

IMECE2004-62545

LINEAR AND NONLINEAR NUMERICAL INVESTIGATIONS OF REGULAR OPEN CELL STRUCTURES

Mathias H. Luxner*

Institute of Lightweight Design and
Structural Biomechanics,
Vienna University of Technology
Vienna, Austria
Email: luxner@ilsb.tuwien.ac.at

Juergen Stampfl

Institute of Materials Science
and Testing
Vienna University of Technology
Vienna, Austria

Heinz E. Pettermann

Institute of Lightweight Design and
Structural Biomechanics,
Vienna University of Technology
Vienna, Austria

ABSTRACT

Linear and nonlinear Finite Element simulations of various regular three-dimensional cellular solids (lattice structures) with relative densities ranging from 10% to 20% are presented. The structures consist of polymeric struts with circular cross sections.

Two different Finite Element modeling techniques are employed. Beam element based models and continuum element based models are utilized and their applicability is assessed. Beam element based models compromise about the numerical model size and the detail resolution of the problem. Continuum element based models are used for highly detailed unit cell analyses.

For simulations of the overall behavior the structures are treated as infinite media by a periodic microfield approach. The entire overall elasticity tensors are computed for the constitutive characterization of the effective mechanical behavior of the micro-structures. Overall stress-strain curves are predicted for uniaxial compressive loading, taking into account finite strains and elasto-plastic strut material. The predicted properties are evaluated with respect to direction dependence and density dependence.

Finite samples of specimens are modeled for comparison to experimentally obtained results.

INTRODUCTION

Highly porous regular cellular solids form the basis of many biological and engineering structures. They gain increasing importance especially in the growing field of scaffold engineering. The main advantages of cellular solids are their high specific stiffness and high specific strength as well as the possibility of tailoring their mechanical properties by designing appropriate cell architectures. This requires knowledge of the relation between their micro-structure and their overall mechanical properties.

Several analytical and numerical approaches establishing relationships between the cell architecture of cellular solids and their mechanical properties can be found in the literature. Analytical models for the description of the relationship between relative density and effective mechanical properties are given by Gibson and Ashby [1]. Analytical and numerical techniques for two- and three-dimensional regular cellular solids are presented in [2]. Modelling approaches based on tetrakaidecahedron unit cells [3,4] and hollow-sphere arrangements [5] are also available.

The present study is embedded in a larger project concerned with cellular solids, where regular cellular structures are designed, fabricated by rapid prototyping, experimentally characterized, and computationally simulated [6, 7]. The aim of the project is to gain knowledge about the mechanical behavior of regular cellular solids with relative densities in the range between 10% and 20%. The governing deformation mechanisms for different structures are to be identified and understood.

This paper deals with numerical simulation of three-

*Address all correspondence to this author.

Table 1. PROPERTIES OF THE POLYMERIC BULK MATERIAL.

Young's modulus	2.5 GPa
Poisson's ratio	0.3
Yield stress	40 MPa
Hardening modulus	40 MPa

dimensional cellular solids. Regular open cell structures are simulated by means of the Finite Element Method (FEM). Two different FEM modeling techniques are utilized and their applicability is assessed with respect to modeling cost and quality of the results. As a computationally cheap approach higher order beam elements are used for modeling of the structures. Continuum elements are employed for highly detailed analyses, being computationally more expensive and requiring high modeling effort.

Using these two FEM techniques the structures can be treated as infinite or finite media, employing unit cell models or finite size models, respectively. The latter consist of a given number of base cells. Comparability of these structural models is discussed and a comparison of simulation results with experimental results is shown.

Constitutive characterization of the three-dimensional cellular structures in the linear elastic regime is done by determination of the entire elasticity tensors, from which the directional and density dependences of the mechanical properties are derived. The investigated cellular solids can easily undergo large deformations and strains that cannot be described by a linear-elastic theory. Nonlinear uniaxial stress-strain curves are predicted for the different structures regarding finite strains and elasto-plastic bulk material.

INVESTIGATED STRUCTURES

The investigated structures in the present paper are regular open cell structures. The bulk material of the structures is a polymer with mechanical properties as shown in Tab. 1. J2 plasticity with isotropic hardening (and constant hardening modulus) is assumed.

Five generic three-dimensional micro-structures are discussed. They are chosen to possess different connectivities leading to different deformation mechanisms. All structures exhibit spatial periodicity and can be generated by duplicating the appropriate base cells in the three principal directions. In Fig. 1 the investigated structures (upper row) and the appropriate base cells (lower row) are shown. They consist of strut arrangements with circular cross sections. The strut diameter is constant within each individual structure.

FINITE ELEMENT MODELS

All numerical investigations are carried out by means of the FEM. In this section two different FEM modeling techniques are presented. Beam element based models are utilized for the simulation of large structures, such as finite samples. They are computationally cheap, but the representation of the structures suffers from several simplifications. For a highly detailed representation of the structures topologies continuum elements are used, giving rise to high modeling effort and high computational cost. They are used to assess the applicability of the beam element based models and for highly resolved unit cell analyses.

Beam Element Models

This FEM modeling technique employs 3D beam elements for the modeling of the structures. Timoshenko beam elements with quadratic interpolation formulations are used to allow for transverse shear deformations.

Simple modeling of a vertex by beam elements suffers from two approximations. First, there is no account for multiple volumes at overlapping sections. Second, ideal beam models do not account for constrained deformation in the vicinity of the vertices, caused by the material aggregation in these domains. Thus, the distribution of the material in the intersections of the struts is considered in terms of stiffness and density.

A 'correction' of the stiffness in the vicinity of the vertices is introduced by using rigid elements in this domain. All elements located within a spherical domain around the vertices with a radius equal to the strut radius are treated as rigid elements. The correction of the stiffness is done only for vertices connecting more than two struts. In addition, the struts' diameter are determined so that the mass represented by the 'corrected' beam model equals the mass of the real structure.

Continuum Element Models

Tetrahedron elements with quadratic interpolation formulations are employed for the continuum element based approach. This has the advantage of all features of the structures geometries being captured with high resolution. Unlike the beam element based models, the fillets between the struts, which emerge during the rapid prototyping process, can be modeled with the continuum element approach. Furthermore it is possible to study highly resolved stress and strain fields in the vicinity of the vertices.

The drawback of the continuum element based models is the high number of degrees of freedom and the resulting high computational cost and high modeling effort.

STRUCTURAL MODELS

In the previous section two different FEM modeling techniques have been presented. This section deals with two differ-

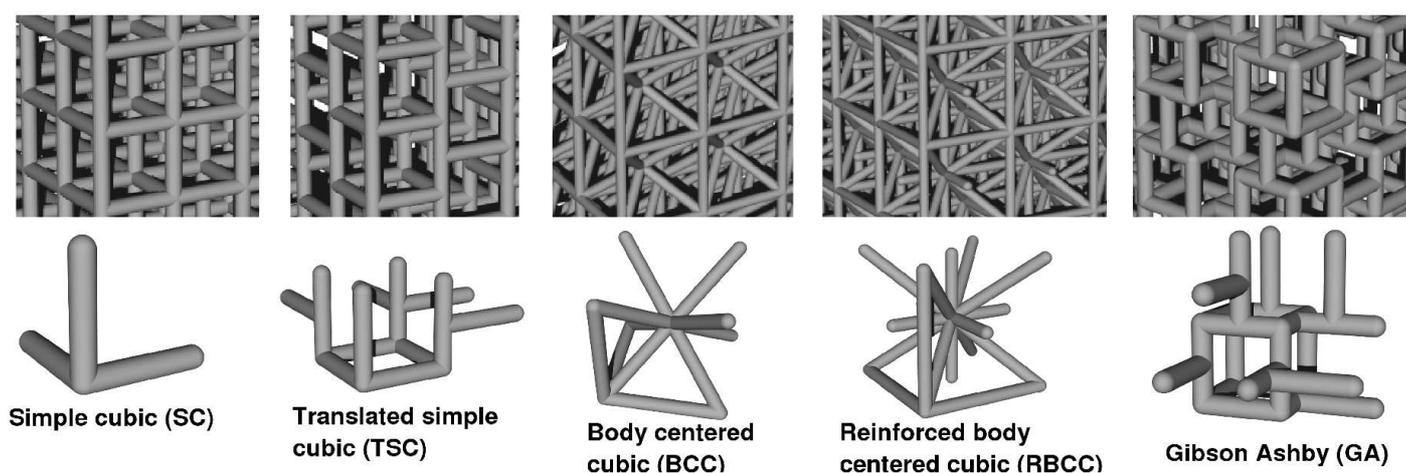


Figure 1. INVESTIGATED CELLULAR STRUCTURES AND THEIR BASE CELLS [8].

ent approaches for treating the structures. The structures may be considered as infinite or finite media. Treating the structures as infinite media has the benefit that the mechanical behavior of the structures can be described by looking only at the periodically repeating part of the structures, resulting in rather small numerical models. To achieve correspondence with the experimental setup the structures are treated as finite media, resulting in larger numerical models.

Infinite Medium

The investigated structures exhibit spatial periodicity. Thus, the mechanical behavior of a three-dimensional periodic structure can be described by modeling a space filling, three-dimensional unit cell [9]. Unit cells of different size and shape may be chosen for each structure. Here, the base cells shown in Fig. 1 are taken as the appropriate unit cells for the simulations.

For analyzing the mechanical behavior of the infinite periodic arrangement under far field mechanical loads homogenization via a periodic microfield approach is employed. An FEM based homogenization concept, also known as ‘macroscopic degrees of freedom’ (concept of master nodes) is employed [9]. The standard Finite Element package ABAQUS (*Version 6.4.1, HKS, Pawtucket, Rhode Island*) is used. Appropriate boundary conditions are applied on the FEM unit cell model to achieve spatial periodicity of the deformation field. Far field mechanical loads are applied to the masternodes. For the FEM modeling of the unit cell both the beam element approach and the continuum element approach are used. A comparison of the two FEM modeling techniques will be presented in the results section.

In order to characterize structures by means of their effective mechanical properties the entire overall elasticity tensors of the structures are determined. For the most general case this is done

by applying six unit load cases to the unit cells and assembling the overall elasticity tensors by means of the unit cell response.

Nonlinear Investigations The stress-strain behavior under uniaxial compression in one of the principal directions is predicted by means of the continuum element based unit cell models.

The investigated cellular solids can exhibit large deformations and large strains. Thus, geometrical nonlinearity is considered in the analyses. In addition, the nonlinear behavior of the bulk material (see Tab. 1) has to be taken into account.

The effective deformation gradients, the effective Green-Lagrange strain tensors, and the effective second Piola-Kirchhoff stresses of the unit cells are determined in every increment of the nonlinear FEM simulation.

Finite Medium

Specimens fabricated with rapid prototyping are tested. To assess the comparability of the results of the unit cell simulations with the experiments, finite samples of different sizes are simulated and compared to the unit cell simulations. This section deals with the experimental setup and the modeling of the finite samples.

Finite Structure Modelling For the testing of the structures finite samples consisting of $n \times n \times n$ base cells (see Fig. 2) are fabricated by rapid prototyping [6]. Testing is done by means of uniaxial compression. A bottom and top plate allow for a well-defined load application and a clear representation of the boundary conditions in the finite sample FEM models.

To achieve correspondence between the experimental setup

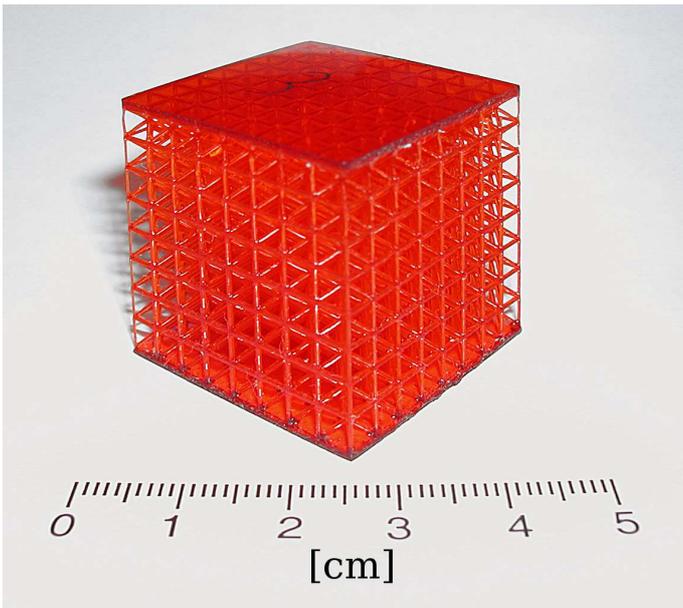


Figure 2. 8×8×8 TESTING SAMPLE, FABRICATED WITH SLA.

and the simulation models finite structures are modeled. With these models free-surface effects are captured and the load application is represented more realistically. Beam element based FEM models are used because of the high number of degrees of freedom. Continuum element based FEM models of finite structures are beyond computational limits, in particular for non-linear simulations.

The top face plate is modeled by coupling the translatory degrees of freedom in the top face. This means that all FEM nodes of the top face will undergo the same translatory displacements during deformation. The rotatory degrees of freedom in the top face are locked, assuming that the top plate does not deform. All degrees of freedom are locked for the nodes in the bottom face.

RESULTS

Linear Investigations

The elasticity tensors of the structures and for different relative densities are determined. This is done with both FEM modeling techniques. The normalized Young's moduli for all directions of the investigated structures are derived from the elasticity tensors.

Comparison of FEM Modelling Techniques The two FEM modeling techniques are compared by means of the normalized Young's moduli for all directions of the investigated structures.

It is shown that for the Simple Cubic (SC) structure, the

Translated Simple Cubic (TSC) structure, and the Gibson Ashby (GA) structure the beam element models react more stiffly than the continuum element models, especially in the principal directions. This is caused by the stiffness correction in the vertices of the beam element based models. The rigid domains of the straight thru struts in the considered directions lead to an increase of the normalized Young's modulus in these directions. For the Body Centered Cubic (BCC) structure and the Reinforced Body Centered Cubic (RBCC) structure the continuum element based models are more stiffly than the beam element based models. In all cases the deviation increases with relative density.

Directional Dependence The directional dependence of the elastic properties of the structures is derived from their elasticity tensors. This is done for structures with a relative density of 10%. Continuum element based unit cell models are used. Figures 3 and 4 show a comparison of the directional dependence of the Young's moduli $E^*(\underline{\phi})$ normalized by the Young's modulus E_s of the bulk material in the 001-010 and 001-110 plane, respectively. The directions are described by the vector $\underline{\phi}$.

It is shown that the Simple Cubic (SC) structure exhibits high stiffness in the principal directions, whereas the normalized Young's modulus decreases rapidly as the loading direction deviates from the principal directions. The Translated Simple Cubic (TSC) structure shows tetragonal behavior. For two principal directions the normalized Young's moduli are equal. In the third direction the stiffness is nearly the same as for SC. This difference can be explained by the material distributions of the two structures. Different behavior is shown by the Body Centered Cubic (BCC) structure. The stiffest direction of this structure is the 111 direction. The direction dependence of the normalized Young's modulus for the Reinforced Body Centered Cubic (RBCC) structure is less pronounced than for all other investigated structures. The directions with the highest value for the normalized Young's modulus are the principal directions. For the bending dominated Gibson Ashby (GA) structure the principal directions are also the stiffest directions, but the normalized Young's modulus is lower for all directions compared to the other structures.

Density Dependence In this section the density dependence of the normalized Young's modulus of the different structures is discussed. Several beam element based unit cell models with various relative densities in the range between 10% and 20% are simulated and the correlation between the density ρ^* and the Young's modulus $E^*(\underline{\phi})$ of the structure is described by the following relation [1],

$$E^*(\underline{\phi}) \propto (\rho^*)^{\beta(\underline{\phi})}, \quad (1)$$

where β denotes the density exponent and $\underline{\phi}$ is the considered direction of the structure.

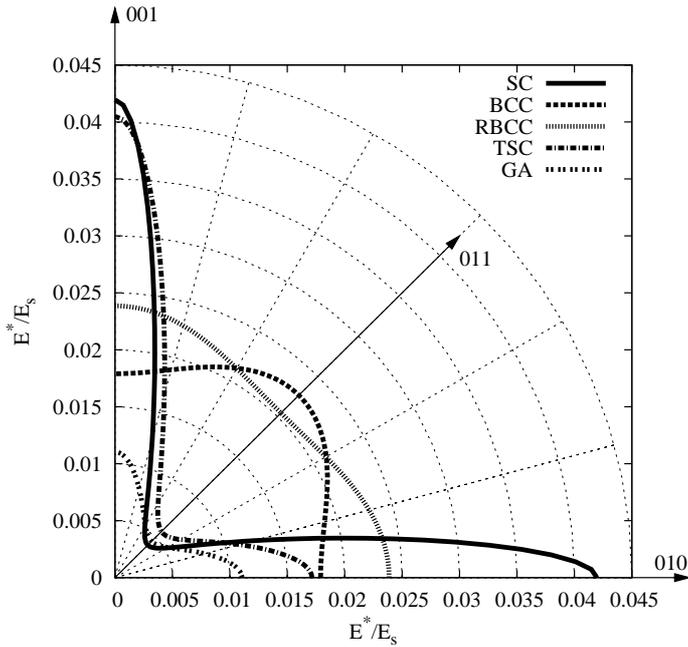


Figure 3. COMPARISON OF THE NORMALIZED YOUNG'S MODULUS IN THE 001-010 PLANE OF ALL STRUCTURES WITH A RELATIVE DENSITY OF 10%.

The value of the density exponent depends on the governing deformation mechanisms in the considered directions. For stretching controlled structures the Young's modulus of the structure changes linearly with the density of the structure, whereas the Young's modulus changes with the second power of the density for bending dominated structure directions [2].

Table 2 shows the determined values in three different directions for the density exponent. It is obvious that the governing deformation mechanisms in the SC structures change with direction. In the 001 direction stretching is the prevailing deformation mechanism, because of the struts that pass through the base cell in these directions. For the 011 and 111 direction the density exponent increases, pointing to a different deformation mechanism.

The BCC and RBCC structures exhibit a density exponent close to one for the three investigated directions. Stretching is the principal mechanism for these directions, which is a consequence of the diagonal struts.

For the GA structure the density exponent is nearly two in the 001 direction, reflecting the fact that in this direction the structure is bending dominated. The density exponent decreases for the 011 and 111 directions to 1.74 and 1.76, respectively. Thus bending is not the exclusive deformation mechanism in these directions.

Determination of the Specimen Size Finite samples of different sizes are simulated and compared to the unit cell

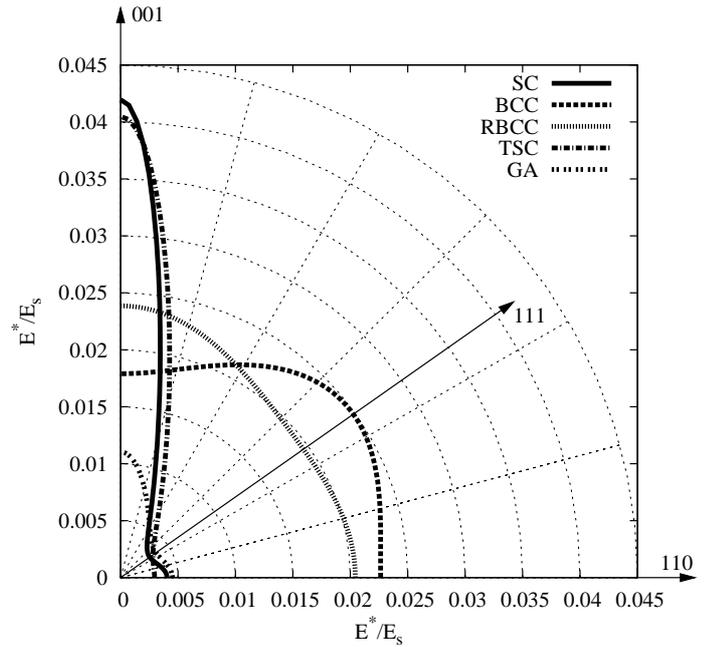


Figure 4. COMPARISON OF THE NORMALIZED YOUNG'S MODULUS IN THE 001-110 PLANE OF ALL STRUCTURES WITH A RELATIVE DENSITY OF 10%.

Table 2. DENSITY EXPONENT FOR DIFFERENT STRUCTURES AND DIFFERENT DIRECTIONS IN THE RELATIVE DENSITY RANGE BETWEEN 10% AND 20%.

Structure	[001]	[011]	[111]
SC	1.00	1.83	1.85
BCC	1.03	1.02	1.01
RBCC	1.01	1.02	1.02
GA	1.98	1.74	1.76

models in order to assess the necessary testing specimen size for suppressing excessive free surface effects. This is done for three directions by means of the normalized Young's modulus. Figure 5 shows this comparison for the BCC structure. It is shown that in terms of the normalized Young's moduli even for finite samples as small as $5 \times 5 \times 5$ base cells the results correspond well with the results of the unit cell approach for the BCC structure. A general rule regarding the size of testing specimen cannot be derived from these results, since the surface effects depend strongly on the structure's architecture.

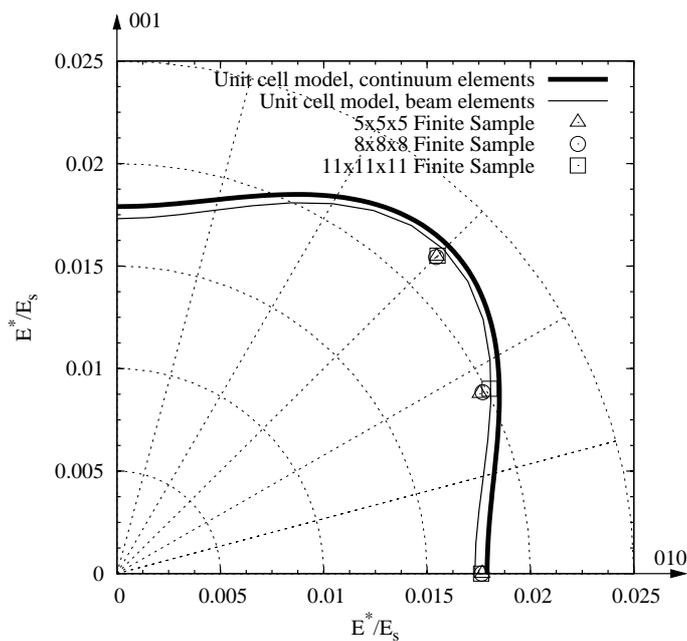


Figure 5. COMPARISON OF FINITE AND INFINITE APPROACHES IN TERMS OF THE NORMALIZED YOUNG'S MODULUS OF THE BCC STRUCTURE WITH A RELATIVE DENSITY OF 10% .

Comparison to Experimental Results

Figure 6 shows the results of the continuum element based unit cell simulations in comparison with experimental results [6]. Testing is done by means of uniaxial compression in one of the principal directions. The testing specimens are fabricated using two different rapid prototyping techniques and consist of $8 \times 8 \times 8$ base cells, with the exception of the GA structure, which consists of $4 \times 4 \times 4$ base cells. Stereolithography (SLA) and Selective Laser Sintering (SLS) [6, 7] are utilized.

For structures with rather low anisotropy such as the BCC and RBCC structures the results correspond well, whereas for structures with higher anisotropy such as the SC and TSC structures deviation of the simulation results is notable. For the latter the geometric imperfections that occur during the fabrication process, play a significant role. As can be seen in Fig. 3 and 4, small changes in direction lead to a pronounced decrease of the Normalized Young's modulus.

Nonlinear Investigations

Continuum element based unit cell models are employed to predict the nonlinear behavior of the structures. Figure 6 shows the stress-strain curves for uniaxial compression in a principal direction. The slope at the origin, of course, coincides with the predicted Young's moduli from the previous section. Note that the elastic regime is not linear because of the chosen stress and strain measures.

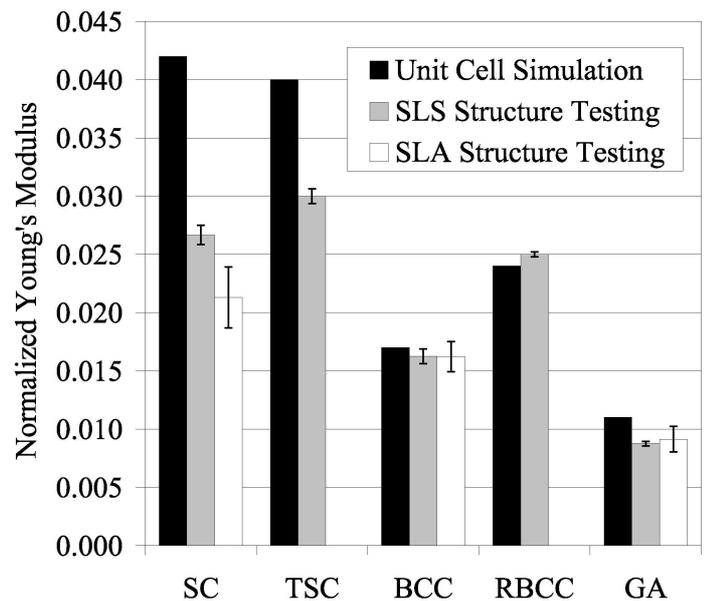


Figure 6. COMPARISON OF EXPERIMENTAL AND SIMULATION RESULTS REGARDING UNIAXIAL COMPRESSION IN ONE OF THE PRINCIPAL DIRECTIONS, RELATIVE DENSITY OF THE STRUCTURES IS 10%.

Overall yielding as well as overall hardening is quite different depending on the structure. For the investigated systems and the modeling approach, the curves are reliable up to moderate plastic strains. At higher strains the presented results are beyond the limit of validity, since additional nonlinearities come into effect.

Preliminary studies [8] show that substantial plasticity in the struts is present before nonlinear buckling may happen. The tendency to buckle is pronounced only for particular combinations of loading scenario and structural architectures, e.g. SC structure compressed in principal direction. Furthermore, there is found, that the buckling mode with the lowest eigenvalue is the one captured by the $1 \times 1 \times 1$ unit cell. Mechanical aspects not treated here are localization of the deformation as seen in experiments, and local stress states beyond the ultimate strength of the bulk material.

CONCLUSIONS

Modelling and constitutive characterization of regular three-dimensional cellular solids are presented. Two different FEM techniques are discussed. Beam element based models and continuum element based models are utilized and their applicability is assessed. It is shown that the beam element based models reveal the basic mechanical behavior of the structures. They are a

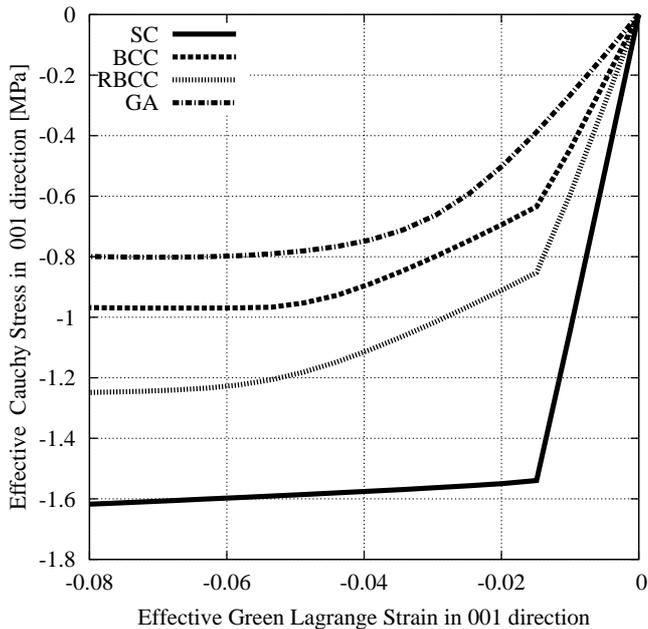


Figure 7. COMPARISON OF THE NONLINEAR STRESS-STRAIN BEHAVIOR BY MEANS OF UNIAXIAL COMPRESSION IN 001 DIRECTION AND THE CONTINUUM ELEMENT BASED UNIT CELL MODEL, RELATIVE DENSITY OF THE STRUCTURES IS 10%.

good trade off in terms of computational cost, modeling effort, and quality of the results. Thus, they are used for large models such as the finite samples models. The computationally more expensive continuum element based models are used for analysis of highly resolved deformation patterns and highly resolved stress and strain fields.

A unit cell approach is used for treating the structures as infinite medium models. Finite structural samples of different sizes are modeled and compared to the infinite medium approach. For the Body Centered Cubic (BCC) structure it is shown that in terms of the normalized Young's moduli even for finite samples as small as $5 \times 5 \times 5$ base cells the results correspond well with the results of the unit cell approach, but no general rule regarding the specimen size is derived from these results.

Constitutive characterization of the effective mechanical behavior of various generic regular three-dimensional cellular structures is done. Their entire overall elasticity tensors are evaluated and the directional dependence and density dependence of the elastic properties are derived. The governing deformation mechanisms are identified. Both the mechanical properties and the deformation behavior are found to be strongly dependent on the structure's cell architecture, as well as on the loading scenario.

The results of the unit cell simulations are compared with ex-

perimental results by means of uniaxial compression tests. The results agree very well, except for structures with rather high anisotropy. Here, the imperfections of the testing specimens, fabricated with rapid prototyping, play a significant role.

Overall stress-strain curves by means of the unit cell approach are predicted for the structures, considering finite strains and nonlinear behavior of the bulk material. The investigated structures show quite different overall yielding as well as overall hardening. The results for the stress-strain behavior are found to be valid up to moderate plastic strains, since additional nonlinearities occur at higher strains.

ACKNOWLEDGMENT

This work was financially supported by the Austrian Science Fond under contract P15852. The authors would like to thank Thomas Daxner, Alexander Woesz, and Peter Fratzl for many useful suggestions, comments, and discussions.

REFERENCES

- [1] Gibson, L., and Ashby, M., 1997. *Cellular Solids: Structure and properties, second ed.* Cambridge University Press.
- [2] Grenestedt, J., 1999. "Effective elastic behaviour of some models for 'perfect' cellular solids". *Int. J. Sol. Struct.*, **36**, pp. 1471–1501.
- [3] Zhu, H., Knott, J., and Mills, N., 1997. "Analysis of the elastic properties of open-cell foams with tetrakaidecahedral cells". *J. Mech. Phys. Solids*, **45**, pp. 319–343.
- [4] Warren, W., and Kraynik, A., 1997. "Linear elastic behavior of a low-density kelvin foam with open cells". *J. Appl. Mech.*, **64**, pp. 787–794.
- [5] Sanders, W., and Gibson, L., 2003. "Mechanics of BCC and FCC hollow-sphere foams". *J. Mat. Sci. and Eng.*, **A352**, pp. 150–161.
- [6] Stampfl, J., Seyr, M., Luxner, M., Pettermann, H., Woesz, A., and Fratzl, P., 2004. "Regular, low density cellular structures - rapid prototyping, numerical simulation, mechanical testing". In Proceedings of 2004 MRS Spring Meeting - Biological and Bioinspired Materials and Devices, J. Aizenberg, C. Orme, W. Landis, and R. Wang, Eds., vol. 823.
- [7] Woesz, A., Stampfl, J., and Fratzl, P., 2004. "Cellular solids beyond the apparent density - an experimental assessment of mechanical properties". *Advanced Eng. Mat.*, **6**, pp. 134–138.
- [8] Seyr, M., 2003. Numerische Simulation regelmässiger zellulärer Strukturen. Master's thesis, Vienna University of Technology.
- [9] Anthoine, A., 1995. "Derivation of the in-plane elastic characteristics of masonry through homogenization theory". *Int. J. Sol. Struct.*, **32(2)**, pp. 137–163.