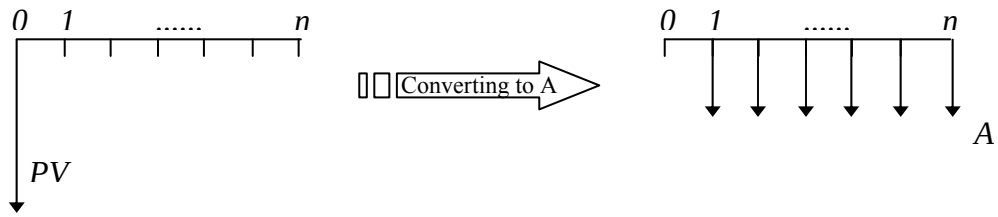


Figure 31. Conversion from present value to annuity

3.1.5 Exchange rates

When a company make investment, it could be also foreign. Typically, these foreign investments are characterised by two or more translation of currencies: (1) when the initial investment is made and (2) when cash flows are returned to the company. Exchange rates between currencies fluctuate, sometimes dramatically. These changes over time are analogous to changes in the general inflation rate.

For the project the currency adopted is the dollar. Any other information obtained in a different currency must be then converted to the dollars making use of the respective exchange rate.

3.2 Economical data for the energy systems analysed

For the modelling of the systems it is necessary to adequate all the economical data obtained. Here are included the cost of equipments, of fuels and electricity.

3.2.1 Updating of the costs

Owing to the information for costs of equipments comes from different sources, published at different years, it is necessary to update these values to a year of reference, which has been defined as the year 2002. Additionally, it is necessary specify a place, a context for the cogeneration projects. The United States has been selected, because the most information used comes from this country.

The methodology used to update lies in increase the costs of the equipments based on the inflation year to year, till the reference year 2002. For this, it will be used the inflation calculator available in the web site of the Bureau of Labour Statistics of the U.S. Department of Labour (BLS).

In addition, some costs are not originally specified in dollars. For this reason it is necessary to convert the currency using the respective exchange rate, and then update the value to the year 2002.

Example:

The installation cost per kilowatt hour for the absorption chillers was obtained from the Good Practice Guide, prepared by the Best Practice Programme of the United Kingdom, and published in the year 1999. All the data of prices are in pounds. To update these costs, it is necessary first of all make the respective conversion from British pounds to dollars from 1999. The exchange rate of a pound at the end of that year was of 1.596 US Dollars.

So, if a kWh installed cost £ 70, the equivalent in dollars of the 1999 is:

$$\begin{aligned} KWh_{Dollar1999} &= KWh_{Pound1999} * 1.596 \\ &= 70 * 1.596\ Dollars \\ &= 111.7\ Dollars \end{aligned}$$

Now the costs can be updated using the conversion factor from the BLS:

$$\begin{aligned} KWh_{Dollar2002} &= KWh_{Dollar1999} * 1.08 \\ &= 111.7 * 1.08\ Dollars \\ &= 120.6\ Dollars \end{aligned}$$

3.2.2 Prediction of the prices

Even though the evaluation of alternatives of configuration for the energy systems is made taken one year as the reference, is must be taken into account the power plant will be operating during several years, that is, during its life cycle, which will be of 25 years. Nevertheless during this time the prices for fuels and electricity vary and in spite of the fact that is not possible to specify the exact behaviour of these prices, it is possible to make use of predictions.

For this project, both the electricity and fuel costs are from the USA; predictions were extracted from the Energy Information Administrator (EIA), which has the official energy statistics from the U.S. Government. The prices are given in Dollars per kWh.

As an example, here is presented the forecast for the electricity price for the Residential sector in the United States. The Prices are given in Dollars per Million Btu and the base year is the 2000.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Electricity Price	24.49	25.35	23.96	23.10	22.96	22.83	22.43	22.28	22.17	22.38	22.34	22.51	22.49
Year	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**
Electricity Price	22.62	22.66	22.66	22.76	22.70	22.77	22.74	22.93	22.87	23.07	23.02	23.22	23.07

Table 3. Electricity prices prediction for the Residential sector in the United States [6].

3.2.3 Annualising of the costs

Because of the fact that the project involves, on the one hand investments that represent one-time costs, and on the other hand operational costs, which are repetitive costs but that change every year and even during the year. Therefore, in order to make possible the comparison of alternatives taking as reference one year, it is necessary to annualise all the costs, during the life cycle of the plant.

The values adopted by the variables included in the equations (48) and (49), for the specific cases analysed are:

n: The cycle life of the power plant is equal to 25 years;

i: The interest rate used for the annualising of all the capital costs was assumed constant, and its value is 4,5 %, and it includes an inflation of 2 % . The source used to get the information for this interest was the Global Financial Data Inc. [4].

PV: The Present values of investment and operational costs were taken, as it was before mentioned, from the year 2002.

The annualised factor a for the capital costs can be calculated very easily, as follows:

$$\begin{aligned} a &= \left(\frac{i*(1+i)^n}{(1+i)^n - 1} \right) \\ &= \left(\frac{0.045*(1+0.045)^{25}}{(1+0.045)^{25} - 1} \right) \\ &= 0.06743 \end{aligned} \qquad (50.)$$

And now, with this factor, it can be determined, for example, the annualised cost for the steam turbines, which is given in Dollars per installed kW:

$$\begin{aligned} A_{ST} &= PV_{ST} * a \\ &= \frac{\$4033}{kW} * 0.06743 \\ &= \frac{\$271.9842}{kW} \end{aligned} \qquad (51.)$$

References

[1] Sullivan, W. Bontadelli, J. Wicks, Elin.: Engineering economy. Prentice Hall. New Jersey, (2000)

[2] Li, K.W. Priddy, A.P.: Power Plant System Design. Wiley. (1985)

[3] Bejan, A. Tsatsaronis, G. Moran, M.: Thermal Design and Optimisation. Wiley. (1996)

[4] Global Financial Data, Inc. http://www.globalfindata.com/

[5] X-Rates. www.x-rates.com

[6] Energy Information Administration. http://www.eia.doe.gov

[7] Bank of England. http://www.bankofengland.co.uk

[8] Vélez Pareja, I. Decisiones de Inversión enfocado a la valoración de empresas. CEJA, 3a edición. Bogotá. (2002)

[9] www.wordreference.com

[10] http://www3.gov.ab.ca/cio/costbenefit/cb_capit.htm

[11] http://www.m-w.com/cgi-bin/dictionary

Chapter 4

Theory of optimisation

Optimisation is the action of finds the best, or optimal, solution to a mathematical problem. Optimisation problems are made up of three basic ingredients, [1][2]:

- An objective function, which we want to minimize or maximize. For instance, in a manufacturing process, to maximize the profit or minimize the cost. The objective function expresses the optimisation criterion as a function of the dependent and independent variables.
- A set of unknowns or variables**,** which affect the value of the objective function. In the manufacturing problem, the variables might include the *amounts of different resources used* or the *time spent on each activity***.** In selecting the variables for the energy system, it is necessary: to include all the important variables that affect the performance and cost effectiveness of the system; not to include fine details or variables of minor importance; and distinguish among independent variables whose values are amenable to change, that is, the decision variables and the parameters, respectively.
- One set of constraints that allow the unknowns to take on certain values but exclude others. For the manufacturing problem, it doesn't make sense to spend a negative amount of time on any activity, so we constrain all the "time" variables to be non-negative. This is an example of constraint that shows how some constraints are related to physical limits.

These elements compose a mathematical model, which is a description in terms of mathematical relations, invariably involving some idealisation, of functions of a physical system. The optimisation problem is then oriented to find the values of the variables that minimize or maximize the objective function while satisfying the constraints. For the case of the energy systems, the optimisation is oriented to minimise the total annual costs of the system. In a wide meaning, this optimisation can involve aspects like materials, financial resources, protection of the environment, and governmental regulation, together with safety, operability, reliability, availability and maintainability of the system [2].

4.1 Linear programming

The term linear programming refers to the optimisation procedure applied to problems in which both the objective function and the constraints are linear. A Linear Program (LP) is a problem that can be expressed as follows (the so-called Standard Form):

$$\begin{aligned} &\textit{Minimise} && c \cdot x \\ &\textit{subject to} && A \cdot x = b \\ & && x \geq 0 \end{aligned} \qquad (52.)$$

Where x is a vector of variables to be solved for, A is a matrix of known coefficients, and c and b are vectors of known coefficients. The expression "c·x" is the objective function, and the equations "A·x=b" are the constraints. All these entities must have consistent dimensions. The matrix A is generally not square; it usually has more columns than rows, and Ax=b is therefore quite likely to be under-determined, leaving great latitude in the choice of x with which to minimize c·x.

The importance of linear programming derives in part from its many applications (see further below) and in part from the existence of good general-purpose techniques for finding optimal solutions. These techniques take as input only an LP in the above Standard Form, and determine a solution without reference to any information concerning the LP's origins or special structure. They are fast and reliable over a substantial range of problem sizes and applications.

Two families of solution techniques are in wide use today. Both visit a progressively improving series of trial solutions, until a solution is reached that satisfies the conditions for an optimum. Simplex methods, introduced by Dantzig about 50 years ago, and Barrier or interior-point methods, derived from techniques for non-linear programming that were developed and popularised in the 1960s by Fiacco and McCormick, even though their application to linear programming dates back only to Karmarkar's innovative analysis in 1984 [1].

4.2 Nonlinear programming

Whenever the assumption of linearity is not warranted, the problem might be expressed as a non-linear programming problem. Non-linear programming deals with those problems in which the objective function and/or constraints are non-linear. A Non-linear Program (NLP) is a problem that can be put into the form

$$\begin{aligned} &\textit{Minimise} \quad F(x) \\ &\textit{subject to} \quad g_i(x) = 0, \quad \textit{for } i = 1,\ldots,m1 \quad \textit{where } m1 \geq 0 \\ &\qquad\qquad\quad h_j(x) \geq 0, \quad \textit{for } j = m1+1,\ldots,m \quad \textit{where } m \geq m1 \end{aligned} \tag{53.}$$

That is, there is one scalar-valued function F, of several variables (x here is a vector), that we seek to minimize subject (perhaps) to one or more other such functions that serve to limit or define the values of these variables. F is called the "objective function", while the various other functions are called the "constraints". (If maximization is sought, it is trivial to do so, by multiplying F by -1).

Because NLP is a difficult field, researchers have identified special cases for study. A particularly well-studied case is the one where all the constraints g and h are linear. The name for such a problem, unsurprisingly, is "linearly constrained optimisation". If, as well, the objective function is quadratic at most, this problem is called Quadratic Programming (QP). A further special case of great importance is where the objective function is entirely linear; this is called Linear Programming (LP. Another important special case, called unconstrained optimisation, is where there are no constraints at all.

One of the greatest challenges in NLP is that some problems exhibit "local optimal", that is, spurious solutions that merely satisfy the requirements on the derivatives of the functions. Think of a near-sighted mountain climber in a terrain with multiple peaks, and you'll see the difficulty posed for an algorithm that tries to move from point to point only by climbing uphill. Algorithms that propose to overcome this difficulty are termed "Global Optimisation" [1].

4.3 Integer programming

Linear programming models assume divisibility. That is, all the variables are assumed to be continuous, and hence they can be assigned any non-negative integer as well as fractional value. However, in many problems the assumption of divisibility does not reflect the realities of life. For example, it cannot be built 2.2 plants or 3.4 machines. In these problems, the decision variables must assume only integer values. Thus there is the need to impose, on the general linear programming model, an additional constraint that some or all of the variables can assume only integer values. When this is done, the resulting model is termed the integer (linear) programming model. It consists of four components: a linear objective function, a set of linear constraints, a set of no negativity constraints, and integer value constraints for some or all the variables. When all variables are restricted to integer values, the model is called all-integer. If only some of the variables are restricted to integer values, it is a mixed-integer model [3].

Integer linear programming problems have the general form:

$$\begin{aligned} &\textit{Minimise} && c \cdot x \\ &\textit{subject to} && A \cdot x = b \\ & && x \geq 0 \\ & && x \in Z^n \end{aligned} \qquad (54.)$$

Where Z^n is the set of n-dimensional integer vectors. In *mixed-integer programs*, some components of x are allowed to be real.

Integer programming problems, such as the *fixed-charge network flow* problem and the famous *travelling salesman* problem are often expressed in terms of binary variables [4].

Integer programs (IPs) often have the advantage of being more realistic than LPs, but the disadvantage of being much harder to solve. The most widely used general-purpose techniques for solving IPs use the solutions to a series of LPs to manage the search for integer solutions and to prove optimality. Thus most IP software is built upon LP software.

4.4 Optimisation for the energy systems

The formulation of an optimisation model for an energy system arises for answering some typical questions during the design of it, like:

- What processes or equipment items should be selected and should they be arranged?
- What is the preferred size of a component or group of components?
- Should some equipment items be used in parallel in specific processes to increase the overall system availability?
- What is the best temperature, pressure, flow rate and chemical composition of each stream in the system?

Appropriate problem formulation is usually the most important and sometimes the most difficult step of a successful optimisation study [2]. Based on the theory described above, the mathematical models for different configurations of the energy system were made. The relevant information for the formulation and the solution of these models is described next.

4.4.1 Mixed Integer-Non-linear Model

The models used for the optimisation of the energy systems studied include integer and continuous variables, and the objective function and some constraints are non-linear. Such a formulation is called a Mixed Integer-Non-linear Model.

For example, in one of these model it can be found: integer variables, like the number of units of an auxiliary equipment; continuous variables, like the power output of a turbine; non-linear terms in the objective function, like the relation between the cost and the size of an equipment; and non-linear terms in the constraints, like product between the number of units and the size of an equipment.

Even though a the validity of the results obtained after solving the model, depends to a large extent on how faithful to the reality the mathematical model is, it is necessary to take into account that the more complex is the model the more difficult is to solve it. The models formulated for the energy systems have a certain grade of complexity, but also some assumptions have been made in order to make easer the solution of them.

4.4.2 Software used for solving the models

There are several software packets for solving mathematical models, linear, non-linear, integer or mixed. Many of them are even freeware. For the solution of the models constructed for the energy systems, it was used LINGO, a product of LINDO Systems.

LINDO Systems is a specialist in providing tools for mathematical optimisation of Linear, Integer, Non-linear and Quadratic programming problems.

LINGO is a comprehensive tool designed to make building and solving linear, non-linear and integer optimisation models. It provides an integrated package that includes a language for expressing optimisation models, a full featured environment for building and editing problems, and a set of built-in solvers. LINGO's modelling language allows expressing models in an intuitive manner using summations and subscripted variables. Models are easy o build and to understand. The software has internally different options for configuration of the solver. The non-linear solver has the following options [5]:

Crash initial solution: it invokes a heuristic for generating a "good" starting point when the model is being solved. If this initial point is relatively good, subsequent solver iterations should be reduced along with overall runtimes.

Global Solver: Many non-linear models are non-convex and/or non-smooth. In a convex function, geometrically defined, for any two points on the function, a straight line connecting the two points lies entirely on or above the function is a function; a smooth function have a unique defined first derivative (slope or gradient) at every point.

Non-linear solvers that rely on local search procedures (as does LINGO's default non-linear solver) will tend to do poorly on these types of models. Typically, they will converge to a local, sub-optimal point that may be quite distant from the true, global optimal point. Global solvers overcome this weakness through methods of range bounding (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branch-and-bound framework to find the global solutions to non-convex models.

The only drawback to the global solver is that it runs considerably slower than the default local solver. Therefore, the preferred option is to always try and write smooth, convex non-linear models, so the faster, default local solver can successfully solve them.

Quadratic Recognition: with this option, LINGO will use algebraic pre-processing to determine if an arbitrary non-linear model is actually a quadratic programming (QP) model. If a model is found to be a QP model, then it can be passed to the faster quadratic solver.

Selective Constraint Evaluation: the software will only evaluate constraints on an as needed basis. Thus, not every constraint will be evaluated during each iteration. This generally leads to faster solution times, but can also lead to problems in models that have functions that are undefined in certain regions.

SLP Directions: with this option, LINGO's non-linear solver will use successive linear programming (SLP) to compute new search directions. This technique uses a linear approximation in search computations in order to speed iteration times. In general, however, the number of total iterations will tend to rise when SLP Directions are used.

Steepest Edge: the solver will use the steepest-edge strategy when selecting variables to iterate on. When LINGO is not in steepest-edge mode, the non-linear solver will tend to select variables that offer the highest absolute rate of improvement to the objective, regardless of how far other variables may have to move per unit of movement in the newly introduced variable. The problem with this strategy is that other variables may quickly hit a bound, resulting in little gain to the objective.

With the steepest-edge option, the non-linear solver spends a little more time in selecting variables by looking at the rate that the objective will improve relative to movements in the other nonzero variables.

For the solution of the models, the default options of the non-linear module were used, that is, using the Successive Linear Programming directions. Several trial were made with the global solver option, and it was noticed that the optimal solution was the same that the obtained with the default configuration. Moreover, the global solver runs very slowly.

The other parameters for the configuration of the LINGO's solver were neither modified; these options are mostly related to tolerances for the values adopted by the variables, and some of these have no effect in the solution of non-linear models.

The SLP method is a constrained multivariable optimisation procedure, and it is considered one of the most successful methods to solve constrained non-linear optimisation problems.

This procedure was called the *method of approximate programming* (MAP) by Griffith and Stewart of Shell Oil Company who originally proposed and tested the procedure on petroleum refinery optimisation.

As the name implies, the method uses linear programming as a search technique. A starting point is selected, and the non-linear economic model and constraints are linearised about this point to give a linear problem, which can be solved by the Simplex Method or its extensions. The point from the linear programming solution can be used as a new point to linearise the non-linear problem, and this can be continued until a stopping criterion is met. As shown by Reklaitis et al., this procedure works without safeguards for functions that are mildly non-linear.

However, it is necessary to bound the steps taken in the iterations to insure that the economic model improves, the values of the independent variables remain in the feasible region and the procedure converges to the optimum. These safeguards are bounds on the independent variables specified in advance of solving the linear programming problem. The net result is that the bounds are additional constraint equations. If the bounds are set too small, the procedure will move slowly toward the optimum. If they are set too large, infeasible solutions will be generated. Consequently, logic is incorporated into computer programs to expand the bounds when they hamper rapid progress and shrink them so that the procedure may converge to a stationary point solution [6]. More detailed information about the Successive Linear Programming method and examples of the procedure to solve non-linear problems with this method, as well as additional bibliography, is available in the reference number [6].

References

[1] Fourer, R. Optimisation Technology Center of Northwestern University and Argonne National Laboratory: Optimisation Frequently Asked Questions. (2000) http://www-unix.mcs.anl.gov/otc/Guide/faq

[2] Bejan, A. Tsatsaronis, G. Moran, M.: Thermal Design and Optimisation. Wiley. (1996)

[3] Narendra, P.: Linear Programming: A Managerial Perspective. Macmillan. New York. (1976)

[4] Optimisation Technology Center of Northwestern University and Argonne National Laboratory: NEOS Guide. http://www-fp.mcs.anl.gov/otc/Guide/OptWeb

[5] LINGO 8.0: software help.

[6] Pike, R. W.: Optimization for Engineering Systems. Minerals Processing Research Institute. Louisiana State University. Chapter 6. 2001. http://www.mpri.lsu.edu/textbook/Chapter6-b.htm#constrained

Chapter 5

Energy demand data

The design and the operation of a cogeneration plant are defined on the basis of the size and the behaviour of the energy demand, that is, the rate of heating and electrical power consumption by the user per unit of time. The demand is typically measured in watts (W) or kilowatts (kW)

For the evaluation of the energy systems modelled, six cases of demand were defined, in order to see how the kind of user conditions the definition of the optimal design and operation of the plant. The estimation or the determination of the energy demand involve some factors that not always is easy to determine. Even though the theory about energy demand management is relatively extensive, some of these concepts are briefly explained below.

5.1 Fundamentals

Energy demand is typically presented through the load profiles, also called load curves or energy demand profiles. A specified user can define a load profile as the estimated use of energy for each period of time. This period is, for example, one hour, one day, one month or one year, so the load profile can be hourly, daily, monthly, or annual.

A load profile should be characterised by [1]: it should represent a relatively homogenous group of users; each profile should be distinctly different for the others; the identifying characteristics for assigning user load to a profile should be readily determined; the accuracy of estimated load profiles should be judged primarily on how well they perform over a trading period (typically one year). Figure 32 illustrates an example of a load profile for heating and electricity demand.

The method used for determining the profile is very important, because depending on it, a reasonable level of reliability can be ensured. The load profiling method should be considered in terms of costs, accuracy and predictability [1]. The most common methods used by the companies that supply energy for load profiling are:

• ***Dynamic metering or profiling.*** A dynamic profile is produced when hourly load levels are assigned on a daily basis-using interval metering for a sample of costumers from each profile group. These hourly profiles reflect actual load based on actual conditions. Dynamic metering is the most accurate method, but it is also the most costly when you consider the financial impact of installing interval meters (including the actual cost of each meter) and gathering data from these meters (either manually or via phone lines). Additionally, interval meters create interval data that now must be managed, creating the need for costly personnel and information technology

resources. It is important to note that although dynamic profiling uses hourly load levels, the profiling is not done in "real time."

• ***Dynamic modelling.*** A dynamic model represents the correlation between load and an external factor such as weather. This type of modelling generally uses historical regression analysis of customer load as it relates to the weather that prevailed during the time of the load. The regression could be either a daily regression or an hourly regression for any season and day type of combination. In dynamic modelling, some part of the data is "derived" using formulas, instead of being represented by the actual historical data.

• ***Proxy day (or "same day") profiling.*** Proxy day profiling is accomplished by selecting a day in history that most closely matches the day being estimated. The proxy day can be chosen based on either system load or weather. Actual data from the sample for the selected proxy day is then used to create the profile.

• ***Static profiling.*** Static profiles are typical-day representations for any season and day type of combination.

• ***Calendar rotation.*** Calendar rotation is simply rotating a calendar of historical interval data to reflect the calendar of the time being estimated.

• ***Deemed profiles.*** Deemed profiles are engineering estimates and are typically used for very predictable loads such as streetlights or area lights. These will normally be based on type of light and number of hours the light stays on.

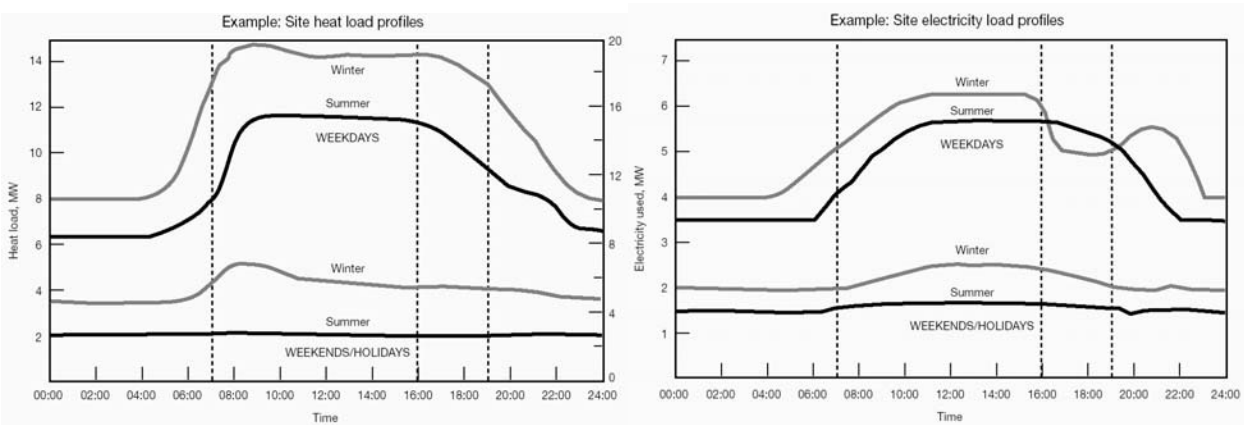


Figure 32. Example of a load profile for energy demand (heat and power) [4].

The weather plays also an important role in affecting energy consumption, being the temperature the most important climatic variable. For many years utility companies and the electric power industry have been interested in the relation between energy consumption and climate, and have developed empirical weather normalization algorithms aimed at improving load forecasting subject to variations in regional climate, and several researchers have recently published estimates of climatic influences on energy consumption. [2]. Computational help results very useful in estimating the load energy demand of a specific user.

Commercially, the load curve allows the supplier to know his customers' behaviour relative to his product for a well-determined length of time. Knowledge of the loads in particular allows sizing of the network, computation of energy, which is not distributed due to power outage, and identification of the peak usage times. Calculation of the load curve requires control of the parameters describing the energy demand [3], like the size, kind of user, location, and period of the year.

5.2 Cases of demand

For this project six cases of demand are defined. Each one of the cases has its respective load profile, which is necessary to get the information related to the energy demand, whether it is electric, heating or cooling demand. The annual energy requirements of the user is divided in four periods, that is, the year is divided in four trimesters as follows: the first from January to March; the second from April to June; the third from July to August; and the fourth from September to December. An average value for each period is calculated using the load profiles. In the same way, an average for the climate conditions (temperature and relative humidity) is calculated.
The reason to divide the year in four periods is the limitations of the software used for the simulation, since it is a limited demo version. Increasing the number of periods it is also increased the number of variables for the model, making more difficult the solution of it.

It is important to mention that the different cases can be classified into three sectors: Residential, commercial and industrial. The Residential sector is related to activities related to use as a dwelling for one or more households; the commercial sector includes service businesses, such as retail and wholesale stores, hotels and motels, restaurants, and hospitals, as well as a wide range of facilities that would not be considered "commercial" in a traditional economic sense, such as public schools, correctional institutions, and religious and fraternal organizations. Excluded from the sector are the goods-producing industries: manufacturing, agriculture, mining, forestry and fisheries, and construction [5], that is, the industrial sector.

The heating water supplies and return temperatures are assumed constant during the year and are taken from a typical district heating system of a medium sized community in southern Sweden.

For all the cases studied in this work the heating water temperature supply is assumed as 100°C, the heating water temperature return is assumed as 65°C, and the heating water pressure as 16 bar. [12]

The six cases defined for the project are described next.

5.2.1 Hospital

A hospital, which is a commercial building, belongs to the Health Care category, that is, to the buildings used for diagnostic and treatment facilities for both inpatient (used for diagnosis and treatment requiring overnight care) and outpatient (diagnosis and treatment in which services are not required overnight) care.

This is the case of demand used in the project for the simulation of the trigeneration energy system. The information for heating, electricity and cooling corresponds to the demand of the General Hospital of Vienna (AKH) of the year 1999 [6].

The information of this user is summarised next:

- Type of user: Commercial
- Information source [6]: Vienna General Hospital (AKH)
- Number of beds: 2000
- Energy demand per period:

Period	Demand (kW)		
	Heating	Electricity	Cooling
1st Period	32474.0783	15000.9601	2521.76139
2nd Period	15350.9558	14652.0311	9063.62007
3rd Period	8781.36201	14808.8411	13784.3489
4th Period	27147.5508	14674.4325	3207.8853

Table 4. Energy demand per period for the hospital

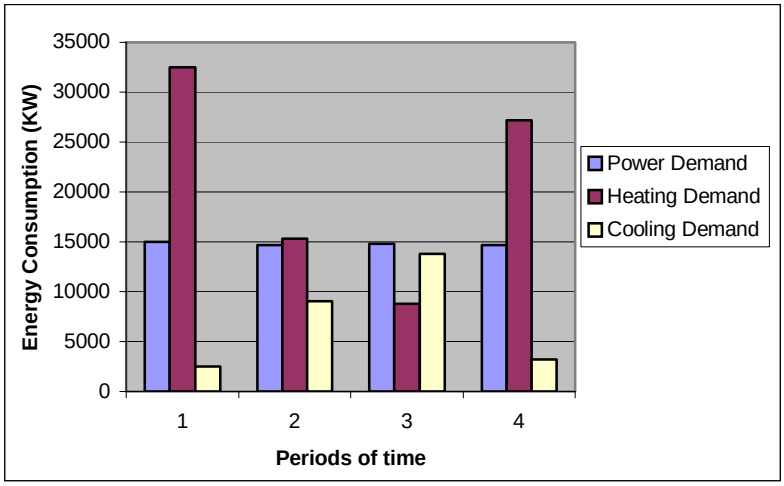


Figure 33. Load profile of the General Hospital of Vienna

5.2.2 Households

Household can be defined as a family, an individual, or a group of up to nine unrelated persons, occupying the same housing unit. Household members include babies, lodgers, boarders, employed persons who live in the housing unit, and persons who usually live in the household but are away travelling or in a hospital. For this project, the number of households is the same as the number of occupied housing units [7].

A housing unit is a house, an apartment, a group of rooms, or a single room if it is either occupied or intended for occupancy as separate living quarters by a family, an individual, or a group of one to nine unrelated persons. Housing units do not include group quarters where 10 or more unrelated persons live [7].

For this case, the same way as for the offices, the load profiles were acquired from the Regional Economic Research (RER) homepage [8], a company specialized in the providing of solutions and in the source of knowledge for collecting, analyzing and applying electric, gas and water usage data. The load profiles correspond to the demand of a house in the USA, in the region called Central Industrial. In order to apply one of the CHP technologies raised, it was necessary to define the demand for a group of houses – like a neighbourhood -, based on the data for one housing unit.

The information about the households is the following:

- Type of user: Residential
- Information source [8]: Regional Economic Research. Households situated in the Central Industrial region, in the United States.
- Number of housing units: 250
- Energy demand per period:

Period	Demand (kW/Household)	
	Heating	Electricity
1st Period	8.46181891	0.86793628
2nd Period	2.59260027	0.82623507
3rd Period	1.27601356	0.81047555
4th Period	5.40831964	0.85455657

Table 5. Energy demand per period for a household

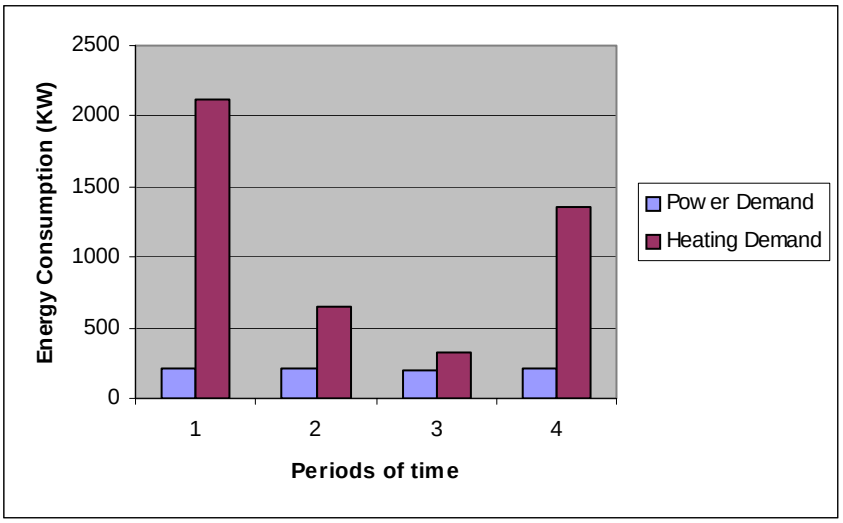


Figure 34. Load Profile of a Household

5.2.3 Hotel

The hotels can be included into the commercial buildings used for lodging, that is, buildings used to offer multiple accommodations for short-term (Convention hotel, inn, motel, tourist home, etc) or long-term (Boarding house, orphanage, Dormitory/sorority/fraternity, etc) residents, including nursing homes.
The load profile used for this project belongs to a medium size hotel. The information was taken from the report of the project *Assessment of CHP Implementation Possibilities in the Tourist Sector,* from the National Technical University of Athens [9].

The information used for the hotel is listed next:

- Type of user: Commercial
- Information source [7]: National Technical University of Athens.
- Energy demand per period:

Period	Demand (kW)	
	Heating	Electricity
1st Period	823.892729	454.957117
2nd Period	611.857826	1117.98088
3rd Period	300.179211	1724.01434
4th Period	744.623656	758.213859

Table 6. Energy demand per period for a hotel

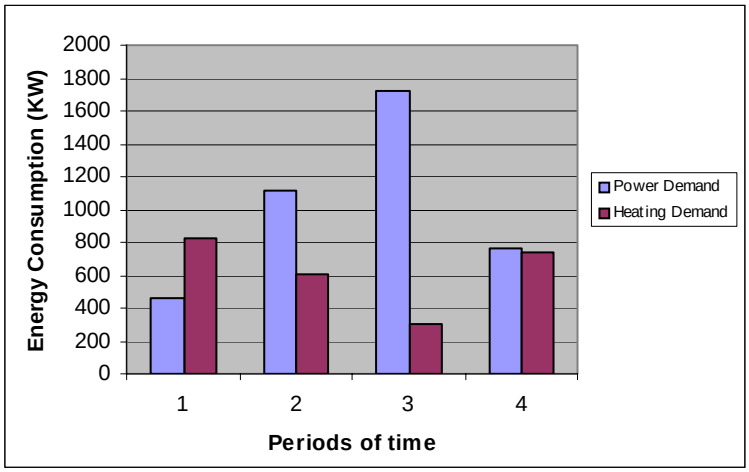


Figure 35. Load Profile of a Hotel

5.2.4 Office building

An office building is a typical example of a commercial building. For these buildings, the size plays a particular role, as depending on it; the energy use characteristics can be quite different. In smaller buildings, heating and cooling systems are employed primarily to moderate outside air. In large buildings, outside air conditions have less impact on heating and cooling systems than do activities within the buildings—equipment used, lighting levels, number of people, and hours of

operation. For example, one part of a building might need to be heated and ventilated to provide comfortable conditions for employees, while a computer room might need to be cooled be cause of excess heat given off by the computer equipment [5].

The load profile presented by RER is given as a function of the area of each office. For the specification of the cases of demand, two different building load profiles were made, that is, for a medium and a large office building. These profiles correspond also to building demand in the Central industrial zone of the USA.

This information is presented in the tables 18 and 19, and in the Figure 53.

- Type of user: Commercial
- Information source [8]: Regional Economic Research. Office building situated in the Central Industrial region, in the United States.
- Number of offices: 1000
- Office area (sqft): 300
- Energy demand per period:

Period	Demand (kW)	
	Heating	Electricity
1st Period	1593.97096	385.367205
2nd Period	257.532329	499.42402
3rd Period	157.939318	552.735519
4th Period	784.986562	405.817041

Table 7. Energy demand per period for an office building

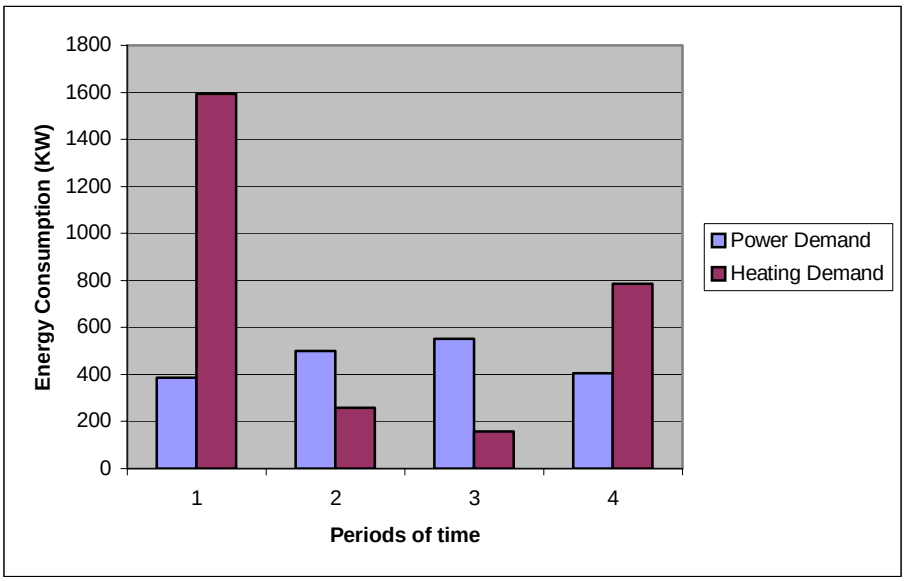


Figure 36. Load Profile of an Office Building

5.2.5 Cities A and B

The load profiles for the cities were elaborated based on the load profiles mentioned above. It is proposed to "build" a city, with a specific number of houses, medium and large office buildings, hospitals and hotels, and the total demand is equal to the addition of the demand of each one of these. In this way, it is possible to define different cases, according with the size of the city. It is clear that the profile can't reproduce in detail the demand of a real city, but it can be considered as a good approximation of an urban city. This method has also the advantage of making the load profiling more flexible and not subject to a specific profile of a particular city.
Additionally this is suitable for the analysis of sensitivity of the optimization model. Nevertheless a maximum size, in terms of number of inhabitants, was established, because of the size limits for the capacity of the cogeneration plants defined for the project, that is; that means that for cities with a size larger than the limit, it is not possible to supply the demand of the city with the largest possible power plant established for the model. The limit established was of 200000 inhabitants.

Taking all this into account, two cities were "created". Even though the number of inhabitants is the same for they both, the climatic conditions defined for each one is different, and hence the energy demand as well. The climatic data – temperature and relative humidity – were extracted from the historical weather database available on the Washington Post homepage [10]. In particular, two cities from two different regions of the United States were selected (See Map on Figure 37). From the South Central Region, Oklahoma (designed as City A); and from the Central Industrial, Detroit (designed as City B).

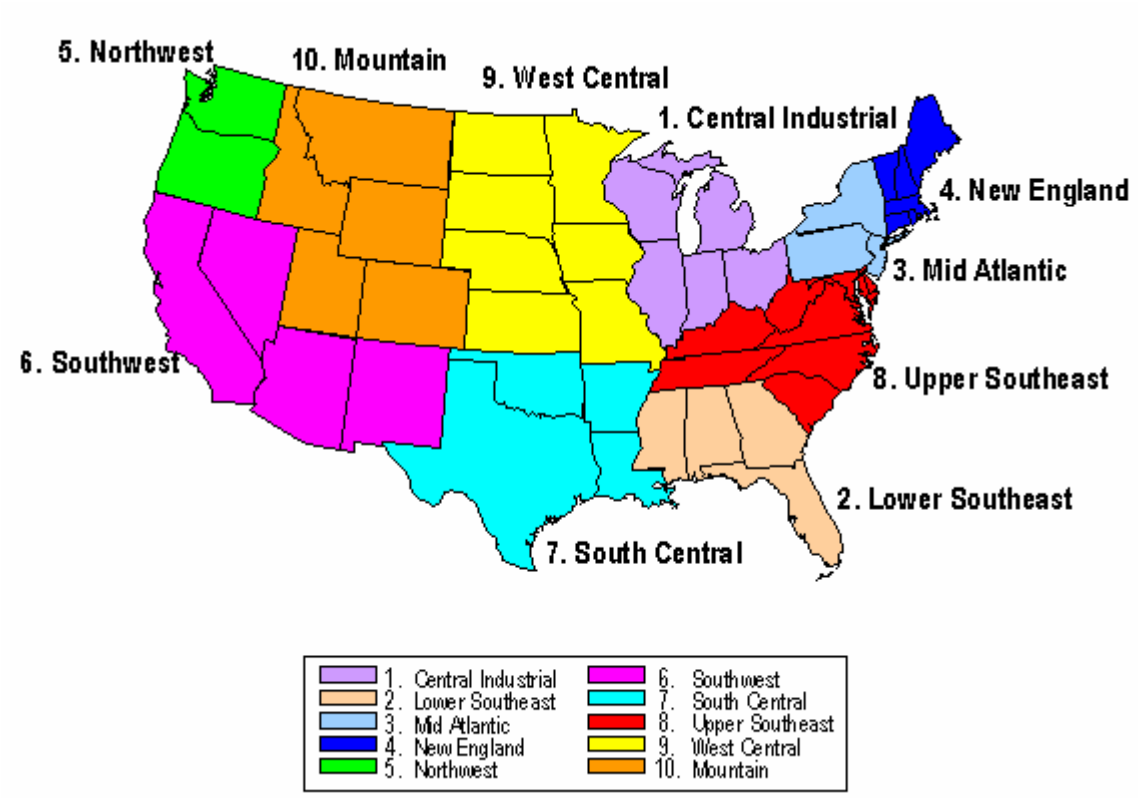


Figure 37. Regional Map of the United States of America [8].

The size of the city as well as the composition was taken from the data of the city of Hoyerswerda, a city situated in the federal state of Saxony, in East Germany [11].

The data used for the city A is the following:

- Type of user: Industrial
- Information source for urban distribution [11]: SRC International CS. Data from the city of Hoyerswerda (Ger)
- Inhabitants: 54000
- Housing units: 6700
- Buildings: 330 (Medium), 5 (Large)
- Hospital (Number of beds): 2000
- Hotels: 5
- Climate data per period:

CITY: Detroit, Michigan	Year Av.	Period 1	Period 2	Period 3	Period 4
Relative Humidity (%)	**60**	65	54	55.6667	64.3333
Temperature (°C)	**9**	-2	13.6667	20	4

Table 8. climate data per period for the city of Detroit [10]

- Energy demand per period:

Period	Demand (MW)	
	Heating	Electricity
1st Period	122.498334	33.1270811
2nd Period	37.4058518	39.538191
3rd Period	20.6210292	44.3509462
4th Period	78.3580272	34.9098502

Table 9. Energy demand per period for the city A

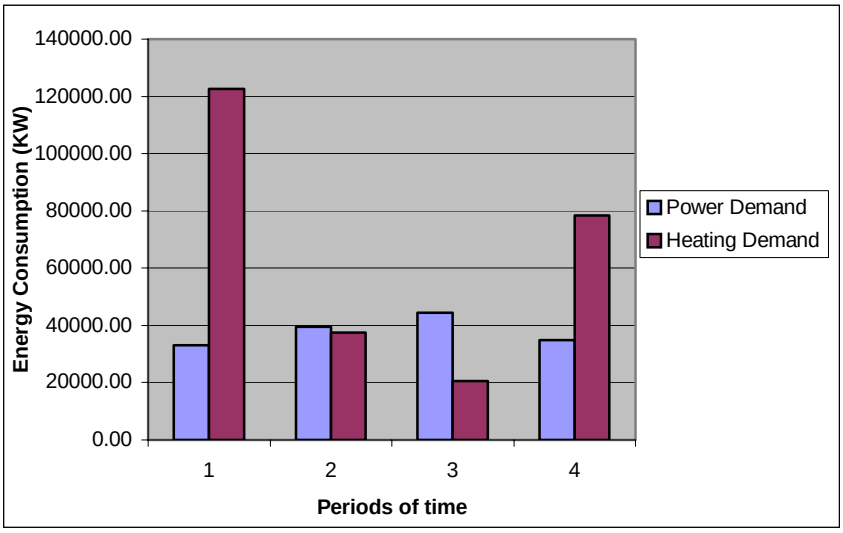


Figure 38. Load Profile of City A

The data used for the city B is the following:

- Type of user: Industrial
- Information source for urban distribution [11]: SRC International CS. Data from the city of Hoyerswerda (Ger)
- Inhabitants: 54000
- Housing units: 6700
- Buildings: 330 (Medium), 5 (Large)
- Hospital (Number of beds): 2000
- Hotels: 5
- Climate data per period:

CITY: Oklahoma City, Okla.	Year Av.	Period 1	Period 2	Period 3	Period 4
Relative Humidity (%)	**55**	57	55.3333	51.6667	55.3333
Temperature (°C)	**15**	5.333333	19.6667	25.3333	9.66667

Table 10. climate data per period for the city of Oklahoma [10]

- Energy demand per period:

Period	Demand (MW)	
	Heating	Electricity
1st Period	64.9252514	33.31059053
2nd Period	27.2244485	48.11116482
3rd Period	17.9425654	55.60982375
4th Period	49.0810188	37.56054456

Table 11. Energy demand per period for the city B

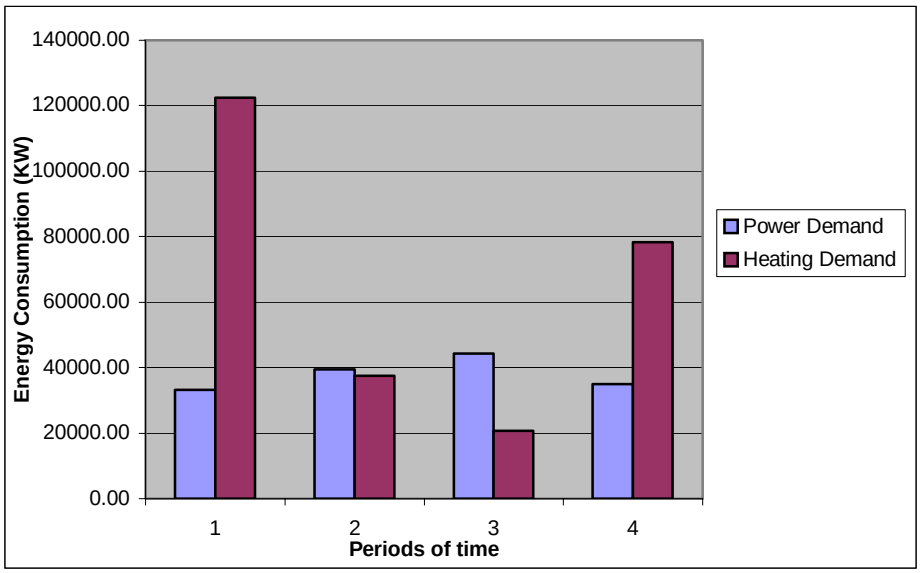


Figure 39. Load Profile of City B

References

[1] Bailey, J. Load Profiling for Retail Choice: Examining a Complex and Crucial Component of Settlement. The Electricity Journal. (December 2000).

[2] Sailor, D.: Relating residential and commercial sector electricity loads to climate—evaluating state level sensitivities and vulnerabilities. Energy, Vol. 26. (2001)

[3] Tatiétsé, T. Villeneuve, P. Ngundam, J. Kenfack, F: Contribution to the analysis of urban residential electrical energy demand in developing countries. Energy, Vol. 27. (2002)

[4] ENERGY EFFICIENCY BEST PRACTICE PROGRAMME. Good Practice Guide 227. How to appraise CHP. United Kingdom Government.

[5] Energy Information Administration, Office of Energy Markets and End Use U.S. Department of Energy of the U.S. Government: A Look at Commercial Buildings in 1995: Characteristics, Energy Consumption, and Energy Expenditures. Washington DC. (October 1998)

[6] VAMED-KMB Krankenhausmanagement und Betriebsführungsgesellschaft m. b. H.: WIENER ALLGEMEINES KRANKENHAUS, Energiebericht. Vienna. (2000)

[7] Energy Information Administration, Office of Energy Markets and End Use U.S. Department of Energy of the U.S. Government: Household Energy Consumption and Expenditures 1993. Washington DC. (October 1995)

[8] Regional Economic Research, Inc. (RER). EShapes™. http://www.rer.com/eshapes/index.htm

[9] Zervos, A. Frangopoulos, Ch. Assessment of CHP Implementation Possibilities in the Tourist Sector. National Technical University of Athens. (June 2001)

[10] The Weather Post, The Washington Post: Historical Weather Database. http://www.washingtonpost.com/wp-srv/weather/historical/historical.htm

[11] SRC International CS: Analysis of CHP Potentials in ERN http://www.srci.cz/nisa/Germany_3.ppt

[12] Carcasci C., Cormacchione N.A. Colito. Part load operation strategies for gas turbines in district heating applications. Journal of Power and Energy. Proceedings of the I MECH E Part A. Volume 215, Number 5. October 2001.

Chapter 6

Thermodynamics of Energy systems

6.1 Selection of technologies

Taking into account that the technologies of cogeneration used in energy systems studies are numerous and the respective thermodynamically and technical analysis of all those is laborious, the present work makes a selection between all the cogeneration technologies.

The selection was based in three main reasons:

- At least two different cogeneration technologies must be determined, in order to compare different possibilities for a given user
- The technologies determined must be available for different users and different energy demands
- The technologies determined must be as simple as possible, in order to may successful calculations in the normal software available

The criterions used for the selection of the technologies for the optimisation studies are summarized in the Figure 40.

The selection of the technology is based mainly in the Table 38, which presents the comparison between the principal characteristics of each cogeneration technology.

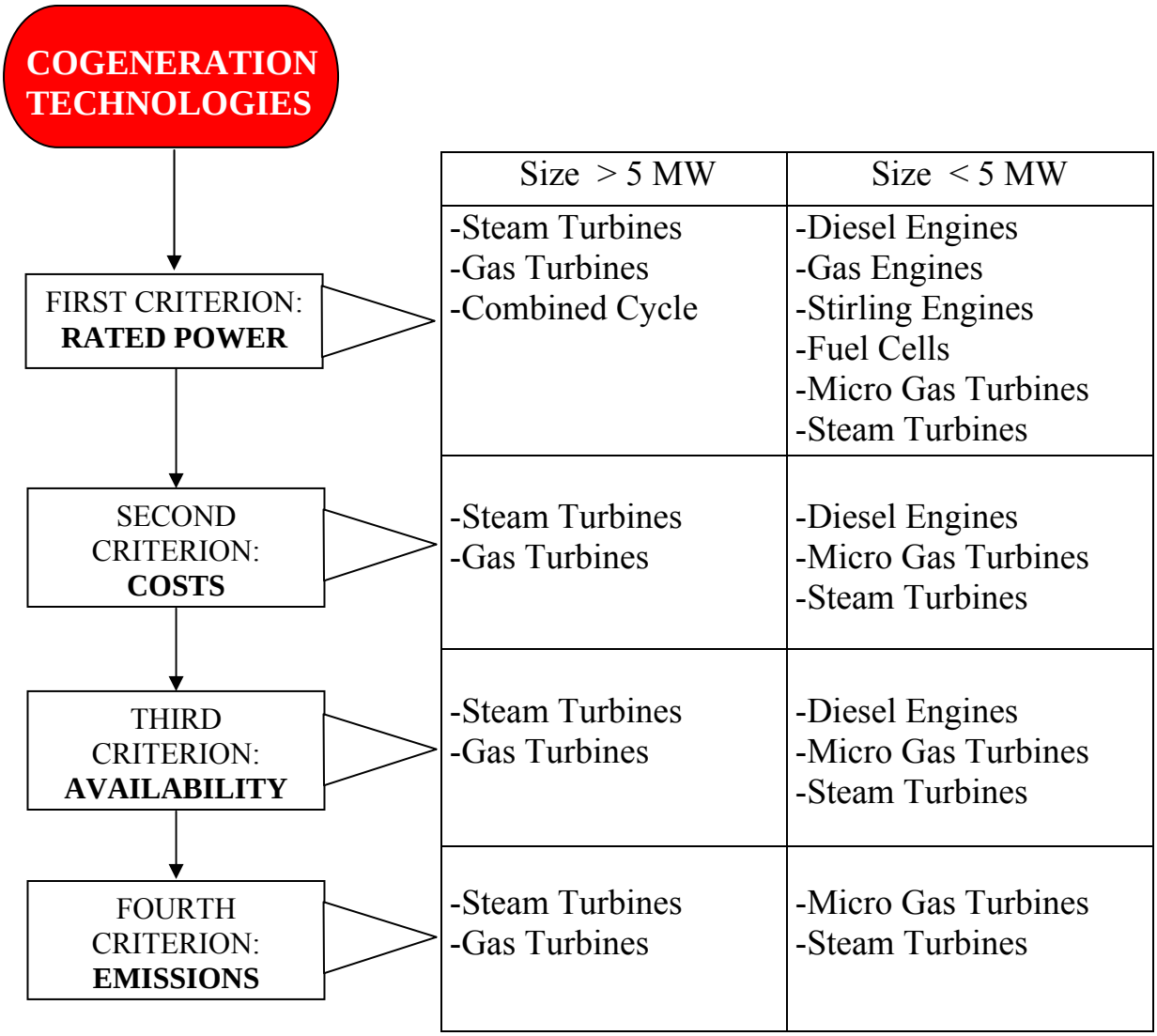


Figure 40. Selection of cogeneration technologies

The cogeneration technologies selected in this work in order to compare different possibilities for a given user, are divided in two groups depending on the size of the user.

For users with an electric capacity lower than 5 MW, the technologies used are micro gas turbines and steam turbines.

For users with an electric capacity higher than 5 MW, the technologies used are gas turbines and steam turbines.

Those selections, present the lowest costs (installation costs and maintenance costs), the highest availability and lowest contaminant emissions.

6.2 Steam Turbine Cogeneration Systems

The selection of the different options of steam turbine cogeneration systems is effectuated between the backpressure steam turbine cogeneration systems and the condensing steam turbine cogeneration systems.

The steam turbines with bottoming systems are not included in this comparison, because they need an industrial process.

The Table 12, present a comparison of the principal characteristics of both options.

Type of System	Advantages	Disadvantages
Back-pressure steam turbine systems	Simple configuration with few components	The steam turbine is larger for the same power output, because it operates under lower enthalpy difference of steam
	The cost of expensive low pressure stages of the turbine are avoided	The control of the thermal load is dependently of the electrical power control
	Low capital cost	
	Reduced or even no need of cooling water	
	High total efficiency, because there is no heat rejection to the environment through a condenser	
Condensing steam turbine systems	It can control the electrical power independently of the thermal load, by a proper regulation of the steam flow rate through the turbine	Higher capital cost
		Lower total efficiency

Table 12. Comparison of steam turbine cogeneration systems [1] [2] [4]

The backpressure steam turbine cogeneration system is selected in this work, because it offers a simple design, with lower capital cost and higher total efficiency than the condensing steam turbine systems.

Although the control of the heat and the electricity generation is not independently, and therefore, there is need of a two-way connection to the grid for purchasing supplemental electricity or selling excess electricity generated.

6.2.1 Thermodynamics of backpressure steam turbine cogeneration systems

As was mentioned in Chapter 3, the backpressure steam turbine cogeneration systems are based on a heat source (Boiler), a steam turbine, a heat exchanger (Feed water heater) and a feed pump.

The Figure 41 shows the configuration used in this work for a backpressure steam turbine cogeneration systems.

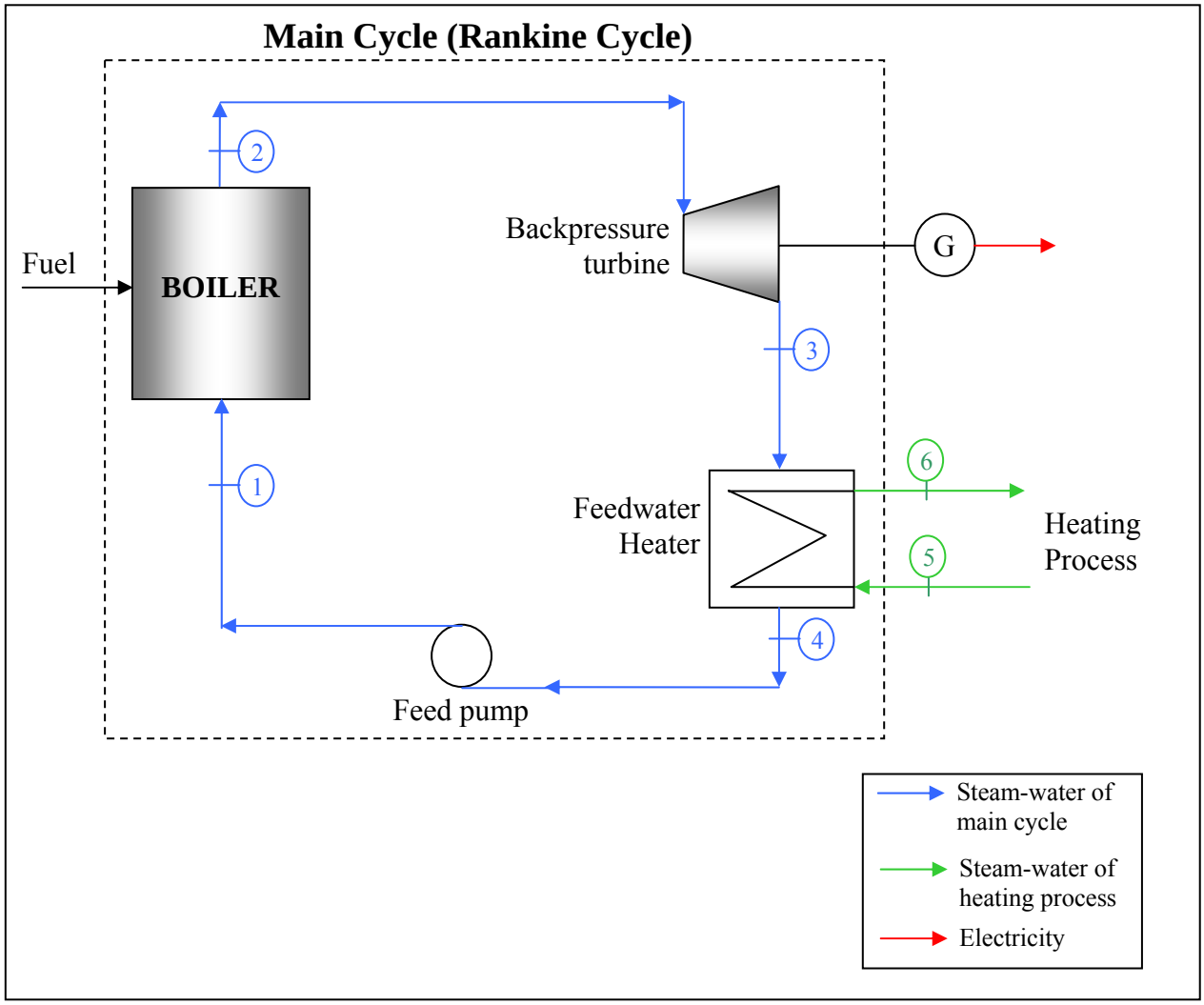


Figure 41. Backpressure steam turbine cogeneration system

It can distinguish two different work fluids, a work fluid for the main Rankine cycle (steam-water of main process), and a work fluid for the heating process (steam-water of heating process). The main cycle (Rankine cycle) is composed by four steps:

1. Constant pressure heat addition process in the boiler (1-2)
2. Expansion process in the turbine (2-3)
3. Constant pressure heat rejection process in the feed water heater (3-4)
4. Compression process in the pump (4-1)

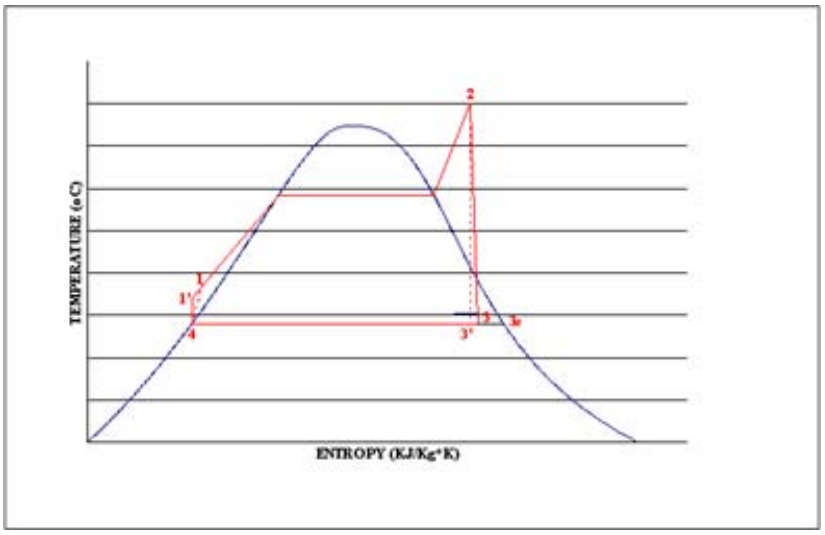


Figure 42. Temperature-entropy diagram for a Rankine cycle (Backpressure steam turbine)

6.2.1.1 Constant pressure heat addition process in the boiler (1-2)

The boiler is a steam generating system that transfers the heat from the products of combustion to water and produces hot water or steam. The combustion is accomplished in a furnace. Using the first law of thermodynamics, the Boiler efficiency can be expressed in the next form:

$$\eta_B = \frac{\text{Heat Produced}}{\text{Fuel Energy Required}} = \frac{Q_B}{\overset{*}{m}_{f_{ST}} H_U} \qquad (55.)$$

$$\eta_B = \frac{\overset{*}{m}_{st}(h_2 - h_1)}{\overset{*}{m}_f H_U} \qquad (56.)$$

In this work the efficiency of the boiler is assumed as 80% [1] [3]. From the previous equation, is possible obtain the fuel mass flow,

$$\overset{*}{m}_f = \frac{\overset{*}{m}_{st}(h_2 - h_1)}{\eta_B H_U} \qquad (57.)$$

The pressure and the temperature determine the enthalpies of the steam-water in the states 1 and 2, like is expressed in the next equations (In this cycle $p_1 = p_2$)

$h_1 = f(p_1 , T_1)$ (58.)
$h_2 = f(p_2 , T_2)$ (59.)

6.2.1.2 Expansion process in the turbine (2-3)

A backpressure steam turbine requires process steam at an appreciable pressure and temperature; its distinguishing characteristic is that its "back pressure" or outlet pressure is above atmospheric. In this strictest interpretation the term applies to any turbine not equipped with a condenser for condensing the exhaust, but common usage has given it the connotation of higher than atmospheric exhaust pressure.

The isentropic efficiency of the backpressure steam turbine can be expressed by the next equation (The isentropic efficiency is taken as 86% as a practical value [2] [3]).

$$\eta_{ST} = \frac{h_2 - h_3}{h_2 - h_{3'}} \qquad (60.)$$

The backpressure and the steam qualities determine the enthalpies of the steam in the states 3 and 3',

$h_3 = f(p_3 , \chi_3)$ (61.)
$h_{3'} = f(p_3 , \chi_{3'})$ (62.)

The steam qualities in the different states can be represented by the equations,

$$\chi_3 = \frac{h_3 - h_4}{h_{3s} - h_4} = \frac{s_3 - s_4}{s_{3s} - s_4} \qquad (63.)$$

$$\chi_{3'} = \frac{h_{3'} - h_4}{h_{3s} - h_4} = \frac{s_{3'} - s_4}{s_{3s} - s_4} = \frac{s_2 - s_4}{s_{3s} - s_4} \qquad (64.)$$

Therefore,

$h_3 = f(p_2 , p_3, T_2, \eta_{ST})$ (65.)

Finally, the work of the backpressure steam turbine can be expressed as,

$$W_{ST} = \dot{m}_{st}\left(h_2 - h_3\right) \qquad (66.)$$

Combining the previous equations is possible express the fuel mass flow in the main boiler as a function of the turbine net output power.

$$\dot{m}_f = \left(\frac{\dot{W}_{ST}}{h_2 - h_3}\right)\left(\frac{h_2 - h_1}{\eta_B H_U}\right) \qquad (67.)$$

6.2.1.3 Constant pressure heat rejection process in the feed water heater (3-4)

The heating process is realised in a feed water heater where the steam-water of the Rankine cycle transfer energy to the water-steam of the heating process. These two different work fluids are employed in order to maintain a good quality of the water-steam used in the Rankine cycle and to avoid problems of the directly use of the heating feed water in the Rankine cycle, like poor quality water and/or hard corrosion in the backpressure steam turbine.

In this work it is assumed that for the practical feed water heater there is a temperature difference of 10°C between the outlet steam from the turbine and the highest temperature in the heating process, the next equation express the mentioned relation. [7]

$$T_3 = T_6 + 10K \qquad (68.)$$

The Figure 43 shows the temperature profiles of the feed water heater.

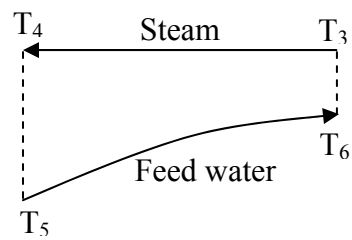


Figure 43. Temperature profiles of the feed water heater

Using the energy conservation law,

$$\overset{*}{m}_{st}(h_4 - h_3) = \overset{*}{m}_{fw}(h_6 - h_5) \qquad (69.)$$

$$\frac{\overset{*}{m}_{fw}}{\overset{*}{m}_{st}} = \frac{h_4 - h_3}{h_6 - h_5} \qquad (70.)$$

$$Q_{ST} = \overset{*}{m}_{st}(h_3 - h_4) = \overset{*}{m}_{fw}(h_6 - h_5) \qquad (71.)$$

6.2.1.4 Compression process in the pump (4-1)

The feed pump increases the pressure of the condensate from the heating process. The isentropic efficiency of the feed pump is expressed in the next equation (The isentropic efficiency of the pump is assumed as 86% [2]).

$$\eta_P = \frac{h_4 - h_{1'}}{h_4 - h_1} \qquad (72.)$$

And the work required for the pump is,

$$W_P = \overset{*}{m}_{st}\left(h_4 - h_1\right) \qquad (73.)$$

6.2.2 Performance Characteristics of the backpressure steam turbine cogeneration system

6.2.2.1 Heat to power ratio

The main characteristic of cogeneration systems is the heat to power ration, which express the relation between the useful heat and the total power produced by the cogeneration plant.

The next equation shows the heat to power ratio for a backpressure cogeneration system,

$$\lambda_{ST} = \frac{Q_{ST}}{W_{ST}} = \frac{\overset{*}{m}_{st}\left(h_3 - h_4\right)}{\overset{*}{m}_{st}\left(h_2 - h_3\right)} = \frac{h_3 - h_4}{h_2 - h_3} = \frac{h_2 - h_4}{\eta_{ST} \cdot \left(h_2 - h_{3'}\right)} - 1 \qquad (74.)$$

It can be observed that the heat to power ratio is a function of the next variables,

$$\lambda_{ST} = f(\eta_{ST}, p_2, p_3, T_2) \qquad (75.)$$

It is also possible to express the fuel mass flow in terms of the turbine net output power and the heat to power ratio.

$$\overset{*}{m}_{F_{ST}} = \frac{\overset{*}{m}_{st}\left(h_2 - h_1\right)}{\eta_B H_U} = \frac{\overset{*}{m}_{st}\left(h_2 - h_3 + h_3 - h_4 + h_4 - h_1\right)}{\eta_B H_U} = \frac{\dot{W}_{ST} + \dot{Q}_{ST} - \dot{W}_P}{\eta_B H_U} \qquad (76.)$$

Assuming that the feed pump work is negligible

$$\overset{*}{m}_{F_{ST}} = \frac{W_{ST} + Q_{ST}}{\eta_B H_U} = \frac{W_{ST} + \left(\lambda_{ST} W_{ST}\right)}{\eta_B H_U} = \frac{\dot{W}_{ST} \cdot \left(1 + \lambda_{ST}\right)}{\eta_B H_U} \qquad (77.)$$

6.2.2.2 Power control in backpressure steam turbine

The aim of governing in power-station turbines is to maintain constant rotary speed under varying load. In alternating current (AC) turbine generators the requirements as to constancy of frequency and hence as to constancy of speed are very exacting. The output power of the backpressure turbine is a function of the steam mass flow and the properties of the steam at the inlet and outlet of the turbine. In this work the method used to control the output power is the regulation of the steam mass flow. In the steam mass flow regulation the properties of the steam at the inlet and outlet of the turbine remain constant. The output power control is required in each moment that the user's demand changes.

6.2.2.3 Partial load operation

The isentropic efficiency of the turbine decreases when the steam mass flow decreases, that means, that when output power control is realised, in order to supply the power demand, the efficiency of the turbine lows. In this work the relation between the isentropic efficiency of the turbine at partial load and the output power is assumed as shows the next equation. [12]

$$[\eta_{ST}]_{pl} = \frac{(J)\cdot[W_{ST}]_{pl}}{[W_{ST}]_{full} + (J)\cdot[W_{ST}]_{pl}} \qquad (78.)$$

$$J = \frac{[\eta_{ST}]_{full}}{1-[\eta_{ST}]_{full}} \qquad (79.)$$

The next figure shows graphically the relation between the isentropic efficiency at partial load.

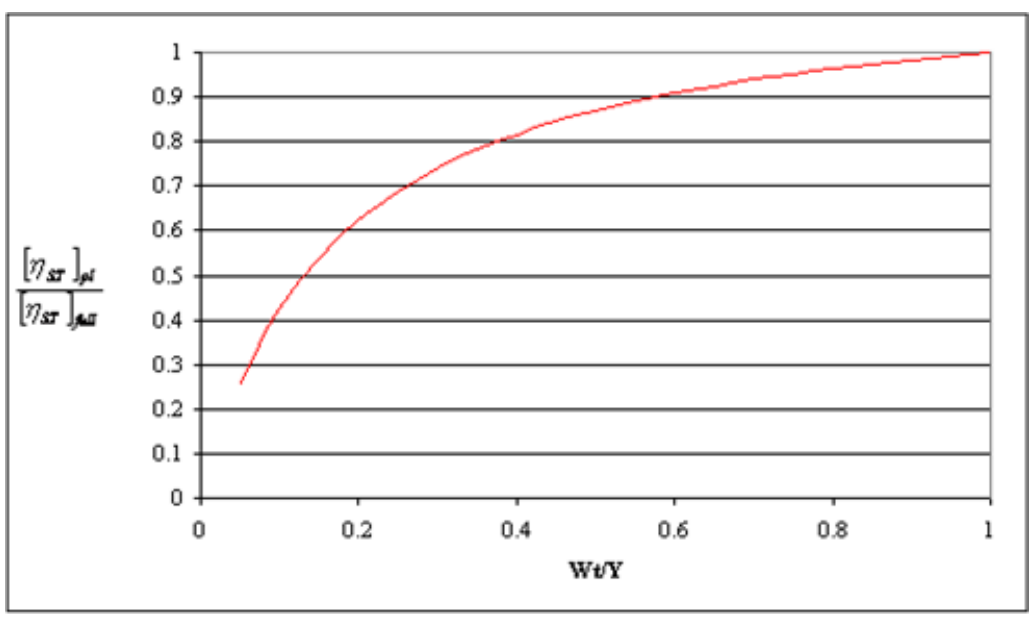


Figure 44. The isentropic efficiency at partial load

6.3 Gas Turbine Cogeneration Systems

The selection of the gas turbine technology used in this work is effectuated between the open cycle gas turbine systems and the closed cycle gas turbine system.

The Table 13 presents a comparison of the advantages and disadvantages of the simple cycle (without regeneration or additional improvements) of both systems, open cycle and close cycle.

Type of system	Advantages	Disadvantages
Open Cycle	This is the most currently available gas turbine system in any sector of operation	Thermal efficiency drop offs as the load is reduced below the design point
	Simple design	Considerable contaminant emissions
	Higher temperatures of heat addition and higher thermal efficiencies	
Closed Cycle	The working fluid (usually helium or air) remains clean and it does not cause corrosion or erosion	Special working fluid gas management system
	Part-load operation with high efficiency	Very high cost of investment and maintenance
		Components may be structurally suitable for high system pressures and temperatures
		High temperatures in the heat exchanger
		Lower maximum allowable heat addition temperature of a closed cycle gas turbine compared to an open cycle gas turbine, this limits the maximum power conversion efficiency

Table 13. Comparison of gas turbine cogeneration systems [1] [2] [3] [7]

The simple open cycle gas turbine cogeneration system is considered in this work as the best option, because it offers the most available technology, the simplest design, the highest thermal efficiency and the lowest capital and maintenance costs.

Although, the open cycle present contaminant emissions, in comparison with the closed cycle, where the emissions are null and the working fluid remains clean.

6.3.1 Thermodynamics of simple open cycle gas turbine cogeneration systems

The simple open cycle gas turbine system is based in the Brayton cycle, where a gas-air mix is the working substance. Figure 62 shows a simple gas turbine cogeneration system that consists of a main cycle (compressor, combustion chamber and turbine) and a heat recovery steam generator (HRSG).

The process consists of the next steps:

1. Compression process in the air compressor (1-2)
2. Mixing (air-gas) and burning process in the combustion chamber (2-3)
3. Expansion process in the gas turbine (3-4)
4. Heat rejection in the heat recovery steam generator (4-5)
5. Heat rejection in the Feed water heater (8-9)

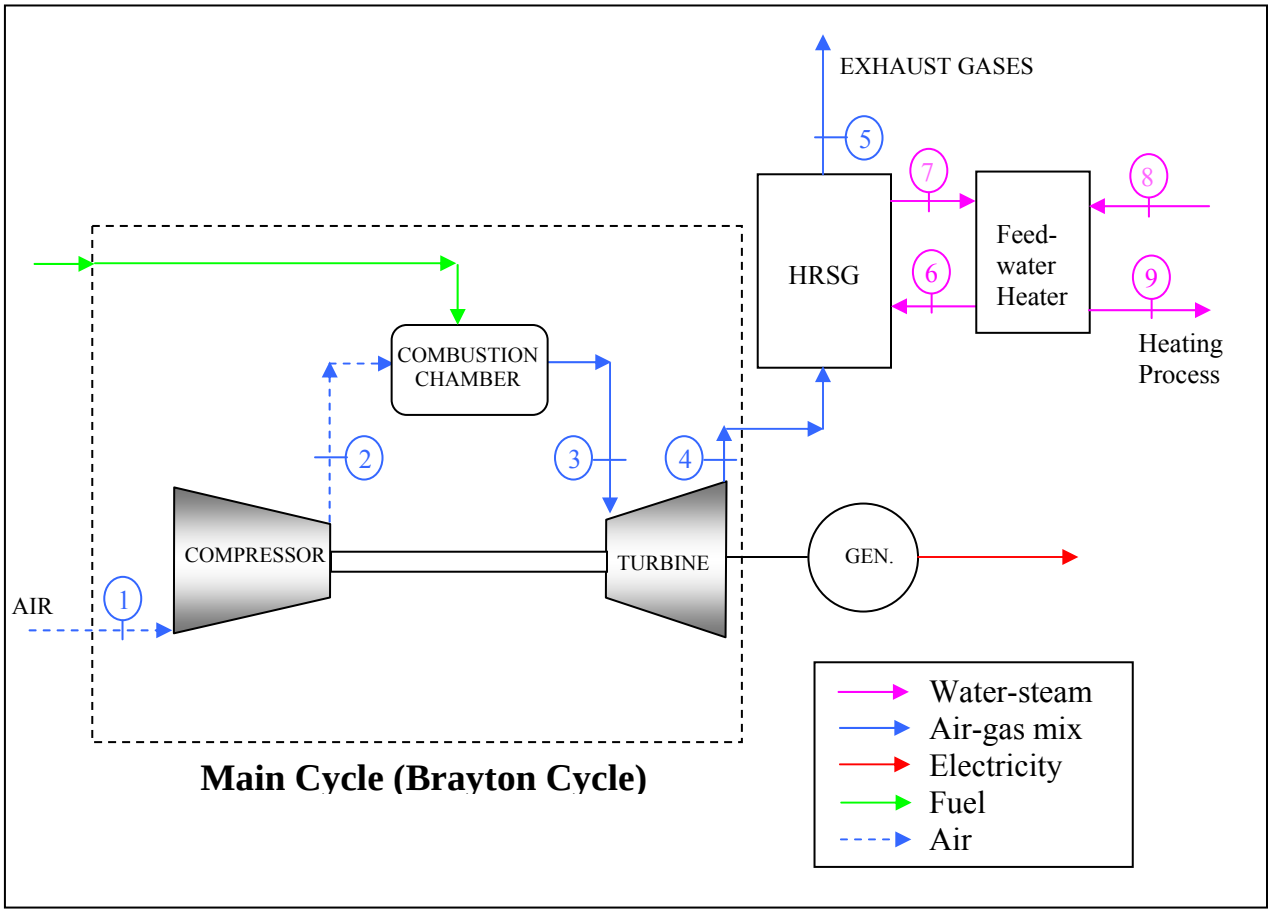


Figure 45. Simple gas turbine cogeneration system

The Figure 46 illustrates the events of the main cycle (Brayton cycle) on a temperature-entropy basis.

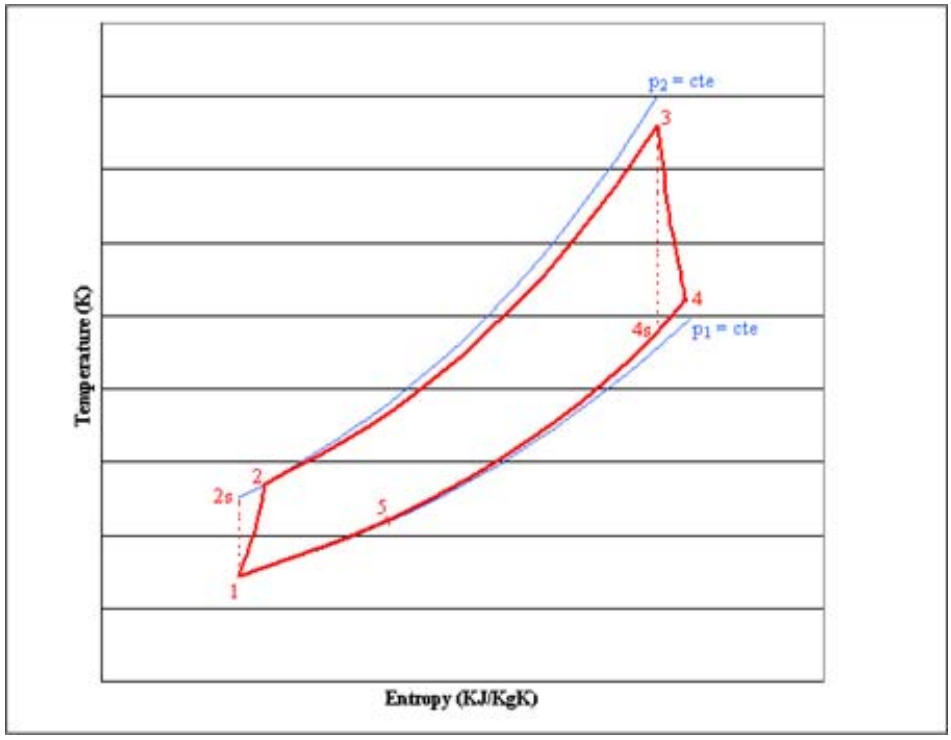


Figure 46. Temperature-entropy diagram for Brayton Cycle (Simple gas turbine)

6.3.1.1 Compression process in the air compressor (1-2)

The compressor draw in air from the environment and by adding mechanical energy, bring the system to the overpressure. The ideal isentropic compression is described by the process 1-2s, which means, that there are not losses and the process is internally reversible.
In the practical cycle, there is deterioration in the performance of the compressor due to aerodynamic losses and pressure losses.

In this work is treated the working substance as the air of fixed composition. The air is an ideal gas and during the compression have a constant specific heat and a constant specific heat ratio.
The compressor inlet temperature and the compressor inlet pressure are assumed as the ambient temperature and pressure, respectively.

The work required for the compressor is determined as follows,

$$W_C = \overset{*}{m}_a (h_2 - h_1) \qquad (80.)$$

But for ideal gases, $\Delta h = C_{pa} \Delta T$ (81.)

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma_a - 1}{\gamma_a}} = (r_c)^{\frac{\gamma_a - 1}{\gamma_a}} = \rho_a \qquad (82.)$$

Where r_c is the pressure ratio or the relation between the pressure at the outlet and at the inlet of the compressor. The isentropic efficiency of the compressor is given by,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad (83.)$$

In this work the next assumptions are made for the compressor:

- Isentropic efficiency = 87% [16]
- Specific heat at constant pressure of the air as (c_{pa}) as 1.004 KJ/(kgK) [14]
- Specific heat ratio of the air (γ_a) as 1.4 [14]
- Gas constant of the air (R_a) as 0.287 KJ/(KgK) [14]

Finally is possible to express the temperature in the state 2 with the next equation,

$$T_2 = T_1 \left\{ 1 + \frac{1}{\eta_c} \left[(r_c)^{\frac{\gamma_a - 1}{\gamma_a}} - 1 \right] \right\} \qquad (84.)$$

6.3.1.2 Combustion chamber

The air leaving the compressor is raised to the preset turbine inlet temperature by the burning of fuel in a combustor chamber. The processes that take place within the combustor are so complex that they cannot be adequately described in a purely mechanical treatment. In the ideal Brayton cycle the combustion process is an isobaric addition of heat, but in the practical cycle occur a loss in pressure owing to the aerodynamic resistance of the baffle and mixing devices and also on account of momentum changes produced by the exothermic reaction. The combustion chamber used in this work is shown in the Figure 47.

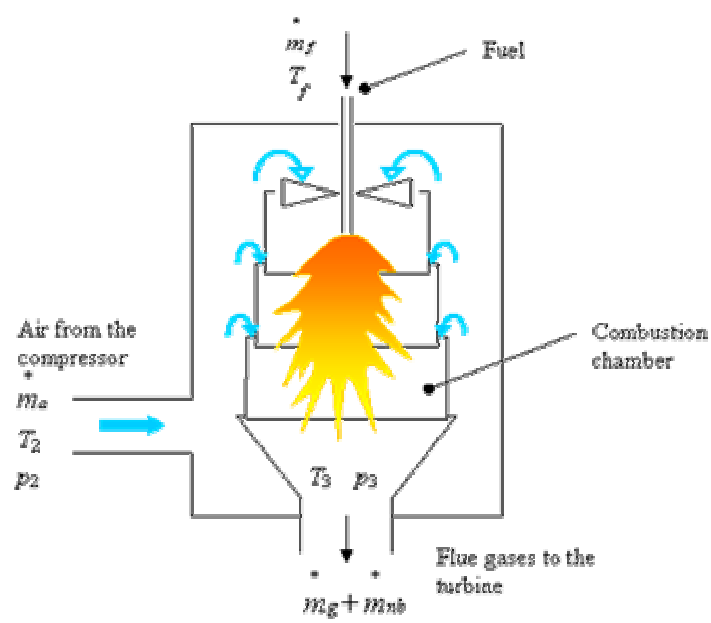


Figure 47. Combustion chamber scheme [29]

The stoichiometric quantity of oxidizer (in this case air) is the amount needed to completely burn a quantity of fuel. The stoichiometric oxidizer fuel ratio (mass) is determined by writing simple atom balances, assuming that the fuel reacts to form an ideal set of products.

For a hydrocarbon fuel given by C_xH_y, the stoichiometric relation can be expressed as:

$$C_xH_y + a(O_2 + 3.76N_2) \rightarrow xCO_2 + \left(\frac{y}{2}\right)H_2O + 3.76aN_2 \qquad (85.)$$

Where

$$a = x + \frac{y}{4} \qquad (86.)$$

For simplicity, it is assumed throughout this work that the simplified composition for air is 21% O_2 and 79% N_2 (by volume), that means, that for each mole of O_2 in air, there are 3.76 moles of N_2. [17]

The stoichiometric air-fuel ratio needed is:

$$L = \left(\frac{\text{Kg. Air required}}{\text{Fuel}}\right) = \left(\frac{4.76a}{1}\right)\left(\frac{MW_a}{MW_f}\right) = \left(\frac{\dot{m}_{a1}}{\dot{m}_f}\right) \qquad (87.)$$

Where MW_a and MW_f are the molecular weights of the air and fuel, respectively.

Therefore the stoichiometric gas mass flow is, [29]

$$\dot{m}_g = L\dot{m}_f + \dot{m}_f = \dot{m}_f(L+1) \qquad (88.)$$

Then making mass continuity in the combustion chamber,

$$\dot{m}_a + \dot{m}_f = \dot{m}_g + \dot{m}_{nb} = \dot{m}_t \qquad (89.)$$

Using the previous equations can be obtained:

$$L = \frac{\dot{m}_a - \dot{m}_{nb}}{\dot{m}_f} \qquad (90.)$$

$$L + 1 = \frac{\dot{m}_a - \dot{m}_{nb} + \dot{m}_f}{\dot{m}_f} = \frac{\dot{m}_g}{\dot{m}_f} \qquad (91.)$$

Now the second law of thermodynamics (Energy conservation law) is expressed in the next equation for the combustion chamber:

$$\overset{*}{m}_a c_{pa} T_2 + \overset{*}{m}_f \left(H_u + h_f\right) = \overset{*}{m}_g c_{pg} T_3 + \overset{*}{m}_{nb} c_{pa} T_3 + \overset{*}{m}_f \left(1 - \eta_{CCH}\right)\left(H_u + h_f\right) \qquad (92.)$$

Where $\overset{*}{m}_f \left(1 - \eta_{CCH}\right)\left(H_u + h_f\right)$ are the heat losses that occur in the combustion chamber.

And finally the fuel mass flow can be obtain, [29]

$$\overset{*}{m}_f = \frac{\overset{*}{m}_a C_{pa}\left(T_3 - T_2\right)}{\eta_{CCH}\left(H_u + h_f\right) - \left(L+1\right)C_{pg} T_3 + L C_{pa} T_3} \qquad (93.)$$

The pressure loss in the combustion chamber is expressed as

$$\Delta p_{CCH} = \frac{p_3 - p_2}{p_2} \qquad (94.)$$

In this work it is assumed $\Delta p_{CCH} = 0.05$, and $\eta_{CCH} = 0.98$. [14]

6.3.1.3 Expansion process in the gas turbine (3-4)

The principle of the gas turbine consists of the channelling of hot gases from burning fuel to spin a turbine wheel. The products of combustion enter the turbine and expand to approximately atmospheric pressure. Usually the turbine and the compressor are mounted in the same shaft, and about two-thirds of the turbine output is used to drive the compressor while the remainder is for power generation.

Turbines differ from compressors in having a relatively small number of stages. In the present research, the next assumptions for the expansion process in the turbine are taken into account:

- The expansion process is not isentropic and therefore irreversible
- The isentropic efficiency of the turbine (87% nominal) vary with several parameters, like the ambient temperature, the pressure ratio in the compressor, the air mass flow, the speed, and the gas inlet temperature to turbine
- The turbine speed is equal to the compressor speed, because they are mounted in the same shaft
- The turbine speed is constant along the operation, because this requirement is needed in power generation
- The pressure ratio in the compressor is higher than the pressure ratio in turbine, because of the pressure losses in the combustion chamber and in the heat recovery steam generator
- Specific heat at constant pressure of the combustion gas as (c_{pg}) as 1.100 KJ/(kgK), specific heat ratio of the air (γ_a) as 1.33 and gas constant (R_g) as 0.290 KJ/(KgK). [14]

Applying the second law of thermodynamics (Energy conservation law) to the expansion process in the turbine, [29]

$$\overset{*}{m}_g C_{pg} T_3 + \overset{*}{m}_{nb} C_{pa} T_3 = \left(\overset{*}{m}_g + \overset{*}{m}_{nb} \right) C_{pg} T_4 + W_T \qquad (95.)$$

And for mass continuity in the combustion chamber,

$$\overset{*}{m}_g + \overset{*}{m}_{nb} = \overset{*}{m}_a + \overset{*}{m}_F = \overset{*}{m}_t \qquad (96.)$$

Where the work generated by the turbine is, [29]

$$W_T = \overset{*}{m}_f (L+1) C_{pg} T_3 + \left(\overset{*}{m}_a - L \overset{*}{m}_f \right) C_{pa} T_3 - \left(\overset{*}{m}_a + \overset{*}{m}_f \right) C_{pg} T_4 \qquad (97.)$$

And the work net generated by the whole gas turbine is,

$$W_{GT} = W_T - W_C \qquad (98.)$$

$$W_{GT} = \overset{*}{m}_f (L+1) C_{pg} T_3 + \left(\overset{*}{m}_a - L \overset{*}{m}_f \right) C_{pa} T_3 - \left(\overset{*}{m}_a + \overset{*}{m}_f \right) C_{pg} T_4 - \overset{*}{m}_a C_{pa} (T_2 - T_1) \qquad (99.)$$

But for ideal gases,

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4} \right)^{\frac{\gamma_g - 1}{\gamma_g}} = (r_t)^{\frac{\gamma_g - 1}{\gamma_g}} = \rho_g \qquad (100.)$$

Where r_t is the pressure ratio in the turbine or the relation between the pressure at the outlet and at the inlet of the turbine.

The isentropic efficiency of the turbine is given by,

$$\eta_{GT} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} \qquad (101.)$$

Finally is possible to express the temperature in the state 4, based in the previous equations and in the turbine inlet temperature,

$$T_4 = T_3 \left\{ 1 - \eta_{GT} \left[1 - \frac{1}{(r_t)^{\frac{\gamma_g - 1}{\gamma_g}}} \right] \right\} \qquad (102.)$$

6.3.1.4 Heat rejection in the Feed water heater (8-9)

For convenience is explained first the process in the feed water heater, because the requirements of the feed water set the standard to design the feed water heater and subsequently the heat recovery steam generator (HRSG). The gases leaving the gas turbine are too warm to use directly to heating space, because of that is required a surface heater (feed water heater), which heat the feed water with the thermal energy of the steam-water from the heat recovery steam generator.

Other additional advantages of the feed water are:

- It increases the temperature of the feed water using for heating process
- It avoids problems of the directly use of the heating feed water in the heat recovery steam generator, like poor quality water and/or losses of working fluid

The feed water used in this work is a surface heater type, where the water flows inside heater tubes and absorbs the heat from the steam condensing outside. After condensing, steam leaves the heater either as saturated water. The feed water heater is a three-zone type as shown in Figure 48.

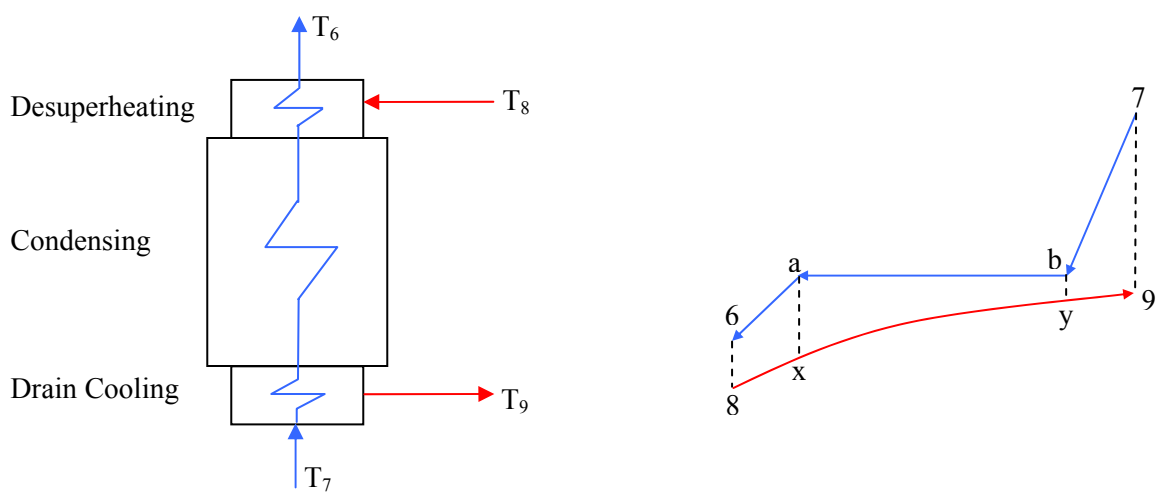


Figure 48. Flow diagram and temperature profile of feed water heater

A desuperheating zone exploits the steam superheat (7-b), a condensing zone functions like a normal heater (b-a), and a drain-cooling zone cools the drained water to a temperature close that of the incoming feed water.

The heating effect of the heater is defined by,

$$\Delta T_{heater} = T_a - T_9 = T_b - T_9 \qquad (103.)$$

And in the drain cooling zone,

$$\Delta T_{cooling} = T_6 - T_8 \qquad (104.)$$

In this work it is assumed [7]

- $\Delta T_{heater} = 1°C$
- $\Delta T_{cooling} = 7°C$

And therefore,

T_8 = 65°C, T_9 = 100°C, T_a = T_b = 101°C and T_6 = 72°C.

Applying the second law of thermodynamics (Energy conservation law) to the feed water heater,

$$\overset{*}{m}_s \left(h_7 - h_6\right) = \overset{*}{m}_d \left(h_9 - h_8\right) \qquad (105.)$$

Where,

- h_9 = 420.2250 KJ/Kg (T_9 = 100°C, p_9 = 16 bar)
- h_8 = 273.384 KJ/Kg (T_8 = 65°C, p_8 = 16 bar)

The heating process is considered at constant pressure,

$p_6 = p_a = p_b = p_7 = p_{sat}$ @ 101°C = 0.105091 Mpa

And then is possible find the enthalpies of the water-steam from the HRSG,

- h_a = 423.32 KJ/Kg (Saturated liquid, T_a = 101°C, p_a = 0.105091 Mpa)
- h_b = 2677.15 KJ/Kg (Saturated steam, T_b = 101°C, p_b = 0.105091 Mpa)
- h_6 = 301.4558 KJ/Kg (T_6 = 72°C, p_6 = 0.105091 Mpa)

6.3.1.5 Heat rejection in the heat recovery steam generator (4-5)

The heat recovery steam generator (HRSG) is one of the few components of combined cycle gas turbine power plants tailored for each specific application. The selection will affect not only the initial cost, but also the operating cost.

Water enters the HRSG in form of compressed liquid. As water receives heat from the hot exhaust gases, it becomes saturated, evaporated, and eventually superheated. On the hot side, the exhaust gases leaving the gas turbine enter the HRSG and release thermal energy.

Many design parameters must be evaluated, some based upon experience and other upon economic considerations.

In this work the assumptions listed below are taken into account in the design of the heat recovery steam generator.

- The pinch point of the heat recovery steam generator is assumed as 20 K as a practical value [3] [15]
- The temperature of the exhaust gases at exit of the heat recovery steam generator is assumed between 6 K higher than the dew point temperature, in order to avoid the humidity in the flue gases and the possible corrosion. [3] [15]
- The temperature approach in the economizer (TAE) is assumed 5 K in order to avoid the dangers of water hammer and steam blanketing in the economizer [3]
- According with General Electric, the temperature of the exhaust gases leaving the turbine and entering to the HRSG is optimal between 538°C - 566°C, in this work the temperature of the exhaust gases is assumed as 550°C [16]

The Figure 49 shows the scheme of the heat recovery steam generator used in this research and the respective assumptions.

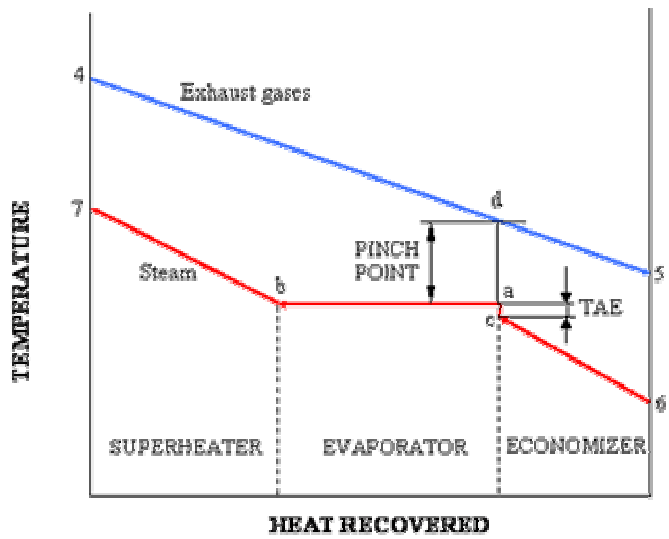


Figure 49. Temperature profiles of the Heat Recovery Steam Generator

The dew point is calculated with the next equation [30]:

$$T_{Dew} = \frac{237.7\alpha}{17.27 - \alpha} = [°C] \qquad (106.)$$

Where

$$\alpha = \frac{17.27 T_{ambient}}{237.7 + T_{ambient}} + \ln(RH) \qquad (107.)$$

These equations are valid for ambient temperatures between 0°C-60°C and a relative humidity (RH) between 0-1.

In this work the dew point is calculated for the standard ambient conditions, that is, ambient temperature as 15°C and a relative humidity as 60%.

T_{Dew} = 100.15 °C

And therefore,

$T_5 = T_{Dew} + 6°C = 106.15$ °C (108.)

The pressure of the gases leaving the HRSG is assumed as atmospheric pressure, that is

$p_5 = p_1 = p_{atm} = 101325$ Pa (109.)

The steam-water temperature in the evaporator is known,

$T_a = T_b = 101$ °C (110.)

The pinch point is known and also the temperature approach in the economizer, so is possible to know,

- T_c = 96 °C, p_c = 0.105091 Mpa
- h_c = 402.2452 KJ/Kg
- T_d = 121 °C

Now making the second law of thermodynamics (Energy conservation law) to the different sections of the Heat Recovery Steam Generator,

$$\overset{*}{m}_g C_{pg}(T_d - T_5) = \overset{*}{m}_s (h_c - h_6) \quad (111.)$$

$$\overset{*}{m}_g C_{pg}(T_4 - T_5) = \overset{*}{m}_s (h_7 - h_6) \quad (112.)$$

And also is possible express the heat transferred to the feed water, or heat generated for the gas turbine cycle as,

$$Q_{GT} = \overset{*}{m}_g C_{pg}(T_4 - T_5) = \overset{*}{m}_s (h_7 - h_6) = \overset{*}{m}_d (h_9 - h_8) \quad (113.)$$

Using the previous equations it is possible obtain,

$T_7 = 417°C$, $h_7 = 3313.9389$ KJ/Kg, $T_e = 456.17°C$.

$$\frac{\overset{*}{m}_s}{\overset{*}{m}_g} = 0.16207 \qquad (114.)$$

$$\frac{\overset{*}{m}_d}{\overset{*}{m}_s} = 20.5152 \qquad (115.)$$

6.3.2 Performance characteristics of the simple open cycle gas turbine cogeneration systems

6.3.2.1 Heat to power ratio

The next equation shows the heat to power ratio for a simple open cycle gas turbine cogeneration system, [2]

$$\lambda_{GT} = \frac{Q_{GT}}{W_{GT}} \qquad (116.)$$

$$\lambda_{GT} = \frac{\left(\overset{*}{m}_a + \overset{*}{m}_f\right) C_{pg}\left(T_4 - T_5\right)}{\left[\overset{*}{m}_f (L+1) C_{pg} T_3 + \left(\overset{*}{m}_a - L\overset{*}{m}_f\right) C_{pa} T_3 - \left(\overset{*}{m}_a + \overset{*}{m}_f\right) C_{pg} T_4 - \overset{*}{m}_a C_{pa}\left(T_2 - T_1\right)\right]} \qquad (117.)$$

In this equation is shown that the heat to power ratio depends on the temperatures of the whole cycle, on the fuel and air mass flow, and on the stoichiometric ratio.

6.3.2.2 Power control in gas turbine cogeneration system

The temperature at the turbine inlet (T_3) is reduced by reducing the amount of fuel burnt in the combustion chamber. This is the obvious way to reduce the power output of a gas turbine, and as such it has been implemented systematically on most engines.

The air mass flow remains constant and the fuel mass flow is directly proportional to the power output. The rotary speed of the gas turbine must remain constant under variation of the load, in order to reach the requirements of alternating current generation of the utility grid, as constant frequency.

6.3.2.3 Partial load operation

For definition, the operating condition where an engine will spend most time has been traditionally chosen as the engine design point. In this work, the term *design point* refers to the gas turbine design point in the concept design phase, which is taken to be coincident with the component design points. [31]

The standard ambient conditions are taken as the ambient conditions in the concept design phase, which means, an ambient temperature as 15°C and a relative humidity as 60% [31]. With the gas turbine components are fixed by the design point calculation, the performance of the gas turbine at other key operating conditions can be evaluated, and the procedure is the off design performance calculation. Here the gas turbine components are fixed and operating conditions are changing. The operating conditions change mainly for two reasons:

- The ambient conditions change during the year
- The power load of the gas turbine change during the year, because the power demand changes also

Many variables are required to describe numerically engine performance throughout the operational envelope. In the practice, the dimensionless parameters are used to reduce the number of variables that influence the behaviour of the gas turbine. To a first order the off design performance may be defined via charts showing the interrelationship of the referred dimensionless parameters. For a given operational condition, knowledge of the values of inlet pressure and temperature allows actual performance parameter values to be easily calculated.

In this work the off design performance map used to predict the behaviour of all the gas turbines is the "Dimensionsloses Kennfeld eines Verdichters" presented by Technische Universität Darmstadt, Fachgebiet Gasturbinen und Flugantriebe in the Seminar in Flugantriebe (See Figure 50). [33]

The work area of the gas turbine in the off design performance map (red lines) is delimited by:

- The maximal and minimal temperatures ratio (T_3/T_1) are 1 and 0.45 respectively
- The dimensionless parameter $\frac{n}{n_n\sqrt{\Theta_1}}$ depends only on the ambient temperature changes, because the rotary speed remains constant ($n = n_n$), and the limits are 1.1 and 0.95 in accord with the limits of temperature changes
- The stall line

The relation between the isentropic efficiency of the turbine and the partial load is determined by the equation 137. [20]

$$\frac{[\eta_t]_{pl}}{[\eta_t]_{Full}} = 1 - K_1\left(\frac{[r_t]_{Full}}{[r_t]_{pl}} - 1\right)^2 \qquad (118.)$$

K_1 for modern turbines is 0.2. [20]

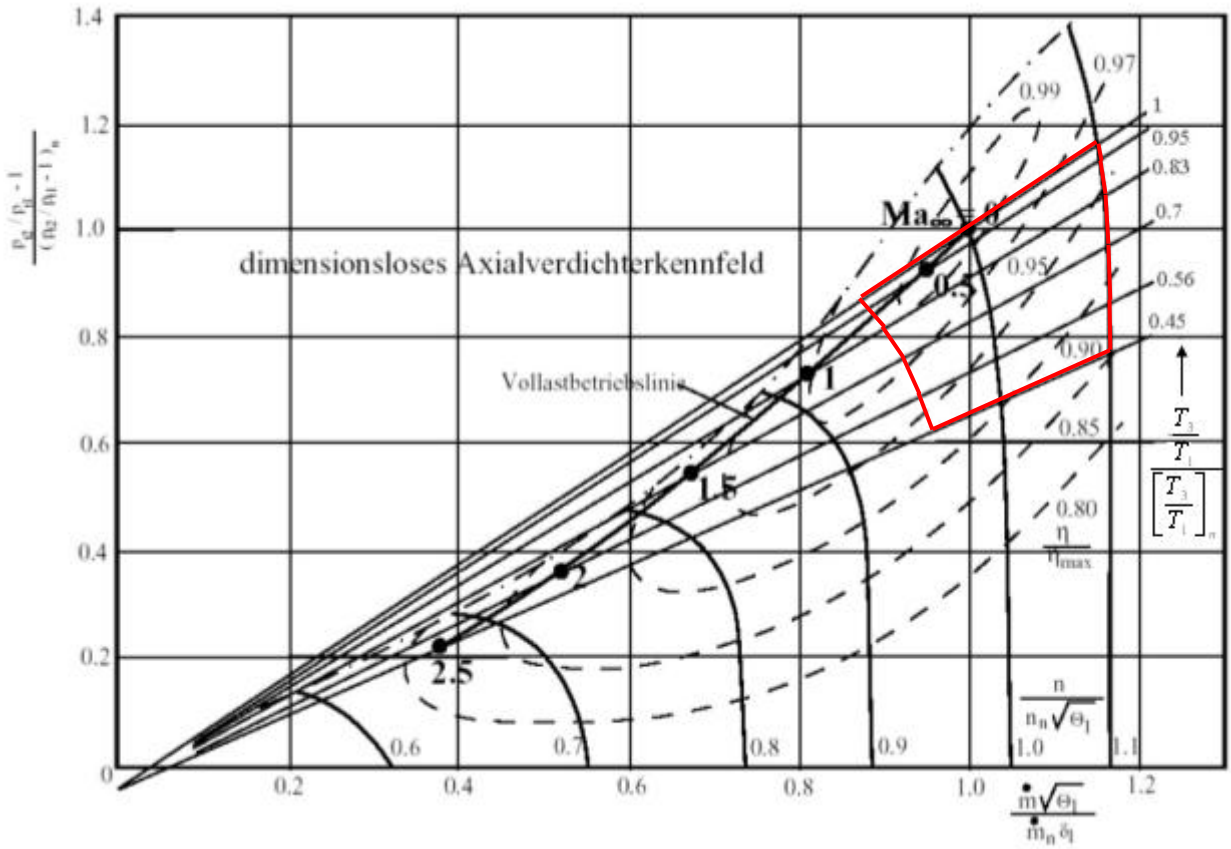


Figure 50. Off design compressor-turbine performance map [33]

With the off design performance map, the work area and the previous equation is possible obtain relations between the different dimensionless parameters in function of the temperature ratio (T_3/T_1), which is the most important parameter to control the output power.

The dimensionless parameters are redefined as:

$$ND1 = \left[\frac{T_3}{T_1}\right]_{pl} \Big/ \left[\frac{T_3}{T_1}\right]_{Full} \qquad (119.)$$

$$ND2 = \left[\overset{*}{m}_a\right]_{pl} \sqrt{\Theta_1} \Big/ \left[\overset{*}{m}_a\right]_{Full} \delta_1 \qquad (120.)$$

$$ND3 = \frac{n}{n_{full}\sqrt{\Theta_1}} = \frac{1}{\sqrt{\Theta_1}} \qquad (121.)$$

$$ND4 = \frac{[r_c - 1]_{pl}}{[r_c - 1]_{full}} \qquad (122.)$$

$$ND5 = \frac{[\eta_C]_{pl}}{[\eta_C]_{full}} \qquad (123.)$$

The relations between these parameters are presented graphically in the next figures.

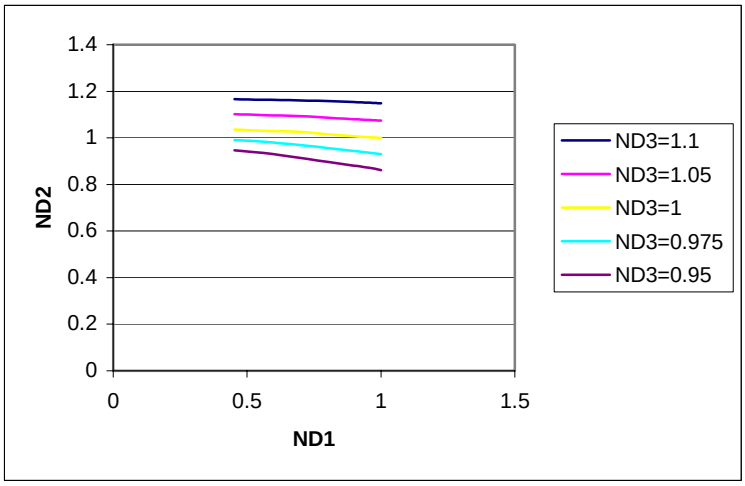


Figure 51. Relation of parameters ND1 and ND2

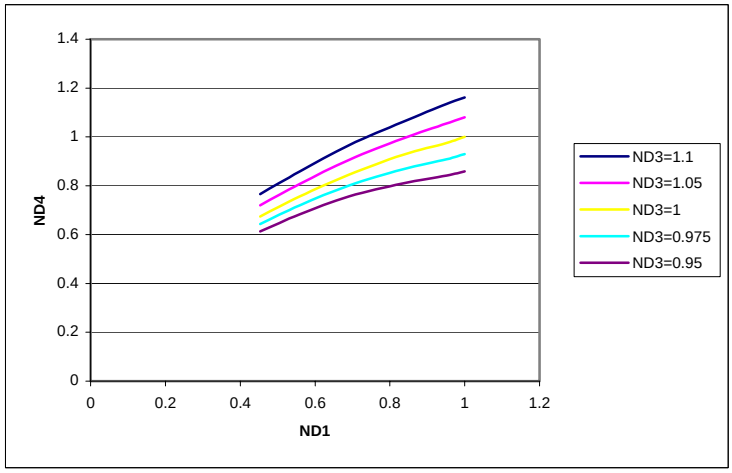


Figure 52. Relation of parameters ND1 and ND4

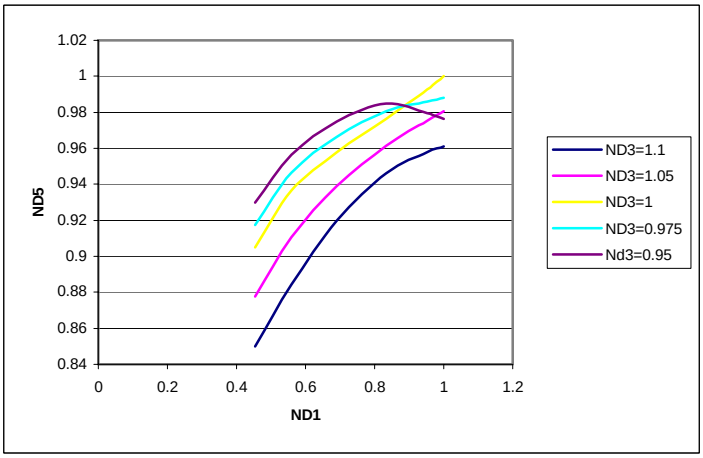


Figure 53. Relation of parameters ND1 and ND5

6.4 Micro-gas turbine cogeneration systems

Mini and micro-gas turbines offer a number of potential advantages compared to other technologies for small-scale power generation, for example, compact size and low-weight per unit power leading to reduced civil engineering costs, a small number of moving parts, lower noise, multi-fuel capabilities as well as opportunities for lower emissions. In addition, gas turbines enjoy certain merits relative to diesel engines in the context of mini and micro-power generation. They have high-grade waste heat, low maintenance cost, low vibration level and short delivery time. Although there are some technical and non-technical barriers for the implementation of the technology, the market potential could increase substantially if the cost, efficiency, durability, reliability and environmental emissions of the existing designs are improved. [27] [36]

Micro gas turbines, for the most part, single-stage, single-shaft, low pressure ratio. Systems may be either simple cycle or recuperated. In this work are analysed the micro gas turbines with a recuperated cycle (heat recovered from the exhaust for preheating of air), because doubles the electrical efficiency of the unit whilst reducing the amount of recoverable heat from the boiler.[27]

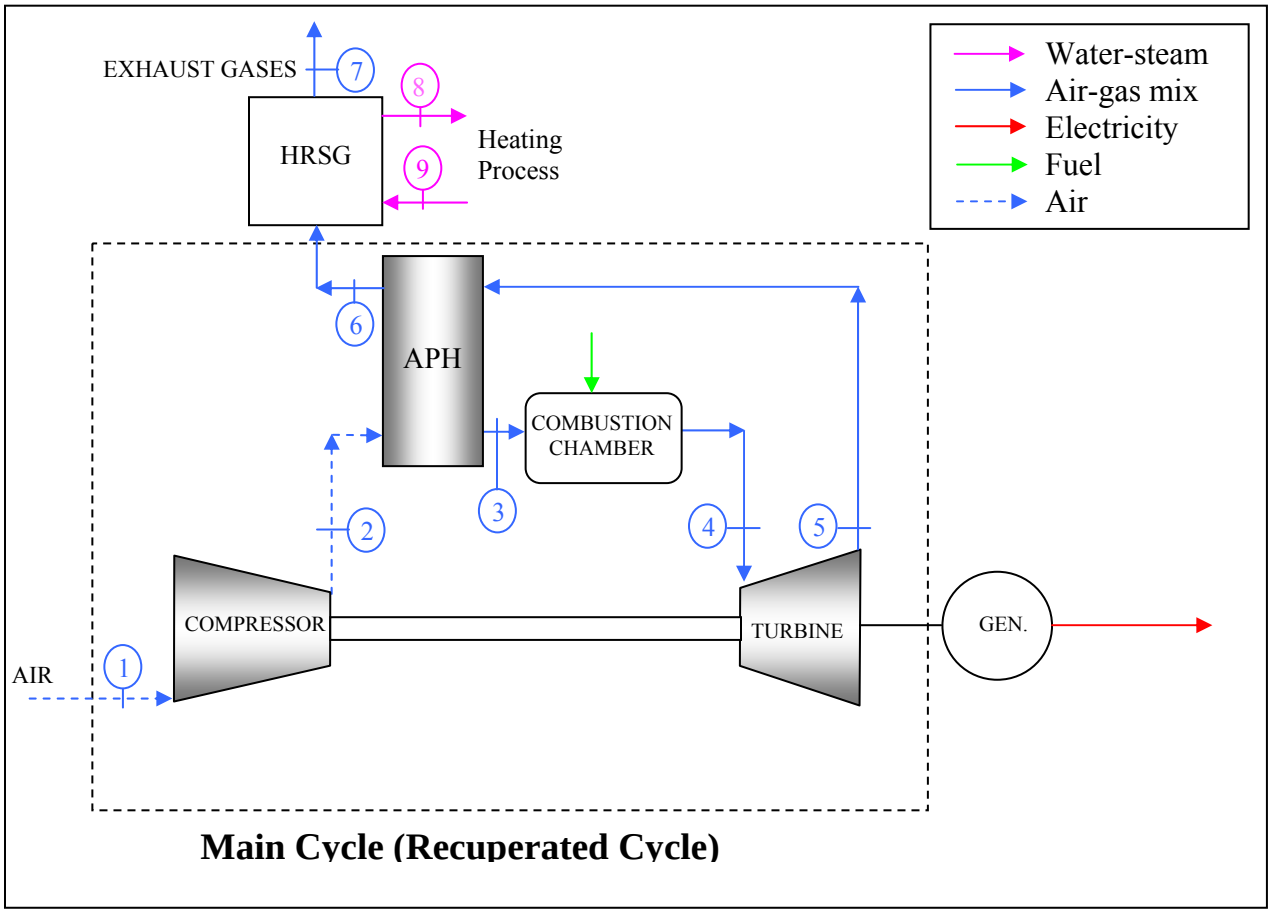


Figure 54. Micro gas turbine regenerated cycle

6.4.1 Micro-gas turbine cogeneration systems

The recuperated Brayton cycle consists of the next steps:

1. Compression process in the air compressor (1-2)
2. Air heating in the air preheater (2-3)
3. Mixing (air-gas) and burning process in the combustion chamber (3-4)
4. Expansion process in the gas turbine (4-5)
5. Heat rejection in the air preheater (5-6)
6. Heat rejection in the heat recovery steam generator (6-7)

The Figure 55 shows the recuperated cycle in a diagram temperature versus entropy.

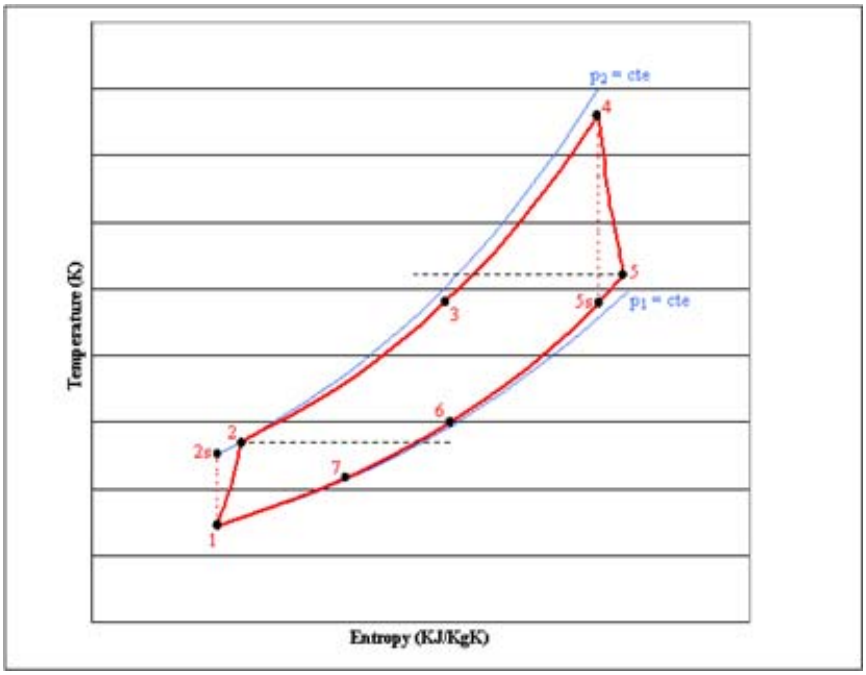


Figure 55. Temperature-entropy diagram for the recuperated Brayton cycle (Micro gas turbine cycle)

6.4.1.1 Compression process in the air compressor (1-2)

The process of compression is exactly the same that the used for normal gas turbines, but the pressure ratio is smaller than for those, normally between $3 < r_c < 5.5$, therefore the complexity of axial compressors is not justified and the simpler to design and construct radial compressor is universally used. [27] [36]

The work required for the compressor is,

$$W_{MC} = \overset{*}{m}_a (h_2 - h_1) \qquad (124.)$$

The isentropic efficiency of the compressor is given by,

$$\eta_{MC} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad (125.)$$

In this work the next assumptions are made for the micro gas turbine compressor:

- Isentropic efficiency = 87% [16]
- Specific heat at constant pressure of the air as (C_{pa}) as 1.004 KJ/(kgK) [14]
- Specific heat ratio of the air (γ_a) as 1.4 [14]
- Gas constant of the air (R_a) as 0.287 KJ/(KgK) [14]

All the assumptions are made for ideal gases and it is possible obtain the next relation between the temperatures at the inlet and outlet of the compressor,

$$T_2 = T_1 \left\{ 1 + \frac{1}{\eta_{MC}} \left[(r_c)^{\frac{\gamma_a - 1}{\gamma_a}} - 1 \right] \right\} \qquad (126.)$$

6.4.1.2 Air heating-Heat rejection in the air preheater (2-3),(5-6)

The cycle efficiency of the simple Brayton cycle can be improved by introducing a regenerator or air preheater. In the air preheater heat is transferred from the turbine exhaust gas to the compressed air leaving the compressor. The temperature of compressed air leaving the regenerator is always less than the exhaust gas temperature and the temperature of compressed air entering the regenerator is always less than the gases leaving the air preheater.

$$T_3 < T_5 \qquad (127.)$$

$$T_2 < T_6 \qquad (128.)$$

The air preheater efficiency is defined by,

$$\eta_{\text{Preh}} = \frac{h_3 - h_2}{h_5 - h_2} = \frac{C_{pa} T_3 - C_{pa} T_2}{C_{pg} T_5 - C_{pa} T_2} \qquad (129.)$$

The regenerator in the micro gas turbine system is relatively inefficient. The heat transfer surface area is usually large in order to keep the pressure drop across it as small as possible.

In this work, the assumptions made in the regenerator are,

- The air preheater efficiency between 70% - 80% [3]
- T_6 between 300° - 310° C [27] [38] [39]
- Pressure loss in the air side as 5% [14]
- Pressure loss in the gases side as 3% [14]

Applying the second law of thermodynamics (Energy Conservation Law),

$$\overset{*}{m}_a C_{pa}(T_3 - T_2) = \left(\overset{*}{m}_a + \overset{*}{m}_f\right) C_{pg}(T_5 - T_6) \qquad (130.)$$

Can be deduced the temperature of the compressed air entering the combustion chamber,

$$T_3 = (1 - \eta_{\mathrm{PH}}) T_2 + \eta_{\mathrm{PH}} T_5 \left(\frac{C_{pg}}{C_{pa}}\right) \qquad (131.)$$

The pressure losses can be expressed as,

$$\frac{p_6}{p_5} = 0.97 \qquad (132.)$$

$$\frac{p_3}{p_2} = 0.95 \qquad (133.)$$

6.4.1.3 Combustion chamber (3-4)

The air leaving the air preheater is raised to the preset turbine inlet temperature by the burning of fuel in a combustor chamber.

For a hydrocarbon fuel given by C_xH_y, the stoichiometric relation can be expressed as:

$$C_x H_y + a(O_2 + 3.76 N_2) \rightarrow x CO_2 + \left(\frac{y}{2}\right) H_2O + 3.76 a N_2 \qquad (134.)$$

Where

$$a = x + \frac{y}{4} \qquad (135.)$$

For simplicity, it is assumed throughout this work that the simplified composition for air is 21% O_2 and 79% N_2 (by volume), that means, that for each mole of O_2 in air, there are 3.76 moles of N_2. [29]

The stoichiometric air-fuel ratio needed is:

$$L = \left(\frac{\text{Kg. Air required}}{\text{Fuel}}\right) = \left(\frac{4.76a}{1}\right)\left(\frac{MW_{AIR}}{MW_{FUEL}}\right) = \left(\frac{\overset{*}{m}_{a1}}{\overset{*}{m}_f}\right) \qquad (136.)$$

Where MW_{AIR} and MW_{FUEL} are the molecular weights of the air and fuel, respectively.

Therefore the stoichiometric gas mass flow is:

$$\overset{*}{m}_g = L\overset{*}{m}_f + \overset{*}{m}_f = \overset{*}{m}_f (L+1) \qquad (137.)$$

Then making mass continuity in the combustion chamber, [29]

$$\overset{*}{m}_a + \overset{*}{m}_f = \overset{*}{m}_g + \overset{*}{m}_{nb} = \overset{*}{m}_t \qquad (138.)$$

Where $\overset{*}{m}_{nb}$ is the portion of the total air that is not burned, and that can be expressed so, [29]

$$\overset{*}{m}_{nb} = \overset{*}{m}_a - L\overset{*}{m}_f \qquad (139.)$$

Using those equations can be obtained:

$$L+1 = \frac{\overset{*}{m}_a - \overset{*}{m}_{nb} + \overset{*}{m}_f}{\overset{*}{m}_f} = \frac{\overset{*}{m}_g}{\overset{*}{m}_f} \qquad (140.)$$

Now the second law of thermodynamics (Energy conservation law) is expressed in the next equation for the combustion chamber:

$$\overset{*}{m}_a C_{pa} T_3 + \overset{*}{m}_f (H_u + h_f) = \overset{*}{m}_g C_{pg} T_4 + \overset{*}{m}_{nb} C_{pa} T_4 + \overset{*}{m}_f (1-\eta_{CCH})(H_u + h_f) \qquad (141.)$$

And finally the fuel mass flow can be obtained, [29]

$$\overset{*}{m}_f = \frac{\overset{*}{m}_a C_{pa}(T_4 - T_3)}{\eta_{CCH}(H_u + h_f) - (L+1)C_{pg}T_4 + LC_{pa}T_4} \qquad (142.)$$

The pressure loss in the combustion chamber is expressed as

$$\Delta P_{CCH} = \frac{p_4 - p_3}{p_3} \qquad (143.)$$

In this work is assumed $\Delta P_{CCH} = 0.05$, and $\eta_{CCH} = 0.98$. [14]

6.4.1.4 Expansion process in the gas turbine (4-5)

The principle of the micro gas turbine is practically equivalent to the concept of the normal gas turbines. The products of combustion enter the turbine and expand to approximately atmospheric pressure. In this case, the turbine and the compressor are mounted in the same shaft with an alternator of high speed device (typically rotating at 75.000-100.000 rpm) [27]. The maximal turbine inlet temperature is approximately 1400°C, according with the state-of-the art. [27] [36]

In the present research, the assumptions made for the simple gas turbine are also taken into account for the micro gas turbine.

Applying the second law of thermodynamics (Energy conservation law) to the expansion process in the micro gas turbine,

$$\overset{*}{m}_g C_{pg} T_4 + \overset{*}{m}_{nb} C_{pa} T_4 = \left(\overset{*}{m}_g + \overset{*}{m}_{nb} \right) C_{pg} T_5 + W_T \qquad (144.)$$

Where the work generated by the turbine is,

$$W_{MT} = \overset{*}{m}_f (L+1) C_{pg} T_4 + \left(\overset{*}{m}_a - L \overset{*}{m}_f \right) C_{pa} T_4 - \left(\overset{*}{m}_a + \overset{*}{m}_f \right) C_{pg} T_5 \qquad (145.)$$

And the net power generated by the whole micro gas turbine is,

$$W_{MGT} = W_{MT} - W_{MC} \qquad (146.)$$

$$W_{MGT} = \overset{*}{m}_f (L+1) C_{pg} T_4 + \left(\overset{*}{m}_a - L \overset{*}{m}_f \right) C_{pa} T_4 - \left(\overset{*}{m}_a + \overset{*}{m}_f \right) C_{pg} T_5 - \overset{*}{m}_a C_{pa} (T_2 - T_1) \qquad (147.)$$

The isentropic efficiency is assumed for all the micro gas turbines in this work as 87%, specific heat at constant pressure of the combustion gas as (c_{pg}) as 1.100 KJ/(kgK), specific heat ratio of the air (γ_a) as 1.33 and gas constant (R_g) as 0.290 KJ/(KgK). [14] [16] [27]

Finally is possible to express the temperature in the state 4, based in the previous equations and in the turbine inlet temperature,

$$T_5 = T_4 \left\{ 1 - \eta_{MGT} \left[1 - \frac{1}{(r_t)^{\frac{\gamma_g - 1}{\gamma_g}}} \right] \right\} \qquad (148.)$$

6.4.1.5 Heat rejection in the heat recovery steam generator (6-7)

Water enters the HRSG in form of compressed liquid. As water receives heat from the hot exhaust gases, it becomes saturated liquid. As it was explained before, the temperature of the feed water for heating process at the inlet and outlet are respectively 65°C and 100°C, at pressure 16 bar. The Figure 56 shows the scheme of the heat recovery steam generator used in the micro gas turbine cycle.

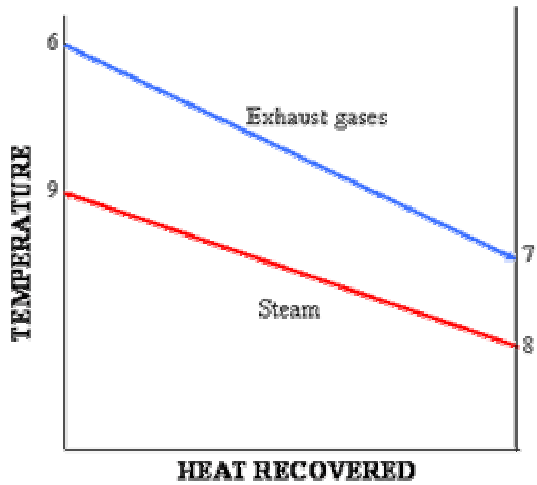


Figure 56. Temperature profiles of the Heat Recovery Steam Generator for micro gas turbines

The dew point is calculated for the standard ambient conditions, that is, ambient temperature as 15°C and a relative humidity as 60% and it is equal to T_{Dew} = 100.15 °C. [30]

The temperature of the exhaust gases is assumed as 10°C higher than the dew point, [3]

$$T_7 = T_{Dew} + 10°C = 110.15\ °C \qquad (149.)$$

The pressure of the gases leaving the HRSG is assumed as atmospheric pressure, that is

$$p_7 = p_1 = p_{atm} = 101325\ Pa \qquad (150.)$$

Now making the second law of thermodynamics (Energy conservation law) to the Heat Recovery Steam Generator,

$$\overset{*}{m}_t C_{pg}(T_6 - T_7) = \overset{*}{m}_s (h_9 - h_8) \qquad (151.)$$

And also is possible express the heat transferred to the feed water, or heat generated for the gas turbine cycle as,

$$Q_{MGT} = \overset{*}{m}_t C_{pg}(T_6 - T_7) = \overset{*}{m}_s (h_9 - h_8) \qquad (152.)$$

6.4.2 Performance characteristics of the micro gas turbine cogeneration systems

6.4.2.1 Heat to power ratio

The next equation shows the heat to power ratio for a simple open cycle gas turbine cogeneration system,

$$\lambda_{MGT} = \frac{Q_{MGT}}{W_{MGT}} \qquad (153.)$$

$$\lambda_{MGT} = \frac{\left(\overset{*}{m}_a + \overset{*}{m}_f\right) C_{pg}\left(T_6 - T_7\right)}{\left[\overset{*}{m}_f\left(L+1\right)C_{pg}T_4 + \left(\overset{*}{m}_a - L\,\overset{*}{m}_f\right)C_{pa}T_4 - \left(\overset{*}{m}_a + \overset{*}{m}_f\right)C_{pg}T_5 - \overset{*}{m}_a\, C_{pa}\left(T_2 - T_1\right)\right]} \qquad (154.)$$

6.4.2.2 Partial load operation

In this research, the micro gas turbines work all the time at full load, and it is assumed that the *design point* is taken at the ambient conditions, which means, an ambient temperature as 15°C and a relative humidity as 60%. The isentropic efficiency of the compressor and the turbine, the pressure ratio, the fuel mass flow and the air mass flow remains constant and no changes with the ambient temperature.

6.5 Heat Pumps

The heat pump used in this research work is a vapour compression heat pump in cascade (Cascade Heat Pump System), which takes waste heat, applies work to the operating fluid and provides heat at a higher temperature than the waste heat source.

The cascade cycle is commonly used for industrial application that incorporates two or more heat pump cycles in series. This is done to acquire high temperatures, which cannot be achieved with a single heat pump cycle. The Figure 57 shows a schema of a cascade heat pump system, where the source of motive power for the compressors could be electric motor, internal combustion engine, reciprocating steam engine or turbine. [10]

Heat pump systems offer much more efficient means of producing heat than traditional combustion or electrical short circuit technologies. Heat pump systems are therefore becoming more common as the prices of fuels and electricity increase. The heat pump systems are used here specially as auxiliary equipments to generate an excess of heat, in peak demand cases. In all the cases the heat pumps are droved with an electric motor, using the power generated by the main cycle driver, as for example a gas turbine or a steam turbine.

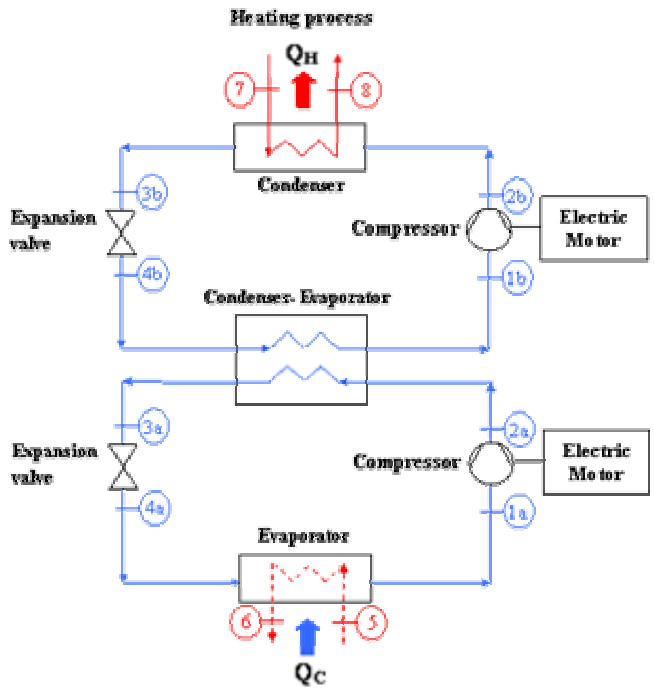


Figure 57. Diagram of a compression heat pump

6.5.1 Thermodynamics of Heat Pumps

In order to make a simple explanation of the system, here there are explained two heat pumps in cascade. Eight steps compose the basic heat pump cycle in cascade:

1. Compression process in the first cycle (1a-2a)
2. Constant pressure heat rejection in the condenser-evaporator in the first cycle (2a-3a)
3. Expansion process in the valve in the first cycle (3a-4a)
4. Constant pressure heat addition in the evaporator in the first cycle (4a-1a)
5. Compression process in the second cycle (1b-2b)
6. Constant pressure heat rejection in the condenser-evaporator in the second cycle (2b-3b)
7. Expansion process in the valve in the second cycle (3b-4b)
8. Constant pressure heat addition in the evaporator in the second cycle (4b-1b)

The eight steps are also presented in a temperature-entropy diagram in the Figure 58, where is possible observe the advantages of use two or more heat pumps in cascade, because there are an decrease in the ideal compressor work but also an increase in the heat required in the first cycle (Q_L).

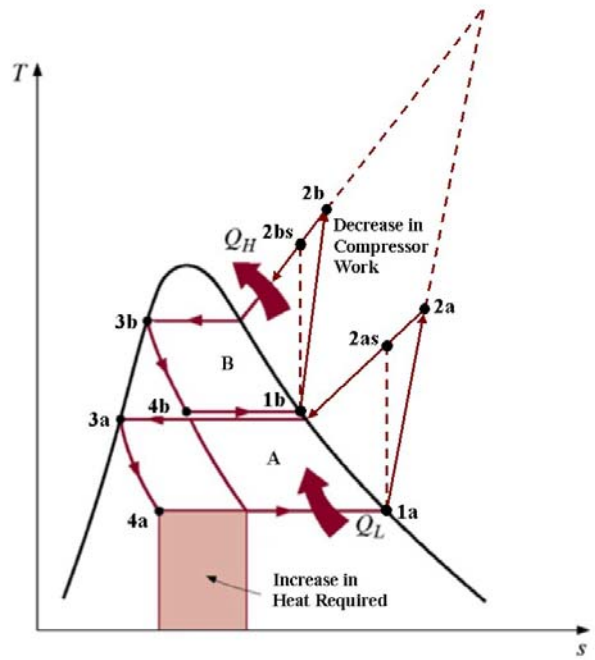


Figure 58. Temperature-entropy diagram for a Heat pump cycle [10]

6.5.1.1 Compression process in the first cycle (1a-2a)

The compressor receives refrigerant saturated vapour and compress it increasing the temperature, in general the compression of saturated vapours results in superheating of the vapour.
However the thermodynamic properties of some working fluids imply that partial condensation should result on compression of the saturated vapour over certain pressure ranges.

The compressor is directly droved by an electric motor with negligible mechanical losses. The compressor has an isentropic efficiency determined by the expression, [9]

$$\eta_{C_{HP1}} = 1 - 0.05\left(\frac{p_{2a}}{p_{1a}}\right) \qquad (155.)$$

The isentropic efficiency of the compressor can be expressed also in terms of the enthalpies, [9]

$$\eta_{C_{HP1}} = \frac{h_{2as} - h_{1a}}{h_{2a} - h_{1a}} \qquad (156.)$$

Where h_{2s} is the entropy of the pressurised vapour with an isentropic compression (isentropic efficiency of the compressor as the unit). The work required to increase the pressure of the saturated vapour from 1 to 2 is determined as,

$$W_{HP1} = \overset{*}{m}_{\mathrm{Ref1}}\left(h_{2a} - h_{1a}\right) \qquad (157.)$$

The pressure and the temperature determine the enthalpies of the refrigerant in the states 1a and 2a,

$h_{1a} = f\,(p_{1a}, T_{1a})$ (158.)

$h_{2a} = f\,(p_{2a}, T_{2a})$ (159.)

6.5.1.2 Constant pressure heat rejection in the condenser-evaporator in the first cycle (2a-3a)

The compressed refrigerant vapour leaving the compressor passes through the condenser, in which the refrigerant vapour of the first cycle transfers energy to the wet vapour refrigerant of the second cycle.

In this work it is assumed that for the practical condenser there is a temperature difference of 10°C between the maximal temperature of the refrigerant in the first cycle and the highest temperature of the refrigerant in the second cycle, this is expressed with the next equation.

$$T_{2a} = T_{1b} + 10K \qquad (160.)$$

Using the second law thermodynamics,

$$\overset{*}{m}_{\mathrm{Ref1}}\left(h_{2a} - h_{3a}\right) = \overset{*}{m}_{\mathrm{Ref2}}\left(h_{1b} - h_{4b}\right) \qquad (161.)$$

$$Q_{HP1} = \overset{*}{m}_{\mathrm{Ref1}}\left(h_{2a} - h_{3a}\right) \qquad (162.)$$

6.5.1.3 Expansion process in the valve in the first cycle (3a-4a)

The high pressure saturated liquid at the outlet of the condenser enters to the expansion valve, where the expansion takes place isenthalpically between the condenser and the evaporator. The working fluid at the outlet of the expansion valve is a low pressure wet vapour and enters to the condenser.

No work is added or taken out from the expansion valve, therefore

$$W_{3a4a} = 0 \qquad (163.)$$

$$Q_{3a4a} = 0 \qquad (164.)$$

$$h_{3a} = h_{4a} \qquad (165.)$$

6.5.1.4 Constant pressure heat addition in the evaporator in the first cycle (4a-1a)

The low pressure wet vapour from the expansion valve enters to the evaporator and it becomes saturated vapour. In this process, the source heat transfers heat to the refrigerant in order to convert it in a saturated vapour state. In this research the source heat is water and it is assumed that the plant count with a source of water at 20°C. For the practical evaporator there is a temperature difference of 10°C between the maximal temperature of the water source and the temperature of the refrigerant, this is expressed with the next equation.

$$T_5 = T_{1a} + 10K \tag{166.}$$

$$T_{1a} = T_{4a} \tag{167.}$$

Using the second law thermodynamics,

$$\overset{*}{m}_{\mathrm{Ref1}}\left(h_{1a} - h_{4a}\right) = \overset{*}{m}_{whs}\left(h_5 - h_6\right) \tag{168.}$$

$$Q_L = \overset{*}{m}_{\mathrm{Ref}}\left(h_1 - h_4\right) \tag{169.}$$

6.5.1.5 Compression process in the second cycle (1b-2b)

The compressor of the second heat pump cycle presents the same behaviour that the compressor of the first cycle. The equations that express thermodynamic characteristics of the compressor are enumerated next. [9]

$$\eta_{C_{HP2}} = 1 - 0.05\left(\frac{p_{2b}}{p_{1b}}\right) \tag{170.}$$

$$\eta_{CH_{P2}} = \frac{h_{2bs} - h_{1b}}{h_{2b} - h_{1b}} \tag{171.}$$

$$W_{HP2} = \overset{*}{m}_{\mathrm{Ref2}}\left(h_{2b} - h_{1b}\right) \tag{172.}$$

$$h_{1b} = f\,(p_{1b}, T_{1b}) \tag{173.}$$

$$h_{2b} = f\,(p_{2b}, T_{2b}) \tag{174.}$$

6.5.1.6 Constant pressure heat rejection in the condenser-evaporator in the second cycle (2b-3b)

The compressed refrigerant vapour leaving the compressor of the second cycle passes through the condenser, in which the refrigerant vapour transfers energy to the water-steam of the heating process. It is assumed that there is a temperature difference of 10°C between the maximal temperature of the refrigerant in the second cycle and the highest temperature of water-steam of the heating process, which is expressed in the next equations.

$$T_{2b} = T_8 + 10K \tag{175.}$$

Using the second law thermodynamics,

$$\overset{*}{m}_{\mathrm{Ref}\,2}\left(h_{2b} - h_{3b}\right) = \overset{*}{m}_d\left(h_8 - h_7\right) \tag{176.}$$

$$Q_{HP2} = \overset{*}{m}_{\mathrm{Ref}\,2}\left(h_{2b} - h_{3b}\right) \tag{177.}$$

6.5.1.7 Expansion process in the valve in the second cycle (3b-4b)

The expansion valve in the second cycle has the same performance that the expansion valve in the first cycle, therefore

$$W_{3b4b} = 0\,,\ Q_{3b4b} = 0\,,\ h_{3b} = h_{4b}$$

6.5.1.8 Constant pressure heat addition in the evaporator in the second cycle (4b-1b)

The compressed refrigerant vapour leaving the compressor of the second cycle passes through the evaporator-condenser, in which the refrigerant vapour of the first cycle transfers energy to the wet vapour refrigerant of the second cycle.
The equations that describe the evaporator-condenser are the same that the for the constant pressure heat addition in the evaporator in the first cycle (4a-1a) process.

6.5.1.9 Coefficient of performance (COP)

The coefficient of performance (COP) of heat pumps is one index used to describe their thermodynamic efficiency. The COP is the heat transferred per unit of energy input. The next equation shows the COP for the first cycle heat pump and second cycle heat pump

$$\left(COP\right)_1 = \frac{Q_{HP1}}{W_{HP1}} = \frac{h_{2a} - h_{3a}}{h_{2a} - h_{1a}} \tag{178.}$$

$$(COP)_2 = \frac{Q_{HP2}}{W_{HP2}} = \frac{h_{2b} - h_{3b}}{h_{2b} - h_{1b}} \qquad (179.)$$

And the coefficient of performance for the cascade heat pump system,

$$(COP)_t = \frac{Q_{HP2}}{W_{HP1} + W_{HP2}} = \frac{\overset{*}{m}_{\mathrm{Ref2}}(h_{2b} - h_{3b})}{\overset{*}{m}_{\mathrm{Ref1}}(h_{2a} - h_{1a}) + \overset{*}{m}_{\mathrm{Ref2}}(h_{2b} - h_{1b})} \qquad (180.)$$

6.5.2 Work fluid selection

The critical temperature of the working fluid provides the upper limit at which a condensing vapour heat pump can deliver heat energy. The working fluid should be condensed at a temperature sufficiently below the critical temperature to provide an adequate amount of latent heat per unit mass.

According with the Kyoto Protocol against the backdrop of global efforts to protect the environment, natural refrigerants are the obvious choice as a sustainable and ecological alternative to the conventional refrigerants as chlorofluorocarbons (CFCs) and the hydrochlorofluorocarbons (HCFCs).

The natural refrigerants used commonly are the ammonia and the carbon dioxide. Carbon dioxide (CO_2) refrigerant is incombustible, non-toxic and is more environment-friendly in nature than the fluorocarbon refrigerant; as well CO_2 refrigerant performs efficiently under the low-temperature condition and is suitable for heating when there is a large temperature difference. The disadvantage of carbon dioxide for heat pumps is the low critical temperature (31.1°C), which is usable for refrigeration, but makes impossible to reach the desirable temperature of the heating process (100°C).

The ammonia is environmentally compatible; it does not deplete the ozone layer and does not contribute to global warming. The disadvantage of ammonia is that is highly toxic and that mixed with water is advisable for refrigeration but not for heating.

Because of the disadvantages presented for the natural refrigerants, it was decided in this research, to work with hydrochlorofluorocarbons (HCFCs) and the hydrofluorocarbons (HFCs), which are refrigerant alternatives to the chlorofluorocarbons and present less deplete ozone layer and global warming that the CFCs.

The Table 14 summarized the most common HCFC and the Table 15 the most common HFC and the principal physical properties of each one of those.

Property	**HCFC-123**	**HCFC-124**	**HCFC-408a**	**HCFC-402B**	**HCFC-401a**	**HCFC-401b**
Critical Temperature (°C)	183.70	122.20	83.50	82.60	108.00	106.000
Critical Pressure (kPa)	3668.00	3614.00	4340.00	4445.00	4604.00	4682.000
ODP, CFC-12=1	0.02	0.02	0.03	0.03	0.03	0.035
GWP, CO2=1	93.00	470.00	2649.00	1964.00	973.00	1062.000

Table 14. Most common HCFC's [42]

Property	**HFC-134a**	**HFC-236fa**	**HFC-404a**	**HFC-407c**
Critical Temperature (°C)	101.10	124.92	75.50	86.74
Critical Pressure (kPa)	4060.00	3200.00	4135.00	4619.00
ODP, CFC-12=1	0.00	0.00	0.02	0.00
GWP, CO2=1	1300.00	6300.00	2250.00	1526.00

Table 15. Most common HFC's [42]

It was observed that the alternative refrigerant that present the best characteristics for the cascade heat pump systems is the HCFC-123, because it presents a high critical temperature (183.68°C) at which the heat pump can reach the desirable temperature in the heating process (100°C), has a high heat latent value at this temperature and the Ozone Depletion Potential (ODP) and the Global Warming Potential (GWP) are lower than the another hydrochlorofluorocarbons and hydrofluorocarbons. Therefore the HCFC-123 is selected as the working fluid for the cascade heat pump systems used in this research. Other important characteristics of the refrigerant HCFC-123 are presented in the Table 16 and Figure 86.

Properties	**HCFC-123**
Chemical Formula	$CHCl_2CF_3$
Molecular Weight	152.93
Boiling Point at 1 atm	27.80
Liquid Density at 25°C (kg/m3)	1463.00
Vapour Pressure at saturated liquid at 25°C (kPa)	91.29
ASHRAE Safety Classification	B1
Refrigerant Number	R-123

Table 16. Characteristics of the refrigerant HCFC-123 [24]

6.5.3 Theoretical Heat Generated By Cascade Heat Pump Systems

Using the previous analysis of the thermodynamics of cascade heat pump systems is possible to make a theoretical calculation, which predict:

- The heat generated for each heat pump in the cascade system and the total heat generated as function of the refrigerant mass flow
- The power consumption of each heat pump in the cascade system and the total power consumption as function of the refrigerant mass flow
- The temperatures, pressures and enthalpies of the refrigerant in each step of the cascade heat pump system

These thermodynamic characteristics of the refrigerant for different number of heat pumps in cascade are presented in the Table 41.

It was observed in the analysis of the thermodynamics, that heat generated and the power consumed by a heat pump can be related with the refrigerant mass flow.

But it is clear also, that for cascade heat pump systems, the heat produced by a bottoming heat pump is the heat used by the next topping heat pump, so it is possible relates the heat generated and the power consumed by any heat pump with the immediately next heat pump in the cascade system.

Therefore for a whole cascade heat pump system, the heat generated and the power consumed by any heat pump can be related with the refrigerant mass flow of any heat pump and the properties of the refrigerant in each step. In this research, the refrigerant mass flows of all the heat pumps in the cascade system are expressed in terms of the refrigerant mass flow of the heat pump, which produces the highest temperature.

The Table 17 presents the heat produced and the power consumed by each heat pump in the cascade in terms of the refrigerant mass flow of the heat pump, which produces the highest temperature.

In this table is shown the total heat generated and the total power consumed for cascade heat pump systems with 2, 3, 4 and 5 heat pumps.

According with the current literature, the purchasing cost of a compression heat pump is approximately 516 US/KW_{thermal}$ [25].

Number of heat pumps is cascade	Steps	Refrigerant mass flow (In terms of the mass flow of the last heat pump)	$\frac{W_{CONSUMED}}{\left[\dot{m}_{Ref}\right]_{\substack{\text{Highest}\\\text{Heat Pump}}}}\left[\frac{KJ}{Kg}\right]$	$\frac{Q_{GENERATED}}{\left[\dot{m}_{Ref}\right]_{\substack{\text{Highest}\\\text{Heat Pump}}}}\left[\frac{KJ}{Kg}\right]$
2	2	1	24,9767749	124,776775
	1	0,608058873	26,6505176	99,8
	TOTAL	-	51,6272925	224,576775
3	3	1	17,9572546	123,657255
	2	0,732706852	16,3830348	105,7
	1	0,541187464	15,3907576	89,3169652
	TOTAL	-	49,731047	318,67422
4	4	1	13,8985111	123,998511
	3	0,793280216	12,8438455	110,1
	2	0,635811139	12,0574619	97,2561545
	1	0,510721797	11,3993929	85,1986926
	TOTAL	-	50,1992114	416,553358
5	5	1	11,6549177	124,354918
	4	0,828404078	10,8891388	112,7
	3	0,692033038	10,2548903	101,810861
	2	0,580266403	9,68038148	91,5559709
	1	0,486756137	9,25157388	81,8755895
	TOTAL	-	51,7309021	512,297339

Table 17. Heat Produced and Power Consumed by each heat pump in a cascade system with the refrigerant HCFC-123

This research uses the previous information and the purchase cost of compression heat pumps units to conclude that:

1. The best option from viewpoint of thermodynamics is the cascade heat pump system with two heat pumps, because it offers the lowest consumption of power and the highest generation of heat in the last step, in comparison with other cascade systems at the same conditions
2. The best option from economical viewpoint is the cascade heat pump system with two heat pumps, due two reasons. The first one is that consumes less power than the other options and the second one is that the operational costs and maintenance costs are lower than the other options
3. The best option from practical viewpoint is the cascade heat pump system with two heat pumps, due two reasons. The first one is that it requires a plan much smaller than the other options, because the net heat generation of the last heat pump is the biggest one. The second one is that it requires less space for the cascade system as a whole. Although the compressor of these units are the biggest of all the options.

6.6 Absorption Chillers

Most refrigeration plants operate on the basis of two well-known physical phenomena:

- When a liquid evaporates (or boils), it absorbs heat, and when it condenses it gives up that heat.
- Any liquid will boil (and condense) at a low temperature at one pressure, and at a higher temperature at higher pressure.

A refrigerant is simply a liquid where the pressures at which boiling and condensing occurs are in the normal engineering ranges.

In the conventional, mechanical vapour compression cycle, the refrigerant evaporates at a low pressure, producing cooling. It is then compressed in a mechanical compressor to a higher pressure, where it condenses. In absorption cooling, the evaporator and the condenser are essentially the same, but a chemical absorber and generator replace the compressor with a pump to provide the pressure change. As a pump requires much less power than a compressor, electrical power consumption is much lower, the heat source provides most of the energy.

Absorption cooling works because some pairs of chemicals have a strong affinity to dissolve in one another. For example, a strong solution of lithium bromide in water will draw water vapour from its surroundings to dilute the solution. This affinity is used in absorption cooling to draw water (which is the refrigerant) from a conventional evaporator into the absorber. From there, the weakened solution is pumped to a higher pressure, to the generator. Here, heat is applied, and the water is driven off to a conventional condenser. The re-strengthened solution can then be recycled to the absorber. Heat is rejected from the absorber (it gets hot as it absorbs the refrigerant), and from the normal condenser [5].

The diagram shows a single effect absorption chiller. In double and triple effect units, some of the heat is recycled internally to improve efficiency, but these units require a higher temperature heat source. In this work were assumed that the absorption chillers for the trigeneration plants are single effect units. The work fluid selected for the absorption chillers is water-LiBr because it offers longer limits of cooling power (until 5000 KW) than absorption chillers with Ammonia-water or with water-silica gel. In other hand the use of water-LiBr implies a higher coefficient of performance and although the temperature of cooling is not so low (6°C), this temperature is enough for the purposes of this research.

6.6.1 Thermodynamics of Absorption Chillers

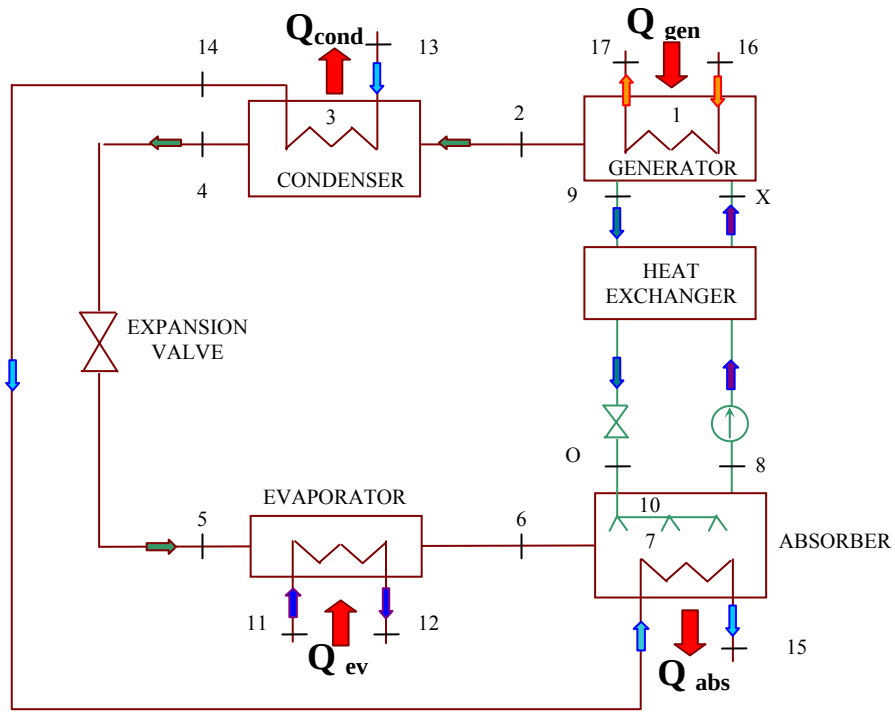


Figure 59. Single stage absorption cycle on Temperature-Entropy diagram

The representation of this cycle in the Temperature-Entropy diagram is showed in Figure 60.

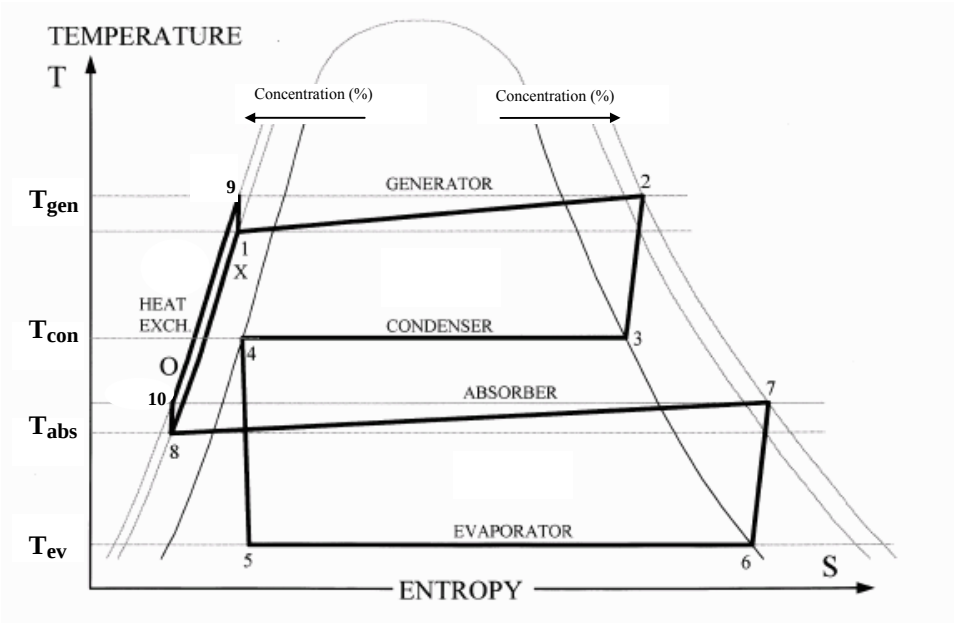


Figure 60. Single stage absorption cycle on Temperature-Entropy diagram [43]

Two circuits compose the scheme for the thermodynamics of absorption chillers: the solution circuit and the refrigeration circuit.

The processes in the solution circuit are [44]:

- 1-9: heating of weak solution in generator as solution concentrates due refrigerant removal.
- 9-O: cooling of strong solution in heat exchanger.
- O-10: cooling of strong solution in absorber at constant concentration.
- 10-8: cooling of strong solution in absorber as solution dilutes due to refrigerant addition.
- 8-X: heating of weak solution in heat exchanger.
- X-1: heating of weak solution in generator at constant concentration.

The processes in the refrigerant circuit are:

- 1-2: heating and evaporation of refrigerant introduced into the generator. The heat is equal to the sum of the heat of evaporation plus the heat of solution.
- 2-3: cooling of the superheated vapour produced in the generator, at constant pressure down to the saturation temperature T_c.
- 3-4: condensation.
- 4-5: isenthalpic expansion of the refrigerant.
- 5-6: heating of weak solution in heat exchanger.
- 6-7: heating of weak solution in generator at constant concentration.
- 7-8: absorption of the superheated vapour (of state 7) where both the heat of solution and the heat of evaporation are liberated.

Since the thermodynamics analysis for absorption cycles results more extensive than the analysis for other cycles, here are presented in a summarised way the relevant equations for the different components of the whole system. A more detailed description of the cycle can be found in the references [43], [44] and [45].

Condenser

$$Q_{con} = \overset{*}{m}_{\mathrm{Ref}} \left(h_{14} - h_{13}\right) = \eta_{con} \overset{*}{m}_{\mathrm{Ref}} \left(h_2 - h_4\right) \qquad (181.)$$

Evaporator

$$Q_{ev} = \overset{*}{m}_{\mathrm{Ref}} \left(h_6 - h_5\right) = \eta_{ev} \overset{*}{m}_{chw} \left(h_{11} - h_{12}\right) \qquad (182.)$$

Generator

$$Q_{gen} = \overset{*}{m}_{gen} \left(h_{16} - h_{17}\right) = \eta_{gen} \overset{*}{m}_{cs} h_{12} + \overset{*}{m}_{ref} h_2 + \overset{*}{m}_{ds} h_X \qquad (183.)$$

Absorber

$$\overset{*}{m}_{ds} = \overset{*}{m}_{cs} + \overset{*}{m}_{ref} \qquad (184.)$$

$$\overset{*}{m}_{ds}\,\xi_{ds} = \overset{*}{m}_{cs}\,\xi_{cs} \qquad (185.)$$

$$Q_{abs} = \overset{*}{m}_{cs}\,h_O + \overset{*}{m}_{ref}\,h_6 + \overset{*}{m}_{ds}\,h_8 = \overset{*}{m}_{cool}\left(h_{15} - h_{14}\right) \qquad (186.)$$

Heat exchanger

$$\overset{*}{m}_{cs}\left(h_9 - h_O\right) = \overset{*}{m}_{ds}\left(h_X - h_8\right) \qquad (187.)$$

(Note: a useful graphic for the calculation on enthalpies of Lithium bromide as a function of the temperature and the concentration is presented in the reference [45])

6.6.1.1 Coefficient of Performance (COP) and Heat Dissipation Ratio (HDR)

Making an idealisation of the absorption cycle, formulae for the COP can be raised. The Figure 61 is the representation of an idealised absorption cycle.

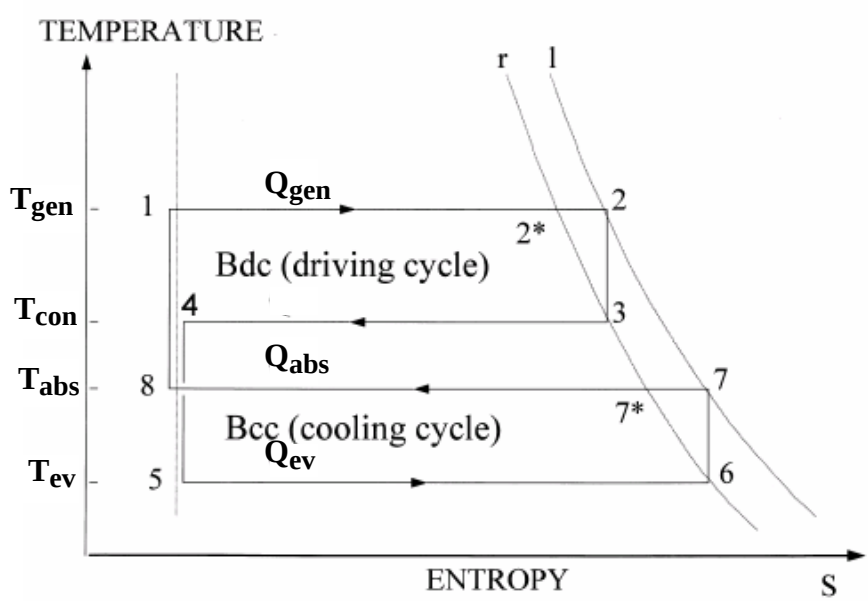


Figure 61. Idealised absorption cycle on Temperature-Entropy diagram [43]

It was demonstrated that the ideal absorption cycle is the combination of a Carnot driving cycle with a reverse Carnot cooling cycle, from where the COP can be derived in terms of absolute temperatures of the cycle [43].

$$COP = \frac{Q_{con}}{Q_{gen}} \tag{188.}$$

$$COP = \frac{T_{ev}}{T_{abs}} = \frac{T_{con}}{T_{gen}} \tag{189.}$$

For the energy systems used in this project, it was made the assumption of a fixed value of 0.68 for the COP of the chiller, which is a typical value for single effect absorption chillers.
Another important parameter for the chillers is the Heat Dissipation Ratio or Heat Rejection Ratio. All cooling systems take heat from one place (the item being cooled) and reject it elsewhere, usually to atmosphere by way of cooling towers or dry air coolers. Absorption cooling is no exception, but absorption units reject more heat than conventional alternatives [5]

The Heat Dissipation Ratio can be defined as the ratio of heat dissipated from the condenser and absorber with respect to the evaporator load. It is used to analyse cooling requirements of different absorption and vapour compression chillers [43].

$$HDR = \frac{Q_{abs} + Q_{con}}{Q_{gen}} = 1 + \frac{1}{COP} \tag{190.}$$

References

[1] EDUCOGEN. *The European Educational Tool on Cogeneration*. 2001. (http://www.cogen.org/projects/educogen.htm)

[2] Horlock J.H. *Cogeneration: combined heat and power*. Pergamon Press. New York. 1987

[3] Li Kam W., Priddy A. Paul. *Power Plant System Design.* John Wiley & Sons, Inc. New York.1985

[4] Protermo. Manual for calculating CHP electricity and heat. Suomen Kaukolämpö. January 2000

[5] An introduction to absorption cooling. Good Practice Guide 256. Energy Efficiency Enquiries Bureau. 1999

[6] Elliot Thomas C. *Standard Handbook of Powerplant Engineering.* Mc Graw Hill Publishing Company. 1989

[7] Aschner F. S. *Planning Fundamentals of Thermal Power Plants.* John Wiley & Sons, Inc. New York.1978

[8] H. L. von Cube, F. Steimle. *Wärmepumpen*. VDI-Verlag GmbH. Düsseldorf.1984

[9] F.A. Holland, F.A. Watson, S. Devotta. *Thermodynamic Design Data for Heat Pump Systems*. Pergamon Press. 1982

[10] International Energy Agency (IEA). *IEA Heat Pump Conference*. Graz, Austria. 1984

[11] International Energy Agency (IEA). *Selected Issues on CO_2 as Working Fluid in Compression Systems.* Trondheim, Norway. 2000

[12] Sugartech Calculations. http://www.sugartech.co.za/turbinecalcs/index.php

[13] C.A. Frangopoulos. *Optimal Synthesis and Operation of Thermal Systems by the Thermoeconomic Functional Approach*. Journal of Engineering for Gas Turbines and Power. Vol. 114. October 1992

[14] C.A. Frangopoulos. *Application of the thermoeconomic functional approach to the CGAM problem*. Energy. Vol 19, No.3, pp 323-342. 1994

[15] C.A. Frangopoulos, A. Valero, G. Tsatsaronis, M. Spakovsky. *CGAM Problem : Definition and conventional solution.* Energy. Vol 19, No.3, pp 279-286

[16] GE Power Generation. *37th GE Turbine State-of-the-Art Technology Seminar*.1993

[17] S. R. Turns. *An Introduction To Combustion*. McGraw-Hill. 1996

[18] Hausenblas H. *Vorausberechnung des Teillastverhaltens von Gasturbinen*. Springer-Verlag. Berlin.1962

[19] H. G. Münzberg, J. Kurzke. *Gasturbinen Betriebsverhalten und Optimierung. Springer-Verlag*. Berlin. 1977

[20] G. Cordes. *Strömungstechnik der gasbeaufschlagten Axial*turbine. Springer-Verlag. Berlin.1963

[21] Advisory Group for Aerospace Research and Development. *Modern prediction methods for turbomachine performance*. AGARD. 1976

[22] Research And Technology Organisation. *Performance Prediction and Simulation of Gas Turbine Engine Operation*. NATO. 2002

[23] J. Kühnel, G. Gyarmathy, P. Ortmann. *Estimation of the Compressor Map for Incompletely Specified Axial Compressors.* Institute of Energy Technology, Turbomachinery Laboratory

[24] Dupont-Suva Refrigerants. *Thermodynamic Properties of HCFC-123 Refrigerant (2,2 dichloro-1,1,1-trifluoroethane).* Wilmington, USA. 1993

[25] A. De Groot, F. Tillemans, C. E. Grégoire Padró, P. Spath. *Hydrogen Systems For Residential Developments*. World Energy Council 18th Congress, Buenos Aires. October 2001

[26] G. Wall. *Thermoeconomic Optimisation Of A Heat Pump System*. Chalmers University of Technology and University of Göteborg, Sweden.1985

[27] P.A. Pilavachi. *Mini- and micro-gas turbines for combined heat and power*. Applied Thermal Engineering 22. August 2002.

[28] W. Wagner, A. Kruse. *Properties of Water and Steam*. Springer-Verlag. Berlin.1997

[29] Jari Backman, *Jaakko Larjola. Kaasuturbiinikytkennät ja niiden laskenta*. Lecture notes. ISBN 951-764-682-8. Lappeenranta. 2002

[30] Paroscientific Inc. Precision Pressure Instrumentation. *Calculation of Dew Point*. http://www.paroscientific.com/dewpoint.htm

[31] Walsh P. P., Fletcher P. *Gas Turbine Performance*. Blackwell Science. 1998

[32] Kurz Rainer, Brun Klaus. *Gas Turbine Performance-What makes the map?* Solar Turbines Incorporated. San Diego, California. 2000

[33] Wörrlein Karl. *Seminar in Flugantriebe. Fachgebiet Gasturbinen und Flugantriebe*. Technische Universität Darmstadt. 2. Auflage WS 2000/2001

[34] S. Gamou, K Joko, R. Yokoyama, K. Ito. *Economic feasibility study of micro gas turbine cogeneration systems by an optimisation approach*. ASME TURBO EXPO 2001. Louisiana. June 2001

[35] S. Campanari, L. Boncompagni, E. Macchi. *Micro turbines and trigeneration: optimisation strategies and multiple engine configuration effects*. ASME TURBO EXPO 2002. Amsterdam. June 2002

[36] McDonald Colin F. *Low-cost compact primary surface recuperator concept for microturbines.* Applied Thermal Engineering 20. 2000

[37] McDonald Colin F. *Recuperator considerations for future higher efficiency microturbines.* Applied Thermal Engineering 23. 2003

[38] Technical Information. *T100 microturbine CHP system*. Turbec AB. Sweeden.2002

[39] Capstone Low Emissions MicroTurbine Technology. Capstone Turbine Corporation. 2000

[40] Cloutier Matthew. *Heat Pumps*. Chapter 10, Refrigeration Cycles. 2001

[41] Die Elektrische Wärmepumpe. Technik und Anwendung. Wärmepumpentechnologie Band V. Vulkan-Verlag Essen. 1980

[42] Dupont Suva Refrigerants. http://www.dupont.com/suva/

[43] Tozer, R. James, R. W.: Heat powered refrigeration cycles. Applied Thermal Engineering. Vol 18. pp. 731-743. (1998)

[44] Tozer, R. James, R. W.: Fundamental thermodynamics of ideal absorption cycles. International Journal of Refrigeration. Vol 20. No. 2. pp. 120-135. (1997)

[45] Herold, K.: Examination and Reconciliation of Aqueous LiBr Thermodynamic Property Data. Center for Environmental Energy Engineering (CEEE).

Chapter 7

Electricity data

The information related to the electricity costs and forecasting used for this document was, like the fuel data, obtained from official sources of the government of the United States. This chapter includes a brief description of the electricity framework and presents the electricity data used for the simulation of the optimisation models.

7.1 Electricity and Energy Systems

Electricity use is the foundation for much of the human way of life. Most communication is based on it. Most machines, other than vehicles, are powered by electric motors. Lighting is universally electric. Less visible are the myriad industrial processes that use electricity. Most recent is the rising tide of electronic devices, including computers, as essential elements of living [1]. The electricity is a fundamental aspect of the energy systems. It can be considered as a product of the energy system, but also as input of it, as it can be seen on Figure 3.

According to the requirements of the user, the energy system can purchase electricity if the demand is higher than the maximum electric output of the system or if the cost of purchase is lower than those of production of electricity; in the same way it is possible to sell electricity when the power produced exceeds the demand.

Moreover, depending on if the electricity is produced far away of the site where it is consumed, or it is close to the load being served, the supply of energy for a specific demand can be classified as Centralised Generation or as Distributed (or Decentralised) Generation, respectively. This last is typical for cogeneration applications. That is the case, for example, of commercial applications like hospitals and hotels.

Distributed generation brings benefits to the energy users as well as to the utility grid provider. Some of these advantages are [4]:

- Benefits for the user**:**
    - Increase of the reliability
    - Reduction of the number of interruptions of the energy supply
    - Efficient use of the energy
    - Lower cost of the energy
    - Easiness to adapt to the site conditions
    - Reduction of emissions of contaminants

- Benefits for the utility grid provider
    - Reduction of losses in transmission and distribution
    - Supply of energy in distant places
    - Better regulation of the tension
    - Reduction of the investment cost
    - Support to the utility grid
    - Reduction of the failure index

In general terms, the projects for decentralised generation try to increase the quality of the energy, that is, to have electric energy without interruptions, with the right electrical parameters defined according to the needs (voltage, current, frequency, etc).

Even though most of the transmission and distribution grids have a reliability of 99.9%, equivalent to 8.7 hours per year out of service, the failures in the supply of electricity cause considerable economical losses. For example, the losses in the United States generated by these failures are close to 119000 million dollars per year, and in Latino America between 10000 and 15000 million dollars annually.

Cogeneration systems with distributed generation and the option to buy and sell electricity need to have interconnection with the utility grid. Some of the technical aspects to consider for the interconnection systems are [4]:

- Relays of protection.
- Connexions for transformers
- Systems to earth the equipments
- Monitoring, control and communication systems
- Maintenance

7.2 Electricity data for the cases of demand

The data related to electricity prices – current prices and forecast- were extracted from the Bureau of Labor Statistics (BLS) of the United States Department of Labor [2] and from the Energy Information Administration (EIA) of the United States government [3]. All the prices are given for the residential, commercial and industrial sector. Like with the fuels, the prices of year 2002 are considered as the current prices of electric power. The forecasting used gives an estimation of the energy prices for 25 years, beginning on 2000, and it was also made the assumption that the trend of increase or drop the prices is the same from the year 2002.

The prices listed below are given in dollars per kilowatt-hour and for four periods along the year: from January to March, from April to June, From July to September, and from October to December. Some conversion factors were used in order to present the original information that was not given in these units. The list includes the prices for buying and for selling electricity. In the present work, it was assumed that the price of one kilowatt-hour of electricity sold is the 80% of the price for buying. The annualised electricity prices are shown in 0.

Sector	Price ($/kWh)							
	Period 1		Period 2		Period 3		Period 4	
	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell
Residential	0.11267	0.09013	0.11300	0.09040	0.11700	0.09360	0.11133	0.08907
Commercial	0.10140	0.08112	0.10170	0.08136	0.10530	0.08424	0.10020	0.08016
Industrial	0.05577	0.04462	0.05594	0.04475	0.05792	0.04633	0.05511	0.04409

Table 18. Electricity prices in the year 2002 in the United States [2]

Sector	Price ($/kWh)							
	Period 1		Period 2		Period 3		Period 4	
	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell
Residential	0.10939	0.08751	0.10971	0.08777	0.11360	0.09088	0.10810	0.08648
Commercial	0.10026	0.08020	0.10055	0.08044	0.10411	0.08329	0.09907	0.07926
Industrial	0.05593	0.04474	0.05609	0.04487	0.05808	0.04646	0.05526	0.04421

Table 19. Annualised electricity prices in the United States

References

[1] Bydesign: Fossil Fuels. http://www.bydesign.com/fossilfuels/links

[2] Bureau of Labor Statistics (BLS). United States Department of Labor. www.bls.gov

[3] Energy Information Administration (EIA). United States Government. www.eia.doe.gov

[4] Comision Nacional para el Ahorro de Energia (CONAE). http://www.conae.gob.mx

Chapter 8

8.1 Fuel data

Power plants operate on a variety of fuels, including wood, coal, natural gas, oils, municipal solid waste, and sludge. The fuel handling, storage, and preparation equipment needed for solid fuels add considerably to the cost of an installation. Thus, such fuels are used only when a high annual capacity factor is expected of the facility, or when the solid material has to be disposed of to avoid an environmental or space occupancy problem[2]. The information used for this document involves only some fossil fuels. Even though other fuels can be used for power plants, it is out of the scope of this project to make a deep analysis of all the possible fuels for the cogeneration technologies studied. This chapter includes a brief description of the main kinds of fossil fuels and presents updated data of the fuels used for the simulation of the optimisation models.

8.2 About fossil fuels

After food, fossil fuel is humanity's most important source of energy. There are three major fossil fuels: coal, oil and natural gas. Many of the benefits derived from the human's way of life, and his high standard of living, are due to fossil fuel use. Light, heats, food, communication, travel, community, all are based on the ability to produce and use energy. And most of the used energy, about 85%, comes from fossil fuel. (Another 8% comes from nuclear power, and 7 % from all other sources, mostly hydroelectric power and wood.) [3].

8.2.1 Environmental Aspects of Fossil Fuel Use

Pollution: Almost all fossil fuel use is by burning (or "combustion"). Burning produces waste products due to impurities in the fuel, especially particulates and various gases such as sulphur dioxide, nitrogen oxides, and volatile organic compounds. These waste products may affect the environment or people, in harmful ways. Then too, there are serious disagreements over whether some effects of fossil fuel use are harmful at all. In some cases the amount of waste is so small that the effect, if any, is difficult to detect. Mercury from coal burning is an example [3].

Climate change: At the other extreme, all burning produces carbon dioxide and water vapour as by-products. This is because carbon is part of what makes fossil fuel useful. But whether these by-products are harmful, or beneficial, is a matter of intense public debate. Some argue that they are beneficial, because water and carbon dioxide are necessary for plant life on earth, which is the basis for all life. Some people believe, however, that the carbon dioxide emissions contribute to harmful global warming and climate change, either now or in the future. Those who fear climate change have proposed new government policies to drastically reduce the use of fossil fuels. Those who do not fear climate change are sceptical of these proposed policies. There is also great debate about the science of climate change [3].

8.2.2 Classification of Fossil Fuels

8.2.2.1 Coal

Coal is our most abundant fossil fuel resource. It is an organogenic sedimentary rock, formed of converted residual plant matter and solidified below overlying strata [5]. Coal is a complex mixture of organic chemical substances containing carbon, hydrogen and oxygen in chemical combination, together with smaller amounts of nitrogen and sulphur. This organic part of coal has associated with it various amounts of moisture and minerals. Coalification is the name given to the development of the series of substances known as peat, lignite or brown coal, sub-bituminous coal, bituminous coal, and anthracite. The degree of coalification, also called the rank of the coal, increases progressively from lignite to low rank coal, to high rank coal, to anthracite. The carbon content increases, while the oxygen and hydrogen contents decrease throughout the series. The hardness increases, while the reactivity decreases. Different amounts of heat and pressure during the geochemical stage of coal development cause these differences in rank [3].

Differences in the kinds of plant material and its biochemical alteration before burial control the type of coal. Differences in the geological conditions of temperature and, to a lesser degree, pressure during coalification control the rank of the coal. Table 20 shows typical values of the coal composition, according to the rank, and the Low Heating Value:

Rank	Moisture	Volatile matter	Fixed carbon	Ash	Specific energy (KJ/kg)	Physical appearance
Peat	80%	9%	5%	6%	16000	Soft, brown to black, poorly compacted plant debris.
Lignite	55%	20%	17%	8%	23000	Tough, dull, brown with obvious woody material.
Sub-bituminous	20%	36%	40%	4%	28000	Black with faint brown tinge and dull shine.
Bituminous	2%	36%	60%	2%	35500	Black with bright, shining lustre.
Semi-anthracite	1%	9%	87%	3%	34500	Dark grey with metallic lustre.

Table 20. Coal Properties according to the rank [10]

8.2.2.2 Oil

Petroleum is a mixture of liquid hydrocarbons (chemical compounds containing only hydrogen and carbon) plus various impurities such as sulphur. Unprocessed petroleum is usually called crude oil, although it has been called mineral oil, or simply oil. The name petroleum is from a combination of Latin words meaning "rock oil". As found in the earth, oil may have a variety of properties. Some forms are black; others dark green, and some light like kerosene. The liquid ranges from very viscous to easy flowing. Crude oil usually consists of a mixture of hydrocarbons having varying molecular weights and differing from one another in structure and properties.

These various species are separated into groups, or fractions, by a process of distillation called refining. Oil fuel, in all of its usable forms, is a refined product, unlike coal and natural gas, which can often be burned in their natural condition [3]. Crude oils are classified according to their density or their API gravity as [5]:

1. Light: less than 870 kg/m^3 (higher than 31,1 °API)
2. Medium: 920-870 kg/m^3 (22,3-31,1 °API)
3. Heavy: 1000-920 kg/m^3 (10-22;3 °API)
4. Extra heavy: greater than 1000 kg/m^3 (less than 10°API)

In an average crude oil the fractions, beginning with the lightest, are: (1) dissolved gases, (2) petroleum ether, (3) gasoline, (4) kerosene, (5) gas oil, (6) lubricating oils, (7) fuel oils, and (8) asphalt. A barrel of crude oil makes:

Product	Gallons per barrel
Gasoline	19.5
Distillate fuel oil (includes home heating oil and diesel fuel)	9.2
Kerosene-type jet fuel	4.1
Residual fuel oil (heavy oils used as fuels in industry, marine transportation and for electric power generation)	2.3
Liquefied refinery gasses	1.9
Still gas	1.9
Coke	1.8
Asphalt and road oil	1.3
Petrochemical feedstocks	1.2
Lubricants	0.5
Kerosene	0.2
Other	0.3

Table 21. Derivates of one barrel of Petroleum [4].

As a fuel, oil was originally used as kerosene for lighting, replacing animal, vegetable and coal oils. It also came to be used in furnaces. Its biggest use, however, came with the development of the automobile. Today almost all forms of locomotion cars, trucks, buses, trains, ships and airplanes are fuelled by oil, diesel or gasoline. Fuel oil has also been burned to produce electricity [3].

8.2.2.3 Fuel Oil

Fuel oils are complex and variable mixtures of alkanes and alkenes, cycloalkanes and aromatic hydrocarbons, containing low percentages of sulphur, nitrogen and oxygen compounds. Kerosene fuel oils are manufactured from straight-run petroleum distillates from the boiling range of kerosene. Other distillate fuel oils contain straight-run middle distillate, often blended with straight-run gas oil and light vacuum distillates, and light cracked distillates.

The main components of residual fuel oils are the heavy residues from distillation and cracking operations; various refinery by-products and heavy distillates may be added. In fuel oils consisting mainly of atmospheric distillates, the content of three- to seven-ring polycyclic aromatic hydrocarbons is generally less than 5%. In fuel oils that contain high proportions of heavy atmospheric, vacuum and cracked distillates or atmospheric and vacuum residues, the content of three- to seven-ring polycyclic aromatic hydrocarbons may be as high as 10%; if large quantities of cracked components are incorporated, levels may approach 20%. Fuel oils are used mainly in industrial and domestic heating, as well as in the production of steam and electricity in power plants.

Skin and inhalation exposures to fuel oil may occur during its production, storage, distribution and use and during maintenance of heating equipment. During the cleaning of fuel oil tanks, high, short-term exposures to total hydrocarbon vapours have been measured at levels ranging from 100-1600 mg/m^3 [1]. Typical values of the fuel oil content and some important characteristics are listed below [8][9]:

Content of	**%**
Moisture	0.2
Ash	0.05
Carbon	86.0
Hydrogen	10.0
Nitrogen	0.4
Sulphur	3.0
Oxygen	0.35
Low heating Value (kJ/kg)	41500
Specific gravity	0.995
Viscosity for fuel oil No. 2 (centistokes)	7.7 to 11

Table 22. Characteristics of the fuel oil

8.2.2.4 Natural Gas

Natural gas is a methane-rich combustible coming from natural fields. It contains varying amounts of heavier hydrocarbons that become liquefied at atmospheric pressure. It may also contain water vapour, sulphuric compounds such as hydrogen sulphide and other nonhydrogen gases such as carbon dioxide, nitrogen or helium [5]. Natural gas is a highly flammable. The gas is found entrapped in the earth's crust at varying depths beneath impervious strata, such as limestone, and may or may not be in association with oil. If oil is present it is called wet gas, else dry gas.

As a fuel, natural gas is convenient and efficient. It is used primarily for heat, in industrial, commercial and residential settings. In many homes the house and water are heated by gas, the food is cooked with it and clothes dried. It is also used to produce electricity, in many cases using gas fired turbines that are similar to jet engines. Gas has the great advantage of producing no smoke or ash on burning, although it is usually much more expensive than coal as a fuel [3].

Typical values of the Natural gas composition and some important characteristics are listed below:

Component	Typical Analysis (mole %)	Range (mole %)
Methane	94.9	87.0 - 96.0
Ethane	2.5	1.8 - 5.1
Propane	0.2	0.1 - 1.5
iso - Butane	0.03	0.01 - 0.3
normal - Butane	0.03	0.01 - 0.3
iso - Pentane	0.01	trace - 0.14
normal - Pentane	0.01	trace - 0.04
Hexanes plus	0.01	trace - 0.06
Nitrogen	1.6	1.3 - 5.6
Carbon Dioxide	0.7	0.1 - 1.0
Oxygen	0.02	0.01 - 0.1
Hydrogen	trace	trace - 0.02
Specific Gravity	0.585	0.57 - 0.62
Gross Heating Value (MJ/m^3), dry basis *	37.8	36.0 - 40.2

Table 23. Composition and properties of Natural Gas [11].

8.3 Fuel used for the cases of demand

8.3.1 Selection of the fuel

8.3.1.1 Selection of the fuel for steam turbines cogeneration systems

The principal characteristics of the most commonly fuels used in steam turbine cogeneration systems is summarised in the Table 24.

Fuel	H_u (kJ/kg)	Estimated Annualised Heating Cost ($/KWh)	CO_2 (kg CO2/ kg fuel)	$\frac{CO_2}{H_U} * 10^5$
Natural Gas	50000	0.025622256	2.75	5.5
Fuel Oil	41500	0.023261012	3.17	7.6385542
Coal	30000	0.00401101	2.93	9.7666667

Table 24. Principal characteristics of fuels used in steam turbine cogeneration systems [3]

It is possible observe that the cheapest fuel in any case is the coal, in terms of annualised heating cost, but the use of these fuel represents a larger and more complex unit design than the units designed for fuel oil or natural gas. With a larger and complex unit design increase the investment costs, the maintenance costs, the personal costs and also the time of construction and running up the plant. On other hand the use of coal implies the use of equipments to receive, transport, storage and manipulates the coal, as well as machinery to handle and control the ash produced by the coal. Finally is clear in the comparison table, that the coal represents the most contaminant option in terms of emissions of CO_2 per heating value.

8.3.1.1.1 Natural gas vs. Fuel oil

The natural gas is the fuel with the less contaminant emissions per heating value and represents an ecological, safe and cheap option in steam turbine system. Liquid fuels need good filtration before entering the heater and a viscosity lower than 20 cS for mechanical atomisation. Steam atomisation can handle oils of up to 60 cS. Oils must be atomised to very tiny droplets of 10-50 microns in size, with a huge increase in the surface-to-mass ratio, to enable their almost instantaneous (microseconds) vaporization. However, for burning to occur, there must be an ample supply of heat to the atomizer discharge.

For yet non-existing fired heaters, liquid fuels tend to need more excess air than gaseous fuels. This has to do with efficiency of contacting air and liquids for a chemical reaction (combustion) to take place over a short period of time, and the need to add reactants to push the reaction towards the desired products of combustion. This is only a tendency when using natural draft. Excess air acts as coolant or heat sink in the furnace. When there is too much excess air the flame-gas temperature falls, and more fuel should be fired to compensate. However, the optimum amount of excess air may be finally determined by the heaters design, and not by the type of fuel.

Liquid fuels also may contain water and mineral (ash-forming, fouling) matter that contribute less heat than the hydrocarbons per unit weight, and may result in air pollution and in metallurgical and refractory attack, such as from V2O5. This oxide can even become a catalyst to convert SO2 into SO3 accelerating sulphuric acid corrosion.

Gas firing, on the other hand, needs careful PCV sizing. What ash-content is to liquid fuel, the % of inert and H2S are to fuel gases. "Afterburning" effects due to "shortages" of air can lead to high heat fluxes in places where they are not wanted. Most start-up explosions have happened in furnaces heated with gas, when the heaters haven't been purged with air or steam. Sometimes gas-fired heaters show low frequency vibrations and noise, because of resonance of the whole system: airbox, burner, radiant cell, flueducts.

Flame stability and impingement depend on many factors: type of burner, type of fuel, amount of fuel, type of atomisation, flame position in the radiant zone, aerodynamic factors, size and number of burner throats relative to the volume of the radiation zone.Therefore the decision between fuel oil and natural gas as fuel of steam turbine systems is a hard choice that depends on many variables and principally on the type of plant user. In this work is decided use the fuel oil as the fuel for all the steam turbine cogeneration systems, because is cheaper than the natural gas, is in the steam turbine practice most used than the natural gas and is generally easier to handle and control than the natural gas.

8.3.1.2 Selection of the fuel for gas turbines cogeneration systems

The main characteristics of the fuel options used commonly in gas turbines cogeneration systems are presented in the figure Table 25.

Fuel	H_u (kJ/kg)	Estimated Annualised Heating Cost ($/KWh)	CO_2 (kg CO2/ kg fuel)	$\frac{CO_2}{H_U} * 10^5$
Natural Gas	50000	0.025622256	2.75	5.5
Fuel Oil	41500	0.023261012	3.17	7.6385542
Liquefied Petroleum Gas	46000	0.04539699	2.7324	5.94

Table 25. Principal characteristics of fuels used in gas turbine cogeneration systems [3]

Liquefied petroleum gas (LPG) has lower particle emissions and lower noise levels relative to the other fuels, making it more attractive for urban areas; as well LPG is easily transportable and offers "stand-alone" storage capability with simple and self-contained dispensing facilities, with minimum support infrastructure. Although LPG has a relatively high energy content per unit mass, its energy content per unit volume is lower than other liquid fuels, therefore LPG tanks take more space that the conventional. But the main disadvantage of the liquefied petroleum gas is the estimated annualised heating cost, because is almost the double of the fuel oil price or natural gas price.

For gas turbine applications, one important requirement is for as cheap a fuel as possible in order to counterbalance the relatively low thermal efficiencies at present obtainable from gas turbines as compared with other technologies.

As is shown the fuel oil presents in average a cheaper price per unit of energy than the natural gas, but in some cases, like in industrial cases is high cheaper than fuel oil or other fuels. This makes attractive the natural gas for large cases of energy generation, as cities, hospitals, universities, industries, etc.

Natural gas is the most commonly fuel used in gas turbines systems for electricity generation, environmentally natural gas is a relatively clean-burning, high efficiency and excellent combustibility fuel. As well, natural gas presents advantages like lower maintenance costs and no storage facilities and manning required.

Because of this reasons, and also taking into account that the natural gas is improving it availability around the world, it is selected in the present research this fuel for all the cases of gas turbines, including the micro-gas turbines.

8.3.2 Prices of the fuel selected

The data related to fuel prices – current prices and forecast- were extracted from the Bureau of Labour Statistics (BLS) of the United States Department of Labour [6] and from the Energy Information Administration (EIA) of the United States government [7]. All the prices are given for the residential, commercial and industrial sector.

As the year of reference used for the optimisation is 2002, the prices of this year will be considered as the current prices. The forecasting used gives an estimation of the fuel costs for 25 years, beginning on 2000. Even though the initial value doesn't match with the year of reference, it was assumed the same trend of increase or drops the prices from the year 2002.

The prices listed below are given in dollars per kilowatt-hour. It was taken an annual average of the fuel cost. Some conversion factors were used in order to present the original information that was not given in these units.

Sector	**Annual price ($/kWh)**	
	Natural Gas	**Fuel Oil**
Residential	0.03158	0.02881
Commercial	0.02716	0.02161
Industrial	0.01494	0.02161

Table 26. Natural gas and fuel oil prices in the year 2002 in the United States [6]

Taking into account the forecast for the natural gas [7] and the economical concepts described in the chapter of economics, the prices has been annualised, as it is showed in Table 27.

Sector	**Annualized price ($/kWh)**	
	Natural Gas	**Fuel Oil**
Residential	0.03295	0.02791
Commercial	0.02833	0.02093
Industrial	0.01558	0.02093

Table 27. Annualised natural gas and fuel oil prices in the United States [6]

References

[1] The International Agency for Research on Cancer IARC: Monographs Programme on the Evaluation of Carcinogenic Risks to Humans. Vol 45 (1989)

[2] Energy Nexus Group: Technology Characterization: Steam Turbines. Virginia. (March 2002)

[3] Bydesign: Fossil Fuels. http://www.bydesign.com/fossilfuels/links

[4] American Petroleum Institute www.api.org/links/map.htm

[5] World Energy Council: Energy Dictionary. Jouve. (1992)

[6] Bureau of Labor Statistics (BLS). United States Department of Labor. www.bls.gov

[7] Energy Information Administration (EIA). United States Government. www.eia.doe.gov

[8] Educogen. June 2003

[9] British Electricity International: Modern Power Station Practice. Oxford. (1992)

[10] Crown Minerals, Ministry of Economic Development of New Zealand. http://www.med.govt.nz/crown_minerals/

[11] Union Gas Limited. http://www.uniongas.com

[12] Engineering Tips. http://www.eng-tips.com/

Chapter 9

Modelling and Simulation

The optimization of the energy systems studies is defined in terms of costs; that means that the models are constructed with the objective to minimise the total annualised costs of the plant. The model is composed of two parts: an objective function and a group of constraints. A series of variables are defined for each model. This function cost is composed of two parts: the operational costs (Φ_O) and the annualized design costs (Φ_D). The operational costs are those derived of producing the electrical energy and the heat necessary to supply the demand during the different periods of time of the year. The design costs are the investment costs, which means, those that depend of the size of the equipments; here are included the costs of installation and the costs of the personal involved in the operation of the plant.

Depending on the technology modelled, the objective function, the constraints and the variables involved can be different. Here is presented a general description of the models, and after this, each model will be described in detail.

9.1 Simulation

The simulation is a theoretical approximation of the physical, economical and thermal performance of an energy system, using mathematical models based on three main aspects:

- Thermodynamics information about the energy systems technologies
- Economics information about the operational and design costs of the energy systems technologies, as well as the costs of fuels, electricity, personal and maintenance
- Statistics information about the energy consumption of the demand cases

The optimization is a method to find the optimal solution of the mathematical model from the economical viewpoint; this method includes two kind of optimization: design optimization, that defines the optimal size and the number of all the equipments of the plant, and the operational optimization, which defines how must operate each equipment of the plant along the year in order to supply the energy demand required. The mathematical model is solved with commercial software for non-linear and mixed integer programming.

This research combines the simulation of energy systems with the method of optimization, in order to find the cheapest option of a specific energy system, which supplies heat, power and cooling to the user.

9.2 Modelling

Modelling is the process to express the physical, thermal and economical characteristics of the energy systems in mathematical language, which describes the system in terms of functions, equations, relations, variables and restrictions. As was mentioned before, the modelling process is made based in three main aspects, thermodynamics information, economics information and statistics information.

9.2.1 Main Components of the Models

9.2.1.1 Objective function

The objective function is the mathematical expression, which represents the total annualized costs of the energy system and depends on a finite number of variables. This variables are determined by the characteristics of the specific energy system, and could be independent among themselves, or could be related through one ore more restrictions.

The objective function is defined as Φ:

$$\min \Phi = \Phi_O + \Phi_D \qquad (191.)$$

This function cost is composed of two parts: the operational costs (Φ_O) and the annualized design costs (Φ_D). The operational costs are those derived of producing the electrical energy and the heat necessary to supply the demand during the different periods of time of the year. The design costs are those that depend of the size of the equipments; an additional factor is included for the costs of installation and the costs of the personal involved in the operation of the plant.

9.2.1.1.1 Operational costs

These are composed by the costs required for the energy system to produce or to buy different forms of energy, as heat, power and cooling.

In this case, the operational costs are: cost of producing electric power or heat –or cooling for the trigeneration option - (Γ_P), cost of the electricity bought from the utility grid (Γ_{BY}), negative cost (profit) of power sold to the utility grid (Γ_S), cost of producing additional heat (Γ_{AH}) and cost of producing additional refrigeration (Γ_{AR}), during each period of time of the year.

$$\Phi_O = \sum_{n=1}^{4} \left(\dot{\Gamma}_P + \dot{\Gamma}_{BY} - \dot{\Gamma}_S + \dot{\Gamma}_{AH} + \dot{\Gamma}_{AR} \right) * \Delta t_n \qquad (192.)$$

Δt_n represents the periods of time in which the year is divided according to the demand. For each period, the duration (in hours), the heat, power and cooling demand, and the cost of buying and selling electricity, are variables. It is assumed the cost of fuel as a constant value.

9.2.1.1.2 Design costs

The design costs are the annualised investment costs of all the equipments of the energy system. The design costs are expressed with a non-linear function, which relate the annualised investment cost of a equipment with the size and the physical characteristics of the same.

The function can be defined by the following expression:

$$\Phi_D = a_1 \cdot Y_1^{b_1} + a_2 \cdot Y_2^{b_2} + \ldots + a_m \cdot Y_m^{b_m} \qquad (193.)$$

The coefficients a_i and b_i are constant values that involve thermodynamic and economic parameters, which have been determined through several studies [4][5][6]. Y_i represents the size of each one of the equipments of the energy system, for example, the size of a turbine.

9.2.1.2 Constraints

For the solving of the optimization problem it is necessary to establish several constraints that delimitate the problem, according to the real behaviour of the cogeneration system. These constraints can be classified in some groups.

9.2.1.2.1 Non negativity

All the variables, operational and of design, must be higher or equal to zero. The opposite doesn't have any sense in the real conditions of the plant.

9.2.1.2.2 Demand requirements

The variables must take values that satisfy the demand of heat, power and cooling.

9.2.1.2.3 Capacity of the equipments

The solution of the model must be restricted in the concerning to the capacity of the equipments. Otherwise, it would be possible that the final results of the optimization could give very big sizes of equipments as the optimal for the power plant, which can't be feasible in the real conditions of the user.

9.2.1.2.4 Value of the operational variables

The production of the power plant is subject to the available equipments. In this way, the operational variables, like the power production in a specific period of time, can not take values superior to the capacity of the respective equipment. In this group of constraints takes place the interrelation between the operational and the design variables.

9.3 Definition of the models

9.3.1 Gas turbine

The simple open cycle gas turbine system is based in the Brayton cycle, where a gas-air mix is the working substance. This model simulates gas turbine systems with a net electrical power higher than 5 MW, for capacities lower than 5 MW is used the micro gas turbine systems.

9.3.1.1 Steps

This model is realised in two steps, one first pre-model defines the size and the operation of the gas turbine components and of the auxiliary equipments, assuming that the gas turbine works all the time at full load. This first model calculates the main characteristics of the gas turbine, as the work temperatures and pressures, the heat to power ratio, the air mass flow, the pressure ratio, the net power and net heat produced.

The second model uses the information calculated in the first model and defines the actual operation of the gas turbine components and of the auxiliary equipments along the year, taking into account that the gas turbine works at partial load and that the weather vary the performance of the same. Both models are built in order to find the optimal design and operation of the gas turbine components and auxiliary equipment from the economical viewpoint.

9.3.1.2 Pre-model

The Figure 62 shows the model of simple open cycle gas turbine that represents the pre-model.
In this model, it is assumed that the gas turbine works at full load along the year, but the system counts with an auxiliary boiler and with equipment to buy or sell electricity in case of peak of heat and/or power.

The main target of the pre-model is to find the nominal characteristics of the gas turbine:

- Air mass flow
- Pressure ratio
- Compressor outlet temperature
- Turbine inlet temperature
- Turbine outlet temperature
- Heat to power ratio
- Net Power generated
- Net Heat generated
- Fuel mass flow

These variables represent the nominal characteristics of the gas turbine, required to supply the demand of heat and power in the most economical way.

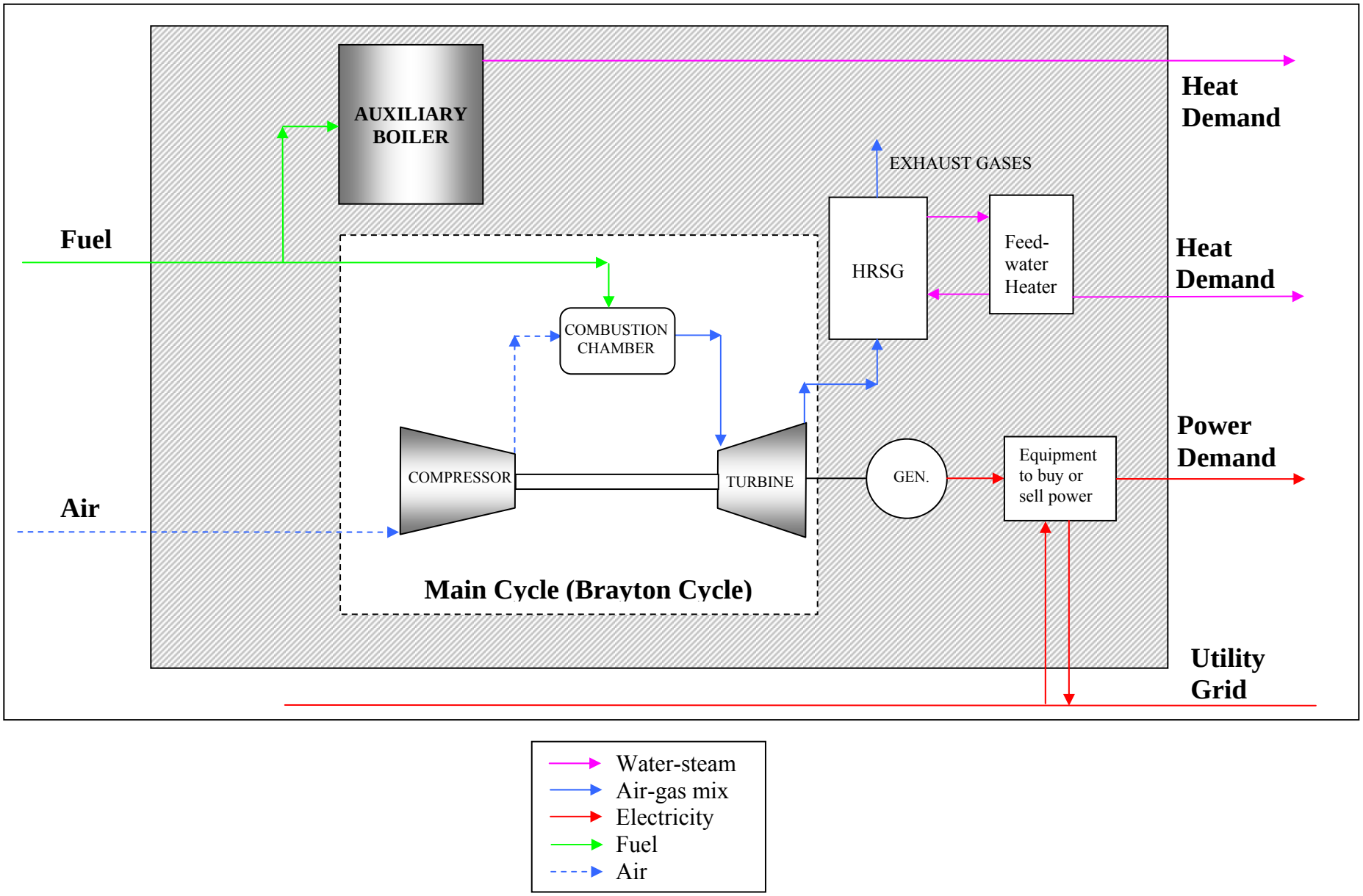


Figure 62. Pre-model of simple open cycle gas turbine

9.3.1.2.1 Assumptions

- The gas turbine works all the time at full load
- The weather does not affect the performance of the gas turbine
- According with the literature, the purchase costs should be expressed in terms of each component of the gas turbine system (air compressor, combustion chamber, gas turbine, heat recovery steam generator and feed water heater) [3]
- Use of auxiliary boiler in a peak of heat demand
- Possibility of buy or sell electricity along the year
- Use of natural gas as fuel
- The life of the plant is assumed as 25 years
- The fuel price is constant during the year, but it is assumed a fuel price variation along the life of the plant

9.3.1.2.2 Objective Function

The objective function of this model represented in the next equation,

Minimize:

Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O) (194.)

Design Costs

The design cost function is defined as follows:

$$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{ACO} + Z_{CCH} + Z_{GT} + Z_{HRSG} + Z_{FH} + Z_{AB} + Z_{CG} \right] \quad (195.)$$

The maintenance factor (φ) is assumed as 1.06 for all components of the gas turbine system, and the annual fixed charge rate (Ψ) as 0.06743903, according with the economics calculations.

The expressions for obtaining the purchase costs of the components of the gas turbine systems are presented in the Table 42, and the constants used in this set of equations are presented in the Table 43. [3]

A set of constants is also added to this equation, because this approximation method proposed by Frangopoulos was made in 1992-1993, therefore was needed to use factors of update, presented in the Table 44. [10] [11]

Design Variables

The design variables of this model are some of the thermal characteristics of the components of the system, the Table 45 resume the design variables for the gas turbine model.

Operational Costs

The operation cost function is defined as:

$$\Phi_O = \left[\left(C_f \overset{*}{m}_f\right)(8000 \text{ hours})\right] + \sum_{n=1}^{4}\left[C_B \overset{*}{W}_{BY} + C_S \overset{*}{W}_S + C_{AH} \overset{*}{Q}_{AH}\right]\Delta t_n \qquad (196.)$$

The time intervals Δt_n are each one of length equal to 2000 hours, which means that the total period of operation is 8000 hours, because the number of time intervals is four, like the year seasons.

The operational costs represent:

- The cost of fuel required to generate heat and power: $\left(C_f \overset{*}{m}_f\right)(8000 \text{ hours})$
- The cost of buy power: $\sum_{n=1}^{4}(C_{BY} W_{BY})\Delta t_n$
- The negative cost of sell power: $\sum_{n=1}^{4}(C_S W_S)\Delta t_n$
- The cost of generate additional heat $\sum_{n=1}^{4}(C_{AH} Q_{AH})\Delta t_n$

These operational costs depend on the type of user that will be use the energy system. The costs of fuel, buy and/or sell power and the cost of generate additional heat change if the user is residential, commercial or industrial. The coefficients of these equations for all the type of user are summarized in the Table 46. [11] [12]

Operational Variables

The operational variables of the gas turbine pre-model are:

- The fuel mass flow consumed by the system $\left(\overset{*}{m}_f\right)$ in all the periods of time (8000 hours)
- The power bought by the system (W_{BY}) in each period of time, totally 4 variables
- The power sold by the system (W_S) in each period of time, totally 4 variables
- The auxiliary heat generated by the system (Q_{AH}) in each period of time, totally 4 variables

It is possible observe, that the total number of operational variables is 13.

9.3.1.2.3 Constraints

Non negativity
Both the design variables and the operational variables must be higher or equal to zero.

Demand requirements
The energy system should satisfy the next energy equations in order to supply the energy demand of the user in each period of time, it is important notify that the power generated by the gas turbine system is constant for all the periods of time.

Power Demand:
$$[W_{GT} - W_{BY} + W_S]_n = [W_{DEMAND}]_n \qquad n = 1,...,4 \tag{197.}$$

Heat Demand:
$$[Q_{GT} + Q_{AH}]_n \geq [Q_{DEMAND}]_n \qquad n = 1,...,4 \tag{198.}$$

It is assumed in this work that the heat generated for the system plus the auxiliary heat could be bigger than the heat demand, which means that sometimes a portion of the total heat generated is wasted. This restriction is made because in some cases, the relation between heat demand and power demand is very small and a small plant could be more expensive than other with a bigger heat generation capacity.

Capacity of the equipments
With these constraints the sizes of all the equipments of the gas turbine system are restricted.

Thermodynamics Equations
The pre-model contains all the equations presented in the chapter of thermodynamics, which relates the temperatures, pressures, mass flows, power generated, heat generated and heat to power ratio.

Physical Constraints
This model has constraints to the maximal temperatures and pressures along the gas turbine system available with the current technology.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

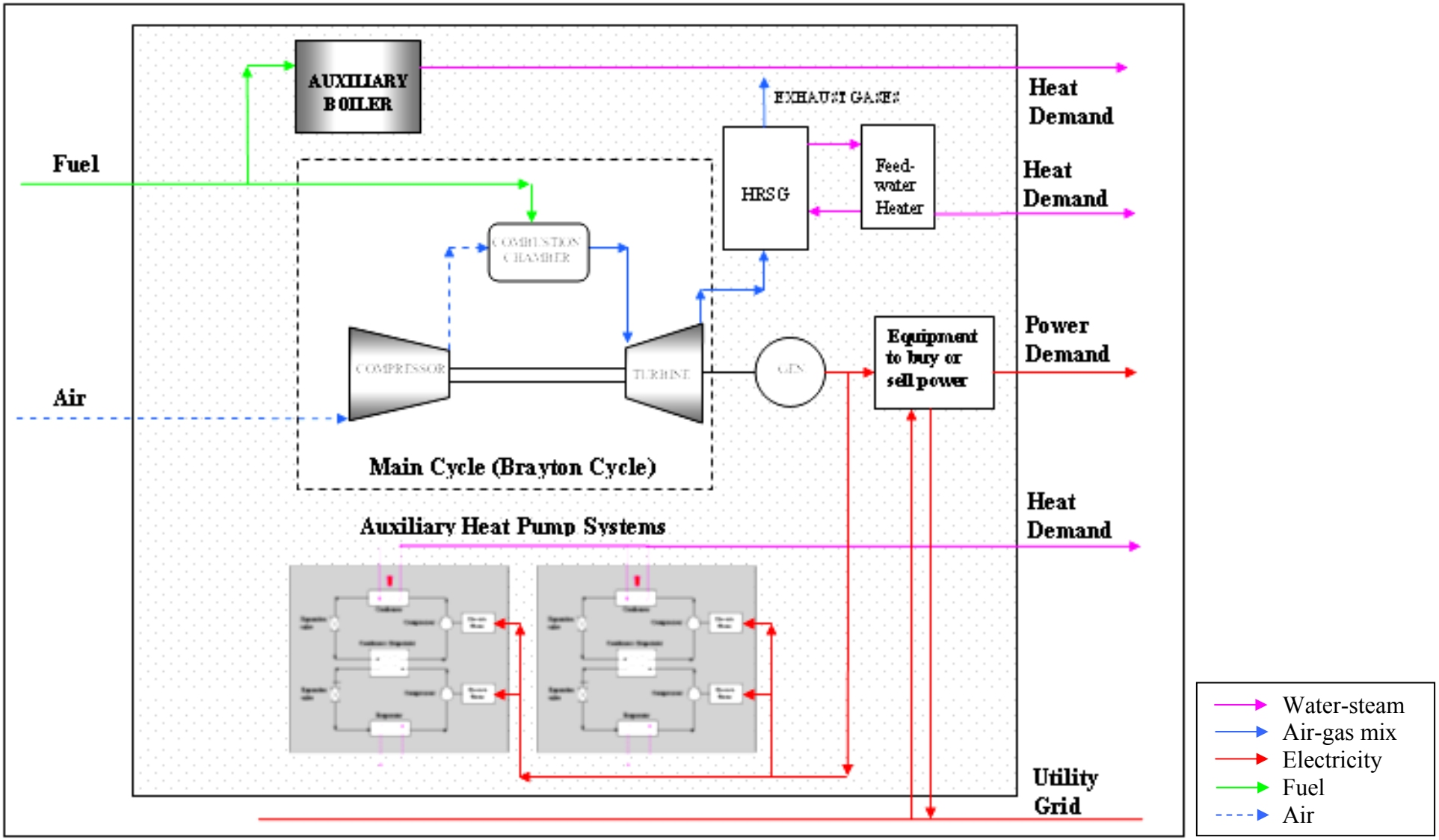


Figure 63. Model of simple open cycle gas turbine with auxiliary equipments

9.3.1.3 Complete model of the gas turbine

The Figure 63 shows the complete model of simple open cycle gas turbine with partial load. The model takes as inlet data the next parameters calculated by the gas turbine pre model:

- Air mass flow
- Pressure ratio
- Turbine inlet temperature
- Turbine outlet temperature
- Compressor outlet temperature
- Heat to power ratio
- Net Power generated
- Fuel mass flow

With these variables is possible to define the *design point* of the gas turbine, and the concept design phase is completed because all the gas turbine components are already fixed, calculated by the pre-model. The standard ambient conditions are taken as the ambient conditions in the concept design phase, which means, an ambient temperature as 15°C and a relative humidity as 60%.

This model evaluates the off design performance of the gas turbine at other key operating conditions:

- The ambient conditions change during the year
- The power load of the gas turbine could change during the year, working at partial load

It is important to notify that the gas turbine could be work at partial load, but the total efficiency of the cycle is lower, because when the rated power generated decrease, the isentropic efficiency of the turbine decrease also, as well as the compressor efficiency, the pressure ratio, the turbine inlet temperature and the air mass flow.

In this model auxiliary boilers and heat pumps are used as auxiliary equipment to generate additional heat in case of energy peak. The results of the heat pump analysis are used for the complete gas turbine model, specially the number of heat pumps in cascade in order to supply a heat demand.

The main targets of the complete gas turbine model are:

- Calculate the energy generated by the gas turbine system in all the periods of time, taking into account the variation of the ambient conditions during the year
- Using the off design method, calculate all the pressures, temperatures, mass flows and efficiencies of the components of the gas turbine system when it is working at partial load
- Define the power sold or bought by the system in all the periods of time
- Define the size of the auxiliary boiler and the additional heat generated in all the periods of time
- Define the size of the heat pump cascade system, the number of heat pump units and additional heat generated in all the periods of time

9.3.1.3.1 Assumptions

- The gas turbine could works at partial load, the lowest limit for the net power output in each period is 75% of the nominal net power output
- The performance at partial load of the gas turbine is predicted using the gas turbine map described in the chapter of thermodynamics
- The gas turbine performance is affected for the ambient conditions
- The total efficiency of the system decrease at partial load
- The standard ambient conditions are the design point conditions (Ambient temperature = 15˚, Relative humidity = 60%)
- The design point is defined in the gas turbine pre model
- According with the literature, the purchase costs should be expressed in terms of each component of the gas turbine system (air compressor, combustion chamber, gas turbine, heat recovery steam generator and feedwater heater)
- Use of auxiliary boiler in a peak of heat demand
- Use of cascade heat pump systems in a peak of heat demand
- The cascade heat pump systems should run with the power generated by the gas turbine
- Possibility of buy or sell electricity along the year
- Use of natural gas and fuel oil as fuels
- The life of the plant is assumed as 25 years
- The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant

9.3.1.3.2 Objective Function

The objective function of this model represented in the next equation,

Minimize:
Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O) (199.)

Design Costs

The design cost function is defined as follows:

$$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{AC} + Z_{CC} + Z_{GT} + Z_{HRSG} + Z_{FH} + Z_{AB} + Z_{CG} + n_{HP} Z_{HP} \right] \quad (200.)$$

The maintenance factor (φ) is assumed as 1.06 for all components of the gas turbine system, and the annual fixed charge rate (Ψ) as 0.06743903, according with the economics calculations. [3]

The expressions for obtaining the purchase costs of the components of the gas turbine systems (Z_{AC}, Z_{CC}, Z_{GT}, Z_{HRSG}, Z_{FH}, Z_{AB}, Z_{CG}) are presented in the Table 42, the constants used in this set of equations are presented in the Table 43, and the factors of update are presented in the Table 44. The expression to obtain the purchase cost of the cascade heat pump systems is defined in the next equation. [13]

$$Z_{HP} = C_{HP} Q_{HP}$$
$$C_{HP} = 516 \frac{\$}{(kW)_{TH}} \qquad (201.)$$

Design Variables

The design variables of the complete gas turbine model are some of the thermal characteristics of the components of the system, the Table 48 resume the design variables for this model. Some of these variables are already defined by the pre model, as the thermodynamic characteristics of the Brayton cycle components (Compressor, Gas Turbine, Combustion Chamber, Heat Recovery Stem Generator and Feedwater Heater), and these are also taken as the nominal characteristics of the plant.

Although the complete gas turbine model revaluates the design variables of the next equipments:

- Auxiliary Boiler
- Connection to the grid
- Cascade Heat Pump System

Operational Costs

The operation cost function for the complete gas turbine model is defined as:

$$\Phi_O = \sum_{n=1}^{4} \left[C_f \overset{*}{m}_f + C_{BY} W_{BY} + C_S W_S + C_{AH} Q_{AH} \right] \Delta t_n \qquad (202.)$$

As in the pre model, The time intervals Δt_n are each one of length equal to 2000 hours, which means that the total period of operation is 8000 hours, because the number of time intervals is four, like the year seasons. As well as for the pre model, the coefficients of these equations for all the type of user are summarized in the Table 46.

Operational Variables

The operational variables of the complete gas turbine model are:

- The fuel mass flow consumed by the system $\left(\overset{*}{m}_f \right)$ in each period of time, totally 4 variables. It muss be clear, that in the pre model the fuel mass flow calculated is the nominal mass flow, and in the complete gas turbine model, for each period of time the fuel mass flow at off design is calculated
- The power bought by the system (W_{BY}) in each period of time, totally 4 variables
- The power sold by the system (W_S) in each period of time, totally 4 variables
- The auxiliary heat generated by the system (Q_{AH}) in each period of time, totally 4 variables

9.3.1.3.3 Constraints

Non negativity
Both the design variables and the operational variables must be higher or equal to zero.

Demand requirements
The gas turbine system should satisfy the next energy equations in order to supply the energy demand of the user in each period of time.

Power Demand:
$$\left[W_{GT} - \left(n_{HP} W_{HP}\right) - W_{BY} + W_S\right]_n = \left[W_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \qquad (203.)$$

Heat Demand:
$$\left[Q_{GT} + \left(n_{HP} Q_{HP}\right) + Q_{AH}\right]_n \geq \left[Q_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \qquad (204.)$$

It is assumed in this model, as well as for the pre model that the total heat generated for the system (which means the heat generated by the gas turbine plus the heat generated by the cascade heat pump systems, plus the heat generated by the auxiliary boiler) could be bigger than the heat demand, which means that sometimes a portion of the total heat generated is wasted. It is also important to make clear, that the number of cascade heat pump systems in service in each period of time can be different, and these variables as well as the total number of cascade systems are integer variables.

Capacity of the equipments
With these constraints the sizes of all the equipments of the gas turbine system are restricted.

On Design Parameters
Some of the parameters calculated in the pre model are now inlet information to the complete gas turbine model, that here are treated as constraints. These parameters are:

- Air mass flow
- Pressure ratio
- Turbine inlet temperature
- Turbine outlet temperature
- Compressor outlet temperature
- Heat to power ratio
- Net Power generated
- Fuel mass flow

Physical Constraints
This model has constraints to the maximal temperatures and pressures along the gas turbine system available with the current technology.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

Off design constraints

In the chapter of Thermodynamics was explained the theory of off design for gas turbines. This research use the gas turbine map shown in the Figure 50, as the prediction map of the gas turbine in off design – part load. [14]

The work area of all the gas turbines studied in this research is represented with a red line. The limits of this area are:

- $1 \leq \frac{T_3}{T_1} \leq 0.45$ (205.)
- $1.1 \leq \frac{n}{n_n \sqrt{\Theta_1}} \leq 0.95$ (206.)

As was explained before the net output heat and power of the gas turbine are controlled with the fuel mass flow. The fuel mass flow in the design point, which means at full load, is calculated with the equations of thermodynamics presented in the chapter 7. But the fuel mass flow in off design is calculated using the gas turbine map, according with the weather conditions and the partial load.

The proceeding used in this research to constraint the fuel mass flow of the gas turbine during all the periods of time is:

- Once the nominal parameters of the gas turbine are already calculated by the pre model, the design point is determined in the gas turbine performance map

- The performance gas turbine map is parameterized using the different variables, and the results are presented in the Table 49.

- Using the parameterized results of the performance gas turbine map and the design point calculated in the pre model, is possible to calculate the design variables of this specific gas turbine in any point at partial load, this is called in this research as *the partial load table* (see an example in Table 52, Appendix B). The partial load table is unique for each turbine and contain all the information about the performance in off design.

- Once the design variables of the gas turbine at partial load are known, it is possible to calculate the fuel mass flow at any point at partial load and therefore the net output heat and power of the plant

- The fuel mass flow is restricted for the work area in the performance map and also for the ambient conditions. In all the cases the maximal fuel mass flow is less than the nominal fuel mass flow, and the maximal fuel mass flow in each period of time depends on the weather conditions. The minimal fuel is the required to produce in each period at least the 75% of the nominal net power output.

The next equations express the constraints for the fuel mass flow of the gas turbine at partial load during all the periods of time.

$$\left(\overset{*}{m}_f\right)_{\substack{\text{PRODUCING 75\% OF}\\ \text{NOMINAL POWER}}} \leq \overset{*}{m}_{f1} \leq \left(\overset{*}{m}_f\right)_{\substack{\text{MAXIMAL IN}\\ \text{WEATHER 1}}}$$

$$\left(\overset{*}{m}_f\right)_{\substack{\text{PRODUCING 75\% OF}\\ \text{NOMINAL POWER}}} \leq \overset{*}{m}_{f2} \leq \left(\overset{*}{m}_f\right)_{\substack{\text{MAXIMAL IN}\\ \text{WEATHER 2}}}$$

$$\left(\overset{*}{m}_f\right)_{\substack{\text{PRODUCING 75\% OF}\\ \text{NOMINAL POWER}}} \leq \overset{*}{m}_{f3} \leq \left(\overset{*}{m}_f\right)_{\substack{\text{MAXIMAL IN}\\ \text{WEATHER 3}}} \qquad (207.)$$

$$\left(\overset{*}{m}_f\right)_{\substack{\text{PRODUCING 75\% OF}\\ \text{NOMINAL POWER}}} \leq \overset{*}{m}_{f4} \leq \left(\overset{*}{m}_f\right)_{\substack{\text{MAXIMAL IN}\\ \text{WEATHER 4}}}$$

The net output heat and power that the plant could generate at partial load in any point of the off design performance map, are functions of the fuel mass flow in each period of time, and these functions are unique for each turbine. The functions which relate the energy production and the fuel mass flow consumed are taken from *the partial load table.* The next equations show the heat and power production as functions of the fuel mass flow.

$$W_{GT1} = f\left(\overset{*}{m}_{f1}\right),\ W_{GT2} = f\left(\overset{*}{m}_{f2}\right),\ W_{GT3} = f\left(\overset{*}{m}_{f3}\right),\ W_{GT4} = f\left(\overset{*}{m}_{f4}\right) \qquad (208.)$$

$$Q_{GT1} = f\left(\overset{*}{m}_{f1}\right),\ Q_{GT2} = f\left(\overset{*}{m}_{f2}\right),\ Q_{GT3} = f\left(\overset{*}{m}_{f3}\right),\ Q_{GT4} = f\left(\overset{*}{m}_{f4}\right) \qquad (209.)$$

Finally, all the constraints used in the complete gas turbine model are presented and summarized in the Table 50 and Table 51.

9.3.2 Micro gas turbine

The model of the recuperated cycle micro gas turbines is presented in the Figure 64. In the chapter of thermodynamics was explained the theory of these turbines, and was assumed that the recuperated cycle micro gas turbine works at full load along the year, which means that the isentropic efficiency of the compressor and the turbine, the pressure ratio, the fuel mass flow and the air mass flow remains constant and no changes with the ambient temperature.

It is also taken as an assumption the *design point* at the ambient conditions, which means, an ambient temperature as 15°C and a relative humidity as 60%.

The method used in this model to control the net heat and power generation is to change the number of micro gas turbines that are in service in each period of time. An auxiliary boiler is used as extra equipment in case of peak heat demand. The energy system allows also buy or sell power in case of excess or deficit of electricity.

The main targets of the micro gas turbines model are:

- Calculate the nominal characteristics of each micro gas turbine:
    - Air mass flow
    - Pressure ratio
    - Compressor outlet temperature
    - Combustion chamber inlet temperature
    - Turbine inlet temperature
    - Turbine outlet temperature
    - Heat to power ratio
    - Net Power generated
    - Net Heat generated
    - Fuel mass flow
- Calculate the total number of micro gas turbines that should be purchased in order to satisfy the heat and power demand
- Calculate the number of micro gas turbines that should operate in each period of time during the year in order to satisfy each period energy demand
- Define the power sold or bought by the system in all the periods of time
- Define the size of the auxiliary boiler and the additional heat generated in all the periods of time

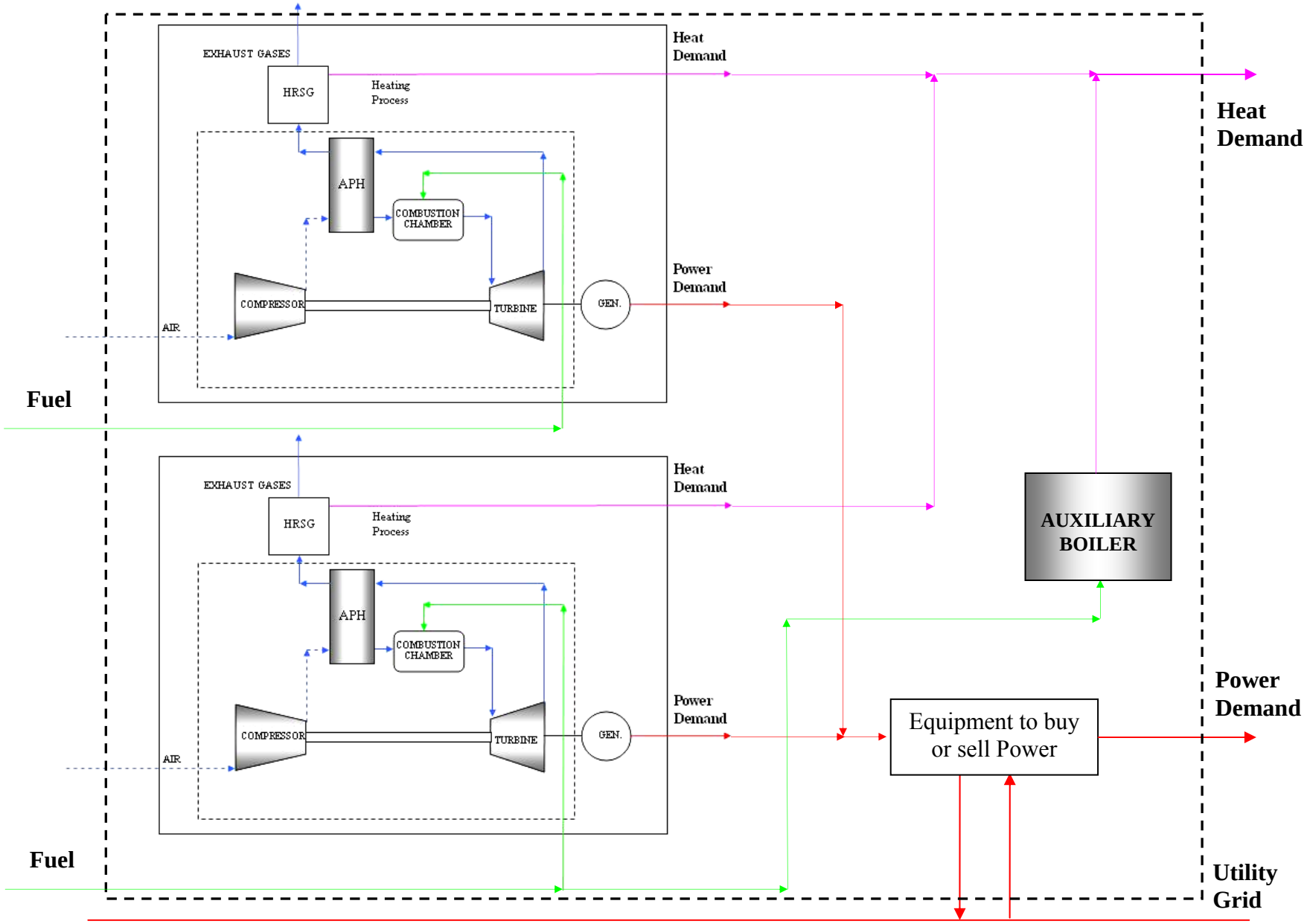


Figure 64. Micro gas turbine model

9.3.2.1 Assumptions

- Each micro gas turbine works all the time at full load
- The ambient conditions do not affect the performance of the micro gas turbine (this assumption was made taking into account the efficiency charts of micro gas turbine manufactured by TURBEC and CAPSTONE) [15] [16]
- The *design point* of the micro gas turbine is taken at the standard ambient conditions (ambient temperature = 15°C, relative humidity = 60%)
- According with the literature, the purchase costs are given in terms of units of micro gas turbines and not in terms of the components (like in the gas turbine models)
- Use of auxiliary boiler in a peak of heat demand
- Possibility of buy or sell electricity along the year
- Use of natural gas as fuel
- The life of the plant is assumed as 25 years
- The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant

9.3.2.2 Objective Function

The objective function is presented in the next equation,

Minimise:
Total **annualised** costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O) (210.)

Design Costs

The design cost function is defined as follows:

$$\Phi_D = \varphi \cdot \psi \cdot \left[n_{MGT} Z_{MGT} + Z_{AB} + Z_{CG} \right] \quad (211.)$$

As well as in the previous models, the maintenance factor (φ) is 1.06 and the annual fixed charge rate (Ψ) as 0.06743903.

The expressions of the purchase costs of the components of the micro gas turbine model are presented in the Table 53. The coefficients expressed in these equations, as well as the factors of update are shown also in the Table 53. [15] [16]

Design Variables

The design variables of the micro gas turbine model are the size of the equipments in terms of energy; although the size of the micro gas turbine (in terms of net output power) is expressed in terms of thermodynamics parameters of the recuperated cycle. These parameters are also design variables, but all they are summarized into the micro gas turbine size. The design variables of the micro gas turbine model are summarized in the Table 54.

Operational Costs

The operation cost function for the micro gas turbine model is expressed in the next equation:

$$\Phi_O = \sum_{n=1}^{4}\left[\left(n_{MGT}\cdot C_f \cdot \overset{*}{m}_f\right) + C_{BY}W_{BY} + C_S W_S + C_{AH}Q_{AH}\right]\Delta t_n \qquad (212.)$$

It should be clear that the number of micro gas turbine units in service could change during the year. The time intervals Δt_n are four (one for each season), each one of length equal to 2000 hours.

The operational costs represent:

- The cost of fuel required to generate heat and power: $\sum_{n=1}^{4}\left(n_{MGT}\cdot C_f \cdot \overset{*}{m}_f\right)\Delta t_n$
- The cost of buy power: $\sum_{n=1}^{4}\left(C_{BY}W_{BY}\right)\Delta t_n$
- The negative cost of sell power: $\sum_{n=1}^{4}\left(C_S W_S\right)\Delta t_n$
- The cost of generate additional heat with the auxiliary $\sum_{n=1}^{4}\left(C_{AH}Q_{AH}\right)\Delta t_n$

The coefficients of these equations for all the type of user are summarized in the Table 46.

Operational Variables

The operational variables of the micro gas turbine model are:

- The number of micro gas turbine units which are in service in each period of time (n_{MGTK}), because this is the way used in this model to control the net energy output. The fuel mass flow consumed by each micro gas turbine unit $\left(\overset{*}{m}_f\right)$ is not considered as a operational variable, because each micro gas turbine works all the time at full load and the fuel mass flow in all the periods of time remains constant
- The power bought by the system $\left(W_{BY}\right)$ in each period of time, totally 4 variables
- The power sold by the system $\left(W_S\right)$ in each period of time, totally 4 variables
- The auxiliary heat generated by the system $\left(Q_{AH}\right)$ in each period of time, totally 4 variables

9.3.2.3 Constraints

Non negativity
Both the design variables and the operational variables must be higher or equal to zero.

Integer Variables
The variables n_{MGT} (total number of micro gas turbine units installed or in service during each period) must be integer variables, because they can't adopt fractional values.

Demand requirements
The micro gas turbines system should satisfy the energy equations in order to supply the energy demand of the user in each period of time.

Power Demand:
$$\left[\left(n_{MGT} \cdot W_{MGT}\right) - W_B + W_S\right]_n = \left[W_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \qquad (213.)$$

Heat Demand:
$$\left[\left(n_{MGT} Q_{MGT}\right) + Q_{AH}\right]_n \geq \left[Q_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \qquad (214.)$$

The total heat generated for all the micro gas turbine units in all the periods, could be bigger than the heat demand, which means that sometimes a portion of the total heat generated is wasted.

Thermodynamics Equations
The micro gas turbine model contains all the thermodynamics equations, which relates the temperatures, pressures, mass flows, power generated, heat generated and heat to power ratio.

Capacity of the equipments
With these constraints the sizes of all the equipments of the micro gas turbine system are restricted.

Physical Constraints
The micro gas turbine model constraint the maximal temperatures and pressures along the recuperated cycle, making use of the literature about current technology.

Value of the operational variables
These constraints impede that the number of micro gas turbine units used in each period of time exceed the maximal number of micro gas turbine units, which possess the energy system. Therefore the maximal number of micro gas turbine units in service in any period of time is equal or lower than the maximal number of units.

All the constraints used in the complete model of the micro gas turbine system are presented in the Table 55 and Table 56.

9.3.3 Back Pressure Steam Turbine

The back pressure steam turbine system was explained in the chapter of thermodynamics, but basically consist on a steam Rankine cycle with a boiler, a steam turbine, a heat exchanger (Feed water heater) and a feed pump. The back pressure steam turbines used in this research are available for the small users (lower than 5 MW net output power), as well as for big users (bigger than 5 MW net output power).

These models simulate back pressure steam turbine systems with other additional equipments (auxiliary boilers, heat pumps and equipment to buy or sell electricity) in order to satisfy the user energy demand.

9.3.3.1 Steps

This model as well as for the gas turbine model, is realised in two steps, one first pre-model defines the size and the operation of the back pressure steam turbine components and of the auxiliary equipments, assuming that the back pressure steam turbine works all the time at full load. This pre model calculates the main thermodynamic parameter of the back pressure steam turbine, the heat to power ratio.

Once the optimal heat to power ratio of the system is found from the economical viewpoint, three different study cases are defined. As was explained in the thermodynamics theory, one very important parameter in the back pressure cogeneration systems is the inlet steam condition to the turbine. Therefore, the study cases are three different inlet steam conditions to the back pressure steam turbine, but taking into account that the three cases should satisfy exactly the same heat to power ratio calculated by the pre model.

The second model uses the heat to power ratio calculated in the first model and all the thermodynamic properties of the turbine inlet steam conditions determined in the research (three different study cases), and defines for each study case the actual design and operation of the system components and of the auxiliary equipments along the year, taking into account that the back pressure steam turbine could work at partial load.

9.3.3.2 Pre-model

The pre model of the back pressure steam turbine is represented in the Figure 65. The back pressure steam turbine works all the periods of time at full load and the system allows the service of auxiliary boiler and equipment to buy or sell electricity in case of an energy peak.

The main objective of the pre model of the back pressure steam turbine system is find the optimal heat to power ratio of the cogeneration system, which satisfy the energy demand of the user in the most economical way possible. Some parameters of the system like the fuel mass flow and the heat generation are expressed in terms of the power generation and the heat to power ratio. After words, this heat to power ratio calculated is used to determine the three study cases and then calculate with the second model, the design and operation of the plant for each study case.

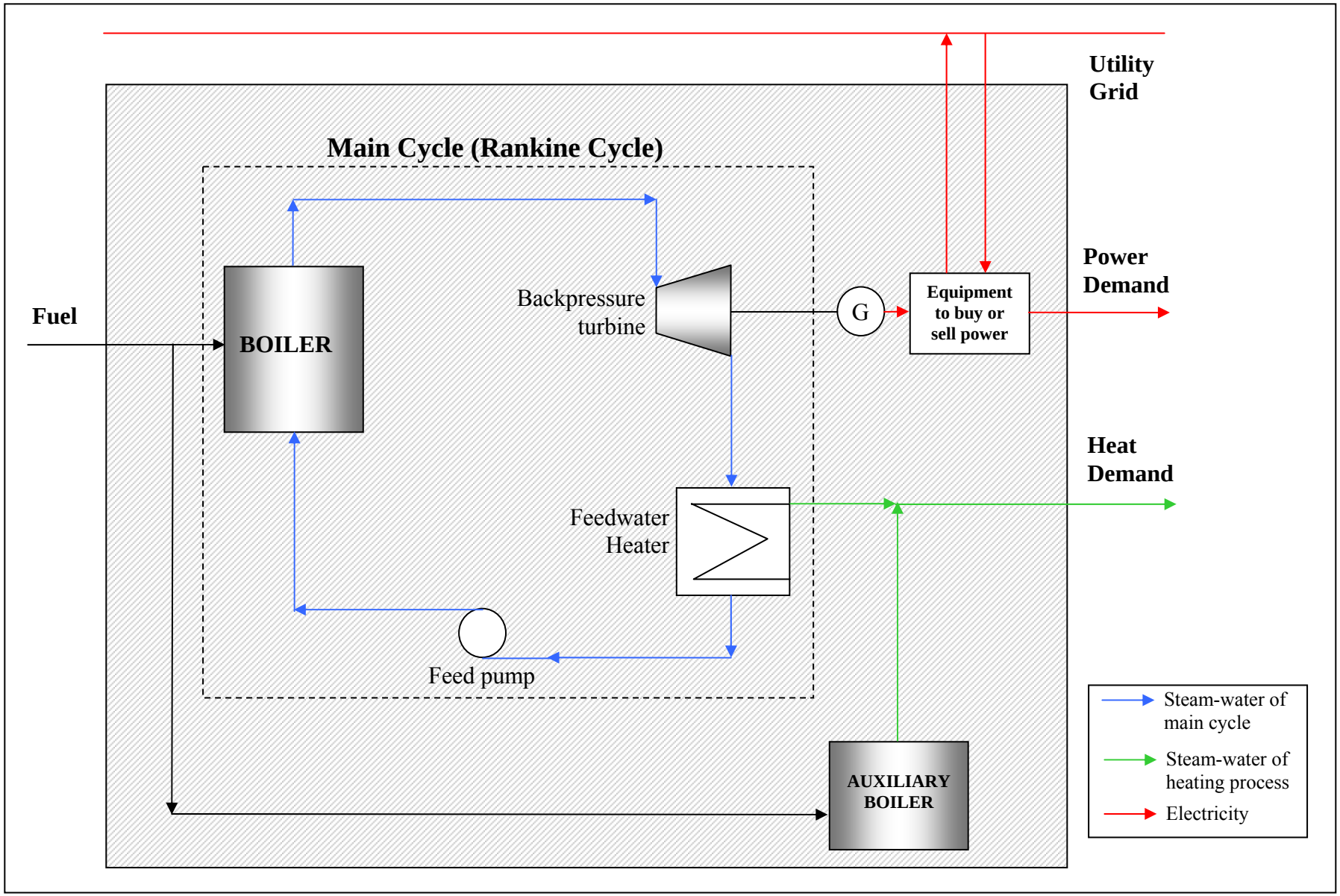


Figure 65. Pre model of the back pressure steam turbine system

9.3.3.2.1 Assumptions

- The back pressure steam turbine works all the time at full load
- The purchase costs are expressed in terms of each component of the back pressure steam turbine system (Main boiler, steam turbine, feedwater heater, feed pump, etc)
- Use of auxiliary boiler in a peak of heat demand
- Possibility of buy or sell electricity along the year
- Use of fuel oil as fuel
- The life of the plant is assumed as 25 years
- The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant

9.3.3.2.2 Objective Function

The objective function of the back pressure steam turbine is,

Minimise:
Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O) (215.)

Design Costs

The design cost function is defined:

$$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{MB} + Z_{ST} + Z_{FH} + Z_{AB} + Z_{CG} \right] \quad (216.)$$

The maintenance factor (φ) is 1.06 for all components of the system, and the annual fixed charge rate (Ψ) is 0.06743903 (life of the plant = 25 years).

The expressions for obtaining the purchase costs of the components of the back pressure steam turbine system are presented in the Table 57, the constants of these equations and the factors of update are presented in the Table 58. The coefficients used for these expressions were calculated based on the studies of Frangopoulus et al. [1][2][3]. Information about fuel and electricity prices was used. The values have been annualised. It was found in the literature, that the purchase cost of the feed pump is already included in the purchase cost of the main boiler.

Design Variables

The design variables of the back pressure steam turbine pre model are the size of the equipments in terms of energy. It should be clear that the thermodynamic parameters like the turbine inlet temperature and pressure are not design variables in this model.

The design variables of the back pressure steam turbine model are summarized in the Table 59.

Operational Costs

The operation cost function is defined as:

$$\Phi_O = \left[\left(C_f \overset{*}{m}_f\right)(8000 \text{ hours})\right] + \sum_{n=1}^{4}\left[C_{BY}W_{BY} + C_S W_S + C_{AH}Q_{AH}\right]\Delta t_n \qquad (217.)$$

The time intervals Δt_n are each one of length equal to 2000 hours. The operational costs represent:

- The cost of fuel required to generate heat and power: $\left(C_f \overset{*}{m}_f\right)(8000 \text{ hours})$
- The cost of buy power: $\sum_{n=1}^{4}(C_{BY}W_{BY})\Delta t_n$
- The negative cost of sell power: $\sum_{n=1}^{4}(C_S W_S)\Delta t_n$
- The cost of generate additional heat $\sum_{n=1}^{4}(C_{AH}Q_{AH})\Delta t_n$

The coefficients of these equations for all the type of user are summarized in the Table 46.

Operational Variables

The operational variables of the back pressure steam turbine pre-model are:

- The fuel mass flow consumed by the system $\left(\overset{*}{m}_f\right)$ in all the periods of time (8000 hours).

  But it was explained in the chapter of thermodynamics that the fuel mass flow in the back pressure steam turbine cycle can be expressed in terms of the turbine net output power, and the heat to power ratio. The expression which relates these variables is shown next.

$$\overset{*}{m}_{fst} = \frac{W_{ST} + Q_{ST}}{\eta_B H_U} = \frac{W_{ST} + (\lambda_{ST}W_{ST})}{\eta_B H_U} = \frac{W_{ST}\cdot(1+\lambda_{ST})}{\eta_B H_U} \qquad (218.)$$

- The power bought by the system (W_{BY}) in each period of time, totally 4 variables
- The power sold by the system (W_S) in each period of time, totally 4 variables
- The auxiliary heat generated by the system (Q_{AH}) in each period of time, totally 4 variables

9.3.3.2.3 Constraints

Non negativity
The design variables and the operational variables must be higher or equal to zero.

Demand requirements
The back pressure steam turbine system should satisfy the next energy equations in order to supply the energy demand of the user in each period of time. The power generated by the system is the same in all the periods. In this model the thermodynamic parameters of the back pressure steam turbine are expressed in terms of the turbine net output power and the heat to power ratio.

Power Demand:
$$\left[W_{ST} - W_{BY} + W_S\right]_n = \left[W_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \tag{219.}$$

Heat Demand:
$$\left[\left(\lambda_{ST} \cdot W_{ST}\right) + Q_{AH}\right]_n \geq \left[Q_{DEMAND}\right]_n \qquad n = 1,\ldots,4 \tag{220.}$$

Capacity of the equipments
With these constraints the sizes of all the equipments of the back pressure steam turbine system are restricted.

Thermodynamics Equations
With these equations is possible express some parameters like the fuel mass flow and the heat generation in function of the power generation and the heat to power ratio, this last one is a very important parameter required to begin with the complete back pressure model.

Physical Constraints
The heat to power ratio is limited by the pressures and temperatures at the inlet and outlet of the back pressure steam turbine. The turbine outlet temperature is already known and with this value is possible to know the minimal and maximal possible values of the pressure and the temperature at the inlet. With all the previous information the limits of the heat to power ratio are defined.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

All the constraints used in the pre model of the back pressure steam turbine system are summarised and presented in the Table 60.

9.3.3.3 Complete model of the back pressure steam turbine system

The Figure 66 shows the complete model of the back pressure steam turbine system with partial load. The complete model takes as inlet data the heat to power ratio calculated in the pre model, which is the optimal from the economical viewpoint. Then three different study cases are defined for the same heat to power ratio. The criteria used to define the three study cases was based in the temperature range observed at the inlet of normal back pressure steam turbines. It was found that the normal turbine inlet temperatures were between 400˚C - 500˚C. Therefore the three study cases are:

First case: Turbine inlet temperature = 400˚C

Second case: Turbine inlet temperature = 450˚C

Third case: Turbine inlet temperature = 500˚C

In order to know all the thermodynamics characteristics of the steam for each study case at the turbine inlet, knowing only the heat to power ratio and the turbine outlet temperature (110˚C), it was made a program using Microsoft Excel and the PWS software (Add-ins), which can solve the next topics:

- Calculate the turbine inlet pressure, knowing the turbine inlet temperature, the heat to power ratio, the turbine outlet temperature and the isentropic efficiency of the steam turbine
- Calculate the enthalpy and entropy at the inlet and at outlet of the steam turbine knowing the pressures and temperatures

With the heat to power ratio (calculated in the pre model), the turbine outlet temperature (known from the thermodynamics chapter) and the pressures and temperatures at the turbine inlet is possible to define the design point of the back pressure steam turbine. The steam turbine could work at partial load, but the total efficiency of the cycle is lower, because when the rated power generated decrease, the isentropic efficiency of the turbine decrease also.

In this model auxiliary boilers and heat pumps are used as auxiliary equipment to generate additional heat in case of energy peak.

The main targets of the complete model of the back pressure steam turbine system are:

- Define the size of all the components of the back pressure steam turbine system
- Calculate the energy generated by the back pressure steam turbine system in all the periods of time, taking into account that the steam turbine could work at partial load
- Define the steam mass flow in each period of time, required to generate the energy at partial load
- Define the power sold or bought by the system in all the periods of time
- Define the size of the auxiliary boiler and the additional heat generated in all the periods of time
- Define the size of the heat pump cascade system, the number of heat pump units and additional heat generated in all the periods of time

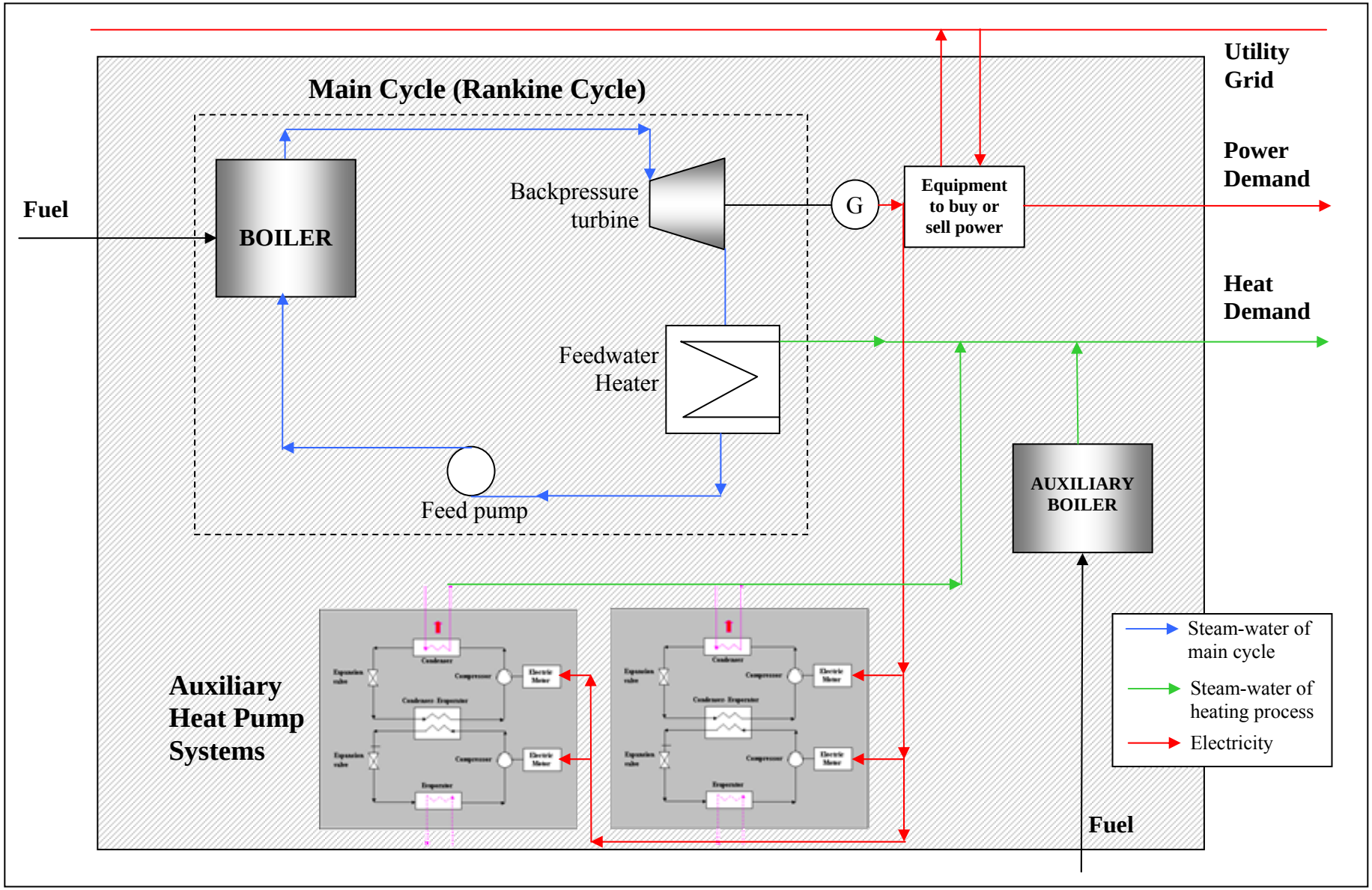


Figure 66. Complete model of the back pressure steam turbine system

9.3.3.3.1 Assumptions

- The back pressure steam turbine could works at partial load, the lowest limit for the net power output in each period is 75% of the nominal net power output
- The performance at partial load of the back pressure steam turbine is assumed as the curve of the **¡Error! No se encuentra el origen de la referencia.**.
- The total efficiency of the system decrease at partial load
- The heat to power ratio of the system is already defined by the pre model
- The design point is defined with the heat to power ratio, the isentropic efficiency of the steam turbine, the outlet turbine temperature (known) and the turbine inlet temperature and pressure, these last are specified for three different study cases
- The purchase costs are expressed in terms of each component of the back pressure steam turbine system (Main boiler, steam turbine, feedwater heater, feed pump, etc)
- Use of auxiliary boiler in a peak of heat demand
- Use of cascade heat pump systems in a peak of heat demand
- The cascade heat pump systems should run with the power generated by steam turbine
- Possibility of buy or sell electricity along the year
- Use of fuel oil as fuel
- The life of the plant is assumed as 25 years
- The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant

9.3.3.3.2 Objective Function

The objective function of the complete model of back pressure steam turbine is,

Minimize:
Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O) (221.)

Design Costs

The design cost function is defined:

$$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{MB} + Z_{ST} + Z_{FH} + Z_{AB} + Z_{CG} + n_{HP} Z_{HP} \right] \quad (222.)$$

The maintenance factor (φ) is 1.06 for all components of the system, and the annual fixed charge rate (Ψ) is 0.06743903 (life of the plant = 25 years).

The expressions for obtaining the purchase costs of the components of the back pressure steam turbine system are shown in the Table 61, the constants of these equations and the factors of update are presented in the Table 62. The purchase cost of the feed pump is already included in the purchase cost of the main boiler.

Design Variables

The design variables of the complete model of the back pressure steam turbine are the size of the equipments in terms of energy.

Some of these variables are already defined by the pre model, as the heat to power ratio, although the complete model of the back pressure steam turbine revaluates other variables, as is explained in the next table.

Operational Costs

The operation cost function for the complete gas turbine model is defined as:

$$\Phi_O = \sum_{n=1}^{4}\left[C_f \overset{*}{m}_f + C_{BY}W_{BY} + C_S W_S + C_{AH}Q_{AH}\right]\Delta t_n \qquad (223.)$$

The time intervals Δt_n are four, each one of 2000 hours of duration.

The operational costs represent:

- The cost of fuel required to generate heat and power: $\sum_{k=1}^{4}\left(C_f \overset{*}{m}_f\right)\Delta t_k$
- The cost of buy power: $\sum_{k=1}^{4}\left(C_{BY}W_{BY}\right)\Delta t_k$
- The negative cost of sell power: $\sum_{k=1}^{4}\left(C_S W_S\right)\Delta t_k$
- The cost of generate additional heat with the auxiliary $\sum_{k=1}^{4}\left(C_{AH}Q_{AH}\right)\Delta t_k$

The coefficients of these equations for all the type of user are summarized in the Table 46.

Operational Variables

The operational variables of the complete gas turbine model are:

- The fuel mass flow consumed by the system $\left(\overset{*}{m}_f\right)$ in each period of time. But the fuel mass flow in the back pressure steam turbine cycle can be expressed for each period of time in terms of the turbine net output power, and the turbine isentropic efficiency.

$$\overset{*}{m}_{fst} = \left(\frac{W_{ST}}{\eta_B H_U}\right)\left(\frac{h_2 - h_1}{h_2 - h_3}\right) = \left(\frac{W_{ST}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right) \qquad (224.)$$

In this equation the enthalpies of the steam states (1,2 and 3´), the efficiency of the boiler and the fuel low heating value, are know. The isentropic efficiency of the back pressure steam turbine is expressed in the next equation.

$$[\eta_{ST}]_{pl} = \frac{(J)\cdot[W_{ST}]_{pl}}{[W_{ST}]_{full} + (J)\cdot[W_{ST}]_{pl}} \qquad (225.)$$

$$J = \frac{[\eta_{ST}]_{full}}{1-[\eta_{ST}]_{full}} \qquad (226.)$$

So it is possible observe that the fuel mass flow consumed in each period of time is a function of the net power generated in each period of time, and of the turbine isentropic efficiency (which is also a function of the power generated in each period of time and of the nominal power of the turbine). The total number of operational variables is 8, four variables of power generation in each period of time and four variables of isentropic efficiency of the steam turbine.

- The power bought by the system (W_{BY}) in each period of time, totally 4 variables
- The power sold by the system (W_S) in each period of time, totally 4 variables
- The auxiliary heat generated by the system (Q_{AH}) in each period of time, totally 4 variables

The operational variables used in the complete model of the back pressure steam turbine system are summarized and presented in the Table 64.

9.3.3.3.3 Constraints

Non negativity
Both the design variables and the operational variables must be higher or equal to zero.

Integer Variables
The variables n_{HP} (total number of cascade heat pump systems installed or operating during each period of time) must be integer variables, because they can't adopt fractional values.

Demand requirements
The back pressure steam turbine system should satisfy the next energy equations.

Power Demand:
$$[W_{ST} - (n_{HP}W_{HP}) - W_{BY} + W_S]_n = [W_{DEMAND}]_n \qquad (227.)$$
$$n = 1,...,4$$

Heat Demand:
$$[(\lambda_{ST}\cdot W_{GT}) + (n_{HP}Q_{HP}) + Q_{AH}]_n \geq [Q_{DEMAND}]_n \qquad (228.)$$
$$n = 1,...,4$$

Capacity of the equipments
With these constraints the sizes of all the equipments of the back pressure steam turbine system are restricted.

On Design Parameters
The heat to power ratio calculated in the pre model is inlet information to the complete model of the back pressure steam turbine, which in this model is treated as constraints. With the heat to power ratio, three different study cases are defined, and for each one the next parameters should be enter to the complete model:

- Enthalpy of the steam at the boiler inlet (state 1)
- Enthalpy of the steam at the turbine inlet (state 2)
- Turbine isentropic efficiency at full load
- Ideal enthalpy of the steam at the turbine outlet (state 3´)
- Enthalpy of saturated steam at the turbine outlet temperature (state 3s)

Physical Constraints
The model has limits for the maximal turbine isentropic efficiency, as well as for the enthalpy of the steam at the turbine outlet.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

Off design constraints
The power generation of the steam turbine at part load decrease the turbine isentropic efficiency. The Figure 44 show the behaviour of the turbine isentropic efficiency when the steam turbine works at partial load.

The relation between the turbine isentropic efficiency and the output power at partial load can be expressed with the equations presented in the section 6.2.2.3, which in this model is treated as a restriction for each period of time.

The heat to power ratio of the system at partial load is also affected by the turbine isentropic efficiency at partial load. So, the heat to power ratio in each period of time at partial load can be expressed with the next equation.

$$\lambda_{ST} = \frac{Q_{ST}}{W_{ST}} = \frac{h_3 - h_4}{h_2 - h_3} = \frac{h_3 - h_4}{\eta_{ST} \cdot \left(h_2 - h_{3'}\right)} \qquad (229.)$$

The lowest limit for the net power generation in each period is 75% of the nominal net power output. All the constraints used in the complete model of the back pressure steam turbine system are summarized in the Table 65 and Table 66.

9.3.4 Trigeneration

The model for trigeneration was made for a backpressure steam turbine system, with the additional equipments of an auxiliary boiler and compression and absorption chillers. Figure 67, shows the trigeneration system.

This model was designed in order to supply three different types of energy, power, heat and cooling. The power is generated by the steam turbine and the system has the possibility to buy or sell electricity from/to the utility grid. The steam leaving the turbine is divided in two ways, one to the feedwater heater and other to the absorption chillers. The feedwater heater is the equipment which supplies the heat to the user.

The cooling demand is reached with two sets of equipments, the set of absorption chillers (which take steam leaving from the turbine) and the set of compression chillers (which take power from the steam turbine).

This model is applied only to one user (The General Hospital of Vienna) in order to compare the performance of the steam turbine and the energy system with and without the use of refrigeration equipment.

As well as for the back pressure steam turbine system, the trigeneration model is composed by two steps. In the first step, the pre model, it is the heat to power ratio calculated and in the second step, the complete model, three different study cases are defined and for each one it is calculated the size and the operation of all the equipments. The main targets of the trigeneration model presented in this research are presented in the following table.

PRE MODEL OF TRIGENERATION	COMPLETE MODEL OF TRIGENERATION
Calculate the optimal heat to power ratio from the economical viewpoint, taking into account that the steam works all the time at full load	Define the size of all the components of the back pressure steam turbine system
	Calculate the energy generated by the steam turbine system in all the periods of time, taking into account the possibility of partial load
	Define the power sold or bought by the system in all the periods of time
	Define the size of the auxiliary boiler and the additional heat generated in all the periods of time
	Define the size of the compression and absorption chillers, as well as the number of each one, which should be in service in each period

Table 28. Targets of the trigeneration model

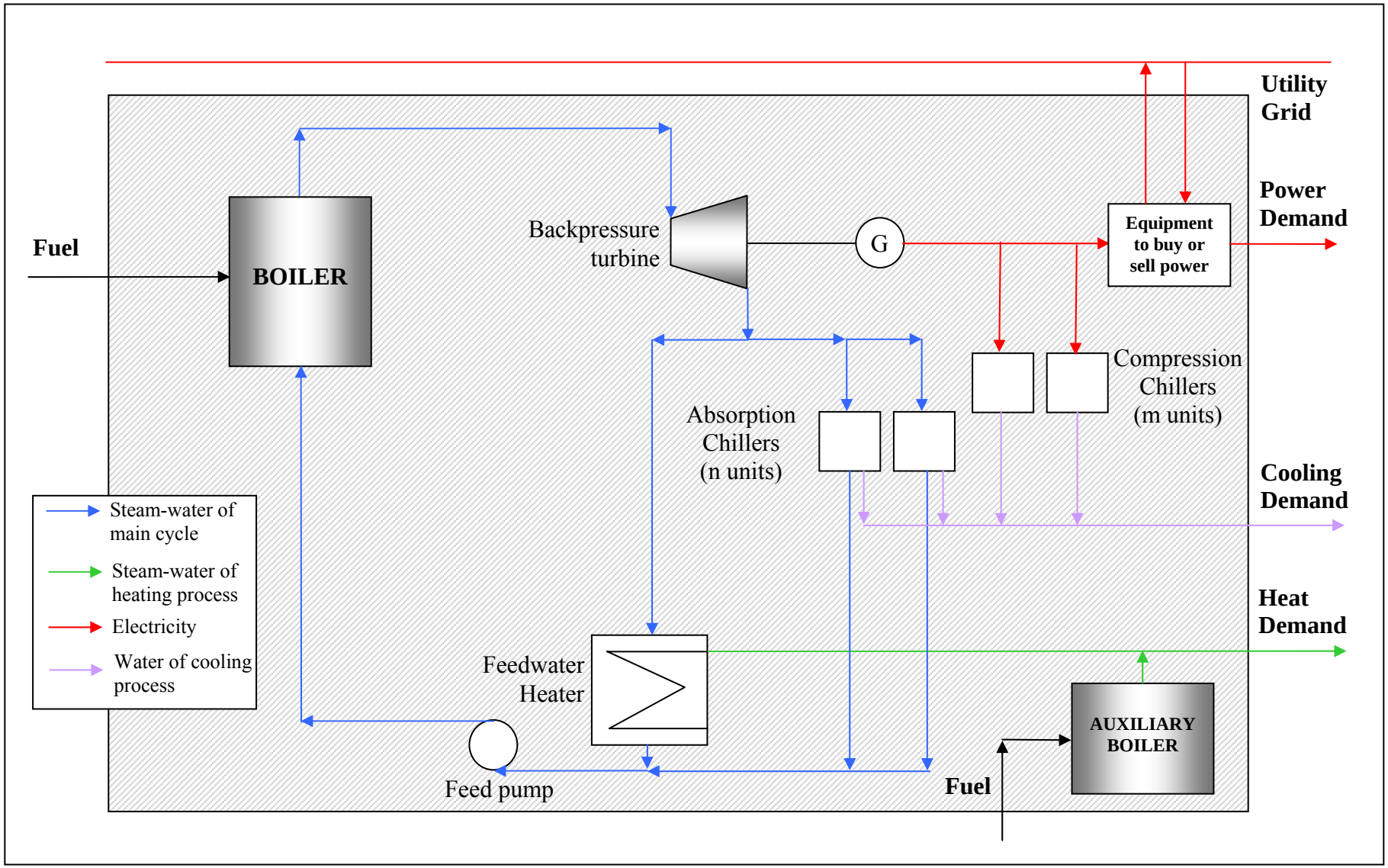


Figure 67. Trigeneration system with a back pressure steam turbine

9.3.4.1 Assumptions

PRE MODEL OF TRIGENERATION	COMPLETE MODEL OF TRIGENERATION
The back pressure steam turbine works all the time at full load	The performance at partial load of the back pressure steam turbine is assumed as the curve of the **¡Error! No se encuentra el origen de la referencia.**
The purchase costs are expressed in terms of each component of the back pressure steam turbine system (Main boiler, steam turbine, feedwater heater, feed pump, absorption and compression chillers, etc)	The design point is defined with the heat to power ratio, the isentropic efficiency of the steam turbine, the outlet turbine temperature (known) and the turbine inlet temperature and pressure, these last are specified for three different study cases
The Coefficient of Performance (COP) was taken fixed for the absorption and the compression chillers. $COP_{ABSORPTION}$ = 0.68, $COP_{COMPRESSION}$ = 4.5	The heat to power ratio of the system is already defined by the pre model
The steam leaving the steam turbine was used for the heat required in the generator of the absorption chillers, and for the heat transfer to the water for heating	The back pressure steam turbine could works at partial load, the lowest limit for the net power output in each period is 75% of the nominal net power output
The cooling demand is supplied with cold water at 5 º C	The Coefficient of Performance (COP) was taken fixed for the absorption and the compression chillers. For absorption it was used 0.68 and for the compression chillers 4.5
Use of auxiliary boiler in a peak of heat demand	The steam leaving the steam turbine was used for the heat required in the generator of the absorption chillers, and for the heat transfer to the water for heating. The cooling demand is supplied with cold water at 5 º C
Possibility of buy or sell electricity along the year	Use of auxiliary boiler in a peak of heat demand
Use of fuel oil as fuel	Possibility of buy or sell electricity along the year
The life of the plant is assumed as 25 years	The life of the plant is assumed as 25 years
The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant	Use of fuel oil as fuel. The fuel price is constant during the year, but it is assumed a fuel price change along the life of the plant

Table 29. Assumptions made for the pre model and the complete model of the trigeneration system

9.3.4.2 Objective Function

The objective function of both the pre model and the complete model of back pressure steam turbine are expressed in theTable 67 and Table 68, where is possible observe that the difference between the objective function of the both models, is the partial load of the steam turbine in the complete model, therefore the power generation of the turbine is variable for each period of time. This is made taking into account that the pre model defines the heat to power ratio and then three different cases are established to study in the complete model of the trigeneration system.

The equations which define the purchase costs of the components of the trigeneration system, which are the same for the pre model and for the complete model, are summarized in the Table 69. The corresponding factors of these equations are also presented for the pre model and the complete model in the Table 70. The design variables used in the pre model and in the complete model of the trigeneration system are summarised and presented in the Table 71. The operational variables of the pre model and of the complete model of the trigeneration system are expressed in the Table 72. As was explained before, the fuel mass flow in the back pressure steam turbine can be expressed as a function of the net output power and of the isentropic efficiency of the turbine.

The difference between the operational variables of the pre model and the operational variables of the complete model, it is that in the pre model the steam turbine work all the time at full load. But in the complete model, the turbine can work at part load, and therefore the power generation in each period is a variable, as well as the isentropic efficiency of the turbine. The coefficients of the operation cost functions for the pre model and for the complete model of the trigeneration system are shown in the Table 73, taking into account that the trigeneration model was applied only for the Vienna General Hospital (Commercial User).

The coefficients C_{CTA} and C_{CTC} represent the running costs of the cooling towers, one for the absorption chillers and the other for the compressor chillers. These running costs include the water consumption, chemical treatment, cleaning and maintenance and fan power. It was found in the literature that the running costs of the cooling towers increase with the cooling generation, with a linear relation.

9.3.4.3 Constraints

9.3.4.3.1 Constraints of the premodel

Non negativity

The design variables and the operational variables must be higher or equal to zero.

Demand requirements

The pre model of the trigeneration system should satisfy the next energy equations in order to supply the energy demand of the user (power, heat and cooling) in each period of time. The power generated by the system is the same in all the periods. In this model the thermodynamic parameters of the back pressure steam turbine are expressed in terms of the turbine net output power and the heat to power ratio.

Power Demand:

$$\left[W_{ST} - \left(n_{AC} \cdot W_{AC}\right) - \left(n_{CC} \cdot W_{CC}\right) - W_{BY} + W_S\right]_n = \left[W_{DEMAND}\right]_n \qquad (230.)$$
$$n = 1,...,4$$

Heat Demand:

$$\left[\left(\lambda_{ST} \cdot W_{ST}\right) - \left(n_{AC} \cdot Q_{AC}\right) + Q_{AH}\right]_n \geq \left[Q_{DEMAND}\right]_n \qquad (231.)$$
$$n = 1,...,4$$

Cooling Demand

$$\left[\left(n_{AC} \cdot Y_{AC}\right) + \left(n_{CC} \cdot Y_{CC}\right)\right]_n = \left[O_{DEMAND}\right]_n \qquad (232.)$$
$$n = 1,...,4$$

Capacity of the equipments

With these constraints the sizes of all the equipments of the back pressure steam turbine system are restricted.

Thermodynamics Equations

With these equations is possible express some parameters like the fuel mass flow and the heat generation in function of the power generation and the heat to power ratio. There are also equations, which relate the capacity of refrigeration of the chillers with the power or heat consumption. Finally there are equations which relate the capacity of the cooling towers with the refrigeration capacities of the corresponding chillers.

Integer Variables

The variables n_{AC} (total number of absorption chillers installed or operating during each period), n_{CC} (total number of compression chillers installed or operating during each period) must be integer variables, because they can't adopt fractional values.

Physical Constraints

The heat to power ratio is limited by the pressures and temperatures at the inlet and outlet of the back pressure steam turbine. The turbine outlet temperature is already known and with this value is possible to know the minimal and maximal possible values of the pressure and the temperature at the inlet. With all the previous information the limits of the heat to power ratio are defined.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

All the constraints used in the pre model of the back pressure steam turbine system are summarized and presented in the Table 74 and Table 75.

9.3.4.3.2 Constraints of the complete model

Non negativity
The design variables and the operational variables must be higher or equal to zero.

Integer Variables
The variables n_{AC} (total number of absorption chillers installed or operating during each period), n_{CC} (total number of compression chillers installed or operating during each period) must be integer variables, because they can't adopt fractional values.

Demand requirements
The complete model of the trigeneration system should satisfy the next energy equations in order to supply the energy demand of the user (power, heat and cooling) in each period of time. The power generated by the system is the same in all the periods. In this model the thermodynamic parameters of the back pressure steam turbine are expressed in terms of the turbine net output power and the heat to power ratio.

Power Demand:

$$\left[W_{ST} - \left(n_{AC} \cdot W_{AC}\right) - \left(n_{CC} \cdot W_{CC}\right) - W_{BY} + W_S\right]_n = \left[W_{DEMAND}\right]_n \qquad (233.)$$
$$n = 1,...,4$$

Heat Demand:

$$\left[\left(\lambda_{ST} \cdot W_{ST}\right) - \left(n_{AC} \cdot Q_{AC}\right) + Q_{AH}\right]_n \geq \left[Q_{DEMAND}\right]_n \qquad (234.)$$
$$n = 1,...,4$$

Cooling Demand

$$\left[\left(n_{AC} \cdot Y_{AC}\right) + \left(n_{CC} \cdot Y_{CC}\right)\right]_n = \left[O_{DEMAND}\right]_n \qquad (235.)$$
$$n = 1,...,4$$

Capacity of the equipments
With these constraints the sizes of all the equipments of the back pressure steam turbine system are restricted.

Thermodynamics Equations
With these equations is possible express some parameters like the fuel mass flow and the heat generation in function of the power generation and the heat to power ratio. There are also equations, which relate the capacity of refrigeration of the chillers with the power or heat consumption. Finally there are equations which relate the capacity of the cooling towers with the refrigeration capacities of the corresponding chillers.

On Design Parameters
The heat to power ratio calculated in the pre model is inlet information to the complete model of the trigeneration system, which in this model is treated as constraints. With the heat to power ratio, three different study cases are defined, and for each one the next parameters should be enter to the complete model:

- Enthalpy of the steam at the boiler inlet (state 1)
- Enthalpy of the steam at the turbine inlet (state 2)
- Turbine isentropic efficiency at full load
- Ideal enthalpy of the steam at the turbine outlet (state 3´)
- Enthalpy of saturated steam at the turbine outlet temperature (state 3s)

Physical Constraints
The heat to power ratio is limited by the pressures and temperatures at the inlet and outlet of the back pressure steam turbine. The turbine outlet temperature is already known and with this value is possible to know the minimal and maximal possible values of the pressure and the temperature at the inlet. With all the previous information the limits of the heat to power ratio are defined.

Value of the operational variables
An operational variable can't adopt a value superior to the capacity of its respective equipment. For example, the auxiliary boiler can not produce in one period more power than the size of it.

Off design constraints
The power generation of the steam turbine at part load decrease the turbine isentropic efficiency. The performance of the back pressure steam turbine used in the trigeneration system and working at part load is treated in the same way that the part load performance of the steam turbine complete model (explained before).

All the constraints used in the complete model of the trigeneration system are summarized and presented in the Table 76, Table 77 and Table 78.

References

[1] C.A. Frangopoulos. *Optimal Synthesis and Operation of Thermal Systems by the Thermoeconomic Functional Approach*. Journal of Engineering for Gas Turbines and Power. Vol. 114. October 1992

[2] C.A. Frangopoulos. *Application of the thermoeconomic functional approach to the CGAM problem*. Energy. Vol 19, No.3, pp 323-342. 1994

[3] C.A. Frangopoulos, A. Valero, G. Tsatsaronis, M. Spakovsky. *CGAM Problem : Definition and conventional solution.* Energy. Vol 19, No.3, pp 279-286

[4] ENERGY EFFICIENCY BEST PRACTICE PROGRAMME. Good Practice Guide 256. An Introduction to Absorption Cooling. United Kingdom Government.(1999)

[5] VAMED-KMB Krankenhausmanagement und Betriebsführungsgesellschaft m. b. H.: WIENER ALLGEMEINES KRANKENHAUS, Energiebericht. Vienna. (2000)

[6] Regional Economic Research, Inc. (RER). EShapes™. http://www.rer.com/eshapes/index.htm

[7] Zervos, A. Frangopoulos, Ch. Assessment of CHP Implementation Possibilities in the Tourist Sector. National Technical University of Athens. (June 2001)

[8] SRC International CS: Analysis of CHP Potentials in ERN http://www.srci.cz/nisa/Germany_3.ppt

[9] The Weather Post, The Washington Post: Historical Weather Database. http://www.washingtonpost.com/wp-srv/weather/historical/historical.htm

[10] Global Financial Data, Inc. http://www.globalfindata.com/

[11] Bureau of Labour Statistics of the U.S. Department of Labour (BLS). http://data.bls.gov

[12] Energy Information Administration. http://www.eia.doe.gov

[13] A. De Groot, F. Tillemans, C. E. Grégoire Padró, P. Spath. *Hydrogen Systems For Residential Developments*. World Energy Council 18th Congress, Buenos Aires. October 2001

[14] Wörrlein Karl. *Seminar in Flugantriebe. Fachgebiet Gasturbinen und Flugantriebe*. Technische Universität Darmstadt. 2. Auflage WS 2000/2001

[15] Technical Information. *T100 microturbine CHP system*. Turbec AB. Sweeden.2002

[16] Capstone Low Emissions MicroTurbine Technology. Capstone Turbine Corporation. 2000

Chapter 10

Results and Analysis

After the models are solved, the software LINGO shows a report of the solution, like this:

Solution report City of Oklahoma

```
Local optimal solution found at iteration:          118
Objective value:                              0.2764572E+08

                    Variable           Value        Reduced Cost
                           F       0.20934 E-01        0.000000
                          H1        473.2950           0.000000
                          H2        3245.441           0.000000
                          H3        2383.158           0.000000
                         H3S        2691.068           0.000000
                          H4        461.3634           0.000000
                           J        5.666666           0.000000
                          NF       0.8500000           0.000000
                          N1       0.8500000           0.000000
                         WT1        23199.99           0.000000
                          YT        23199.99           0.000000
                          N2       0.8095238           0.000000
                         WT2        17399.99           0.000000
                          N3       0.8095238           0.000000
                         WT3        17399.99           0.000000
                          N4       0.8095238           0.000000
                         WT4        17399.99           0.000000
                           K        2.800000           0.000000
                          YB        88125.24           0.000000
                         YBS        38209.83           0.000000
                         YAB        0.000000           1499.873
                         MHP        39.61875           0.000000
                         ZHP        0.000000           334759.2
                         WB1        10110.60           0.000000
                         WS1        0.000000           22.37080
```

NUM

Figure 68. Example of a Solution report showed by the LINGO software

The literals included in the column *Variable* are representations of the variables defined for the modelling, which where described in the preceding chapter. This information is rearranged and presented in organised tables, and it is used also for the graphic representation.

10.1 Obtained Results

The results obtained with the LINGO software for the gas turbine models, the micro gas turbine models and the back pressure steam turbine models are compiled for each user and are presented in the Figure 69 to Figure 85.

The results of the micro gas turbine model for the building, the hotel and household are summarized and presented in the Table 31. The results of the gas turbine model for the City A, the City B and the Vienna General Hospital are summarized and presented in the Table 32. The results of the back pressure steam turbine model for all the users (without trigeneration option) are also compiled in the Table 34.

The results comparison of the study options for the Vienna General Hospital with cooling load and without cooling load, using back pressure steam turbine system are shown in the Table 33. Two tables were built with the complete results of all the technologies for all the users.

The Table 35 illustrates the comparison of the energy production of the system and the energy production of the auxiliary equipments. The parameters evaluated in each technology for each case are:

- Power generation of the system (kW)
- Heat generation of the system (kW)
- Size of the equipment to buy or sell power (kW)
- Size of the auxiliary boiler (kW)
- Total cost ($)

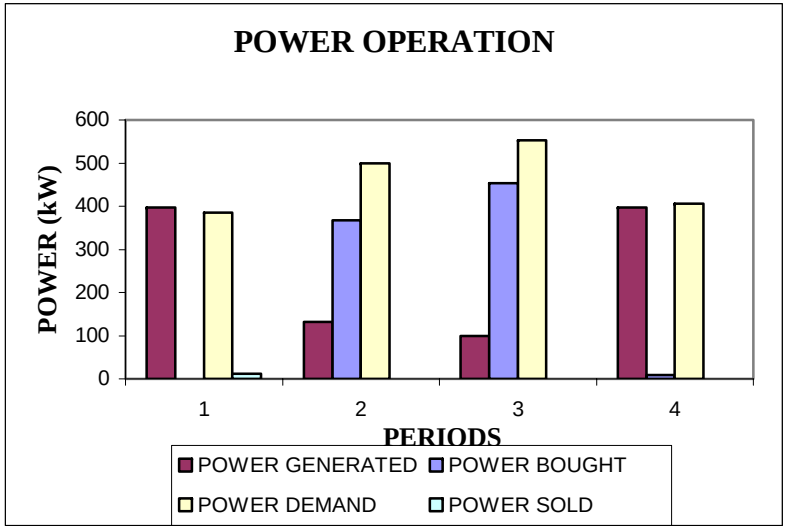
The Table 36 shows the comparison of the design costs and the operational costs for each case. The parameters presented in this comparison are:

- Main cycle purchase cost ($)
- Auxiliary equipments purchase cost ($)
- Fuel cost ($)
- Total operational cost ($)
- Fuel cost per unit of energy generated ($/KW)
- Main cycle purchase cost per unit of energy generated ($/kW)
- Total cost ($)

Afterwards the results are studied making three important analyses. The first analysis is made for each technology revising the technical and economical characteristics of the system applied to each user. The second analysis is for the Vienna General Hospital with and without the trigeneration option. Finally the third analysis is made for each user comparing the different technologies used to supply the specific energy demand.

USER	BUILDING
FUEL USED	NATURAL GAS
TOTAL COST ($)	501948.2

Period	Demand (kW)	
	Heating	Electricity
1	1593.970958	385.3672055
2	257.532329	499.42402
3	157.939318	552.7355191
4	784.9865621	405.8170414



DESIGN SOLUTION

Period	Units In Service
1	12
2	4
3	3
4	12

Y_{CG} (kW)	453.3667
Q_{AB} (kW)	808.9844

W_{MGT} (kW)	33.122950
Q_{MGT} (kW)	65.415550
λ_{MGT}	1.974931
$\dot{m}_a$ (kg/s)	0.303953
$\dot{m}_f$ (kg/s)	0.002824
r_c	5.175563
η_{Reg}	0.8
T_4 (K)	1448.906000
EUF	0.696734293
$\eta_{ELECTRIC}$	0.234202

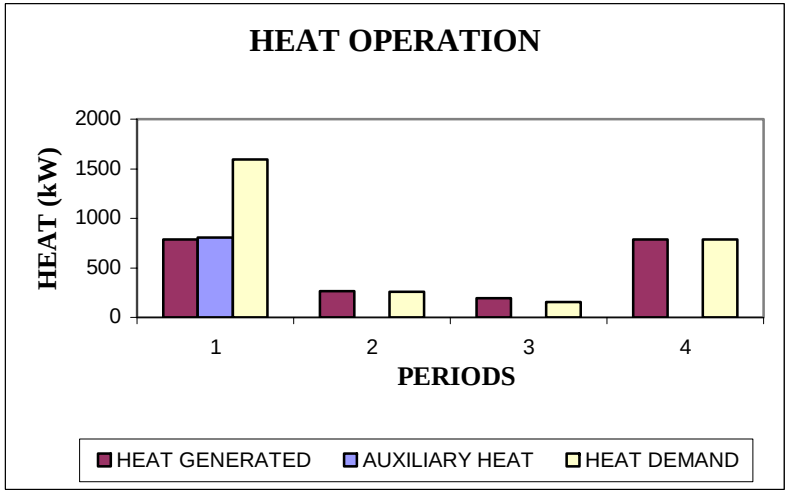
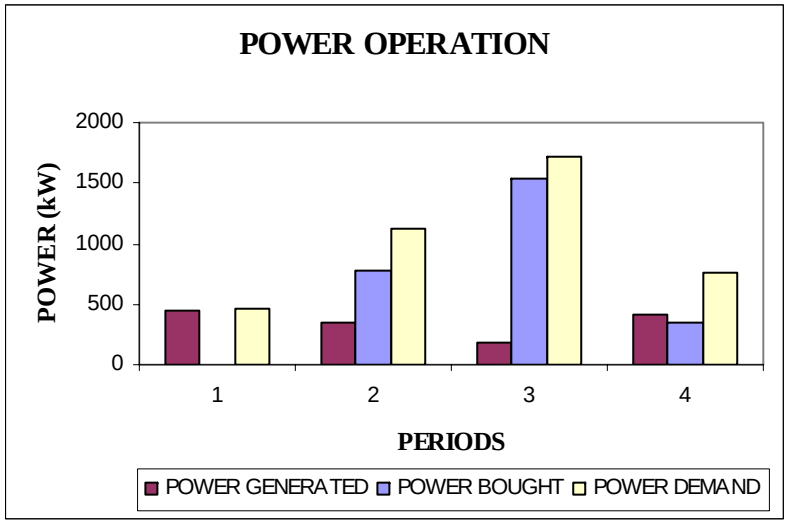


Figure 69. Compiled results for the building using micro gas turbine option

USER	HOTEL
FUEL USED	NATURAL GAS
TOTAL COST ($)	897664.9

Period	Demand (kW)	
	Heating	Electricity
1	823.892729	454.957117
2	611.857826	1117.98088
3	300.179211	1724.01434
4	744.623656	758.213859



DESIGN SOLUTION

Period	Units In Service
1	12
2	9
3	5
4	11

Y_{CG} (kW)	1535.424
Q_{AB} (kW)	0

W_{MGT} (kW)	37.718
Q_{MGT} (kW)	68.657
λ_{MGT}	1.820289
$\dot{m}_a$ (kg/s)	0.3032038
$\dot{m}_f$ (kg/s)	0.0029826
r_c	5.383778
η_{Reg}	0.8
T_4 (K)	1496.172
EUF	0.712037848
$\eta_{ELECTRIC}$	0.252469825

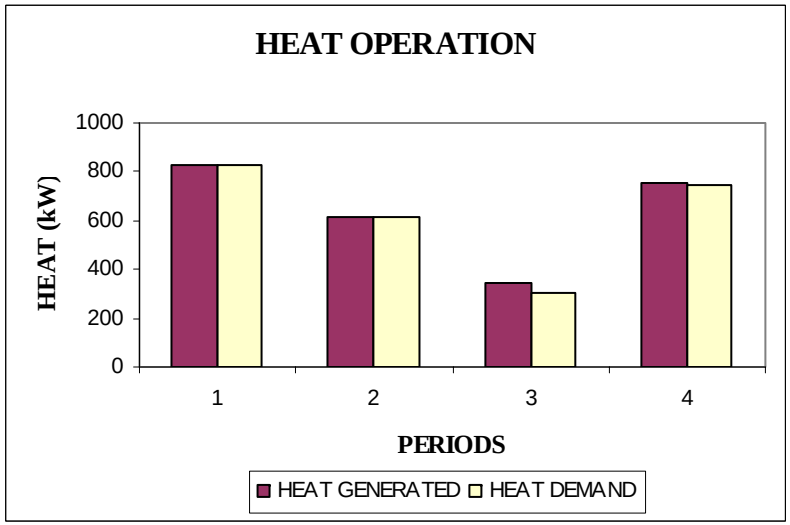
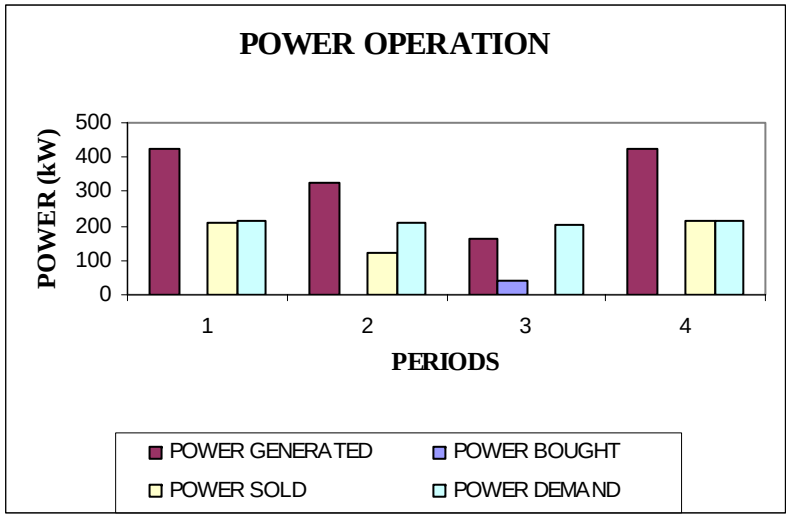


Figure 70. Compiled results for the hotel using micro gas turbine option

USER	HOUSEHOLD
FUEL USED	NATURAL GAS
TOTAL COST ($)	465587.5

Period	Demand (kW)	
	Heating	Electricity
1	2115.45473	216.984071
2	648.150067	206.558767
3	319.003389	202.618887
4	1352.07991	213.639144



DESIGN SOLUTION

Period	Units In Service
1	13
2	10
3	5
4	13

Y_{CG} (kW)	213.0062
Q_{AB} (kW)	1272.86

W_{MGT} (kW)	32.818
Q_{MGT} (kW)	64.815
λ_{MGT}	1.974931
$\dot{m}_a$ (kg/s)	0.3011628
$\dot{m}_f$ (kg/s)	0.002797660
r_c	5.175563
η_{Reg}	0.8
T_4 (K)	1448.906
EUF	0.69673416
$\eta_{ELECTRIC}$	0.23420177

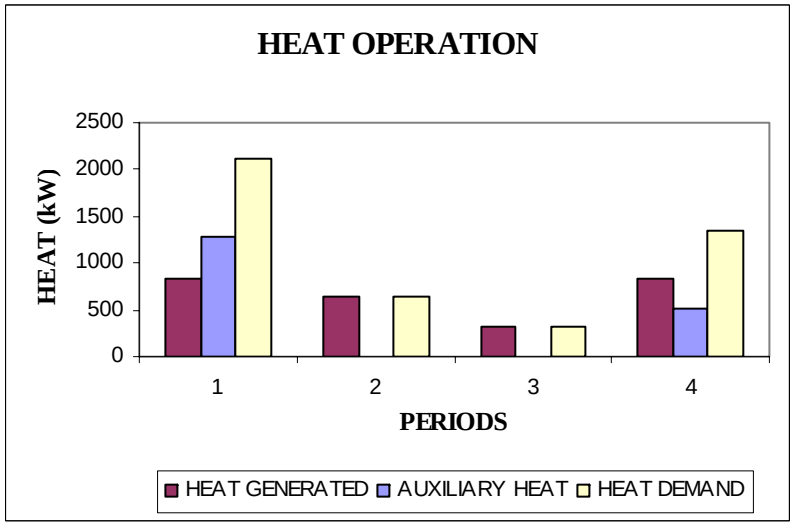
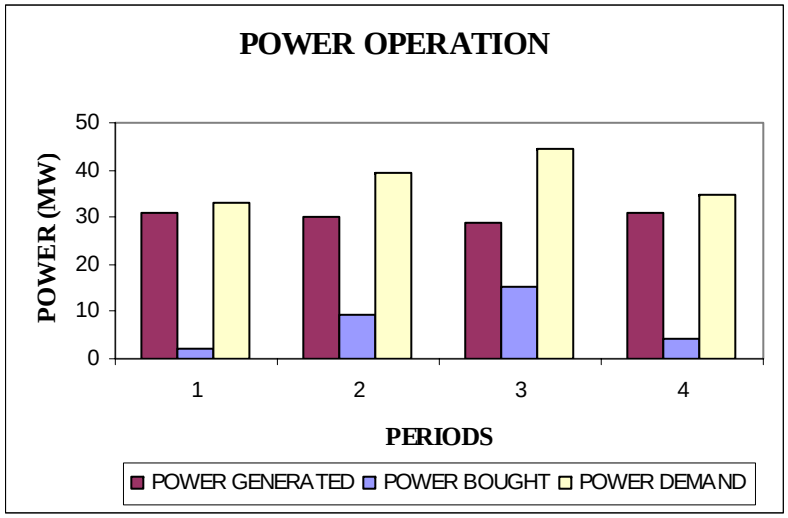


Figure 71. Compiled results for the household using micro gas turbine option

USER	CITY A
FUEL USED	NATURAL GAS
TOTAL COST ($)	24905970

Period	Demand (MW)	
	Heating	Electricity
1	122.498334	33.1270811
2	37.4058518	39.538191
3	20.6210292	44.3509462
4	78.3580272	34.9098502



DESIGN SOLUTION

Period	H.P. Units Service
1	0
2	0
3	0
4	0

W_{GT} (MW)	30.67933
λ_{GT}	2.554098
$\dot{m}_a$ (kg/s)	157.8342
$\dot{m}_f$ (kg/s)	2.658221
r_c	21.66432
T_3 (K)	1507.192
Q_{HP} (kW)	0
n_{HP}	0
EUF	0.818928026
$\eta_{ELECTRIC}$	0.230417933

Y_{CG} (MW)	15.3398
Q(MW)	41.7225

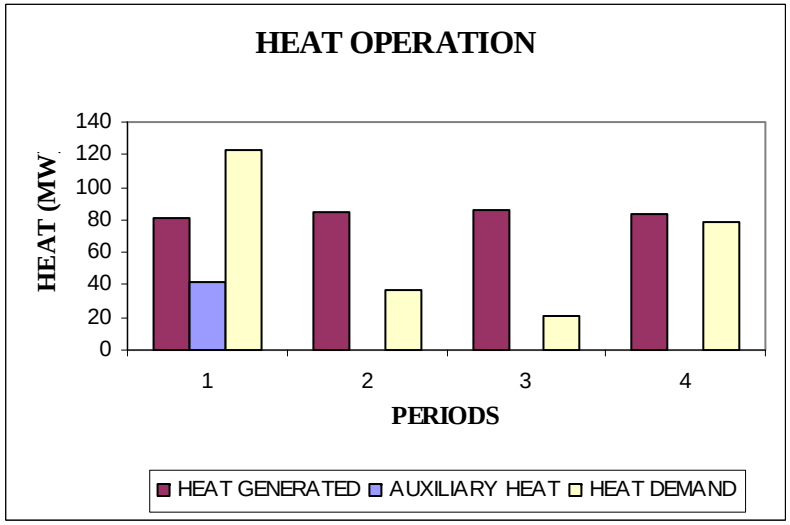
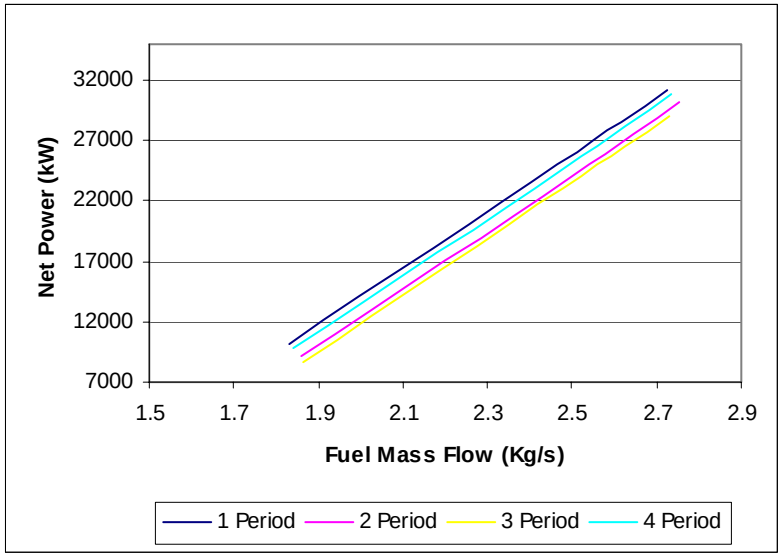
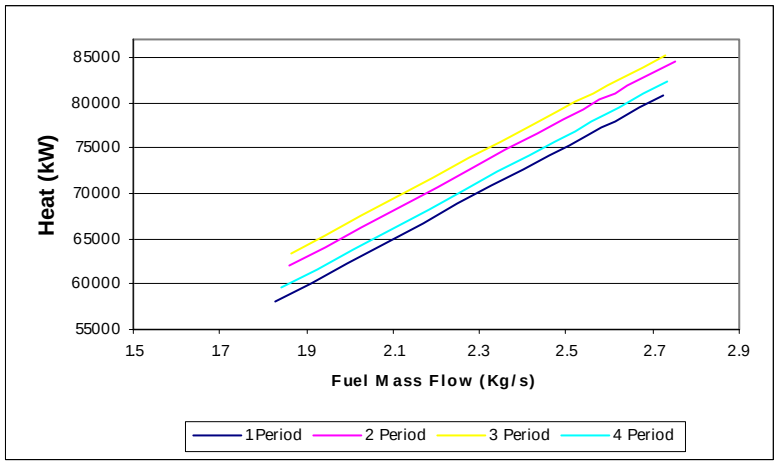


Figure 72. Compiled results for the City A using gas turbine option (First part)





PART LOAD PERFORMANCE

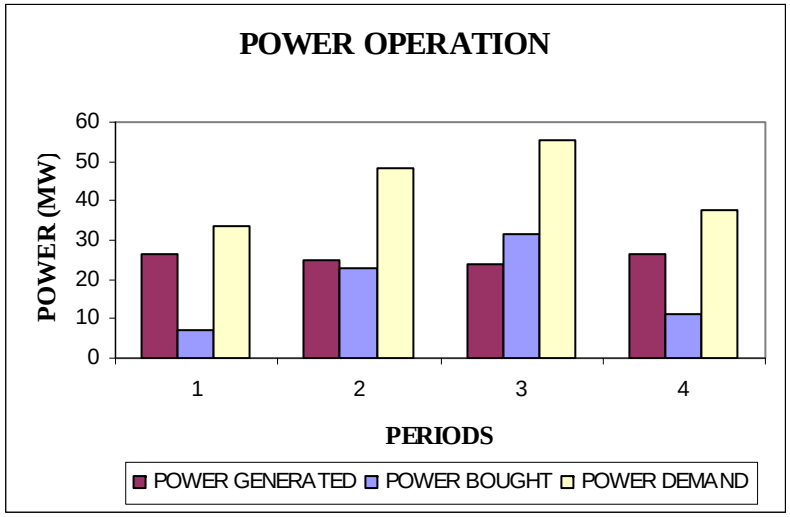
Period	T_3 (K)	T_4 (K)	$\dot{m}_a$ (kg/s)	r_c
1	1443.3176	781.70407	170.17686	22.669605
2	1526.7105	833.25103	158.74686	21.721454
3	1560.4225	856.08086	152.77099	21.143379
4	1475.2553	801.32142	165.63938	22.296994

Period	$\dot{m}_f$ (kg/s)	% Full Load	W_{GT} (MW)	Q_{GT} (MW)
1	2.7229128	1.0158501	31165.639	80738.782
2	2.7542293	0.9854689	30233.562	84573.336
3	2.7310024	0.9472622	29061.406	85336.874
4	2.7354738	1.0040545	30803.758	82259.163

Figure 73. Compiled results for the City A using gas turbine option (Second part)

USER	CITY B
FUEL USED	NATURAL GAS
TOTAL COST ($)	23984990

Period	Demand (MW)	
	Heating	Electricity
1	64.9252514	33.3105905
2	27.2244485	48.1111648
3	17.9425654	55.6098238
4	49.0810188	37.5605446



DESIGN SOLUTION

Period	H.P. Units Service
1	0
2	0
3	0
4	0

Y_{CG} (MW)	31.6228
Q_{AB} (MW)	0

W_{GT} (MW)	25.41854
λ_{GT}	2.554248
$\dot{m}_a$ (kg/s)	130.7770
$\dot{m}_f$ (kg/s)	2.202497
r_c	21.64105
T_3 (K)	1507.149
Q_{HP} (kW)	0
n_{HP}	0
EUF	0.818925644
$\eta_{ELECTRIC}$	0.230407582

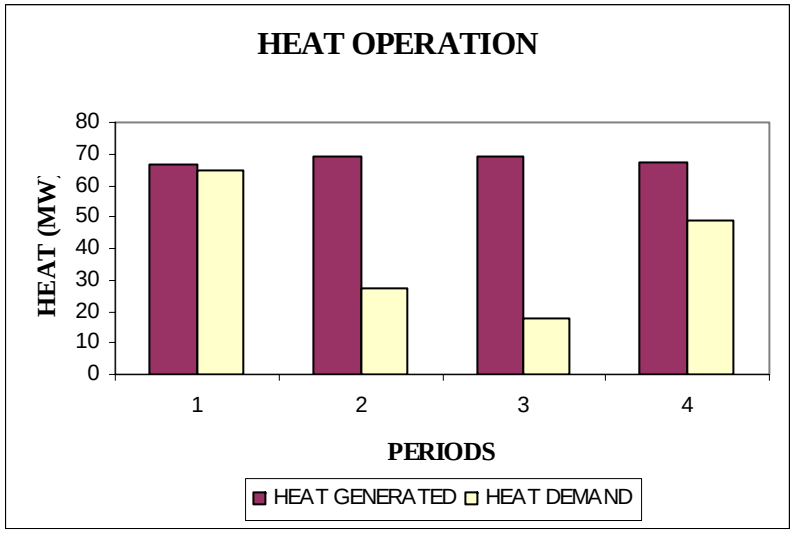
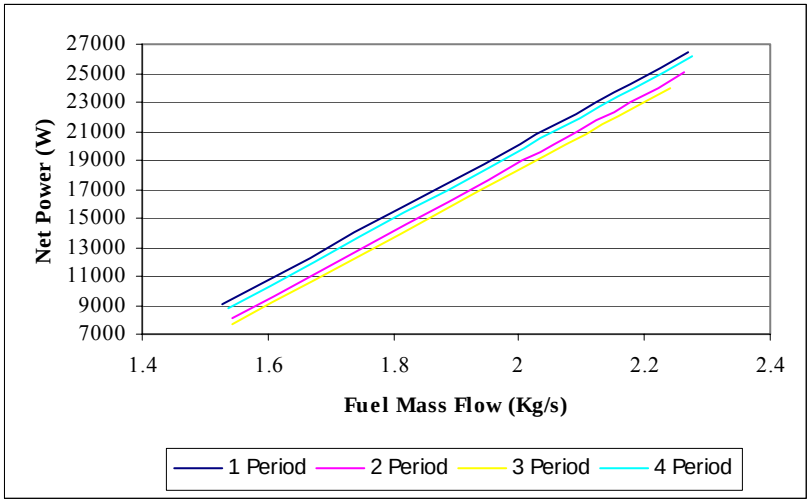
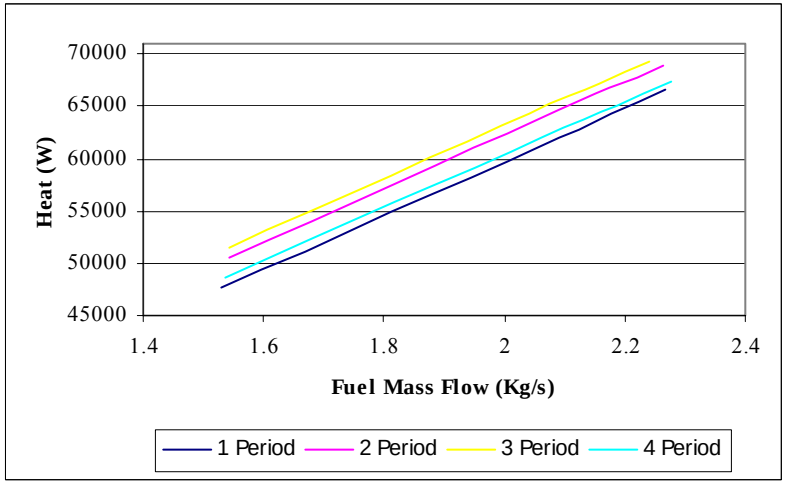


Figure 74. Compiled results for the City B using gas turbine option (First part)





PART LOAD PERFORMANCE

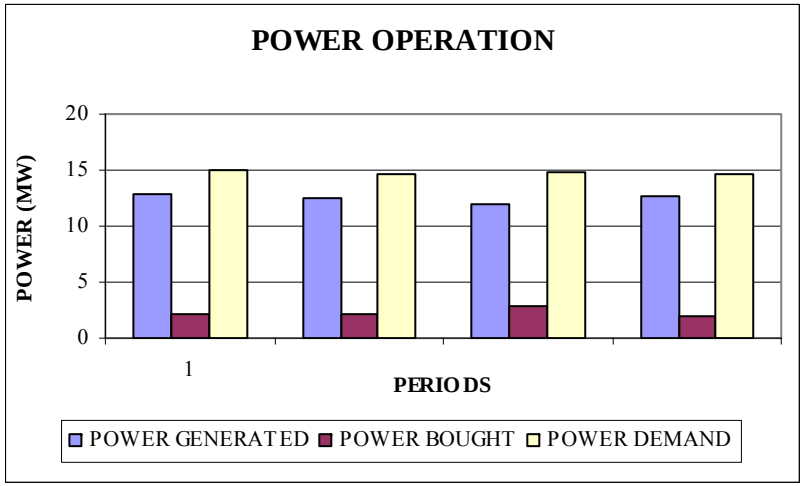
Period	T_3 (K)	T_4 (K)	$\dot{m}_a$ (kg/s)	r_c
1	1482.3093	805.701677	136.424417	22.2124399
2	1558.60267	854.845143	126.849673	21.1731519
3	1588.76517	876.021281	122.273267	20.6198088
4	1505.37474	819.990786	133.83956	21.9526779

Period	$\dot{m}_f$ (kg/s)	% Full Load	W_{GT} (MW)	Q_{GT} (MW)
1	2.26861269	1.04097628	26458.6596	66512.3343
2	2.26409357	0.9863506	25070.2298	68898.0617
3	2.23998172	0.94558368	24034.0505	69343.5011
4	2.27566608	1.03186023	26226.9553	67415.5714

Figure 75. Compiled results for the City B using gas turbine option (Second part)

USER	**Vienna General Hospital**
FUEL USED	**Natural Gas**
TOTAL COST ($)	**8867702**

Period	Demand (MW)	
	Heating	Electricity
1	32.4740783	15.0009601
2	15.3509558	14.6520311
3	8.78136201	14.8088411
4	27.1475508	14.6744325



DESIGN SOLUTION

Period	H.P. Units Service
1	0
2	0
3	0
4	0

Y_{CG} (MW)	2.782795
Q_{AB} (MW)	0

W_{GT} (MW)	12.71736
λ_{GT}	2.553523
$\dot{m}_a$ (kg/s)	65.41150
$\dot{m}_f$ (kg/s)	1.101711
r_c	21.65691
T_3 (K)	1507.358
Q_{HP} (kW)	0
n_{HP}	0
EUF	0.818936042
$\eta_{ELECTRIC}$	0.23045746

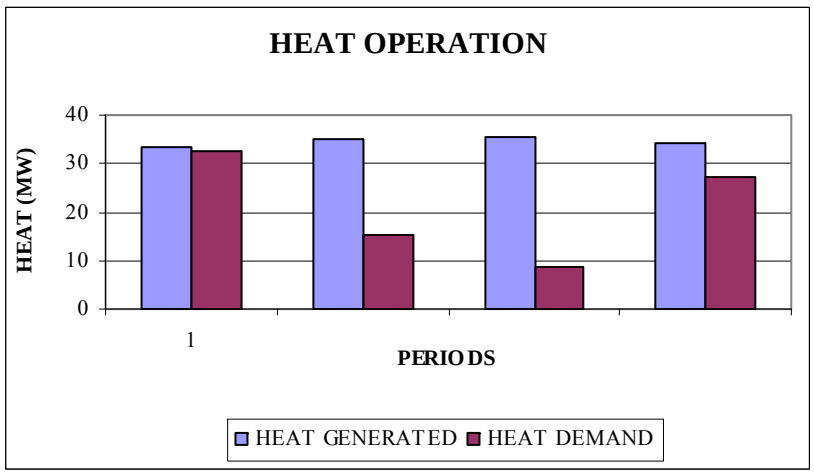
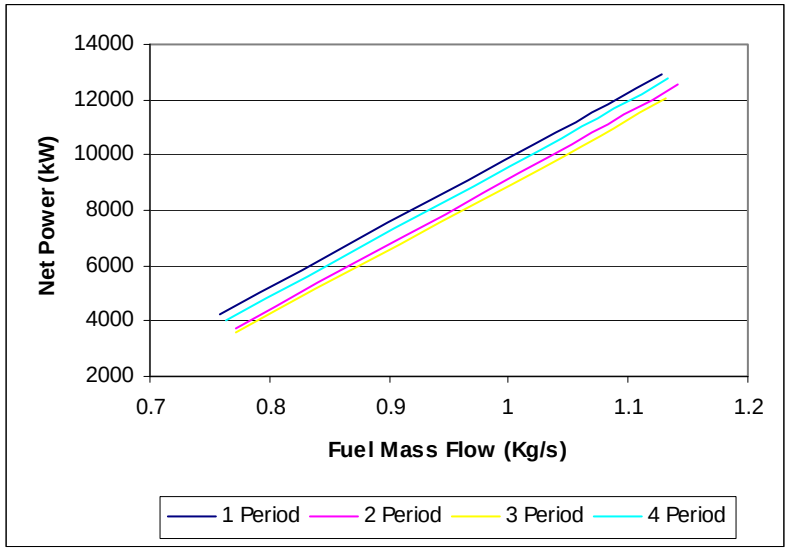
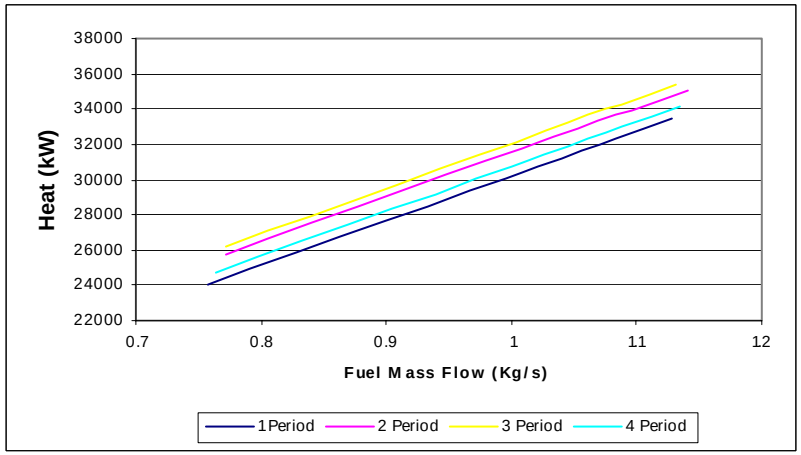


Figure 76. Compiled results for the Vienna General Hospital using gas turbine option (First part)





PART LOAD PERFORMANCE

Period	T_3 (K)	T_4 (K)	$\dot{m}_a$ (kg/s)	r_c
1	1443.4756	781.70404	70.52669	22.682789
2	1526.8776	833.25086	65.789737	21.734061
3	1560.5933	856.08068	63.313143	21.155634
4	1475.4168	801.32132	68.646215	22.309951

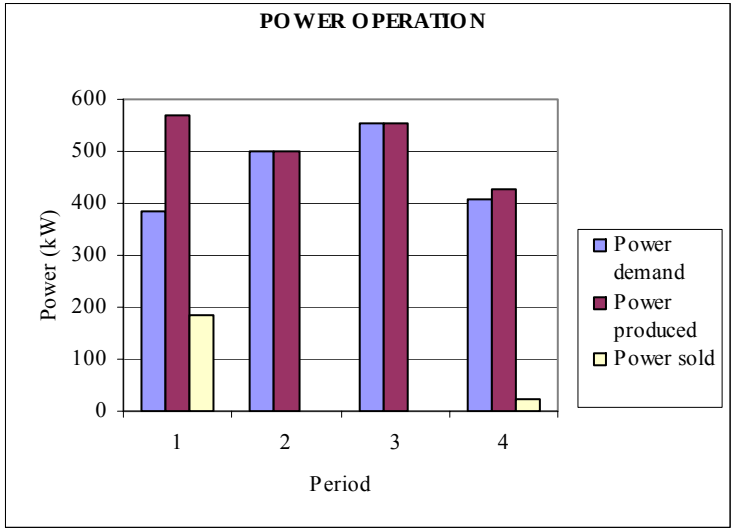
Period	$\dot{m}_f$ (kg/s)	% Full Load	W_{GT} (MW)	Q_{GT} (MW)
1	1.1285197	1.0158426	12918.84	33460.738
2	1.1415037	0.9854893	12532.826	35049.894
3	1.1318783	0.9472932	12047.073	35366.331
4	1.1337276	1.0040581	12768.973	34090.829

Figure 77. Compiled results for the Vienna General Hospital using gas turbine option (Second part)

USER	BUILDING
FUEL USED	FUEL OIL
TOTAL COST ($)	432497.6

Period	Demand (kW)	
	Heating	Electricity
1	1593.97096	385.367205
2	257.532329	499.42402
3	157.939318	552.735519
4	784.986562	405.817041

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (kW)	2163.439	2163.551	2163.363
W_{ST} (kW)	569.4677	569.5797	569.3922
λ_{ST}	2.8	2.8	2.8
Y_{CG} (kW)	184.1005	184.2125	184.025
EUF	0.800199039	0.800314986	0.800121056
$\eta_{ELECTRIC}$	0.210578694	0.210609207	0.210558173
Q_{AB} (kW)	0	0	0
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	437373.3	434512.1	432497.6



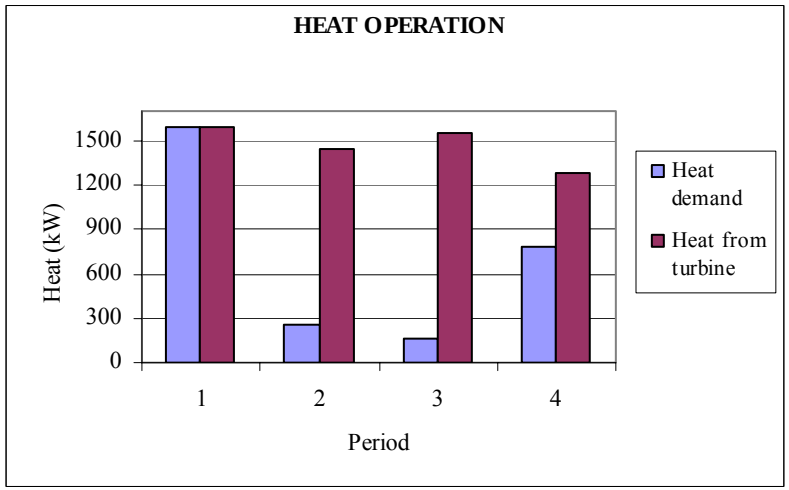
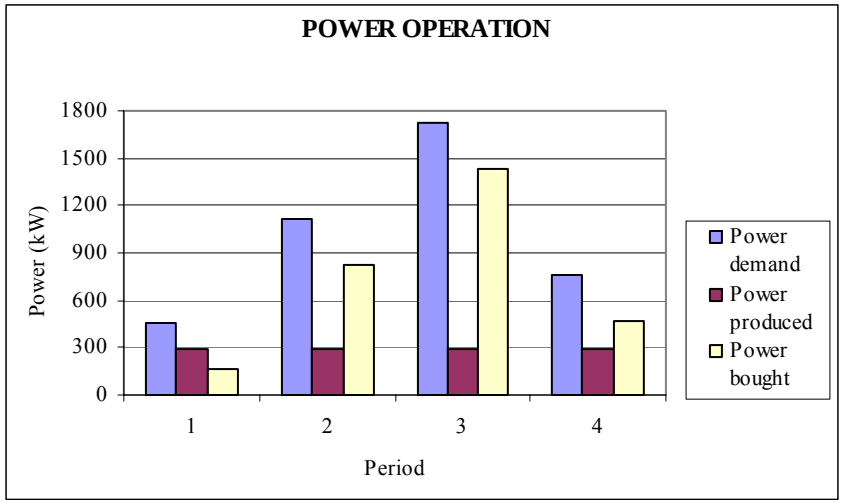


Figure 78. Compiled results for the building using back pressure steam turbine option

USER	HOTEL
FUEL USED	FUEL OIL
TOTAL COST ($)	858915.2

Period	Demand (kW)	
	Heating	Electricity
1	823.892729	454.957117
2	611.857826	1117.98088
3	300.179211	1724.01434
4	744.623656	758.213859

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (kW)	1118.24	1118.297	1118.2
W_{ST} (kW)	294.3469	294.4047	294.3078
λ_{ST}	2.8	2.8	2.8
Y_{CG} (kW)	1429.667	1429.61	1429.707
EUF	0.800199041	0.80031538	0.800121366
$\eta_{ELECTRIC}$	0.210578695	0.21060931	0.210558254
Q_{AB} (kW)	0	0	0
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	861804.7	860096.5	858915.2



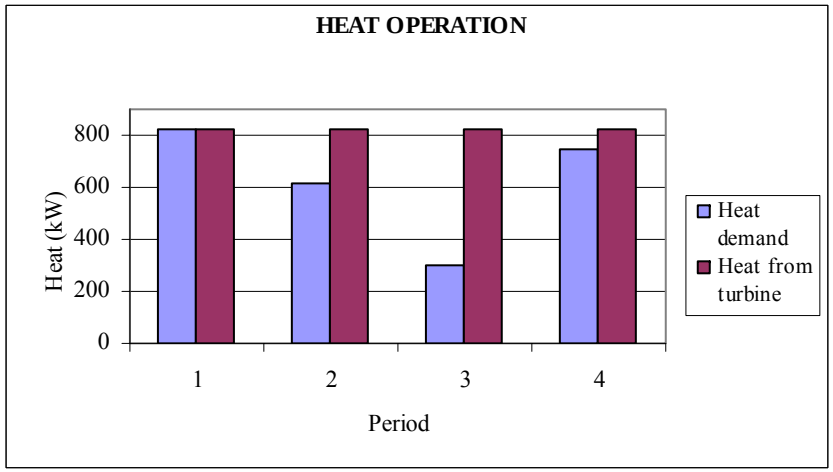
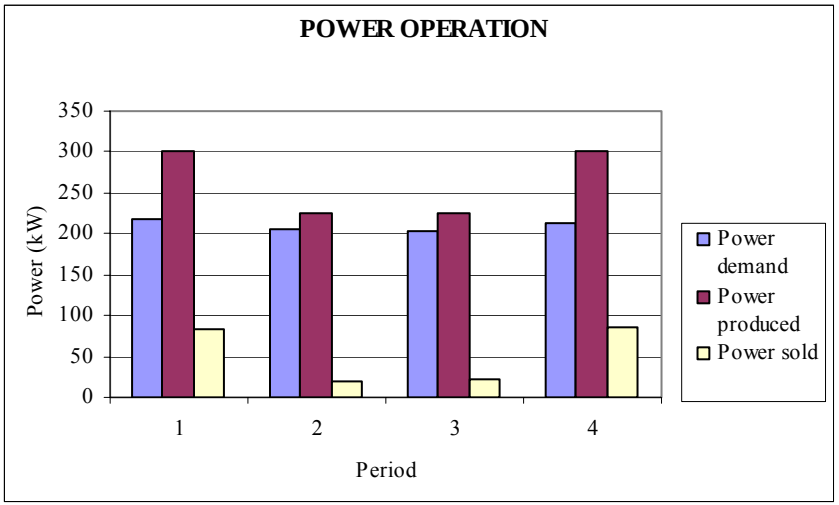


Figure 79. Compiled results for the hotel using back pressure steam turbine option

USER	HOUSEHOLD
FUEL USED	FUEL OIL
TOTAL COST ($)	420415.3

Period	Demand (kW)	
	Heating	Electricity
1	2115.45473	216.984071
2	648.150067	206.558767
3	319.003389	202.618887
4	1352.07991	213.639144

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (kW)	1098.406	1139.551	1139.828
W_{ST} (kW)	289.1262	300	300
λ_{ST}	2.8	2.8	2.8
Y_{CG} (kW)	75.48706	86.3609	86.3609
EUF	0.800199241	0.800315212	0.80012072
$\eta_{ELECTRIC}$	0.210578748	0.210609266	0.210558084
Q_{AB} (kW)	1306.175	1275.904	1275.627
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	423215.7	421599.3	420415.3



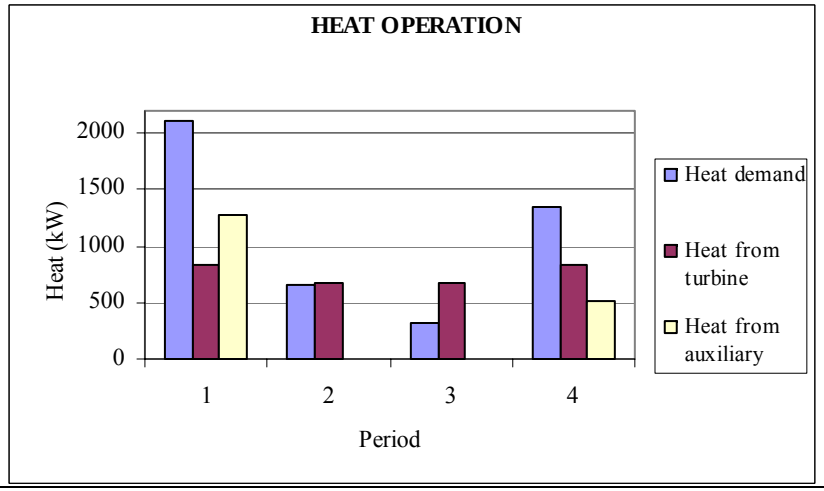
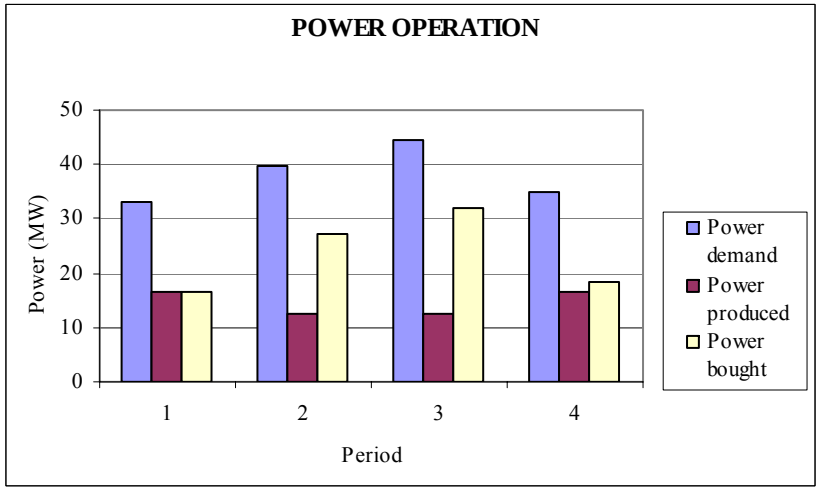


Figure 80. Compiled results for the household using back pressure steam turbine option

USER	CITY A
FUEL USED	FUEL OIL
TOTAL COST ($)	29175370

Period	Demand (MW)	
	Heating	Electricity
1	122.498334	33.1270811
2	37.4058518	39.538191
3	20.6210292	44.3509462
4	78.3580272	34.9098502

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (MW)	63.39089	63.39396	63.38882
W_{ST} (MW)	16.68597	16.68919	16.68379
λ_{ST}	2.8	2.8	2.8
Y_{CG} (MW)	31.83647	31.83405	31.8381
EUF	0.800199347	0.800315008	0.80012093
$\eta_{ELECTRIC}$	0.210578776	0.210609213	0.210558139
Q_{AB} (MW)	75.79341	75.79356	75.79331
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	29236720	29200590	29175370



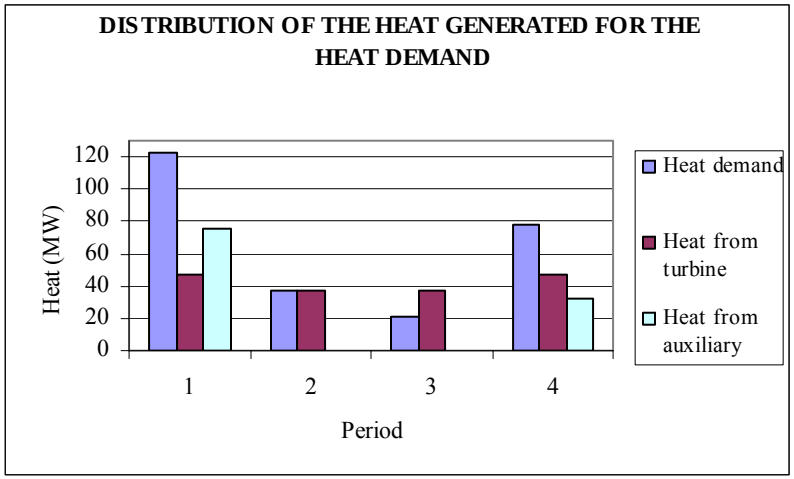
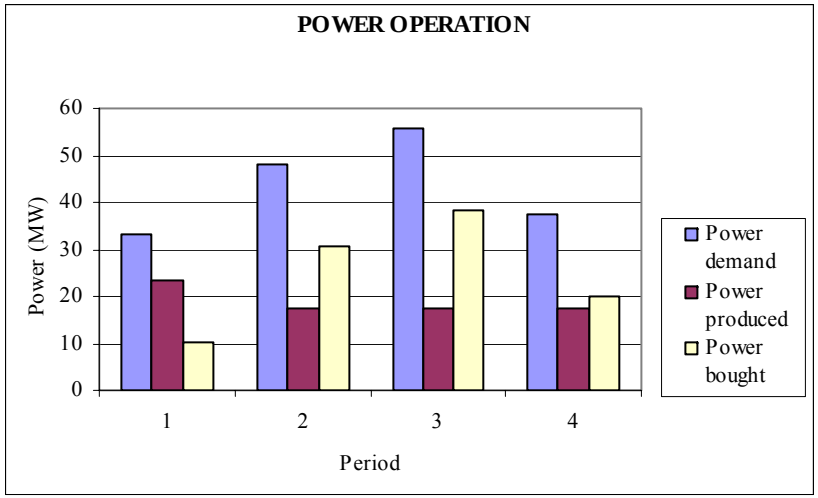


Figure 81. Compiled results for the City A using back pressure steam turbine option

USER	CITY B
FUEL USED	FUEL OIL
TOTAL COST ($)	27612920

Period	Demand (MW)	
	Heating	Electricity
1	64.92525145	33.3105905
2	27.22444847	48.1111648
3	17.9425654	55.6098238
4	49.08101881	37.5605446

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (MW)	88.12068	88.12524	88.1176
W_{ST} (MW)	23.19543	23.19999	23.19235
λ_{ST}	2.8	2.8	2.8
Y_{CG} (MW)	38.21325	38.20983	38.21556
EUF	0.800199308	0.800315206	0.80012102
$\eta_{ELECTRIC}$	0.210578765	0.210609265	0.210558163
Q_{AB} (MW)	0	0	0
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	27692640	27645720	27612920



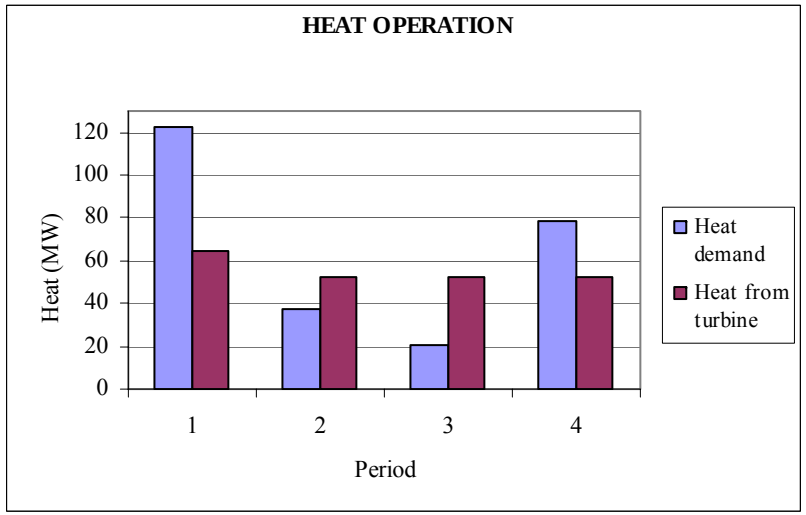
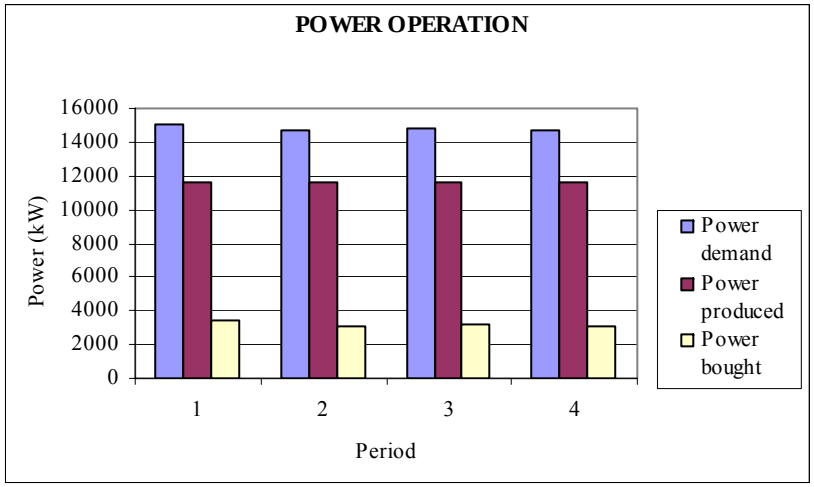


Figure 82. Compiled results for the City B using back pressure steam turbine option

USER	Vienna General Hospital
FUEL USED	Fuel Oil
TOTAL COST ($)	12283550

Period	Demand (MW)	
	Heating	Electricity
1	32.4740783	15.0009601
2	15.3509558	14.6520311
3	8.78136201	14.8088411
4	27.1475508	14.6744325

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	11.6	9.8	8.3
T_2 (ºC)	400	450	500
Q_{MB} (MW)	44.07578	44.07806	44.07424
W_{ST} (MW)	11.60178	11.60406	11.60024
λ_{ST}	2.8	2.8	2.8
Y_{CG} (MW)	3.399183	3.396902	3.400723
EUF	0.800199366	0.800315223	0.800121105
$\eta_{ELECTRIC}$	0.21057878	0.210609269	0.210558185
Q_{AB} (MW)	0	0	0
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
COST ($)	12328110	12300650	12283550



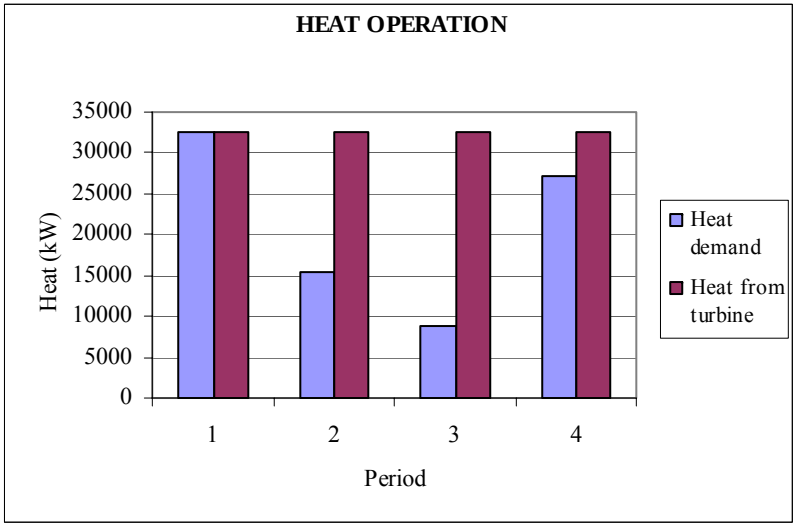


Figure 83. Compiled results for the Vienna General Hospital using back pressure steam turbine option

USER	Hospital (Trigeneration)
FUEL USED	Fuel Oil
TOTAL COST ($)	13674550

Period	Demand (MW)		
	Heating	Electricity	Cooling
1	32.474078	15.000960	2.521761
2	15.350956	14.652031	9.063620
3	8.781362	14.808841	13.784349
4	27.147551	14.674432	3.207885

Period	Absorption Chillers in Service	Compression Chillers in Service
1	7	0
2	25	0
3	38	0
4	9	0

	Study Case 1	Study Case 2	Study Case 3
P_2 (Mpa)	16	11.5	8.3
T_2 (ºC)	350	420	500
Q_{MB} (MW)	49.14357	49.14611	49.14185
W_{ST} (MW)	12.93574	12.93828	12.93402
Q_{ST} (MW)	36.220072	36.227184	36.215256
λ_{ST}	2.8	2.8	2.8
Y_{CG} (MW)	2.71875	2.716208	2.720467
EUF	0.694605207	0.694726423	0.694523117
$\eta_{ELECTRIC}$	0.210578759	0.210609222	0.210558129
Q_{AB} (MW)	0	0	0
Y_{ACH} (MW)	0.3627368	0.3627368	0.3627368
n_{ACH}	38	38	38
Y_{CCH} (MW)	0	0	0
n_{CCH}	0	0	0
TOTAL COST ($)	13722700	13692970	13674550

Figure 84. Compiled results for the Vienna General Hospital using Trigeneration option (First Part)

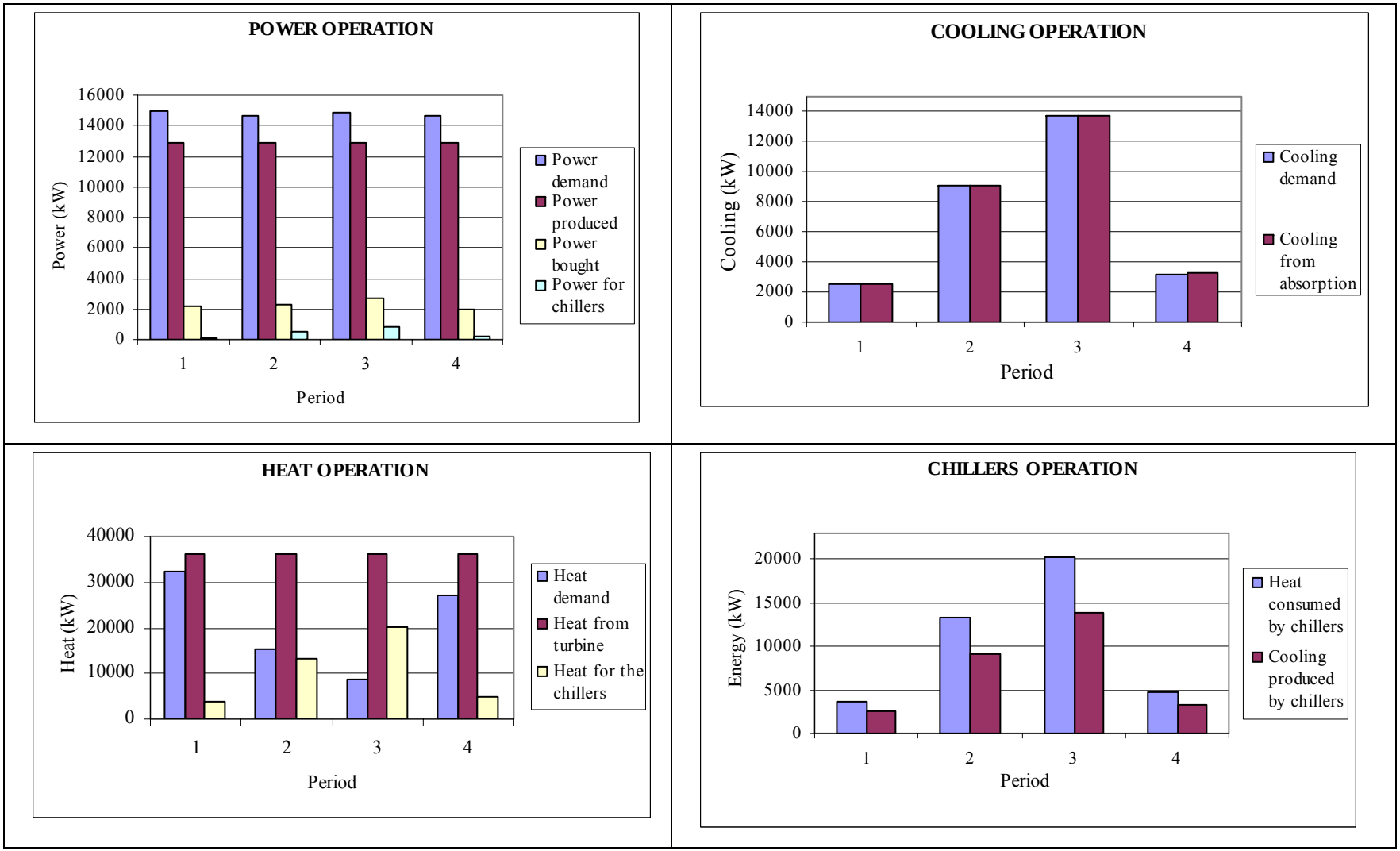


Figure 85. Compiled results for the Vienna General Hospital using Trigeneration option (Second part)

		Building	Hotel	Household	City A	City B	Hospital	Hospital (Trigeneration)
POWER DEMAND (kW)	Period 1	385.3672	454.9571	216.9841	33127.0811	33310.5905	15000.9601	15000.9601
	Period 2	499.4240	1117.9809	206.5588	39538.1910	48111.1648	14652.0311	14652.0311
	Period 3	552.7355	1724.0143	202.6189	44350.9462	55609.8238	14808.8411	14808.8411
	Period 4	405.8170	758.2139	213.6391	34909.8502	37560.5446	14674.4325	14674.4325
	Average	460.83593	1013.7916	209.95023	37981.517	43648.031	14784.066	14784.066
	Min/Max	0.6972	0.263894	0.9337961	0.7469307	0.5990055	0.9767396	0.9767396
HEAT DEMAND (kW)	Period 1	1593.9710	823.8927	2115.4547	122498.3340	64925.2514	32474.0783	32474.0783
	Period 2	257.5323	611.8578	648.1501	37405.8518	27224.4485	15350.9558	15350.9558
	Period 3	157.9393	300.1792	319.0034	20621.0292	17942.5654	8781.3620	8781.3620
	Period 4	784.9866	744.6237	1352.0799	78358.0272	49081.0188	27147.5508	27147.5508
	Average	698.6073	620.13835	1108.672	64720.811	39793.321	20938.487	20938.487
	Min/Max	0.0990854	0.3643426	0.1507966	0.1683372	0.2763573	0.2704114	0.2704114
COOLING DEMAND (kW)	Period 1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2521.7610
	Period 2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	9063.6200
	Period 3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	13784.3490
	Period 4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3207.8850
Average Heat Demand/Power Demand		1.71799391	0.87860116	5.19760151	1.83835823	1.03608033	1.4138681	1.4138681
Maximal Heat Demand/ Maximal Power Demand		2.88378619	0.47789203	9.74935352	2.76202301	1.16751406	2.16479999	2.16479999

Table 30. Summary of the heat, power and cooling demands of the users along the year

	BUILDING	HOTEL	HOUSEHOLD
W_{MGT} (kW)	33.122950	37.718	32.818
Q_{MGT} (kW)	65.415550	68.657	64.815
λ_{MGT}	1.974931	1.820289	1.974931
$\dot{m}_a$ (kg/s)	0.303953	0.3032038	0.3011628
$\dot{m}_f$ (kg/s)	0.002824	0.0029826	0.002797660
r_c	5.175563	5.383778	5.175563
η_{Reg}	0.8	0.8	0.8
T_4 (K)	1448.906000	1496.172	1448.906
Excess of Air	6.251003134	5.903034272	6.251003134
EUF	0.696734293	0.712037848	0.69673416
$\eta_{ELECTRIC}$	0.234202	0.252469825	0.23420177
n_{MGT}	12	12	13
Y_{CG} (kW)	33.122950	37.718	32.818
Q_{AB} (kW)	65.415550	68.657	64.815
COSTO TOTAL ($)	**501948.2**	**897664.9**	**465587.5**
ANALYSIS WITH MICRO GAS TURBINES WITH THE SAME NET OUTPUT POWER, THE SAME PRESSURE RATIO, THE SAME TURBINE INLET TEMPERATURE, BUT WITHOUTH REGENERATION			
W_{MGT} (kW) (Without regeneration)	33.122950	37.718	32.818
Q_{MGT} (kW) (Without regeneration)	152.815	162.436	151.349
λ_{MGT} (Without regeneration)	4.612101	4.3066	4.612101
$\dot{m}_a$ (kg/s) (Without regeneration)	0.2033	0.20795	0.20135
$\dot{m}_f$ (kg/s) (Without regeneration)	0.00436	0.004664	0.004318
Excess of air (Without regeneration)	2.7109975	2.5920168	2.7109975
EUF (Without regeneration)	0.852433601	0.857666	0.852433601
$\eta_{ELECTRIC}$ (Without regeneration)	0.151892069	0.161619	0.151892069

Table 31. Summary of the results obtained for small users with micro gas turbine systems

	CITY A	CITY B	HOSPITAL
$\dot{W}_{GT}$ (MW)	30.67933	25.41854	12.71736
Q_{GT}(MW)	78.358	64.925	32.474
λ_{GT}	2.554098	2.554248	2.553523
$\dot{m}_a$ (kg/s)	157.8342	130.7770	65.41150
$\dot{m}_f$ (kg/s)	2.658221	2.202497	1.101711
r_c	21.66432	21.64105	21.65691
T_3 (K)	1507.192	1507.149	1507.358
Excess of air	3.452083781	3.452135576	3.451898331
Actual air-fuel ratio	59.37584104	59.37673191	59.37265129
Air Compressor Power (MW)	73.86103816	61.19135263	30619037.2
Turbine Power Generation	104.5404063	86.60851149	43336401.56
% Power consumed by air compressor	70.65310033	0.706528164	70.65431392
Steam mass flow in the HRSG (kg/s)	25.58028507	21.19397367	10.60128171
Water mass flow for heating (kg/s)	524.7846643	434.7986086	217.4874145
EUF	0.818928026	0.818925644	0.818936042
$\eta_{ELECTRIC}$	0.230417933	0.230407582	0.23045746
Q_{HP} (kW)	0	0	0
n_{HP}	0	0	0
Y_{CG} (kW)	15.3398	31.6228	2.782795
Q_{AB} (kW)	41.7225	0	0
COSTO TOTAL ($)	**24905970**	**23984990**	**8867702**

Table 32. Summary of the results obtained for the industrial users using gas turbines

	HOSPITAL (NORMAL)	HOSPITAL (TRIGENERATION)
P_2 (Mpa)	8.3	8.3
T_2 (ºC)	500	500
Q_{MB} (MW)	44.07424	49.14185
W_{ST} (MW)	11.60024	12.93402
λ_{ST}	2.8	2.8
Y_{CG} (MW)	3.400723	2.720467
EUF	0.800121105	0.694523117
$\eta_{ELECTRIC}$	0.210558185	0.210558129
Q_{AB} (MW)	0	0
Q_{HP} (MW)	0	0
n_{HP}	0	0
Y_{ACH} (MW)	0	0.3627368
n_{ACH}	0	38
Y_{CCH} (MW)	0	0
n_{CCH}	0	0
TOTAL COST ($)	**12283550**	**13674550**

Table 33. Summary of the results obtained for the Vienna General Hospital with and without trigeneration (using steam turbine)

	BUILDING	HOTEL	HOUSEH.	CITY A	CITY B	HOSPITAL
P_2 (Mpa)	8.3	8.3	8.3	8.3	8.3	8.3
T_2 (ºC)	500	500	500	500	500	500
Q_{MB} (kW)	2163.363	1118.2	1139.828	63388.82	88117.6	44074.24
W_{ST} (kW)	569.3922	294.307	300	16683.79	23192.35	11600.24
λ_{ST}	2.8	2.8	2.8	2.8	2.8	2.8
Y_{CG} (kW)	184.025	1429.70	86.3609	31838.1	38215.56	3.400723
EUF	0.8001211	0.80012	0.80012072	0.8001209	0.800121	0.8001211
$\eta_{ELECTRIC}$	0.2105582	0.21055	0.2105581	0.2105581	0.2105582	0.2105582
Q_{AB} (kW)	0	0	1275.627	75793.31	0	0
Q_{HP} (kW)	0	0	0	0	0	0
n_{HP}	0	0	0	0	0	0
COST ($)	**432497.6**	**858915.2**	**420415.3**	**29175370**	**27612920**	**12283550**

Table 34. Summary of the results for all the users using the back pressure steam turbine option

		BUILDING	HOTEL	HOUSEHOLD	CITY A	CITY B	HOSPITAL	HOSPITAL (TRIG.)
GAS TURBINE	POWER GENERATION (KW)	NON APPLICABLE			30679.33	25418.54	12717.36	
	HEAT GENERATION (KW)				78358.03	64925.25	32474.08	
	EQUIPMENT TO BUY OR SELL POWER (KW)				15339.78	31622.79	2782.795	
	SIZE OF THE AUXILIARY BOILER (KW)				41722.47	0	0	
	TOTAL COST ($)				24905970	23984990	8867702	
MICRO GAS TURBINE	POWER GENERATION (KW)	397.464	452.616	426.645	NON APPLICABLE			
	HEAT GENERATION (KW)	784.9866	823.892	842.59513				
	EQUIPMENT TO BUY OR SELL POWER (KW)	453.366	1535.424	213.006				
	SIZE OF THE AUXILIARY BOILER (KW)	808.984	0	1272.860				
	TOTAL COST ($)	501947.8	897664.7	465587				
BACK PRESSURE STEAM TURBINE	POWER GENERATION (KW)	569.3922	294.3078	300	16683.79	23192.35	11600.24	12934.02
	HEAT GENERATION (KW)	1594.2982	824.06184	840	46714.612	64938.58	32480.672	36215.256
	EQUIPMENT TO BUY OR SELL POWER (KW)	184.025	1429.707	86.3609	31838	38216	3400.723	2720
	SIZE OF THE AUXILIARY BOILER (KW)	0	0	1275.904	75793	0	0	0
	TOTAL COST ($)	432497.57	858915.2	420415.25	29175373	27612920	12283546	13674547

Table 35. Summary of the systems capacities and the auxiliary equipments for all the users and all the technologies

		BUILDING	HOTEL	HOUSEHOLD	CITY A	CITY B	HOSPITAL	HOSPITAL (TRIG.)
GAS TURBINE	Main Cycle Purchase Cost ($)	NON APPLICABLE			1869546	1548622	778840.1	
	Auxiliary Equipments Purchase Cost ($)				286331.3	45859.28	10668.81	
	Fuel Cost ($)				17054570	14101130	7068412	
	Total Operational Cost ($)				22750098	22390510	8078193	
	Fuel Cost Per Unit Of Energy Generated ($/kW)				156.4103	156.083	156.4104	
	Main Cycle Purchase Cost Per Unit of Energy Generated ($/kW)				17.1459	17.1414	17.2342	
	Total Cost ($)				24905970	23984990	8867702	
MICRO GAS TURBINE	Main Cycle Purchase Cost ($)	28413.67	32355.550	30498.89	NON APPLICABLE			
	Auxiliary Equipments Purchase Cost ($)	15271.55	7840.966	18923.31				
	Fuel Cost ($)	248017.9	312697.3	377921.9				
	Total Operational Cost ($)	458262.6	857468.2	416164.84				
	Fuel Cost Per Unit Of Energy Generated ($/kW)	209.7490	244.9630	297.7544				
	Main Cycle Purchase Cost Per Unit of Energy Generated ($/kW)	24.0294	25.3469	24.0294				
	Total Cost ($)	501947.8	897664.7	465587				
BACK PRESSURE STEAM TURBINE	Main Cycle Purchase Cost ($)	50431.24	30650.47	31095.93	666366.3	860011.83	503282.4	547372.4
	Auxiliary Equipments Purchase Cost ($)	2195.627	7512.434	17949.54	482916.3	53948.165	12635	328935
	Fuel Cost ($)	412754	233279.6	283378.3	11819140	15453330	9194792	10252000
	Total Operational Cost ($)	379870.7	820752.3	371369.78	28026090	26698960	11767629	12798240
	Fuel Cost Per Unit Of Energy Generated ($/kW)	190.76389	208.5889	248.57745	186.4264	175.345	208.5889	162.9026
	Main Cycle Purchase Cost Per Unit of Energy Generated ($/kW)	23.3079	27.4059	27.2771	10.5107	9.7583	11.4172	8.6976
	Total Cost ($)	432497.57	858915.2	420415.25	29175373	27612920	12283546	13674547

Table 36. Summary of the costs of all the technologies applied to the users

10.2 Analysis For Each Technology

10.2.1 Back Pressure Steam Turbine

The back pressure steam turbines used in this study have a net output power between 500 kW – 56 MW, with the important characteristic that the steam turbine outlet pressure is higher than the atmospheric pressure. These steam turbines were used for all the users, which means the small users (with a power demand lower than 5 MW) and the big users (power demand higher than 5 MW). The calculations show that the optimal heat to power ratio of the back pressure steam turbines required to supply the energy demand of all the users is 2.8. This implies that the back pressure steam turbine generates 2.8 times more heat than power per unit of fuel consumed.

For all the users the dimensioning of the steam turbine was in some grade related with the maximal heat to maximal power ratio of the users. The user which present the highest maximal heat to maximal power ratio is the household (9.7493) and in this case the back pressure was designed to supply more power that the maximal demand (38% more) buying the excess, but the heat demand is too high that the steam turbine supply only 39.70% of the heat peak, requiring the use of auxiliary boiler. The users which have maximal heat to maximal power ratios lower than the heat to power ratio of the respective plants are the Hospital (2.1648), the City B (1.1675) and the Hotel (this is the lowest ratio, 0.4778, indicating the high power demand in relation to the heat demand). For these users the steam turbine was designed in order to satisfy the maximal heat demand, but taking into account that the power generation is in all the cases different that the maximal power demand, it was needed buy or sell power from/to the grid.

Two users present a maximal heat to maximal power ratios close to the heat to power ratio of the plant (2.8). In the first case, the City A the ratio is a bit lower than the heat to power ratio of the turbine and it is designed smaller than the maximal user power and heat demands, requiring auxiliary equipments. In the second case, the Building the ratio is a bit bigger than the heat to power ratio of the turbine, although the turbine is designed to supply the maximal heat demand and the output power exceeds the power demand. This is because for commercial users the power sold has a better price than for industrial users and therefore it is more expensive for the industrial users design a big turbine and sell power excess than design a small turbine and buy power from the grid.

The calculations demonstrate that for the optimal heat to power ratio defined in the pre model (2.8) the best turbine inlet conditions were 8.3 MPa (Turbine inlet pressure) and 500ºC (Turbine inlet temperature). These optimal turbine inlet conditions were obtained for all the users.
The electrical efficiency of the back pressure steam turbines obtained for all the users was close to 21%. This electrical efficiency is clearly lower than normal steam turbines because in those the turbine outlet pressure is the atmospheric pressure and therefore the steam expansion and the net output power are higher than for the back pressure steam turbine (where the turbine outlet pressure is higher than the atmospheric).
The energy utilization factor was in the same way similar for all the users, taking a value close to 80%. This represents the advantages of use cogeneration against the conventional technologies, because it is explicit that four fifths of the input fuel energy is transformed in different forms of energy like heat and power, against maximal 40-50% of the conventional systems.

The partial load for the back pressure steam turbines is made with the steam flow mass control, this decrease the isentropic efficiency of the turbine, the net output power and increase the heat to power ratio of the plant. For all the cases was observed that the steam turbine worked at partial load only in the periods where the power demand is lower than the maximal power demand, forcing the turbine to generate less electricity. In these cases is cheaper to work at partial load that buy power from the grid. For all the cases the minimal partial load was the 75% of the nominal load. In any case was required the use of heat pumps, and the auxiliary boiler was preferred in all the cases of heat peak. This is due the high purchase cost of the heat pumps in comparison with the auxiliary boilers.

According with the costs of the back pressure steam turbines it is observed that:

- The purchase cost of the main cycle of the plant for the small cases are 3.5% - 12% of the plant total costs (11.665% for the building, 3.56% for the hotel and 7.39% for the household), and for the big cases are 2.2% - 4% of the plant total costs (2.28% for the City A, 3.11% for the City B and 4.09% for the Hospital)
- The main cycle purchase cost per unit of energy generated is a parameter highly related with the capacity of the plant. For small users this parameter was between 23.30-27.27 ($/kW) and for big users between 9.75-11.41 ($/kW), making evident that the purchase cost of the turbine decrease with a non linear tendency when the size increase
- The fuel cost of the system depends on the capacity of the plant and the portion of it in the energy supplied. For users which supply the most of the maximal power demand (>50%), the fuel cost are higher than 67% of the total plant cost (95.43% for the Building, 67.4% for the household and 74.85% for the hospital). For users which supply less than the half maximal power demand the fuel costs are between 27-56% of the total costs (27.16% for the hotel, 40.51% for City A and 55.96% for City B). This explains that the fuel costs are the most important factor if the plant supplies the most portion of the power demand.
- The fuel cost per unit of energy generated (heat and power) is related with the fuel and the capacity of the plant. For industrial users this parameter was between 175.34-208.58 ($/kW), for commercial users between 190.76-208.58 ($/kW) and for residential users 248.77 ($/kW)
- The costs of buy auxiliary energy (heat and power) depend on the capacity of the plant. For users which supply the most of the maximal power demand, these costs are between 0-20.9% of the total cost (0% for the Building, 20.9% for the household and the hospital). For the other users it represents 40-68% of the total costs (68.39% for the hotel, 55.55% for City A and 40.72% for City B).

10.2.2 Gas Turbine

The gas turbines used in this research have a simple open cycle configuration, with natural gas as fuel. The gas turbines were studied for the big demand users, which mean, the Vienna General Hospital and the cities A and B, both of 100.000 inhabitants, where the maximal heat and power demands exceed 122 MW and 55 MW respectively.

The results show the use of gas turbines that supply more than 45% of the power peak, (69% for the city A, 45% for the city B and 85% for the Hospital) and almost the complete heat demand. In the case of the Hospital and the City B, the gas turbine supplies the complete heat demand in all the periods, but in the City A the gas turbine supplies the 64% of the heat peak, requiring the use of an auxiliary boiler.

The heat to power ratios of all the gas turbines exceed the unity, taking values close to 2.55, making clear that the gas turbine system generates more heat than power for a given fuel consumption. This characteristic makes ideal the employ of gas turbines in big demand users (more than 5 MW power demand) where the heat demand is double bigger than the power demand, for example in small cities and towns.

It is interesting observe that for the users which present a maximal heat to maximal power ratio lower than the heat to power ratio of the plant (For City B this factor is 1.1675, for the Hospital is 2.1648 and the heat to power ratio of the plant is 2.5540) the gas turbine is dimensioned to supply the maximal heat demand, and the maximal power demand is satisfied buying power from the grid. In City A, the maximal heat demand to maximal power demand ratio is higher than the heat to power ratio of the gas turbine and therefore it is required the use of auxiliary equipments to buy heat and power.

The electrical efficiency for all the users is lower than 24% due the high power consumption of the air compressor, which was in all the cases bigger than 70% of the power generated by the turbine. The high pressure ratios used in the air compressors (for the entire cases equal and bigger than 21.6) explains the elevated power consumptions and also indicates the necessity of use high technology compressors. The elevated turbine inlet temperatures help to increase the cycle efficiency and the power generation, but in the same way obligate to use high technology materials (special alloys) and air cooling. In all the cases this temperature was bigger than 1230ºC, but it is important make clear that the state of the art in this temperature is approximately 1400ºC.

According with the literature the excess of air coefficients calculated for these gas turbines are between the normal limits 3-15, the excess of air is ever bigger than the unity in order to avoid high gas temperatures, although the excess of air is inversely proportional to the electrical efficiency.

It was observed that in all the cases and during all the periods, the gas turbines work at the maximal load allowed in each period. In any case the gas turbines work at a load lower than 94% of the nominal output power.

The energy utilization factors calculated for all the turbines show that the losses of energy along all the cycle are lower than 19% and are the heat wasted to the ambient at the exit of the heat recovery steam generator, the heat losses in the combustion chamber, the increase of the entropy in the air compressor and the turbine and the pressure losses in the combustion chamber and the heat recovery steam generator.

In any case was required the use of heat pumps, and the auxiliary boiler was preferred in all the cases of heat peak. This is probably due first the gas turbine supplies the complete heat demand and second the high purchase cost of the heat pumps in comparison with the auxiliary boilers.

In relation to the costs of the system, the results exhibit that:

- The main cycle purchase costs represent maximal the 9% of the total cost of the plant (8.65% for City A, 6.65% for City B and 8.9% for the Hospital)

- The main cycle purchase cost per unit of energy generated (heat and power) is in the three cases very similar (17.14 - 17.21 $/kW), demonstrating that for similar thermodynamics conditions (pressure ratio, turbine inlet temperature) the purchase cost of the equipments have a linear relation with the size

- The fuel cost of the systems are in the three cases higher than 58% of the plant total cost (68.475% for the City A, 58.79% for the City B and 79.71% for the Hospital), making evident that the fuel consumption and the fuel cost are the most important factors to take into account in the design of gas turbine cogeneration systems

- The fuel cost per unit of energy generated (heat and power) is other parameter that show cost of the fuel consumption in terms of the plant capacity and allow a clear comparison with other technologies. For the three cases this parameter remains almost constant 156.08 - 156.41 $/kW

- The costs due auxiliary heat or power bought are parameters which describe the capacity of the plant to produce the energy required. In the case of City A these costs are 22.869% of total costs, in the City B are 34.5586% of total costs and in the Hospital 11.3864% of total costs. This indicates that the turbine more adequate to the user energy demand is the gas turbine of the Hospital, followed by the turbine of the City A and finally the turbine of the City B

- The auxiliary equipments purchase cost is the lowest cost among the design and operational cost of the plant. In the case of the city B and the Hospital, which do not use auxiliary boiler, the auxiliary equipments purchase cost is maximal 0.19% of the total cost. In the case of the City A, which needs equipment to buy/sell power and auxiliary boiler, this cost represents the 1.15% of the plant total cost

10.2.3 Micro Gas Turbine (MGT)

The micro gas turbines studied here had a recuperated cycle, with sizes between 30-250 kW and with natural gas as fuel. The micro gas turbines were employed to satisfy the energy demand of the small users (with a power demand lower than 5 MW).

The optimal heat to power ratios calculated for the household and the building was the same value (1.974931), and for the hotel (which present a very high power consumption in comparison with the heat demand) this value was 1.8202089; this explains that the MGT for the hotel generate more power than the MGT for the building or the household, using the same amount of fuel.

The maximal heat to maximal power ratio of the small users (building, hotel and household) is very different and therefore the systems employed are special for each one. The household present a much higher maximal heat to maximal power ratio (9.7493) than the heat to power ratio of the micro gas turbine system (1.9749), and the micro gas turbines are designed in order to satisfy more than the maximal power demand (96% more) but only 39.83% of the heat demand, requiring equipment to sell electricity and auxiliary boiler. The building presents a ratio (2.88) bigger than the heat to power ratio of the MGT, and the plant is designed to supply 71.9% of the power peak and 49.24% of the heat peak, requiring also auxiliary equipment. The hotel is the only case where the maximal heat to maximal power ratio (0.4778) is lower than the same of the MGT (1.820289), and therefore the system is designed to satisfy the complete heat peak and only the 26.25% of the power peak, doing necessary the purchase of power.

The electrical efficiency of the systems are lower than 25.5% due (as well as the gas turbines) the high power consumption of the air compressor, which in all the cases represent between 62-65% of the power generated by the micro turbine. There are losses in the combustion chamber, the air pre heater, the water heater and the turbine which decrease the electric efficiency. The optimal pressure ratios for the MGT were between 5.17-5.38 (5.17 for the building and the household and 5.38 for the hotel) in all the cases lower that the state of the art (5.5-6). The optimal turbine inlet temperatures were between 1175-1223º C, which shows that the MGT should have special alloys or ceramic materials and need the use of cooling (these technologies are the state of the art in MGT).

The air pre heater for all the MGT should have an efficiency of 80%, which signifies a good quality pre heater but not the best one, because the contact area becomes it very expensive. It can be seen that the excess of air in the MGT is at least double higher than it in the industrial gas turbines; this is because the combustion chamber inlet temperature in the MGT is much higher than the same in the gas turbines and therefore less energy from the fuel is needed to produce the equal turbine inlet temperature and energy generation.

The power and heat control used in this system consists in the use of different number of MGT in accord with the energy demand. The size of the MGT calculated were between 32-37 kW and the number of those were 12 for the building and the hotel and 13 for the household. It was found that the number of MGT in service in a period for all the users is related with the heat demand to power demand of the respective period. Then the maximal number of MGT in service was in winter for all the users and the minimal in summer.

The energy utilization factors calculated for the MGT (69-71%) is lower than the EUF for the industrial gas turbines (81%), because a portion of the heat rejected by the cycle is used for itself in the air pre heater making the regeneration and therefore is energy not used for heating, like in the gas turbines, which use almost all the heat rejected.

Making a technical comparison between the results of the MGT obtained with the models and MGT without regeneration (with the same net output power, the same pressure ratio and the same turbine inlet temperature), the next differences were found:

- It is essential the use of the regenerative cycle in MGT because without it the low pressure ratios in the air compressor do not allow to obtain a good electrical efficiency (for the entire cases lower than 16%)
- The fuel consumption for MGT without regeneration is almost double than for MGT with regeneration making it double expensive in operational costs
- The heat to power ratio for MGT without regeneration is 233% higher than for MGT with regeneration, increasing the useful heat and the energy utilization factor but also the energy from the fuel and become this type of MGT in a good technology for users with a high heat demand to power demand ratio

In accord with the costs calculated for the MGT, it is concluded that:

- The MGT purchase cost was 5.66% of the total cost for the building, 3.6% for the hotel and 6.55% for the household
- The MGT purchase cost per unit of energy generated (heat and power) was very close for all the cases, between 24.05-25.34 ($/kW)
- The fuel cost depended on the type of user (commercial or residential) and on the capacity of the plant; for the building this cost was 49.41% of the total cost, for the hotel 34.83% and for the household 81.17% of the total cost
- The fuel cost per unit of energy generated was maximal in the household (residential user) with 297.7544 ($/kW), followed for the hotel (commercial user) with 244.963 ($/kW) and finally the building (commercial user) with 209.749 ($/kW)
- The costs due the purchase of power and the auxiliary boiler operation were 41.88% of the total cost for the building, 60.68% for the hotel and 8.21% for the household. In this last cast are much lower than the others because this system buys almost half of the power generation
- The purchase cost of the auxiliary equipments were for these systems 1-4% of the total costs

10.3 Analysis for Vienna General Hospital with and without Trigeneration (Using Back Pressure Steam Turbine)

The Vienna General Hospital is an user who demands an almost constant power during all the periods (14.6 MW – 15 MW), a variable heat (8.7 MW – 32.5 MW) and a highly changeable cooling (minimal in winter with 2.521 MW and maximal in summer with 13.784 MW). The maximal heat to maximal power ratio is calculated as 2.1648 and the maximal heat to maximal cooling ratio is 2.355.

It was found for both systems that the optimal heat to power ratio is the same 2.8 and the best turbine inlet condition to satisfy this parameter were 8.3 Mpa (Turbine inlet pressure) and 500ºC (Turbine inlet temperature). The electrical efficiency of both systems is equal 21.05%, indicating that the use of trigeneration (with refrigeration equipments) does not increase the power generation in comparison with the fuel consumed.

The sizes calculated for the steam turbine with and without trigeneration are 12.93 MW and 11.6 MW, which supply 86.22% and 77.33% of the power peak respectively, requiring in both cases the use of equipment to buy power from the grid. The two systems were designed to supply the entire heat peak.

The powerful advantages of trigeneration appear if it is observed the use of the heat generated by the steam turbine. In the case without trigeneration, the back pressure steam turbine is designed to supply the entire heat peak (32.5 MW) in winter, and in the other periods the turbine works at full load satisfying completely the heat demand. But it is evident that the heat demand in the other periods is lower than the heat generation and a lot of heat is wasted, especially in summer (0% of the heat generation is wasted in winter, 52% in spring, 73% in summer and 16.41% in autumn).

When trigeneration is applied to this user with the cooling demand it is observed that a portion of the heat generated and a very small percentage of the power generated by the steam turbine is used to generate cooling with the absorption chillers. The most interesting aspect is that the heat available from the steam turbine to let run the absorption chillers is maximal in summer, where the cooling demand is also maximal.

The results show that using the back pressure steam turbine and 38 absorption chillers (each one with 362.74 kW of cooling generation), the power demand, the heat demand and the cooling demand are entirely satisfied (using 7 absorption chillers in the first period, 25 in the second, 38 in the third and 9 in the fourth).

The chillers make use of the heat wasted by the steam turbine, taking in all the period between 53-100% of these heat (100% of the heat wasted is used for the chillers in winter, 64% in spring, 74% in summer and 53% in autumn), which represent between 10-56% of the total heat generated by the steam turbine (the chillers use 10.3% of the total heat generated in winter, 36.82% in spring, 56% in summer and 13.25% in autumn). A small amount of power is also required to run the chillers, 1.2% of the total power generated in winter, 4.3% in spring, 6.5% in summer and 1.5% in autumn.

Therefore the total heat wasted by the trigeneration system is maximal 20% of the total heat generated (0% wasted in winter, 20% in spring, 19.7% in summer and 11.78%). The energy utilization factor is, although lower for the system with trigeneration than the system without it, and that is because the COP for absorption chillers of single effect is lower than the unity. That means that the absorption chillers consume more units of energy (in form of heat) than the units of cooling generated. Therefore is better use absorption chillers of double effect.

About the economy of the trigeneration systems is deducible that:

- The main cycle purchase costs for both systems are close to 4% of the total cost of the plant

- The absorption chillers purchase costs are for the trigeneration system the 2.3% of the total cost of the plant

- The fuel costs are again the most important factor for both system, because these represent more than 74.85% of the total plant costs

- The fuel cost per unit of energy generated (in the case with trigeneration is heat, cooling and power generation) is clearly lower for the trigeneration (almost 22% cheaper) than for the system without absorption chillers and this infers that if both systems generate the same energy the trigeneration system saves the 22% of the fuel cost in comparison with the normal system

- The main cycle cost per unit of energy generated (in the case with trigeneration is heat, cooling and power generation and it does not include the cost of the absorption chillers) is also lower for the trigeneration system (almost 24%) than for the normal system. This demonstrate that if both systems generate the same amount of energy, the main cycle equipments will be cheaper with the use of absorption chillers than without them

- Anyway the absorption chillers purchase costs is more than 58% of the main cycle purchase cost, and that becomes it very expensive in design costs, but at the same time very cheap in operational costs

10.4 Analysis for each user

10.4.1 Small Users

The small users have a power demand lower than 5 MW and the technologies in comparison are the back pressure steam turbine and the micro gas turbine.

10.4.1.1 Building

The building is a commercial user demands almost a moderate constant power during the year (with a minimal to maximal power demand equal to 69.71%) and a power demand average of 460.83 kW. The heat demand has an average of 698.6 kW but it is very changeable (wit a minimal to maximal power demand equal to 10%) and the maximal heat demand to maximal power demand ratio is 2.883.

The micro gas turbine option requires the use of 12 units, which supply 71.9% of the power peak and 49.24% of the heat peak, requiring auxiliary boiler and power purchase. The heat to power ratio of the MGT is 1.97; the electrical efficiency is 23.42% and the EUF 69.67%. The MGT purchase cost was 5.66% of the total cost, the fuel cost was 49.41% of the total cost and the costs due the purchase of power and the auxiliary boiler operation were 41.88% of the total cost. The purchase cost per unit of energy generated (heat and power) was 24.0294 ($/kW) and the fuel cost per unit of energy generated 209.749 ($/kW).

The back pressure steam turbine was designed to supply the maximal heat demand and the output power exceeds the power demand, requiring equipment to sell power to the grid. The heat to power ratio is 2.8, the electrical efficiency is 21.05% and the EUF is 80%, This option has a purchase cost of the main cycle equal to 11.665% of the total cost; the fuel cost is 95.43% of the total cost and does not have costs due auxiliary equipments. The main cycle purchase cost per unit of energy generated is 23.30 ($/kW) and the fuel cost per unit of energy generated (heat and power) is 190.76 ($/kW).
The best option for the building is the steam turbine because it offers the lowest total cost, the most adequate heat to power ratio and it supplies the entire energy demand, generating lower fuel and operational costs than the micro gas turbine option, but it should be clarified that the fuel oil cost is cheaper than the natural gas cost for this user.

10.4.1.2 Hotel

The hotel is a commercial user who has the lowest maximal heat to maximal power ratio for all the users in this research (0.477). It presents a high changing power demand (minimal to maximal power demand equal to 26.38%) with a maximal demand equal to 1.724 MW. The heat demand is also unsteady (minimal to maximal power demand equal to 36.43%) with a maximal demand equal to 744 kW.

The micro gas turbine option (with a heat to power ratio equal to 1.82 and an electric efficiency equal to 25.24%) needs the use of 12 units of 37.718 kW each one; the heat to power ratio of the

MGT is bigger than the maximal heat to maximal power ratio of the user (0.4778), and therefore the system is designed to satisfy the complete heat peak and only the 26.25% of the power peak, doing necessary the purchase of power. The MGT purchase cost was 5.66% of the total cost, the fuel cost was 49.41% of the total and the costs due the purchase of power and the auxiliary boiler operation were 41.88% of the total. The MGT purchase cost per unit of energy generated (heat and power) was 25.34 ($/kW) and the fuel cost per unit of energy generated was 209.749 ($/kW).

The back pressure steam turbine (electrical efficiency equal to 21.05% and heat to power ratio equal to 2.8) was designed to satisfy the maximal heat demand (do not need auxiliary boiler), but the power generation is only the 17% of the power peak, and requires high power purchase. The main cycle purchase cost of the of the plant was 3.56% of the total cost, the fuel cost was 27.16% of the total and the cost of buy auxiliary energy (heat and power) was 68.39% of the total. The main cycle purchase cost per unit of energy was 27.27 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 190.76 ($/kW).

For this user is interesting infer that although the best option from economical viewpoint is the steam turbine, it presents operational costs (in proportion to the total) due the power purchasing and the auxiliary boiler higher than the MGT. But it is important to observe that the operational cost per unit of energy generated is lower for the MGT than for the steam turbine and that the fuel oil is much cheaper for this user than the natural gas.

10.4.1.3 Household

The household is a residential user and has the highest maximal heat to maximal power ratio for all the users of this research (9.749). It has a very constant power demand with an average of 209.95 kW and a minimal to maximal demand of 93.37%. The heat demand is highly unsteady, with a heat peak of 2.115 MW (in winter) and a minimal to maximal demand of 15.07%.

The MGT system used for this user (with a heat to power ratio equal to 1.974931 and an electrical efficiency equal to 23.42%) uses 13 units of 32.818 kW each one to generate more than the maximal power demand (96% more) but only 39.83% of the heat demand, requiring equipment to sell electricity and auxiliary boiler. The MGT purchase cost was 6.55% of the total cost, the fuel cost was 81.17% of the total and the costs due the purchase of power and the auxiliary boiler operation were 8.21% of the total. The MGT purchase cost per unit of energy generated (heat and power) was 24.0294 ($/kW) and the fuel cost per unit of energy generated was in the household (residential user) with 297.7544 ($/kW).

The steam turbine applied in this case (electrical efficiency equal to 21.05% and heat to power ratio equal to 2.8) was designed to supply more power that the maximal demand (38% more) buying the excess but only 39.70% of the heat peak, requiring the use of auxiliary boiler. The main cycle purchase cost was 7.39% of the total cost, the fuel cost was 67.4% of the total and the costs of buy auxiliary energy (heat and power) were 20.9% of the total. The main cycle purchase cost per unit of energy generated was 27.27 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 248.77 ($/kW).

For this user the best option is the back pressure steam turbine because it has the lowest total cost, the fuel cost is much lower than for the MGT and also the operational cost. Additionally the steam turbine has the lowest fuel price per unit of energy generated (although the main cycle purchase cost per unit of energy generated is a bit higher than for the MGT). Both systems supply the maximal power demand, but the MGT generates more power and of course consumes more fuel, that for this user has the highest price.

10.4.2 Big Users

The big users have a power demand bigger than 5 MW and the technologies in comparison are the back pressure steam turbine and industrial gas turbine.

10.4.2.1 City A

The City A has 100.000 inhabitants, the ambient conditions of Detroit City and in this research is considered as an industrial user. The City A has a moderate steady power demand, with a peak power of 44.35 MW and a minimal demand to maximal demand ratio of 74.69%. On the other hand it has a high unsteady heat demand, with a heat peak of 122.498 MW and a minimal demand to maximal demand ratio of 16.83%.

The back pressure steam turbine option (with an electrical efficiency of 21.05% and a heat to power ratio of 2.8) generates less than the maximal user power and heat demands, requiring auxiliary boiler and power purchase. The main cycle purchase cost was 2.28% of the total cost, the fuel cost was 40.51% of the total and the costs of buy auxiliary energy (heat and power) were 55.55% of total. The main cycle purchase cost per unit of energy generated was 10.51 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 186.42 ($/kW).

The gas turbine option (with a heat to power ratio of 2.55 and an electrical efficiency of 23.041%) generates 69% of the power peak and 64% of the heat peak, requiring the use of auxiliary boiler and power purchase. The main cycle purchase costs represent 8.65% of the total cost, the fuel cost 68.475% of the total and the costs due auxiliary heat or power bought 22.869% of total cost. The main cycle purchase cost per unit of energy generated (heat and power) was 17.14 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 156.41 ($/kW).

It is clear that the option which is more adequate to the energy demand of the user is the gas turbine system; this option generates the most energy required and uses smaller auxiliary equipments than the steam turbine option. In addition the natural gas is cheaper than the fuel oil for this user and therefore the fuel cost, the operational cost and the total cost of the system are much lower for the gas turbine than for the steam turbine (the total cost of the gas turbine is 17% cheaper than the cost of the steam turbine).

10.4.2.2 City B

The City B with 100.000 and the climatic conditions of Oklahoma City is considered also an industrial user. It has a variable power demand with a peak of 55.609 MW and a minimal to maximal demand ratio of 59%. The heat demand is highly variable with a peak of 64.925 MW and a minimal to maximal demand ratio of 27.63%. In comparison with the City A, the City B presents a lower heat demand (the average heat demand is 61% of the average heat demand for the City A) because the ambient temperature is higher all the periods and presents also a higher power demand (the average power demand is 15% higher than the average power demand for the City A) probably explained because the use of refrigerant equipments in summer.

The back pressure steam turbine used to serve to City B (with a heat to power ratio equal to 2.8 and an electrical efficiency equal to 21.05%) produces the maximal heat demand, but only 41.7% of the power peak and requires power purchase.

The main cycle purchase cost was 3.11% of the total cost; the fuel cost 55.96% of the total and the cost of power purchase 40.72% of the total cost. The main cycle purchase cost per unit of energy generated was 9.75 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 175.34 ($/kW)

The gas turbine option (with a heat to power ratio equal to 2.55 and an electrical efficiency equal to 23.041%) produces the maximal heat demand and 45.7% of the power peak and requires also power purchase. The main cycle purchase cost was 6.65% of the total cost, the fuel cost was 58.79% of the total and the cost of power purchase was 34.5586% of total costs.

The main cycle purchase cost per unit of energy generated (heat and power) was 17.14 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 156.083 ($/kW).

As well as for the City A, in this case the best option is again the gas turbine, because it produces the maximal heat demand (as well as the steam turbine) but generates more power than the steam turbine because the heat to power ratio is lower. Therefore a smaller power purchase is required and the fuel cost, the operational cost and the total cost is lower for the gas turbine than for the steam turbine. In this case the natural gas is also cheaper than the fuel oil.

10.4.2.3 Hospital

The Vienna General Hospital is a user who demands an almost constant power during all the periods (14.6 MW – 15 MW) and a variable heat (8.7 MW – 32.5 MW).

The back pressure steam turbine (heat to power ratio = 2.8 and electrical efficiency = 21.05%) offers a system which satisfies the maximal heat demand and 77.33% of the power peak. The main cycle purchase cost was 4.09% of the total cost, the fuel cost was 74.85% of the total and the power purchase cost was 20.9% of the total cost. The main cycle purchase cost per unit of energy generated was 11.41 ($/kW) and the fuel cost per unit of energy generated (heat and power) was 208.58 ($/kW).

The gas turbine (heat to power ratio = 2.55 and electrical efficiency = 23.04%) offers a system which supplies 85% of the power peak and the complete heat demand. The main cycle purchase cost was 8.9% of the total cost, the fuel cost was 79.71% of the total and the power purchase cost was 11.3864% of total cost. The main cycle purchase cost per unit of energy generated (heat and power) was 17.23 ($/kW) and the fuel cost per unit of energy generated (heat and power) 156.41 ($/kW).

In this case the gas turbine represents the best option, because taking into account that the natural gas is cheaper than the fuel oil, this system supplies the entire heat demand and more power than the steam turbine. This means a smaller amount of power purchased and lowers operational and fuel costs. In addition the fuel cost per unit of energy and the purchase cost per unit of energy are much lower for the gas turbine than for the steam turbine.

10.5 Conclusions

Analyzing the results, it was interesting to observe that when an energy system is designed to satisfy the specific energy demand (power, heat and cooling) of a user, two aspects are essential. In the first place the specific characteristics of the user and in the second place the economy and the technical attributes of the energy system technology.

The specific characteristics of the user include the size of it in terms of power, heat and cooling consumption during the year, the location of the user in the world, the climatic conditions (moisture and temperature) and the category of the user in the energy generation market (commercial user, industrial user and residential user). In this research the maximal heat to maximal power ratio was a parameter used to classify the users in accord with the priority of the energy required and it was very useful to determine preliminary the most adequate technological option.

The economy of the energy system technology includes the purchase costs of it in relation with the size, the maintenance cost, the fuel cost, the operational costs, the purchase cost of auxiliary equipments and the cost to sell or buy power (although these last depends on the type of user). The technical attributes refers principally to the size of the system, the heat to power ratio of the system, the electrical efficiency, the energy utilization factor and the thermodynamic operation conditions.

Generally was found that for almost all the cases the operational costs due fuel, power purchase and auxiliary boiler represented between 87-97% of the total plant cost demonstrating the importance of the fuel choice and the choice of the technology which save more fuel and buy less power and auxiliary heat.

The prime movers of the energy systems (micro gas turbines, industrial gas turbines and steam turbines) were dimensioned according with the relation between the heat to power ratio specific of the technology and the maximal heat to maximal power ratio of the user. In some cases, if the maximal heat to maximal power ratio of the user was lower than the heat to power ratio of the system, the prime mover was designed to satisfy the entire heat peak and a percentage of the power peak. In the opposite situations, the prime mover was designed to satisfy less than the heat and power peak, needing the use of auxiliary boiler and power purchase.

The optimal micro gas turbines calculated to serve the small users (power demand <5 MW) presented the highest electrical efficiency of all the prime movers (23.42%), the lowest heat to power ratio (1.97) and the lowest energy utilization factor (69.71%). This implies that the MGT are a very good option if the user demands only a bit more heat than power. Although the MGT had the highest fuel cost per unit of energy generated for all the small users and it is because the lower heat to power ratio in comparison with the other technologies. It is essential the use of the regenerative cycle in MGT because without it the low pressure ratios in the air compressor do not make economically attractive this option.

The optimal back pressure steam turbines calculated to serve all the users presented the lowest electrical efficiency of all the prime movers (21.05%), the highest heat to power ratio (2.8) and a good energy utilization factor (80%). The back pressure option is ideal for users which demand more heat than power and it becomes cheaper if the size increase. A disadvantage in comparison with other technologies is that the turbine outlet temperature can not be too high (less than 300-350ºC), because the electrical efficiency and the net output power decreases.

The optimal industrial gas turbines calculated to serve the big industrial users (City A, City B and the Vienna General Hospital) had a normal electrical efficiency (23.041%) a medium heat to power ratio (2.55) and the highest energy utilization factor for all the users (81.89%). This option needs high technology air compressors to make elevated pressure ratios (21.66), special alloys and air cooling to allow high turbine inlet temperatures (higher than 1200ºC) and a heat recovery steam generator (HRSG) to produces steam. The use of this equipments make this option expensive in terms of purchase cost per unit generated in comparison with other options, but it should be observed that the design costs are never higher than 13-13% of the total costs. In terms of the fuel cost per unit of energy generated, the gas turbine for big users is the best option to save fuel and to generate energy with the quality desired.

The small users (power demand <5MW) which are residential users (the household) and commercial users (the hotel and the offices building) have higher fuel and power prices than the industrial users (City A, City B and Vienna General Hospital). The back pressure steam turbine was the technology which best serve the small users from the economical viewpoint. It generates for the three cases the most energy demand (heat and power) and used smaller auxiliary boilers and less power purchase than the micro gas turbine option. In addition the steam turbine presented for all the small cases the lowest total cost, the lowest fuel cost, the lowest fuel cost per unit of energy generated and the lowest operational cost in comparison with the MGT.

For all the big industrial users the best option was the gas turbine, which generates almost all the heat peak and a high percentage of the power peak using smaller auxiliary equipments than the steam turbine. The total cost, fuel cost, fuel cost per unit of energy generated and operational cost

were much lower for the gas turbine than for the steam turbine. The purchase cost per unit of energy generated is although higher for the gas turbine than for the steam turbine. It should be clarified that the low price of the natural gas consumed by the gas turbines is an important factor, which makes that this option become the cheaper and save more fuel than the steam turbine.

Comparing the gas turbine option for the two cities with the same number of inhabitants but with different ambient conditions, it results a bigger turbine for the City A, which had higher energy consumption per inhabitant (heat and power) than the City B. Anyway the thermodynamic conditions of the gas turbine for both cities were almost equal and the purchase cost per unit generated and fuel cost per unit generated remained constant in both cases.

For the Vienna General Hospital the comparison of systems with and without trigeneration option show big advantages of trigeneration to make use of the heat wasted by the steam turbine in a better way than without it. Improving the capacity of the system to satisfy the entire power, heat and cooling demand without the use of auxiliary equipments. The solution of the models indicates that the use of compression chillers is not convenient from the economical viewpoint; this fact can be compared with the models of the energy systems that include electrical heat pumps, which –according tom the results of the models- don't contribute to reduce the total annualised costs. The preference for the absorption chillers can be justified if it is taken into account that there is a big availability from heat in summer and a big demand of cooling power, and the absorption units take profit of both aspects, using the heat that in other case would be wasted and supplying the cooling power required. It was also clear that is more convenient the use of absorption chillers of double effect, because they have a COP bigger than the unity and that contributes to increase the energy utilization factor (EUF).

Appendix A

List of Symbols and Abbreviations

Θ_1	The square root of the relation between the standard ambient temperature and the temperature in any period
Ψ	Annual fixed charge rate
μ_{CO_2}	Carbon dioxide emissions per kilogram of fuel
φ	Maintenance factor
$\dot{m}$	Mass flow rate
ξ	Concentration of absorber fluid
Γ	Operational cost
γ	Specific heat ratio
η_{CHP}	Thermal efficiency of a CHP system
Δt	Duration of each period of time
η_{TC}	Total efficiency of the conventional Energy System
ΔTML	Log mean temperature difference
a	Fuel factor in the stoichiometric relation
a_i	Constant coefficients that involve thermodynamic and economic parameters
C_{ab}	Constant values used for the modelling of the equipments. a and b are numerical subscripts
CHCP	Combined Heating, Cooling and Power
CHP	Combined Heat and Power
COP	Coefficient Of Performance
C_p	Specific heat
C_x	Coefficients of cost for the equipment x
EUF	Energy Utilisation Factor
F_B	Fuel power for a boiler
F_C	Fuel power for a conventional Energy System
F_{CHP}	Fuel power for a CHP system
F_{PP}	Fuel power for a power plant
FU	Factor of update
h	Enthalpy
HDR	Heat Dissipation Ratio
HRSG	Heat Recovery Steam Generator
H_U	Low Heating Value of the fuel
J	Correction factor for partial load operation
L	Stoichiometric air-fuel ratio
MW	Molecular Weight; Megawatts
n	Rotary speed of the gas turbine
n_i	Number of units of the equipment i
O	Cooling (kW)

ORC	Organic Rankine Cycle
p	Pressure
Q	Heat (kW)
R_a	Gas constant of the air
r_c	Pressure ratio
s	Entropy
T	Temperature
W	Work (kW)
x	Steam Quality
Y_i	Size of each one of the equipments of the Energy System
Z_i	Function of design cost for the equipment i
ϕ	Total costs to minimise
ϕ_D	Design costs
ϕ_O	Operational costs
α	Correction factor for the COP; factor for calculating the dew point
η_B	Boiler thermal efficiency
η_e	Electrical efficiency
λ	Heat to Power Ratio

Subscripts

1,2,…	States or points of a cycle
a	Air
abs	Absorption; Absorber
AC	Absorption Chiller
ACO	Air Compressor
AH	Additional heat produced in a period
AR	Absorption Refrigeration
AR	Additional refrigeration produced in a period
atm	Atmospheric
B	Boiler
BY	Electricity bought from the utility grid in a period
C	Conventional System; compressor
CC	Compression Chiller
CCH	Combustion chamber
CG	Connection to the Grid
CHP	Combined Heat and Power system
cond	Condenser
cs	Concentrated solution
CTA	Cooling tower operation, due the absorption units operating during each period. Here are included water consumption, chemical treatment, cleaning and maintenance and fan power.
CTB	Cooling tower operation, due the compression units operating during each period. Here are included water consumption, chemical treatment, cleaning and maintenance and fan power
D	Demand
Dew	Dew Point

ds	Dissolved solution
Elec.	Electrical chiller
ER	Electrical Refrigeration
EV	Evaporator
f	Fuel
FH	Feedwater Heater
full	Full Load
fw	Feed water
g	Gases
gen	Generator
GT	Gas Turbine
HP	Heat Pump
MB	Main Boiler
MC	Compressor of the micro gas turbine
md	Water mass flow from feed water heater for heating (district heating)
MGT	Micro Gas Turbine
MT	Turbine of the Micro gas turbine
N	Net
n	Period of time
nb	Portion of the total air that is not burned
P	Pump
p	Produced electric power, heat or cooling for the trigeneration option in a period
PH	Preheater
pl	Partial Load
PP	Power Plant
Ref	Refrigeration fluid
Refri	Refrigeration
Reg	Regenerator
r_t	Pressure ratio in the gas turbine (relation between the pressure at the outlet and at the inlet of the turbine)
S	Power sold to the utility grid in a period
sat	Saturation
SH	Superheater
st	Steam
ST	Steam Turbine
t	Total
T	Trigeneration; gas turbine (without compressor)
whs	Water heat source

Superscripts

b_i	Constant exponents that involve thermodynamic and economic parameters

Appendix B

Particular Information

	Conventional Energy System	Cogeneration Energy System
Work Demand	1	1
Heat Demand	λ_D	λ_D
Fuel Energy Required	$\frac{1}{\eta_e} + \frac{\lambda_D}{\eta_B}$	$\frac{1}{\eta_{CHP}}$
Fuel Mass Flow Required	$\frac{1}{H_U}\left(\frac{1}{\eta_e} + \frac{\lambda_D}{\eta_B}\right)$	$\frac{1}{\eta_{CHP} H_U}$
Total Efficiency	$\frac{(1+\lambda_D)(\eta_e \eta_B)}{\eta_B + \lambda_D \eta_e}$	$(1+\lambda_D)\eta_{CHP}$
Total Carbon Dioxide Emissions	$\frac{\mu_{CO_2}}{H_U}\left(\frac{1}{\eta_e} + \frac{\lambda_D}{\eta_B}\right)$	$\frac{\mu_{CO_2}}{\eta_{CHP} H_U}$

Table 37. Summary of the comparison between Conventional Energy System and Cogeneration Energy System

	Steam Turbines	Gas Turbines	Micro gas Turbines	Combined Cycle	Diesel Engines	Gas Engines	Stirling Engines	Fuel Cells
Electric Efficiency	15-35%	15-45%	20-30%	35-45%	30-50%	25-45%	21-28%	40-70%
Size (MW)	0.2-100	0.5-200	0.025-0.25	4-100	0.05-5	0.05-5	0.003-1.5	0.2-2
Heat to power ratio	2-10	1.25-3.5	1.25-3	0.5-1.67	0.4-1.25	1.42-2	0.58-0.84	0.5-1.25
Footprint (m^2/KW)	< 0.009	0.0018-0.57	0.014-0.14	-	0.002	0.002-0.029	-	0.056-0.37
CHP Installed cost (€/KW)	800-1000	700-900	500-1.300	500-1500	800-1500	800-1500	2.400	>3000
Operation and Maintenance cost (€/KWh)	0.004	0.002-0.008	0.002-0.01	-	0.005-0.008	0.007-0.015	0.004-0.011	0.003-0.015
Availability	95-100%	90-98%	90-98%	77-85%	90-95%	92-97%	85-90%	> 95%
Hours between overhauls	> 50.000	30.000-50.000	5.000-40.000	-	25.000-30.000	24.000-60.000	5.000-25.000	10.000-40.000
Star-up Time	1 hr.- 1 day	10 min.-1 hr.	60 sec.	-	10 sec.	10 sec.	-	3 hrs.-2 days
Fuel Pressure (Mpa)	-	0.83-3.45	0.275-0.69	-	< 0.035	0.0069-0.31	-	0.0035-0.31
Fuels	All	Natural gas, Biogas, Propane, Distillate oil	Natural gas, Biogas, Propane, Distillate oil	All	Diesel and residual oil	Natural gas, biogas, propane	All	Hydrogen, Natural gas, Propane
Noise	Moderate to high	Moderate	Moderate	Moderate to high	Moderate to high	Moderate to high	Low to moderate	Low
NO_x Emissions (Kg/MWh)	0.82	0.14-1.82	0.18-1	0.82-2	1.35-13.5	1-12.7	-	< 0.009
Uses for Heat Recovery	LP-HP steam, District heating	Direct heat, hot water, LP-HP steam, District heating	Direct heat, hot water, LP steam	Direct heat, hot water, LP-HP steam, District heating	Hot water, LP steam, district heating	Hot water, LP steam, district heating	Hot water, LP-HP steam	Hot water, LP-HP steam
Useable Temperature for CHP (°C)	70-300	260-600	205-345	70-300	80-480	150-260	max. 600	260-370

Table 38. Comparison between the different cogeneration technologies [1][2][3]

	Water - LiBr	Ammonia - Water	Water - Silicagel
Cooling Power (KW)	50-5000	200-1000	50-1050
Working fluids	Water/Water-Libr	Ammonia/Ammonia-water	Water/Water-silicagel
Coefficient of performance (COP)	0.68	0.45-0.6	0.4-0.62
Temperature of heat source (oC)	85-140	80-160	55-100
Temperature of heat rejection (oC)	20-45	20-45	25-50
Temperature of cooling (oC)	6	-30	5
Specific weight (Kg/KW)	8-20	35-50	37-71
Specific area (m^2/KW)	0.01-0.03	0.025-0.04	0.02-0.08
Partial load (%)	10-100	10-100	100
Electricity required (Kwe/KW)	0.06-0.08	0.08-0.1	0.07-0.09
Specific primary energy required (kWhe/kWh)	0.35-0.5	0.55-0.7	0.55-0.7
Specific investment cost (Only the refrigerant equipment) (DM/kW) (Year 1997)	130-300	800-2500	700-3200

Table 39. Trigeneration systems with absorption refrigeration [4]

	Conventional Energy System	Trigeneration (electrical refrigeration)	Trigeneration (absorption refrigeration)
Work Demand	1	1	1
Heat Demand	λ_D	λ_D	λ_D
Cooling Demand	$Q_{Refri.}$	$Q_{Refri.}$	$Q_{Refri.}$
Total Work Produced	$1+\frac{Q_{Refri.}}{(COP)_{Elec.}}$	$1+\frac{Q_{Refri.}}{(COP)_{Elec.}}$	1
Total Heat Produced	λ_D	λ_D	$\lambda_D+\frac{Q_{Refri.}}{(COP)_{AR}}$
Heat to power ratio (Cogeneration System)	-	$\frac{\lambda_D}{1+\frac{Q_{Refri.}}{(COP)_{Elec.}}}$	$\lambda_D+\frac{Q_{Refri.}}{(COP)_{AR}}$
Fuel Energy Required	$\frac{\eta_B\left(1+\frac{Q_{Refri..}}{(COP)_{Elec.}}\right)+\lambda_D\eta_e}{\eta_e\eta_B}$	$\frac{1+\frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP}}$	$\frac{1}{\eta_{CHP}}$
Fuel Mass Flow Required	$\frac{\eta_B\left(1+\frac{Q_{Refri..}}{(COP)_{Elec.}}\right)+\lambda_D\eta_e}{\eta_e\eta_B H_U}$	$\frac{1+\frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP}H_U}$	$\frac{1}{\eta_{CHP}H_U}$
Energy Utilisation Factor	$\frac{(1+\lambda_D+Q_{Refri.})(\eta_e\eta_B H_U)}{\eta_B\left(1+\frac{Q_{Refri.}}{(COP)_{Elec.}}\right)+\lambda_D\eta_e}$	$\frac{1+\lambda_D+Q_{Re\,fri.}}{\left[\frac{1+\frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP}}\right]}$	$\eta_{CHP}(1+\lambda_D+Q_{Re\,fri.})$
Total Carbon Dioxide Emissions	$\frac{\mu_{CO_2}\left[\eta_B\left(1+\frac{Q_{Refri.}}{(COP)_{Elec.}}\right)+\lambda_D\eta_e\right]}{\eta_e\eta_B H_U}$	$\mu_{CO_2}\left[\frac{1+\frac{Q_{Refri.}}{(COP)_{Elec.}}}{\eta_{CHP}H_U}\right]$	$\frac{\mu_{CO_2}}{\eta_{CHP}H_U}$

Table 40. Summary of the comparison between the different options to produce heating, cooling and power

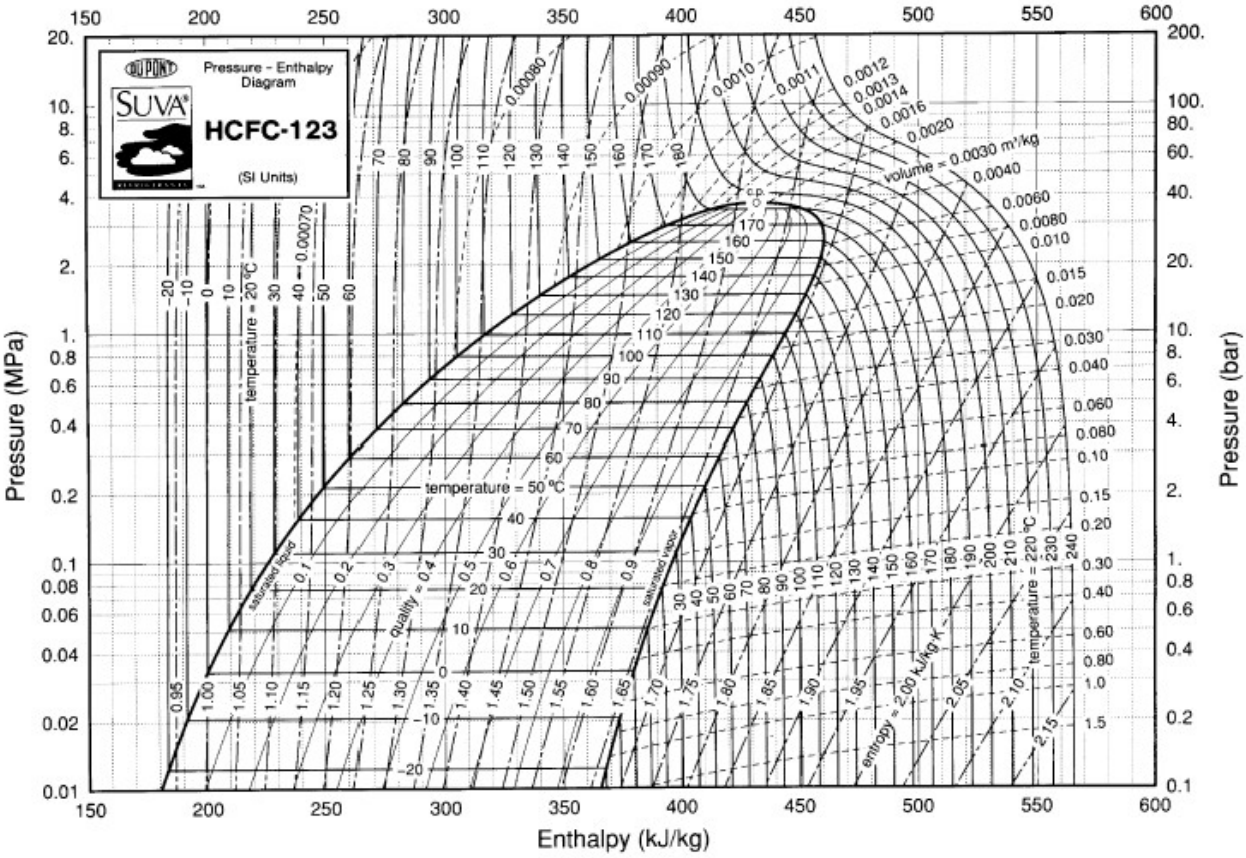


Figure 86. Pressure-enthalpy diagram for the refrigerant HCFC-123 [5]

Heat pumps in cascade	Steps		T (oC)	P (Kpa)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)
2	2	Outlet	110	977,71	316,4	443,9	1,3533	1,6862
		Inlet	55	247,33	260	416,2	1,1974	1,6662
	1	Outlet	65	329,62	265,5	419,2	1,2136	1,6681
		Inlet	10	50,57	209	385,8	1,0323	1,6567
3	3	Outlet	110	977,71	316,4	443,9	1,3533	1,6862
		Inlet	70	377,81	271	422,1	1,2297	1,6701
	2	Outlet	80	489,93	282,2	427,9	1,2614	1,6742
		Inlet	40	154,48	238,7	404,1	1,1316	1,6598
	1	Outlet	50	212,61	249,2	410,2	1,1646	1,6627
		Inlet	10	50,57	209	385,8	1,0323	1,6567
4	4	Outlet	110	977,71	316,4	443,9	1,3533	1,6862
		Inlet	77,5	459,86	279,35	426,5	1,25355	1,6732
	3	Outlet	87,5	589,19	290,6	432,15	1,28485	1,6774
		Inlet	55	247,33	254,6	413,2	1,1811	1,6644
	2	Outlet	65	329,62	265,5	419,2	1,2136	1,6681
		Inlet	32,5	119,665	230,9	399,5	1,10565	1,6582
	1	Outlet	42,5	167,705	241,3	405,6	1,13985	1,6606
		Inlet	10	50,57	209	385,8	1,0323	1,6567
5	5	Outlet	110	977,71	316,4	443,9	1,3533	1,6862
		Inlet	82	515,06	284,4	429,1	1,2677	1,6751
	4	Outlet	92	655,47	295,7	434,6	1,2988	1,6792
		Inlet	64	320,57	264,4	418,6	1,2104	1,6677
	3	Outlet	74	420,04	275,5	424,5	1,2424	1,6717
		Inlet	46	187,63	245	407,8	1,1514	1,6615
	2	Outlet	56	254,77	255,7	413,8	1,1843	1,6647
		Inlet	28	101,92	226,4	396,8	1,0917	1,6574
	1	Outlet	38	144,51	236,6	402,9	1,1249	1,6593
		Inlet	10	50,57	209	385,8	1,0323	1,6567

Table 41. Thermodynamics characteristics of the refrigerant HCFC-123 for different number of heat pumps in cascade

COMPONENTS	PURCHASE COSTS
Air Compressor	$Z_{ACO} = FU_{ACO} \cdot \left(\frac{C_{11} \overset{*}{m}_a}{C_{12} - \eta_{ACO}} \right) \cdot r_c \cdot \ln(r_c)$
Combustion Chamber	$Z_{CCH} = FU_{CCH} \cdot \left(\frac{C_{21} \overset{*}{m}_a}{C_{22} - \frac{p_3}{p_2}} \right) \left[1 + e^{(C_{23} \cdot T_3 - C_{24})} \right]$
Turbine	$Z_{GT} = FU_{GT} \left(\frac{C_{31} \overset{*}{m}_g}{C_{32} - \eta_{GT}} \right) \cdot \ln(r_t) \cdot \left[1 + e^{(C_{33} \cdot T_3 - C_{34})} \right]$
Heat Recovery Steam Generator	$Z_{HRSG} = FU_{HRSG} \cdot C_{41} \left[\left(\frac{\overset{*}{Q}_{SH}}{(\Delta TLM)_{SH}} \right) + \left(\frac{\overset{*}{Q}_{EV}}{(\Delta TLM)_{EV}} \right) + \left(\frac{\overset{*}{Q}_{PH}}{(\Delta TLM)_{PH}} \right) \right] + C_{42} \overset{*}{m}_{st} + C_{43} \overset{*}{m}_g^{1.2}$
Feedwater Heater	$Z_{FH} = FU_{FH} C_{51} \overset{*}{Q}_{FH}$
Auxiliary Boiler	$Z_{AB} = FU_{AB} C_{61} \overset{*}{Q}_{AB}^{0.8} \left[1 + \left(\frac{1 - C_{62}}{1 - \eta_{AB}} \right)^{C_{63}} \right]$
Connection to the grid	$Z_{CG} = FU_{CG} C_{71} Y_{CG}^{0.6}$

Table 42. Equations for calculating the purchase costs for the components of the gas turbine pre-model

COMPONENTS	CONSTANTS			
Air Compressor	$C_{11} = \left(39.5 \frac{\$}{\frac{Kg}{s}}\right)$	$C_{12} = 0.9$		
Combustion Chamber	$C_{21} = \left(25.6 \frac{\$}{\frac{Kg}{s}}\right)$	$C_{22} = 0.995$	$C_{23} = \left(0.018 \frac{1}{K}\right)$	$C_{24} = 26.4$
Turbine	$C_{31} = \left(266.3 \frac{\$}{\frac{Kg}{s}}\right)$	$C_{32} = 0.92$	$C_{33} = \left(0.036 \frac{1}{K}\right)$	$C_{34} = 54.4$
Heat Recovery Steam Generator	$C_{41} = \left[3650 \frac{\$}{\left(\frac{kW}{K}\right)^{0.8}}\right]$	$C_{42} = \left(11820 \frac{\$}{\frac{kg}{s}}\right)$		
	$C_{43} = \left[658 \frac{\$}{\left(\frac{kg}{s}\right)^{1.2}}\right]$			
Feedwater Heater	$C_{51} = \left(10.4 \frac{\$}{kW}\right)$			
Auxiliary Boiler	$C_{61} = \left[560 \frac{\$}{(kW)^{0.8}}\right]$	$C_{62} = 0.9$	$C_{63} = 7$	
Connection to the grid	$C_{71} = \left[1000 \frac{\$}{(kW)^{0.6}}\right]$			

Table 43. Constants used in the equations for the purchase costs of the components of the gas turbine pre-model

COMPONENTS	FACTOR OF UPDATE
Air Compressor	$FU_{ACO} = 1.24$
Combustion Chamber	$FU_{CCH} = 1.24$
Turbine	$FU_{GT} = 1.24$
Heat Recovery Steam Generator	$FU_{HRSG} = 1.24$
Feedwater Heater	$FU_{FH} = 1.28$
Auxiliary Boiler	$FU_{AB} = 1.28$
Connection to the grid	$FU_{CG} = 1.28$

Table 44. Factors of update for the components of the gas turbine pre-model

COMPONENTS	DESIGN VARIABLES
Air Compressor	$\dot{m}_a^*, r_c, \eta_{ACO}$
Combustion Chamber	$\dot{m}_a^*, \frac{P_3}{P_2}, T_4$
Turbine	$\dot{m}_g^*, \mathrm{r_t}, T_4, \eta_{GT}$
Heat Recovery Steam Generator	$\dot{m}_g^*, \dot{m}_{st}^*, \dot{Q}_{PH}^*, \dot{Q}_{EV}^*, \dot{Q}_{SH}^*$
Feedwater Heater	$\dot{Q}_{FH}^*$
Auxiliary Boiler	$\dot{Q}_{AB}^*, \eta_{AB}$
Connection to the grid	$\dot{Y}_{CG}^*$

Table 45. Design variables for the components of the gas turbine pre-model

		C_f ($/KWh)		C_{BY} ($/KWh)	C_S ($/KWh)	C_{AH} ($/KWh) (η_{AB} =0.8, Fuel Oil)
		Natural Gas	**Fuel Oil**			
COMMERCIAL USER	Period 1	0,028334942	0,02093491	0,100256202	0,080204961	0,026168638
	Period 2			0,100552818	0,080442254	
	Period 3			0,104112209	0,083289767	
	Period 4			0,099069738	0,07925579	
RESIDENTIAL USER	Period 1	0,032947607	0,027913214	0,109390229	0,087512183	0,034891517
	Period 2			0,109713868	0,087771095	
	Period 3			0,113597545	0,090878036	
	Period 4			0,10809567	0,086476536	
INDUSTRIAL USER	Period 1	0,015584218	0,02093491	0,055926755	0,044741404	0,026168638
	Period 2			0,056092219	0,044873775	
	Period 3			0,058077784	0,046462227	
	Period 4			0,0552649	0,04421192	

Table 46. Coefficients used in the equation of operation costs for the gas turbine pre-model

TYPE OF CONSTRAINT	CONSTRAINT						
Non negativity	$Y_{CG} \geq 0$ $W_{BY1} \geq 0$ $W_{S1} \geq 0$ $Q_{AH1} \geq 0$	$Q_{AB} \geq 0$ $W_{BY2} \geq 0$ $W_{S2} \geq 0$ $Q_{AH2} \geq 0$	$\overset{*}{m}_a \geq 0$ $W_{BY3} \geq 0$ $W_{S3} \geq 0$ $Q_{AH3} \geq 0$	$\overset{*}{m}_f \geq 0$ $W_{BY4} \geq 0$ $W_{S4} \geq 0$ $Q_{AH4} \geq 0$	$W_{GT} \geq 0$ $T_1 \geq 0$ $T_4 \geq 0$ $r_t \geq 0$	$Q_{GT} \geq 0$ $T_2 \geq 0$ $T_5 \geq 0$	$\lambda_{GT} \geq 0$ $T_3 \geq 0$ $r_c \geq 0$
Thermodynamics Equations	$\overset{*}{m}_f = \dfrac{\overset{*}{m}_a C_{pa}(T_3 - T_2)}{\eta_{CCH}(H_u + h_f) - (L+1)C_{pg}T_3 + LC_{pa}T_3}$						
	$W_{GT} = \overset{*}{m}_f (L+1)C_{pg}T_3 + \left(\overset{*}{m}_a - L\overset{*}{m}_f\right)C_{pa}T_3 - \left(\overset{*}{m}_a + \overset{*}{m}_f\right)C_{pg}T_4 - \overset{*}{m}_a C_{pa}(T_2 - T_1)$						
	$Q_{GT} = \overset{*}{m}_t C_{pg}(T_4 - T_5)$						
	$\lambda_{GT} = \dfrac{Q_{GT}}{W_{GT}}$			$\overset{*}{m}_g + \overset{*}{m}_{nb} = \overset{*}{m}_a + \overset{*}{m}_f = \overset{*}{m}_t$			
	$T_2 = T_1\left\{1 + \dfrac{1}{\eta_c}\left[(r_c)^{\frac{\gamma_a - 1}{\gamma_a}} - 1\right]\right\}$			$T_4 = T_3\left\{1 - \eta_{gt}\left[1 - \dfrac{1}{(r_t)^{\frac{\gamma_g - 1}{\gamma_g}}}\right]\right\}$			
Physical Constraints	$r_c \geq 1.5$			$r_c \leq 30$			
	$T_2 \geq T_1$	$T_3 \geq T_2$		$T_4 \geq T_2$		$T_4 \geq T_9$	
	$T_3 \leq 1400°C$			$\overset{*}{m}_a \geq \overset{*}{m}_f$			
	$(W_{GT} + Q_{GT}) \leq \overset{*}{m}_f (H_u + h_f)$						
Demand Requirements	$[W_{GT} - W_{BY1} + W_{S1}] = [W_{DEMAND}]_1$			$[Q_{GT} + Q_{AH1}] \geq [Q_{DEMAND}]_1$			
	$[W_{GT} - W_{BY2} + W_{S2}] = [W_{DEMAND}]_2$			$[Q_{GT} + Q_{AH2}] \geq [Q_{DEMAND}]_2$			
	$[W_{GT} - W_{BY3} + W_{S3}] = [W_{DEMAND}]_3$			$[Q_{GT} + Q_{AH3}] \geq [Q_{DEMAND}]_3$			
	$[W_{GT} - W_{BY4} + W_{S4}] = [W_{DEMAND}]_4$			$[Q_{GT} + Q_{AH4}] \geq [Q_{DEMAND}]_4$			
Capacity of the equipments	$W_{GT} \leq 56\text{MW}$		$Q_{AB} \leq 130\text{ MW}$			$Y_{CG} \leq 56\text{ MW}$	
Value of operational variables	$W_{BY1} \leq Y_{CG}$ $W_{S1} \leq Y_{CG}$ $Q_{AH1} \leq Q_{AB}$		$W_{BY2} \leq Y_{CG}$ $W_{S2} \leq Y_{CG}$ $Q_{AH2} \leq Q_{AB}$	$W_{BY3} \leq Y_{CG}$ $W_{S3} \leq Y_{CG}$ $Q_{AH3} \leq Q_{AB}$		$W_{BY4} \leq Y_{CG}$ $W_{S4} \leq Y_{CG}$ $Q_{AH4} \leq Q_{AB}$	

Table 47. Constraints used in the gas turbine pre model

COMPONENTS	DESIGN VARIABLES	NOTE
Air Compressor	$\overset{*}{m}_a, r_c, \eta_{AC}$	Defined in the pre model
Combustion Chamber	$\overset{*}{m}_a, \frac{P_3}{P_2}, T_4$	Defined in the pre model
Turbine	$\overset{*}{m}_g, \mathrm{r_t}, T_4, \eta_{GT}$	Defined in the pre model
Heat Recovery Steam Generator	$\overset{*}{m}_g, \overset{*}{m}_{st}, Q_{PH}, Q_{EV}, Q_{SH}$	Defined in the pre model
Feedwater Heater	Q_{FH}	Defined in the pre model
Auxiliary Boiler	Q_{AB}, η_{AB}	To evaluate in the complete model
Connection to the grid	Y_{CG}	To evaluate in the complete model
Cascade Heat Pump System	Q_{HP}, n_{HP}	To evaluate in the complete model

Table 48. Design variables for the complete gas turbine model

$\frac{n}{n_n\sqrt{\Theta_1}}=1.1$			
$\left[\frac{T_3}{T_1}\right]_{PARTIAL} \Big/ \left[\frac{T_3}{T_1}\right]_{FULL}$	$\frac{\left[\overset{*}{m}_a\right]_{PARTIAL}\sqrt{\Theta_1}}{\left[\overset{*}{m}_a\right]_{FULL}\delta_1}$	$\frac{[r_c-1]_{PARTIAL}}{[r_c-1]_{FULL}}$	$\frac{[\eta_C]_{PARTIAL}}{[\eta_C]_{FULL}}$
1	1,148717949	1,160869565	0,961111111
0,951672862	1,151282051	1,134782609	0,957777778
0,832713755	1,156410256	1,060869565	0,94625
0,691449814	1,161538462	0,966666667	0,92
0,559479554	1,164102564	0,858333333	0,884375
0,453531599	1,166666667	0,765217391	0,85

$\frac{n}{n_n\sqrt{\Theta_1}}=1$			
$\left[\frac{T_3}{T_1}\right]_{PARTIAL} \Big/ \left[\frac{T_3}{T_1}\right]_{FULL}$	$\frac{\left[\overset{*}{m}_a\right]_{PARTIAL}\sqrt{\Theta_1}}{\left[\overset{*}{m}_a\right]_{FULL}\delta_1}$	$\frac{[r_c-1]_{PARTIAL}}{[r_c-1]_{FULL}}$	$\frac{[\eta_C]_{PARTIAL}}{[\eta_C]_{FULL}}$
1	1	1	1
0,951672862	1,002564103	0,975	0,9925
0,832713755	1,012820513	0,925	0,97625
0,691449814	1,025641026	0,845833333	0,957777778
0,559479554	1,030769231	0,756521739	0,936666667
0,453531599	1,035897436	0,673913043	0,905

$\frac{n}{n_n\sqrt{\Theta_1}}=0.95$			
$\left[\frac{T_3}{T_1}\right]_{PARTIAL} \Big/ \left[\frac{T_3}{T_1}\right]_{FULL}$	$\frac{\left[\overset{*}{m}_a\right]_{PARTIAL}\sqrt{\Theta_1}}{\left[\overset{*}{m}_a\right]_{FULL}\delta_1}$	$\frac{[r_c-1]_{PARTIAL}}{[r_c-1]_{FULL}}$	$\frac{[\eta_C]_{PARTIAL}}{[\eta_C]_{FULL}}$
1	0,861538462	0,858333333	0,976363636
0,951672862	0,871794872	0,841666667	0,97952381
0,832713755	0,892307692	0,808333333	0,985
0,691449814	0,915384615	0,756521739	0,9748
0,559479554	0,935897436	0,682608696	0,956363636
0,453531599	0,946153846	0,613043478	0,93

Table 49. Parameterization of the performance gas turbine map

TYPE OF CONSTRAINT	CONSTRAINT				
Integer Variables	n_{HP}	n_{HP1}	n_{HP2}	n_{HP3}	n_{HP4}
Non negativity	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$	$Q_{HP} \geq 0$	$W_{HP} \geq 0$	$n_{HP} \geq 0$
	$\dot{m}_{f1} \geq 0$	$\dot{m}_{f2} \geq 0$	$\dot{m}_{f3} \geq 0$	$\dot{m}_{f4} \geq 0$	
	$n_{HP1} \geq 0$	$n_{HP2} \geq 0$	$n_{HP3} \geq 0$	$n_{HP4} \geq 0$	
	$W_{GT1} \geq 0$	$W_{GT2} \geq 0$	$W_{GT3} \geq 0$	$W_{GT4} \geq 0$	
	$Q_{GT1} \geq 0$ $W_{BY1} \geq 0$ $W_{S1} \geq 0$ $Q_{AH1} \geq 0$	$Q_{GT2} \geq 0$ $W_{BY2} \geq 0$ $W_{S2} \geq 0$ $Q_{AH2} \geq 0$	$Q_{GT3} \geq 0$ $W_{BY3} \geq 0$ $W_{S3} \geq 0$ $Q_{AH3} \geq 0$	$Q_{GT4} \geq 0$ $W_{BY4} \geq 0$ $W_{S4} \geq 0$ $Q_{AH4} \geq 0$	
On Design Parameters (Defined in the pre model)	$\dot{m}_a, \dot{m}_f, r_c, T_2, T_3, T_4, W_{GT}, Q_{GT}, \lambda_{GT}$				
Physical Constraints	$(W_{GT1} + Q_{GT1}) \leq \dot{m}_{f1}(H_u + h_f)$		$(W_{GT2} + Q_{GT2}) \leq \dot{m}_{f2}(H_u + h_f)$		
	$(W_{GT3} + Q_{GT3}) \leq \dot{m}_{f3}(H_u + h_f)$		$(W_{GT4} + Q_{GT4}) \leq \dot{m}_{f4}(H_u + h_f)$		
Demand Requirements	$[W_{GT1} - (n_{HP1} \cdot W_{HP}) - W_{B1} + W_{S1}] = [W_{DEMAND}]_1$				
	$[W_{GT2} - (n_{HP2} \cdot W_{HP}) - W_{B2} + W_{S2}] = [W_{DEMAND}]_2$				
	$[W_{GT3} - (n_{HP3} \cdot W_{HP}) - W_{B3} + W_{S3}] = [W_{DEMAND}]_3$				
	$[W_{GT4} - (n_{HP4} \cdot W_{HP}) - W_{B4} + W_{S4}] = [W_{DEMAND}]_4$				
	$[Q_{GT1} + (n_{HP1} \cdot Q_{HP}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$				
	$[Q_{GT2} + (n_{HP2} \cdot Q_{HP}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$				
	$[Q_{GT3} + (n_{HP3} \cdot Q_{HP}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$				
	$[Q_{GT4} + (n_{HP4} \cdot Q_{HP}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$				
Capacity of the equipments	$2\,\text{kW} \leq Q_{HP} \leq 2\,\text{MW}$	$Q_{AB} \leq 130\,\text{MW}$		$Y_{CG} \leq 56\,\text{MW}$	

Table 50. Constraints for the complete gas turbine model (1st Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Off Design Constraints	$1 \le \left[\frac{T_3}{T_1}\right]_{\text{Period 1}} \le 0.45$		$1 \le \left[\frac{T_3}{T_1}\right]_{\text{Period 2}} \le 0.45$	
	$1 \le \left[\frac{T_3}{T_1}\right]_{\text{Period 3}} \le 0.45$		$1 \le \left[\frac{T_3}{T_1}\right]_{\text{Period 4}} \le 0.45$	
	$1.1 \le \left[\frac{n}{n_n\sqrt{\Theta_1}}\right]_{\text{Period 1}} \le 0.95$		$1.1 \le \left[\frac{n}{n_n\sqrt{\Theta_1}}\right]_{\text{Period 2}} \le 0.95$	
	$1.1 \le \left[\frac{n}{n_n\sqrt{\Theta_1}}\right]_{\text{Period 3}} \le 0.95$		$1.1 \le \left[\frac{n}{n_n\sqrt{\Theta_1}}\right]_{\text{Period 4}} \le 0.95$	
Off Design Constraints	$W_{GT1} = f\left(\overset{*}{m}_{f1}\right)$	$W_{GT2} = f\left(\overset{*}{m}_{f2}\right)$	$W_{GT3} = f\left(\overset{*}{m}_{f3}\right)$	$W_{GT4} = f\left(\overset{*}{m}_{f4}\right)$
	$Q_{GT1} = f\left(\overset{*}{m}_{f1}\right)$	$Q_{GT2} = f\left(\overset{*}{m}_{f2}\right)$	$Q_{GT3} = f\left(\overset{*}{m}_{f3}\right)$	$Q_{GT4} = f\left(\overset{*}{m}_{f4}\right)$
	$\left(\overset{*}{m}_f\right)_{\text{PRODUCING 75\% OF NOMINAL POWER}} \le \overset{*}{m}_{f1} \le \left(\overset{*}{m}_f\right)_{\text{MAXIMAL IN WEATHER 1}}$			
	$\left(\overset{*}{m}_f\right)_{\text{PRODUCING 75\% OF NOMINAL POWER}} \le \overset{*}{m}_{f2} \le \left(\overset{*}{m}_f\right)_{\text{MAXIMAL IN WEATHER 2}}$			
	$\left(\overset{*}{m}_f\right)_{\text{PRODUCING 75\% OF NOMINAL POWER}} \le \overset{*}{m}_{f3} \le \left(\overset{*}{m}_f\right)_{\text{MAXIMAL IN WEATHER 3}}$			
	$\left(\overset{*}{m}_f\right)_{\text{PRODUCING 75\% OF NOMINAL POWER}} \le \overset{*}{m}_{f4} \le \left(\overset{*}{m}_f\right)_{\text{MAXIMAL IN WEATHER 4}}$			
Value of operational variables	$n_{HP1} \le n_{HP}$	$n_{HP2} \le n_{HP}$	$n_{HP3} \le n_{HP}$	$n_{HP4} \le n_{HP}$
	$W_{B1} \le Y_{CG}$ $W_{S1} \le Y_{CG}$ $Q_{AH1} \le Q_{AB}$	$W_{B2} \le Y_{CG}$ $W_{S2} \le Y_{CG}$ $Q_{AH2} \le Q_{AB}$	$W_{B3} \le Y_{CG}$ $W_{S3} \le Y_{CG}$ $Q_{AH3} \le Q_{AB}$	$W_{B4} \le Y_{CG}$ $W_{S4} \le Y_{CG}$ $Q_{AH4} \le Q_{AB}$

Table 51. Constraints for the complete gas turbine model (2nd Part)

	$\frac{\left[\frac{T_3}{T_1}\right]_{PARTIAL}}{\left[\frac{T_3}{T_1}\right]_{FULL}}$	$\frac{\left[\dot{m}_a\right]_{PARTIAL}\sqrt{\Theta_1}}{\left[\dot{m}_a\right]_{FULL}\delta_1}$	$\frac{[r_c-1]_{PARTIAL}}{[r_c-1]_{FULL}}$	$\frac{[\eta_C]_{PARTIAL}}{[\eta_C]_{FULL}}$	T_3 (˚C)	$\dot{m}_a$ (kg/s)	η_c	r_c	η_t	T_4 (˚C)	T_2 (˚C)	$\dot{m}_f$ (kg/s)	$\dot{W}_{GT}$ (KW)	$\dot{Q}_{GT}$ (KW)
Period 1	1,000	1,046	1,050	0,988	1443,026	250,603	0,850	23,843	0,860	781,854	741,772	4,08202	35394,316	126542,889
	0,952	1,048	1,024	0,982	1373,288	251,217	0,844	23,291	0,860	747,064	739,455	3,67474	25777,578	116270,606
	0,833	1,057	0,967	0,967	1201,627	253,295	0,832	22,042	0,860	660,412	734,099	2,69026	3068,956	90816,936
Period 2	1,000	1,003	1,004	0,999	1526,402	233,771	0,859	22,843	0,860	833,263	769,052	4,14463	34911,777	132521,207
	0,952	1,006	0,979	0,992	1452,635	234,368	0,853	22,299	0,860	796,587	767,004	3,73588	25065,549	122409,016
	0,833	1,016	0,928	0,976	1271,056	236,730	0,839	21,198	0,859	704,059	763,036	2,74946	1986,646	97190,364
Period 3	1,000	0,976	0,976	0,996	1560,107	224,971	0,857	22,234	0,860	855,999	781,152	4,11542	33597,351	133697,226
	0,952	0,980	0,952	0,990	1484,711	225,865	0,852	21,721	0,860	818,362	778,398	3,72007	24066,001	123878,617
	0,833	0,992	0,905	0,978	1299,122	228,634	0,841	20,695	0,859	723,223	773,081	2,75672	1613,363	99095,856
Period 4	1,000	1,029	1,032	0,992	1474,957	243,921	0,853	23,450	0,860	801,420	752,265	4,10686	35204,807	128913,031
	0,952	1,032	1,006	0,986	1403,676	244,529	0,848	22,901	0,860	765,912	750,057	3,69897	25495,627	118704,563
	0,833	1,041	0,952	0,970	1228,217	246,721	0,835	21,711	0,860	677,047	745,227	2,71366	2634,016	93347,161

Table 52. Example of a part load table of the gas turbine in a case similar to Detroit City

COMPONENTS	PURCHASE COSTS	COEFFICIENTS	FACTORS OF UPDATE
Micro gas turbine unit	$Z_{MGT} = C_{MGT} W_{MGT}$	$C_{MGT} = \left[1000 \frac{\$}{kW}\right]$	1
Auxiliary Boiler	$Z_{AB} = FU_{AB} C_{61} Q_{AB}^{0.8} \left[1 + \left(\frac{1 - C_{62}}{1 - \eta_{AB}}\right)^{C_{63}}\right]$	$C_{61} = \left[560 \frac{\$}{(kW)^{0.8}}\right]$ $C_{62} = 0.9$ $C_{63} = 7$	$FU_{AB} = 1.28$
Connection to the grid	$Z_{CG} = FU_{CG} C_{71} Y_{CG}^{0.6}$	$C_{71} = \left[1000 \frac{\$}{(kW)^{0.6}}\right]$	$FU_{CG} = 1.28$

Table 53. Equations, coefficients and factors of update for calculating the purchase costs for the components of the micro gas turbine model

COMPONENTS	DESIGN VARIABLES
Micro gas turbine unit	$W_{MGT} \left(\overset{*}{m}_f, \overset{*}{m}_a, r_c, T_1, T_2, T_4, T_5\right)$
Auxiliary Boiler	Q_{AB}, η_{AB}
Connection to the grid	Y_{CG}

Table 54. Design variables for the components of the micro gas turbine model

TYPE OF CONSTRAINT	CONSTRAINT						
Integer Variables	n_{MGT}	n_{MGT1}	n_{MGT2}	n_{MGT3}	n_{MGT4}		
Non negativity	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$	$\overset{*}{m}_a \geq 0$	$\overset{*}{m}_f \geq 0$	$W_{GT} \geq 0$	$Q_{GT} \geq 0$	$\lambda_{GT} \geq 0$
	$Q_{GT1} \geq 0$	$Q_{GT2} \geq 0$	$Q_{GT3} \geq 0$	$Q_{GT4} \geq 0$	$T_1 \geq 0$	$T_2 \geq 0$	$T_3 \geq 0$
	$W_{BY1} \geq 0$	$W_{BY2} \geq 0$	$W_{BY3} \geq 0$	$W_{BY4} \geq 0$	$T_4 \geq 0$	$T_5 \geq 0$	$T_6 \geq 0$
	$Q_{AH1} \geq 0$	$Q_{AH2} \geq 0$	$Q_{AH3} \geq 0$	$Q_{AH4} \geq 0$	$r_c \geq 0$	$r_t \geq 0$	$\eta_{\mathrm{Re}g} \geq 0$
	$n_{MGT1} \geq 0$		$n_{MGT2} \geq 0$		$n_{MGT3} \geq 0$		$n_{MGT4} \geq 0$
Thermodynamics Equations	$\overset{*}{m}_f = \dfrac{\overset{*}{m}_a C_{pa}(T_4 - T_3)}{\eta_{CCH}(H_u + h_f) - (L+1)C_{pg}T_4 + LC_{pa}T_4}$						
	$W_{MGT} = \overset{*}{m}_f (L+1)C_{pg}T_4 + \left(\overset{*}{m}_a - L\overset{*}{m}_f\right)C_{pa}T_4 - \left(\overset{*}{m}_a + \overset{*}{m}_f\right)C_{pg}T_5 - \overset{*}{m}_a C_{pa}(T_2 - T_1)$						
	$Q_{MGT} = \overset{*}{m}_t C_{pg}(T_6 - T_7)$			$T_3 = (1-\eta_{\mathrm{Reg}})T_2 + \eta_{\mathrm{Reg}}T_5\left(\dfrac{C_{pg}}{C_{pa}}\right)$			
	$\lambda_{GT} = \dfrac{Q_{GT}}{W_{GT}}$			$\overset{*}{m}_g + \overset{*}{m}_{nb} = \overset{*}{m}_a + \overset{*}{m}_f = \overset{*}{m}_t$			
	$T_2 = T_1\left\{1 + \dfrac{1}{\eta_c}\left[(r_c)^{\frac{\gamma_a - 1}{\gamma_a}} - 1\right]\right\}$			$T_5 = T_4\left\{1 - \eta_{MGT}\left[1 - \dfrac{1}{(r_t)^{\frac{\gamma_g - 1}{\gamma_g}}}\right]\right\}$			
Physical Constraints	$r_c \geq 1.5$			$r_c \leq 6$			
	$\eta_{\mathrm{Re}g} \geq 0.7$			$\eta_{\mathrm{Re}g} \leq 0.8$			
	$T_6 \geq 300°C$			$T_6 \leq 310°C$			
	$T_4 \leq 1400°C$			$\overset{*}{m}_a \geq \overset{*}{m}_f$			
	$T_2 \geq T_1$			$T_3 \geq T_2$			
	$T_5 \geq T_3$			$T_6 \geq T_2$			
	$(W_{GT} + Q_{GT}) \leq \overset{*}{m}_f (H_u + h_f)$						

Table 55. Constraints used in the micro gas turbine model (1st Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Demand Requirements	$[(n_{MGT1} \cdot W_{MGT}) - W_{B1} + W_{S1}] = [W_{DEMAND}]_1$			
	$[(n_{MGT2} \cdot W_{MGT}) - W_{B2} + W_{S2}] = [W_{DEMAND}]_2$			
	$[(n_{MGT3} \cdot W_{MGT}) - W_{B3} + W_{S3}] = [W_{DEMAND}]_3$			
	$[(n_{MGT4} \cdot W_{MGT}) - W_{B4} + W_{S4}] = [W_{DEMAND}]_4$			
	$[(n_{MGT1} \cdot Q_{MGT}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$			
	$[(n_{MGT2} \cdot Q_{MGT}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$			
	$[(n_{MGT3} \cdot Q_{MGT}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$			
	$[(n_{MGT4} \cdot Q_{MGT}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$			
Capacity of the equipments	$30\text{kW} \leq W_{MGT} \leq 250\text{kW}$			
	$Q_{AB} \leq 12\text{ MW}$		$Y_{CG} \leq 3\text{ MW}$	
Value of operational variables	$n_{MGT1} \leq n_{MGT}$	$n_{MGT2} \leq n_{MGT}$	$n_{MGT3} \leq n_{MGT}$	$n_{MGT4} \leq n_{MGT}$
	$W_{B1} \leq Y_{CG}$ $W_{S1} \leq Y_{CG}$ $Q_{AH1} \leq Q_{AB}$	$W_{B2} \leq Y_{CG}$ $W_{S2} \leq Y_{CG}$ $Q_{AH2} \leq Q_{AB}$	$W_{B3} \leq Y_{CG}$ $W_{S3} \leq Y_{CG}$ $Q_{AH3} \leq Q_{AB}$	$W_{B4} \leq Y_{CG}$ $W_{S4} \leq Y_{CG}$ $Q_{AH4} \leq Q_{AB}$

Table 56. Constraints used in the micro gas turbine model (2nd Part)

COMPONENTS	PURCHASE COSTS
Main Boiler	$Z_{MB} = FU_{MB} \cdot C_{81} \cdot (Q_{MB})^{C_{82}} \cdot \left[1+\left(\frac{1-C_{83}}{1-\eta_{MB}}\right)^{C_{84}}\right] \cdot \left[1+C_{85} \cdot e^{\left(\frac{T_2-C_{86}}{C_{87}}\right)}\right] \cdot \left[e^{\left(\frac{p_2-C_{88}}{C_{89}}\right)}\right]$
Back pressure steam turbine	$Z_{ST} = FU_{ST} \cdot C_{91} \cdot (W_{ST})^{C_{92}} \cdot \left[1+\left(\frac{1-C_{93}}{1-\eta_{ST}}\right)^{C_{94}}\right] \cdot \left[1+C_{95} \cdot e^{\left(\frac{T_2-C_{96}}{C_{97}}\right)}\right]$
Feedwater Heater	$Z_{FH} = FU_{FH} C_{51} Q_{FH}$
Auxiliary Boiler	$Z_{AB} = FU_{AB} C_{61} Q_{AB}^{0.8} \left[1+\left(\frac{1-C_{62}}{1-\eta_{AB}}\right)^{C_{63}}\right]$
Connection to the grid	$Z_{CG} = FU_{CG} C_{71} Y_{CG}^{0.6}$

Table 57. Equations for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	CONSTANTS				
Main Boiler	$FU_{MB} = 1.28$	$C_{81} = 360 \frac{\$}{(kW)^{0.8}}$	$C_{82} = 0.8$	$C_{83} = 0.9$	$C_{84} = 7$
	$C_{85} = 5$	$C_{86} = 866K$	$C_{87} = 10.42K$	$C_{88} = 28bar$	$C_{89} = 150bar$
Back pressure steam turbine	$FU_{ST} = 1.28$	$C_{91} = 3000 \frac{\$}{(kW)^{0.7}}$	$C_{92} = 0.7$	$C_{93} = 0.95$	
	$C_{94} = 3$	$C_{95} = 5$	$C_{96} = 866K$	$C_{97} = 10.42K$	
Feedwater Heater	$FU_{FH} = 1.28$		$C_{51} = 10.4\left(\frac{\$}{kW}\right)$		
Auxiliary Boiler	$FU_{AB} = 1.28$	$C_{61} = \left[560 \frac{\$}{(kW)^{0.8}}\right]$	$C_{62} = 0.9$	$C_{63} = 7$	
Connection to the grid	$FU_{CG} = 1.28$		$C_{71} = \left[1000 \frac{\$}{(kW)^{0.6}}\right]$		

Table 58. Coefficients and factors of update for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	DESIGN VARIABLES
Main Boiler	Q_{MB}
Back pressure steam turbine	W_{ST} , λ_{ST}
Feedwater Heater	Q_{FH}
Auxiliary Boiler	Q_{AB} , η_{AB}
Connection to the grid	Y_{CG}

Table 59. Design variables for the components of the back pressure steam turbine pre-model

TYPE OF CONSTRAINT	CONSTRAINT			
Non negativity	$Q_{MB} \geq 0$	$Q_{FH} \geq 0$	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$
	$\overset{*}{m}_{fst} \geq 0$	$W_{ST} \geq 0$	$Q_{ST} \geq 0$	$\lambda_{ST} \geq 0$
	$W_{BY1} \geq 0$	$W_{BY2} \geq 0$	$W_{BY3} \geq 0$	$W_{BY4} \geq 0$
	$W_{S1} \geq 0$	$W_{S2} \geq 0$	$W_{S3} \geq 0$	$W_{S4} \geq 0$
	$Q_{AH1} \geq 0$	$Q_{AH2} \geq 0$	$Q_{AH3} \geq 0$	$Q_{AH4} \geq 0$
Thermodynamics Equations	$\overset{*}{m}_{fst} = \dfrac{W_{ST} \cdot (1+\lambda_{ST})}{\eta_B H_U}$			
	$Q_{ST} = \lambda_{ST} \cdot W_{ST}$		$Q_{ST} = Q_{FW}$	
Physical Constraints	$(W_{ST} + Q_{ST}) \leq Q_{MB}$		$2.8 \leq \lambda_{ST} \leq 5.4$	
Demand Requirements	$[W_{ST} - W_{BY1} + W_{S1}] = [W_{DEMAND}]_1$		$[(\lambda_{ST} \cdot W_{GT}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$	
	$[W_{ST} - W_{BY2} + W_{S2}] = [W_{DEMAND}]_2$		$[(\lambda_{ST} \cdot W_{GT}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$	
	$[W_{ST} - W_{BY3} + W_{S3}] = [W_{DEMAND}]_3$		$[(\lambda_{ST} \cdot W_{GT}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$	
	$[W_{ST} - W_{BY4} + W_{S4}] = [W_{DEMAND}]_4$		$[(\lambda_{ST} \cdot W_{GT}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$	
Capacity of the equipments	$Q_{MB} \leq 130$ MW		$W_{ST} \leq 56$ MW	
	$Q_{AB} \leq 130$ MW		$Y_{CG} \leq 56$ MW	
Value of operational variables	$W_{BY1} \leq Y_{CG}$ $W_{S1} \leq Y_{CG}$ $Q_{AH1} \leq Q_{AB}$	$W_{BY2} \leq Y_{CG}$ $W_{S2} \leq Y_{CG}$ $Q_{AH2} \leq Q_{AB}$	$W_{BY3} \leq Y_{CG}$ $W_{S3} \leq Y_{CG}$ $Q_{AH3} \leq Q_{AB}$	$W_{BY4} \leq Y_{CG}$ $W_{S4} \leq Y_{CG}$ $Q_{AH4} \leq Q_{AB}$

Table 60. Constraints used in the pre model of the back pressure steam turbine

COMPONENTS	PURCHASE COSTS
Main Boiler	$Z_{MB} = FU_{MB} \cdot C_{81} \cdot (Q_{MB})^{C_{82}} \cdot \left[1+\left(\frac{1-C_{83}}{1-\eta_{MB}}\right)^{C_{84}}\right] \cdot \left[1+C_{85} \cdot e^{\left(\frac{T_2-C_{86}}{C_{87}}\right)}\right] \cdot \left[e^{\left(\frac{p_2-C_{88}}{C_{89}}\right)}\right]$
Back pressure steam turbine	$Z_{ST} = FU_{ST} \cdot C_{91} \cdot (W_{ST})^{C_{92}} \cdot \left[1+\left(\frac{1-C_{93}}{1-\eta_{ST}}\right)^{C_{94}}\right] \cdot \left[1+C_{95} \cdot e^{\left(\frac{T_2-C_{96}}{C_{97}}\right)}\right]$
Feedwater Heater	$Z_{FH} = FU_{FH} C_{51} Q_{FH}$
Auxiliary Boiler	$Z_{AB} = FU_{AB} C_{61} Q_{AB}^{0.8} \left[1+\left(\frac{1-C_{62}}{1-\eta_{AB}}\right)^{C_{63}}\right]$
Connection to the grid	$Z_{CG} = FU_{CG} C_{71} Y_{CG}^{0.6}$
Cascade Heat Pump System	$Z_{HP} = C_{HP} Q_{HP}$

Table 61. Equations for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	CONSTANTS				
Main Boiler	$FU_{MB} = 1.28$	$C_{81} = 360 \frac{\$}{(kW)^{0.8}}$	$C_{82} = 0.8$	$C_{83} = 0.9$	$C_{84} = 7$
	$C_{85} = 5$	$C_{86} = 866K$	$C_{87} = 10.42K$	$C_{88} = 28bar$	$C_{89} = 150bar$
Back pressure steam turbine	$FU_{ST} = 1.28$	$C_{91} = 3000 \frac{\$}{(kW)^{0.7}}$	$C_{92} = 0.7$	$C_{93} = 0.95$	
	$C_{94} = 3$	$C_{95} = 5$	$C_{96} = 866K$	$C_{97} = 10.42K$	
Feedwater Heater	$FU_{FH} = 1.28$		$C_{51} = 10.4\left(\frac{\$}{kW}\right)$		
Auxiliary Boiler	$FU_{AB} = 1.28$	$C_{61} = \left[560 \frac{\$}{(kW)^{0.8}}\right]$	$C_{62} = 0.9$	$C_{63} = 7$	
Connection to the grid	$FU_{CG} = 1.28$		$C_{71} = \left[1000 \frac{\$}{(kW)^{0.6}}\right]$		
Cascade Heat Pump System	$C_{HP} = 516 \frac{\$}{(kW)_{TH}}$				

Table 62. Coefficients and factors of update for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	DESIGN VARIABLES	NOTE
Main Boiler	Q_{MB}	To evaluate in the complete model
Back pressure steam turbine	λ_{ST}	Defined in the pre model
	W_{ST}	To evaluate in the complete model
Feedwater Heater	Q_{FH}	To evaluate in the complete model
Auxiliary Boiler	Q_{AB}, η_{AB}	To evaluate in the complete model
Connection to the grid	Y_{CG}	To evaluate in the complete model
Cascade Heat Pump System	Q_{HP}, n_{HP}	To evaluate in the complete model

Table 63. Design variables for the complete gas turbine model

COMPONENTS	OPERATIONAL VARIABLES	DESCRIPTION
Back pressure steam turbine system	$W_{ST1}, W_{ST2}, W_{ST3}, W_{ST4}$	Power generation of the steam turbine in each period at partial load
	$\eta_{ST1}, \eta_{ST2}, \eta_{ST3}, \eta_{ST4}$	Isentropic efficiency of the steam turbine in each period at partial load
Connection to the grid	$W_{BY1}, W_{BY2}, W_{BY3}, W_{BY4}$	Power bought by the system in each period
	$W_{S1}, W_{S2}, W_{S3}, W_{S4}$	Power sold by the system in each period
Auxiliary Boiler	$Q_{AH1}, Q_{AH2}, Q_{AH3}, Q_{AH4}$	Heat generated by the auxiliary boiler
Cascade Heat Pump System	$n_{HP1}, n_{HP2}, n_{HP3}, n_{HP4}$	The number of cascade heat pump system in service (These variables are indirectly related with the power generation of steam turbine in each period, although they do not appear in the operation cost function)

Table 64. Operational variables for the complete model of the back pressure steam turbine system

TYPE OF CONSTRAINT	CONSTRAINT				
Integer Variables	n_{HP}	n_{HP1}	n_{HP2}	n_{HP3}	n_{HP4}
Non negativity	$Q_{MB} \geq 0$	$Q_{FH} \geq 0$	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$	
	$W_{ST} \geq 0$	$Q_{HP} \geq 0$	$W_{HP} \geq 0$	$n_{HP} \geq 0$	
	$\eta_{ST1} \geq 0$	$\eta_{ST2} \geq 0$	$\eta_{ST3} \geq 0$	$\eta_{ST4} \geq 0$	
	$\lambda_{ST1} \geq 0$	$\lambda_{ST2} \geq 0$	$\lambda_{ST3} \geq 0$	$\lambda_{ST4} \geq 0$	
	$n_{HP1} \geq 0$	$n_{HP2} \geq 0$	$n_{HP3} \geq 0$	$n_{HP4} \geq 0$	
	$W_{ST1} \geq 0$ $W_{BY1} \geq 0$ $W_{S1} \geq 0$ $Q_{AH1} \geq 0$	$W_{ST2} \geq 0$ $W_{BY2} \geq 0$ $W_{S2} \geq 0$ $Q_{AH2} \geq 0$	$W_{ST3} \geq 0$ $W_{BY3} \geq 0$ $W_{S3} \geq 0$ $Q_{AH3} \geq 0$	$W_{ST4} \geq 0$ $W_{BY4} \geq 0$ $W_{S4} \geq 0$ $Q_{AH4} \geq 0$	
On Design Parameters	$[\lambda_{ST}]_{full}, [\eta_{ST}]_{full}, h_1, h_2, h_{3s}, h_{3'}, h_4$				
Physical Constraints	$W_{ST1} \cdot (1+\lambda_{ST1}) \leq Q_{MB}$	$W_{ST2} \cdot (1+\lambda_{ST2}) \leq Q_{MB}$			
	$W_{ST3} \cdot (1+\lambda_{ST3}) \leq Q_{MB}$	$W_{ST4} \cdot (1+\lambda_{ST4}) \leq Q_{MB}$			
	$h_2 - [\eta_{ST1} \cdot (h_2 - h_{3'})] \leq h_{3s}$	$h_2 - [\eta_{ST2} \cdot (h_2 - h_{3'})] \leq h_{3s}$			
	$h_2 - [\eta_{ST3} \cdot (h_2 - h_{3'})] \leq h_{3s}$	$h_2 - [\eta_{ST4} \cdot (h_2 - h_{3'})] \leq h_{3s}$			
Demand Requirements	$[W_{ST1} - (n_{HP1} \cdot W_{HP}) - W_{BY1} + W_{S1}] = [W_{DEMAND}]_1$				
	$[W_{ST2} - (n_{HP2} \cdot W_{HP}) - W_{BY2} + W_{S2}] = [W_{DEMAND}]_2$				
	$[W_{ST3} - (n_{HP3} \cdot W_{HP}) - W_{BY3} + W_{S3}] = [W_{DEMAND}]_3$				
	$[W_{ST4} - (n_{HP4} \cdot W_{HP}) - W_{BY4} + W_{S4}] = [W_{DEMAND}]_4$				
	$[(\lambda_{ST1} \cdot W_{ST1}) + (n_{HP1} \cdot Q_{HP}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$				
	$[(\lambda_{ST2} \cdot W_{ST2}) + (n_{HP2} \cdot Q_{HP}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$				
	$[(\lambda_{ST3} \cdot W_{ST3}) + (n_{HP3} \cdot Q_{HP}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$				
	$[(\lambda_{ST4} \cdot W_{ST4}) + (n_{HP4} \cdot Q_{HP}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$				
Capacity of the equipments	$Q_{MB} \leq 130$ MW	$W_{ST} \leq 56$ MW			
	$2\text{ kW} \leq Q_{HP} \leq 2$ MW	$Q_{AB} \leq 130$ MW	$Y_{CG} \leq 56$ MW		

Table 65. Constraints for the complete model of the back pressure steam turbine system (1st Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Off Design Constraints	$\dot{m}^{*}_{fst1} = \left(\frac{W_{ST1}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST1}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$		$\dot{m}^{*}_{fst2} = \left(\frac{W_{ST2}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST2}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$	
	$\dot{m}^{*}_{fst3} = \left(\frac{W_{ST3}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST3}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$		$\dot{m}^{*}_{fst4} = \left(\frac{W_{ST4}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST4}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$	
	$\eta_{ST1} = \frac{J \cdot W_{ST1}}{W_{ST} + (J \cdot W_{ST1})}$		$\eta_{ST2} = \frac{J \cdot W_{ST2}}{W_{ST} + (J \cdot W_{ST2})}$	
	$\eta_{ST3} = \frac{J \cdot W_{ST3}}{W_{ST} + (J \cdot W_{ST3})}$		$\eta_{ST4} = \frac{J \cdot W_{ST4}}{W_{ST} + (J \cdot W_{ST4})}$	
	$\lambda_{ST1} = \frac{h_2 - h_4}{\eta_{ST1} \cdot (h_2 - h_{3'})} - 1$		$\lambda_{ST2} = \frac{h_2 - h_4}{\eta_{ST2} \cdot (h_2 - h_{3'})} - 1$	
	$\lambda_{ST3} = \frac{h_2 - h_4}{\eta_{ST3} \cdot (h_2 - h_{3'})} - 1$		$\lambda_{ST4} = \frac{h_2 - h_4}{\eta_{ST4} \cdot (h_2 - h_{3'})} - 1$	
	$0.75 \cdot W_{ST} \le W_{ST1}$		$0.75 \cdot W_{ST} \le W_{ST2}$	
	$0.75 \cdot W_{ST} \le W_{ST3}$		$0.75 \cdot W_{ST} \le W_{ST4}$	
Value of operational variables	$W_{ST1} \le W_{ST}$	$W_{ST2} \le W_{ST}$	$W_{ST3} \le W_{ST}$	$W_{ST4} \le W_{ST}$
	$\eta_{ST1} \le [\eta_{ST}]_{full}$	$\eta_{ST2} \le [\eta_{ST}]_{full}$	$\eta_{ST3} \le [\eta_{ST}]_{full}$	$\eta_{ST4} \le [\eta_{ST}]_{full}$
	$n_{HP1} \le n_{HP}$	$n_{HP2} \le n_{HP}$	$n_{HP3} \le n_{HP}$	$n_{HP4} \le n_{HP}$
	$W_{BY1} \le Y_{CG}$ $W_{S1} \le Y_{CG}$ $Q_{AH1} \le Q_{AB}$	$W_{BY2} \le Y_{CG}$ $W_{S2} \le Y_{CG}$ $Q_{AH2} \le Q_{AB}$	$W_{BY3} \le Y_{CG}$ $W_{S3} \le Y_{CG}$ $Q_{AH3} \le Q_{AB}$	$W_{BY4} \le Y_{CG}$ $W_{S4} \le Y_{CG}$ $Q_{AH4} \le Q_{AB}$

Table 66. Constraints for the complete model of the back pressure steam turbine system (2nd Part)

OBJECTIVE FUNCTION (MINIMISE)	Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O)
DESIGN COSTS (Φ_D)	$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{MB} + Z_{ST} + Z_{FH} + Z_{AB} + Z_{CG} + \left(n_{AC} \cdot Z_{AC}\right) + \left(n_{CC} \cdot Z_{CC}\right)\right]$
OPERATIONAL COSTS (Φ_O)	$\Phi_O = \left[\left(C_f \overset{*}{m}_f\right)(8000 \text{ hours})\right] + \sum_{n=1}^{4}\left[C_{BY}W_{BY} + C_S W_S + C_{AH}Q_{AH} + \left(C_{CTA} \cdot n_{AC} \cdot Y_{AC}\right) + \left(C_{CTC} \cdot n_{CC} \cdot Y_{CC}\right)\right]\Delta t_n$

Table 67. Objective function of the pre model of the trigeneration system

OBJECTIVE FUNCTION (MINIMIZE)	Total annualised costs (Φ_T) = Design costs (Φ_D) + Operational costs (Φ_O)
DESIGN COSTS (Φ_D)	$\Phi_D = \varphi \cdot \psi \cdot \left[Z_{MB} + Z_{ST} + Z_{FH} + Z_{AB} + Z_{CG} + \left(n_{AC} \cdot Z_{AC}\right) + \left(n_{CC} \cdot Z_{CC}\right)\right]$
OPERATIONAL COSTS (Φ_O)	$\Phi_O = \sum_{n=1}^{4}\left[C_f \overset{*}{m}_f + C_B W_B + C_S W_S + C_{AH}Q_{AH} + \left(C_{CTA} \cdot n_{AC} \cdot Y_{AC}\right) + \left(C_{CTC} \cdot n_{CC} \cdot Y_{CC}\right)\right]\Delta t_n$

Table 68. Objective function of the complete model of the trigeneration system

COMPONENTS	PURCHASE COSTS
Main Boiler	$Z_{MB} = FU_{MB} \cdot C_{81} \cdot (Q_{MB})^{C_{82}} \cdot \left[1 + \left(\frac{1 - C_{83}}{1 - \eta_{MB}}\right)^{C_{84}}\right] \cdot \left[1 + C_{85} \cdot e^{\left(\frac{T_2 - C_{86}}{C_{87}}\right)}\right] \cdot \left[e^{\left(\frac{p_2 - C_{88}}{C_{89}}\right)}\right]$
Back pressure steam turbine	$Z_{ST} = FU_{ST} \cdot C_{91} \cdot (W_{ST})^{C_{92}} \cdot \left[1 + \left(\frac{1 - C_{93}}{1 - \eta_{ST}}\right)^{C_{94}}\right] \cdot \left[1 + C_{95} \cdot e^{\left(\frac{T_2 - C_{96}}{C_{97}}\right)}\right]$
Feedwater Heater	$Z_{FH} = FU_{FH} C_{51} Q_{FH}$
Auxiliary Boiler	$Z_{AB} = FU_{AB} C_{61} Q_{AB}^{0.8} \left[1 + \left(\frac{1 - C_{62}}{1 - \eta_{AB}}\right)^{C_{63}}\right]$
Absorption Chillers	$Z_{AC} = \left(C_{AC1} \cdot Y_{AC} + C_{AC2} \cdot Y_{AC}{}^{C_{AC3}} + C_{CT1} \cdot Y_{CT1}{}^{C_{CT2}}\right)$
Compression Chillers	$Z_{CC} = \left(C_{CC1} \cdot Y_{CC} + C_{CC2} \cdot Y_{CC}{}^{C_{CC3}} + C_{CT1} \cdot Y_{CT2}{}^{C_{CT2}}\right)$
Connection to the grid	$Z_{CG} = FU_{CG} C_{71} Y_{CG}^{0.6}$

Table 69. Equations for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	CONSTANTS				
Main Boiler	$FU_{MB} = 1.28$	$C_{81} = 360 \frac{\$}{(kW)^{0.8}}$	$C_{82} = 0.8$	$C_{83} = 0.9$	$C_{84} = 7$
	$C_{85} = 5$	$C_{86} = 866K$	$C_{87} = 10.42K$	$C_{88} = 28bar$	$C_{89} = 150bar$
Back pressure steam turbine	$FU_{ST} = 1.28$	$C_{91} = 3000 \frac{\$}{(kW)^{0.7}}$	$C_{92} = 0.7$	$C_{93} = 0.95$	
	$C_{94} = 3$	$C_{95} = 5$	$C_{96} = 866K$	$C_{97} = 10.42K$	
Feedwater Heater	$FU_{FH} = 1.28$		$C_{51} = 10.4 \left(\frac{\$}{kW} \right)$		
Auxiliary Boiler	$FU_{AB} = 1.28$	$C_{61} = \left[560 \frac{\$}{(kW)^{0.8}} \right]$	$C_{62} = 0.9$	$C_{63} = 7$	
Absorption Chillers	$C_{AC1} = 23.0179(\$/kW)$	$C_{AC2} = 104.81 \left[\frac{\$}{(kW)^{-0.32}} \right]$	$C_{AC3} = -0.32$		
Cooling Tower	$C_{CT1} = 6.9651 \left[\frac{\$}{(kW)^{-0.33}} \right]$		$C_{CT2} = -0.33$		
Compression Chillers	$C_{CC1} = 20.8077(\$/kW)$	$C_{CC2} = 62.62 \left[\frac{\$}{(kW)^{-0.36}} \right]$	$C_{CC3} = -0.36$		
Connection to the grid	$FU_{CG} = 1.28$		$C_{71} = \left[1000 \frac{\$}{(kW)^{0.6}} \right]$		

Table 70. Coefficients and factors of update for calculating the purchase costs for the components of the back pressure steam turbine pre model

COMPONENTS	PRE MODEL	COMPLETE MODEL
Main Boiler	Q_{MB}	Q_{MB}
Back Pr. steam Turbine	W_{ST} , λ_{ST}	W_{ST} (λ_{ST} is defined in the pre model)
Feedwater Heater	Q_{FH}	Q_{FH}
Auxiliary Boiler	Q_{AB} , η_{AB}	Q_{AB} , η_{AB}
Absorption Chillers	Y_{AC} , n_{AC}	Y_{AC} , n_{AC}
Compression Chillers	Y_{CC} , n_{CC}	Y_{CC} , n_{CC}
Cooling Towers	Y_{CT1}, Y_{CT2}	Y_{CT1}, Y_{CT2}
Connection to the grid	Y_{CG}	Y_{CG}

Table 71. Design variables for the pre model and the complete model of the trigeneration system

COMPONENTS	PRE MODEL	COMPLETE MODEL	DESCRIPTION
Back pressure steam turbine system	W_{ST} (Power generation of the steam turbine in all the periods at full load)	$W_{ST1}, W_{ST2}, W_{ST3}, W_{ST4}$	Power generation of the steam turbine in each period at partial load
		$\eta_{ST1}, \eta_{ST2}, \eta_{ST3}, \eta_{ST4}$	Isentropic efficiency of the steam turbine in each period at partial load
Feedwater Heater	$Q_{FW1}, Q_{FW2}, Q_{FW3}, Q_{FW4}$	$Q_{FW1}, Q_{FW2}, Q_{FW3}, Q_{FW4}$	Heat generated by the feedwater heater in each period
Connection to the grid	$W_{BY1}, W_{BY2}, W_{BY3}, W_{BY4}$	$W_{BY1}, W_{BY2}, W_{BY3}, W_{BY4}$	Power bought by the system in each period
	$W_{S1}, W_{S2}, W_{S3}, W_{S4}$	$W_{S1}, W_{S2}, W_{S3}, W_{S4}$	Power sold by the system in each period
Auxiliary Boiler	$Q_{AH1}, Q_{AH2}, Q_{AH3}, Q_{AH4}$	$Q_{AH1}, Q_{AH2}, Q_{AH3}, Q_{AH4}$	Heat generated by the auxiliary boiler
Absorption Chillers	$n_{AC1}, n_{AC2}, n_{AC3}, n_{AC4}$	$n_{AC1}, n_{AC2}, n_{AC3}, n_{AC4}$	The number of absorption chillers in service in each period of time
Compression Chillers	$n_{CC1}, n_{CC2}, n_{CC3}, n_{CC4}$	$n_{CC1}, n_{CC2}, n_{CC3}, n_{CC4}$	The number of compression chillers in service in each period of time

Table 72. Operational variables for the pre model and the complete model of the trigeneration system

	C_F (\$/kWh)		C_{BY} (\$/kWh)	C_S (\$/kWh)	C_{AH} (\$/kWh) (η_{AB} =0.8, Fuel Oil)	C_{CTA} (\$/kWh)	C_{CTC} (\$/kWh)
	Natural Gas	**Fuel Oil**					
Period 1	0,0283349	0,020934	0,1002562	0,0802049	0,026168	0.0220	0.0059
Period 2			0,1005528	0,0804422			
Period 3			0,1041122	0,0832897			
Period 4			0,0990697	0,079255			

Table 73. Coefficients of the operation cost function for the pre model and the complete model of the trigeneration system

TYPE OF CONSTRAINT	CONSTRAINT			
Non negativity	$Q_{MB} \geq 0$	$Q_{FH} \geq 0$	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$
	$\overset{*}{m}_{fst} \geq 0$	$W_{ST} \geq 0$	$Q_{ST} \geq 0$	$\lambda_{ST} \geq 0$
	$W_{BY1} \geq 0$	$W_{BY2} \geq 0$	$W_{BY3} \geq 0$	$W_{BY4} \geq 0$
	$W_{S1} \geq 0$	$W_{S2} \geq 0$	$W_{S3} \geq 0$	$W_{S4} \geq 0$
	$Q_{AH1} \geq 0$	$Q_{AH2} \geq 0$	$Q_{AH3} \geq 0$	$Q_{AH4} \geq 0$
	$Y_{AC} \geq 0$	$Y_{CC} \geq 0$	$n_{AC} \geq 0$	$n_{CC} \geq 0$
	$n_{AC1} \geq 0$	$n_{AC2} \geq 0$	$n_{AC3} \geq 0$	$n_{AC4} \geq 0$
	$n_{CC1} \geq 0$	$n_{CC2} \geq 0$	$n_{CC3} \geq 0$	$n_{CC4} \geq 0$
Thermodynamics Equations	$COP_{AC} = \frac{Y_{AC}}{Q_{AC}} = 0.68$		$COP_{CC} = \frac{Y_{CC}}{W_{CC}} = 4.5$	
	$Y_{CT1} = Y_{AC} \cdot \left[1 + \left(\frac{1}{COP_{AC}}\right)\right]$		$Y_{CT2} = Y_{CC} \cdot \left[1 + \left(\frac{1}{COP_{CC}}\right)\right]$	
	$\overset{*}{m}_{fst} = \frac{W_{ST} \cdot (1 + \lambda_{ST})}{\eta_B H_U}$		$Q_{ST} = \lambda_{ST} \cdot W_{ST}$	
	$Q_{ST} = Q_{FW1} + (n_{AC1} \cdot Q_{AC})$		$Q_{ST} = Q_{FW2} + (n_{AC2} \cdot Q_{AC})$	
	$Q_{ST} = Q_{FW3} + (n_{AC3} \cdot Q_{AC})$		$Q_{ST} = Q_{FW4} + (n_{AC4} \cdot Q_{AC})$	
Physical Constraints	$(W_{ST} + Q_{ST}) \leq Q_{MB}$		$2.8 \leq \lambda_{ST} \leq 5.4$	
Power Demand Requirements	$[W_{ST} - (n_{AC1} \cdot W_{AC}) - (n_{CC1} \cdot W_{CC}) - W_{BY1} + W_{S1}] = [W_{DEMAND}]_1$			
	$[W_{ST} - (n_{AC2} \cdot W_{AC}) - (n_{CC2} \cdot W_{CC}) - W_{BY2} + W_{S2}] = [W_{DEMAND}]_2$			
	$[W_{ST} - (n_{AC3} \cdot W_{AC}) - (n_{CC3} \cdot W_{CC}) - W_{BY3} + W_{S3}] = [W_{DEMAND}]_3$			
	$[W_{ST} - (n_{AC4} \cdot W_{AC}) - (n_{CC4} \cdot W_{CC}) - W_{BY4} + W_{S4}] = [W_{DEMAND}]_4$			

Table 74. Constraints used in the pre model of the trigeneration system (1st Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Heat Demand Requirements	$[(\lambda_{ST} \cdot W_{ST}) - (n_{AC1} \cdot Q_{AC}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$			
	$[(\lambda_{ST} \cdot W_{ST}) - (n_{AC2} \cdot Q_{AC}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$			
	$[(\lambda_{ST} \cdot W_{ST}) - (n_{AC3} \cdot Q_{AC}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$			
	$[(\lambda_{ST} \cdot W_{ST}) - (n_{AC4} \cdot Q_{AC}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$			
Cooling Demand Requirements	$[(n_{AC1} \cdot Y_{AC}) + (n_{CC1} \cdot Y_{CC})] = [O_{DEMAND}]_1$			
	$[(n_{AC2} \cdot Y_{AC}) + (n_{CC2} \cdot Y_{CC})] = [O_{DEMAND}]_2$			
	$[(n_{AC3} \cdot Y_{AC}) + (n_{CC3} \cdot Y_{CC})] = [O_{DEMAND}]_3$			
	$[(n_{AC4} \cdot Y_{AC}) + (n_{CC4} \cdot Y_{CC})] = [O_{DEMAND}]_4$			
Integer Variables	$n_{AC1}, n_{AC2}, n_{AC3}, n_{AC4}$			
	$n_{CC1}, n_{CC2}, n_{CC3}, n_{CC4}$			
Capacity of the equipments	$Q_{MB} \leq 130\,\text{MW}$		$W_{ST} \leq 56\,\text{MW}$	
	$Q_{AB} \leq 130\,\text{MW}$		$Y_{CG} \leq 56\,\text{MW}$	
	$300\,\text{kW} \leq Y_{AC} \leq 4000\,\text{kW}$		$300\ \text{kW} \leq Y_{CC} \leq 4000\ \text{kW}$	
Value of operational variables	$W_{BY1} \leq Y_{CG}$	$W_{BY2} \leq Y_{CG}$	$W_{BY3} \leq Y_{CG}$	$W_{BY4} \leq Y_{CG}$
	$W_{S1} \leq Y_{CG}$	$W_{S2} \leq Y_{CG}$	$W_{S3} \leq Y_{CG}$	$W_{S4} \leq Y_{CG}$
	$Q_{AH1} \leq Q_{AB}$ $n_{AC1} \leq n_{AC}$ $n_{CC1} \leq n_{CC}$	$Q_{AH2} \leq Q_{AB}$ $n_{AC2} \leq n_{AC}$ $n_{CC2} \leq n_{CC}$	$Q_{AH3} \leq Q_{AB}$ $n_{AC3} \leq n_{AC}$ $n_{CC3} \leq n_{CC}$	$Q_{AH4} \leq Q_{AB}$ $n_{AC4} \leq n_{AC}$ $n_{CC4} \leq n_{CC}$

Table 75. Constraints used in the pre model of the trigeneration system (2nd Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Non negativity	$Q_{MB} \geq 0$	$Q_{FH} \geq 0$	$Y_{CG} \geq 0$	$Q_{AB} \geq 0$
	$\overset{*}{m}_{fst} \geq 0$	$W_{ST} \geq 0$	$Q_{ST} \geq 0$	$\lambda_{ST} \geq 0$
	$W_{ST1} \geq 0$	$W_{ST2} \geq 0$	$W_{ST3} \geq 0$	$W_{ST4} \geq 0$
	$Q_{FW1} \geq 0$	$Q_{FW2} \geq 0$	$Q_{FW3} \geq 0$	$Q_{FW4} \geq 0$
	$\eta_{ST1} \geq 0$	$\eta_{ST2} \geq 0$	$\eta_{ST3} \geq 0$	$\eta_{ST4} \geq 0$
	$\lambda_{ST1} \geq 0$	$\lambda_{ST2} \geq 0$	$\lambda_{ST3} \geq 0$	$\lambda_{ST4} \geq 0$
	$W_{BY1} \geq 0$	$W_{BY2} \geq 0$	$W_{BY3} \geq 0$	$W_{BY4} \geq 0$
	$W_{S1} \geq 0$	$W_{S2} \geq 0$	$W_{S3} \geq 0$	$W_{S4} \geq 0$
	$Q_{AH1} \geq 0$	$Q_{AH2} \geq 0$	$Q_{AH3} \geq 0$	$Q_{AH4} \geq 0$
	$Y_{AC} \geq 0$	$Y_{CC} \geq 0$	$n_{AC} \geq 0$	$n_{CC} \geq 0$
	$n_{AC1} \geq 0$	$n_{AC2} \geq 0$	$n_{AC3} \geq 0$	$n_{AC4} \geq 0$
	$n_{CC1} \geq 0$	$n_{CC2} \geq 0$	$n_{CC3} \geq 0$	$n_{CC4} \geq 0$
	$Q_{ST1} \geq 0$	$Q_{ST2} \geq 0$	$Q_{ST3} \geq 0$	$Q_{ST4} \geq 0$
On Design Parameters	$[\lambda_{ST}]_{full}, [\eta_{ST}]_{full}, h_1, h_2, h_{3s}, h_{3'}, h_4$			
Thermodynamics Equations	$COP_{AC} = \frac{Y_{AC}}{Q_{AC}} = 0.68$		$COP_{CC} = \frac{Y_{CC}}{W_{CC}} = 4.5$	
	$Y_{CT1} = Y_{AC} \cdot \left[1 + \left(\frac{1}{COP_{AC}}\right)\right]$		$Y_{CT2} = Y_{CC} \cdot \left[1 + \left(\frac{1}{COP_{CC}}\right)\right]$	
	$Q_{ST1} = \lambda_{ST1} \cdot W_{ST1}$		$Q_{ST2} = \lambda_{ST2} \cdot W_{ST2}$	
	$Q_{ST3} = \lambda_{ST3} \cdot W_{ST3}$		$Q_{ST4} = \lambda_{ST4} \cdot W_{ST4}$	
	$Q_{ST} = Q_{FW1} + (n_{AC1} \cdot Q_{AC})$		$Q_{ST} = Q_{FW2} + (n_{AC2} \cdot Q_{AC})$	
	$Q_{ST} = Q_{FW3} + (n_{AC3} \cdot Q_{AC})$		$Q_{ST} = Q_{FW4} + (n_{AC4} \cdot Q_{AC})$	
Physical Constraints	$W_{ST1} \cdot (1 + \lambda_{ST1}) \leq Q_{MB}$		$W_{ST2} \cdot (1 + \lambda_{ST2}) \leq Q_{MB}$	
	$W_{ST3} \cdot (1 + \lambda_{ST3}) \leq Q_{MB}$		$W_{ST4} \cdot (1 + \lambda_{ST4}) \leq Q_{MB}$	
	$h_2 - [\eta_{ST1} \cdot (h_2 - h_{3'})] \leq h_{3s}$		$h_2 - [\eta_{ST2} \cdot (h_2 - h_{3'})] \leq h_{3s}$	
	$h_2 - [\eta_{ST3} \cdot (h_2 - h_{3'})] \leq h_{3s}$		$h_2 - [\eta_{ST4} \cdot (h_2 - h_{3'})] \leq h_{3s}$	

Table 76. Constraints used in the complete model of the trigeneration system (1st Part)

TYPE OF CONSTRAINT	CONSTRAINT	
Power Demand Requirements	$[W_{ST} - (n_{AC1} \cdot W_{AC}) - (n_{CC1} \cdot W_{CC}) - W_{BY1} + W_{S1}] = [W_{DEMAND}]_1$	
	$[W_{ST} - (n_{AC2} \cdot W_{AC}) - (n_{CC2} \cdot W_{CC}) - W_{BY2} + W_{S2}] = [W_{DEMAND}]_2$	
	$[W_{ST} - (n_{AC3} \cdot W_{AC}) - (n_{CC3} \cdot W_{CC}) - W_{BY3} + W_{S3}] = [W_{DEMAND}]_3$	
	$[W_{ST} - (n_{AC4} \cdot W_{AC}) - (n_{CC4} \cdot W_{CC}) - W_{BY4} + W_{S4}] = [W_{DEMAND}]_4$	
Heat Demand Requirements	$[(\lambda_{ST1} \cdot W_{ST1}) - (n_{AC1} \cdot Q_{AC}) + Q_{AH1}] \geq [Q_{DEMAND}]_1$	
	$[(\lambda_{ST2} \cdot W_{ST2}) - (n_{AC2} \cdot Q_{AC}) + Q_{AH2}] \geq [Q_{DEMAND}]_2$	
	$[(\lambda_{ST3} \cdot W_{ST3}) - (n_{AC3} \cdot Q_{AC}) + Q_{AH3}] \geq [Q_{DEMAND}]_3$	
	$[(\lambda_{ST4} \cdot W_{ST4}) - (n_{AC4} \cdot Q_{AC}) + Q_{AH4}] \geq [Q_{DEMAND}]_4$	
Cooling Demand Requirements	$[(n_{AC1} \cdot Y_{AC}) + (n_{CC1} \cdot Y_{CC})] = [O_{DEMAND}]_1$	
	$[(n_{AC2} \cdot Y_{AC}) + (n_{CC2} \cdot Y_{CC})] = [O_{DEMAND}]_2$	
	$[(n_{AC3} \cdot Y_{AC}) + (n_{CC3} \cdot Y_{CC})] = [O_{DEMAND}]_3$	
	$[(n_{AC4} \cdot Y_{AC}) + (n_{CC4} \cdot Y_{CC})] = [O_{DEMAND}]_4$	
Integer Variables	$n_{AC1}, n_{AC2}, n_{AC3}, n_{AC4}$	
	$n_{CC1}, n_{CC2}, n_{CC3}, n_{CC4}$	
Capacity of the equipments	$Q_{MB} \leq 130\,\text{MW}$	$W_{ST} \leq 56\,\text{MW}$
	$Q_{AB} \leq 130\,\text{MW}$	$Y_{CG} \leq 56\,\text{MW}$
	$300\,\text{kW} \leq Y_{AC} \leq 4000\,\text{kW}$	$300\ \text{kW} \leq Y_{CC} \leq 4000\ \text{kW}$

Table 77. Constraints used in the complete model of the trigeneration system (2nd Part)

TYPE OF CONSTRAINT	CONSTRAINT			
Off Design Constraints	$\overset{*}{m}_{fst1} = \left(\frac{W_{ST1}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST1}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$		$\overset{*}{m}_{fst2} = \left(\frac{W_{ST2}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST2}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$	
	$\overset{*}{m}_{fst3} = \left(\frac{W_{ST3}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST3}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$		$\overset{*}{m}_{fst4} = \left(\frac{W_{ST4}}{\eta_B H_U}\right)\left(\frac{1}{\eta_{ST4}}\right)\left(\frac{h_2 - h_1}{h_2 - h_{3'}}\right)$	
	$\eta_{ST1} = \frac{J \cdot W_{ST1}}{W_{ST} + (J \cdot W_{ST1})}$		$\eta_{ST2} = \frac{J \cdot W_{ST2}}{W_{ST} + (J \cdot W_{ST2})}$	
	$\eta_{ST3} = \frac{J \cdot W_{ST3}}{W_{ST} + (J \cdot W_{ST3})}$		$\eta_{ST4} = \frac{J \cdot W_{ST4}}{W_{ST} + (J \cdot W_{ST4})}$	
	$\lambda_{ST1} = \frac{h_2 - h_4}{\eta_{ST1} \cdot (h_2 - h_{3'})} - 1$		$\lambda_{ST2} = \frac{h_2 - h_4}{\eta_{ST2} \cdot (h_2 - h_{3'})} - 1$	
	$\lambda_{ST3} = \frac{h_2 - h_4}{\eta_{ST3} \cdot (h_2 - h_{3'})} - 1$		$\lambda_{ST4} = \frac{h_2 - h_4}{\eta_{ST4} \cdot (h_2 - h_{3'})} - 1$	
	$0.75 \cdot W_{ST} \leq W_{ST1}$		$0.75 \cdot W_{ST} \leq W_{ST2}$	
	$0.75 \cdot W_{ST} \leq W_{ST3}$		$0.75 \cdot W_{ST} \leq W_{ST4}$	
Value of operational variables	$W_{ST1} \leq W_{ST}$	$W_{ST2} \leq W_{ST}$	$W_{ST3} \leq W_{ST}$	$W_{ST4} \leq W_{ST}$
	$\eta_{ST1} \leq [\eta_{ST}]_{full}$	$\eta_{ST2} \leq [\eta_{ST}]_{full}$	$\eta_{ST3} \leq [\eta_{ST}]_{full}$	$\eta_{ST4} \leq [\eta_{ST}]_{full}$
	$n_{AC1} \leq n_{AC}$	$n_{AC2} \leq n_{AC}$	$n_{AC3} \leq n_{AC}$	$n_{AC4} \leq n_{AC}$
	$n_{CC1} \leq n_{CC}$	$n_{CC2} \leq n_{CC}$	$n_{CC3} \leq n_{CC}$	$n_{CC4} \leq n_{CC}$
	$W_{BY1} \leq Y_{CG}$ $W_{S1} \leq Y_{CG}$ $Q_{AH1} \leq Q_{AB}$	$W_{BY2} \leq Y_{CG}$ $W_{S2} \leq Y_{CG}$ $Q_{AH2} \leq Q_{AB}$	$W_{BY3} \leq Y_{CG}$ $W_{S3} \leq Y_{CG}$ $Q_{AH3} \leq Q_{AB}$	$W_{BY4} \leq Y_{CG}$ $W_{S4} \leq Y_{CG}$ $Q_{AH4} \leq Q_{AB}$

Table 78. Constraints used in the complete model of the trigeneration system (3rd Part)

References

[1] EDUCOGEN. *The European Educational Tool on Cogeneration*. 2001. (http://www.cogen.org/projects/educogen.htm)

[2] Energytech. Austria. http://www.energytech.at

[3] Protermo. Manual for calculating CHP electricity and heat. Suomen Kaukolämpö. January 2000

[4] H. L. von Cube, F. Steimle. *Wärmepumpen*. VDI-Verlag GmbH. Düsseldorf.1984

[5] Dupont-Suva Refrigerants. *Thermodynamic Properties of HCFC-123 Refrigerant (2,2 dichloro-1,1,1-trifluoroethane).* Wilmington, USA. 1993

Appendix C

List of Figures

Appendix D

List of Tables