The classical analysis of turbulent boundary layers in the limit of large Reynolds number *Re* is characterised by an asymptotically small velocity defect with respect to the external irrotational ¤ow. As an extension of the classical theory and as a £rst step towards a more general approach which also accounts for separating ¤ows, it has been shown, [1], that the defect may become moderately large and essentially independent of *Re* but still remain small compared to the external streamwise velocity component for non-zero pressure gradient boundary layers. That wake-type ¤ow turns out to be characterised by large values of the Rotta–Clauser parameter, which serves as an appropriate measure for the defect and hence as a second perturbation parameter besides *Re*. Most important, it is demonstrated that also this case can be treated by rigorous asymptotic analysis on basis of the Reynolds-averaged Navier–Stokes equations, independent of the choice of a speci£c Reynolds stress closure. The asymptotic investigation predicts the remarkable result that in the special case of quasi-equilibrium ¤ow the transition from classical small-defect to a pronounced wake ¤ow is accompanied by double-valued solutions, a phenomenon seen to agree well with early experimental observations. In addition, a numerical treatment of the nonlinear boundary layer equations, supplemented with an asymptotically correct shear stress closure which provides the well-known logarithmic velocity portion close to the surface, con£rms those theoretically predicted results for £nite values of *Re*. The solutions exhibit good agreement with experimental data taken from the literature and convergence to the asymptotic limit obtained analytically, [2], [3]. As a minor but nevertheless interesting point, certainly relevant for engineering applications, it is found that the in¤uence of the wall curvature, which must be included in the analysis in the case of an asymptotically small velocity defect from a rational point of view, on the irrotational bulk ¤ow associated with double-valued near-equilibrium boundary layers allows for self-similar potential ¤ow solutions. They describe ¤ows along curved diffuser ducts. Such ¤ow con£gurations are seen to withstand the maximum pressure rise possible in order to maintain equilibrium, [3].

The results summarised so far indicate that turbulent boundary layers having a moderately large velocity defect represent the 'missing link' between the classical ¤ow with a defect of $O(1/\ln Re)$ and boundary layers where the defect is a quantity of $O(1)$. The investigation of the latter, [4], suggests the need of a second perturbation parameter, denoted by α, besides *Re* which measures the slenderness of the boundary layer. The asymptotic analysis of that general type of turbulent boundary layers has been completed as far as the limit $\alpha = 0, Re^{-1} = 0$ is concerned. This primary limit gives rise to weakly singular solutions of the resulting shear layer approximation of the wake ¤ow if the prescribed adverse pressure gradient is controlled suitably. The local behaviour of these solutions is studied analytically. They indicate marginally separated ¤ows for £nite values of both α and *Re*. However, a correct treatment of the separation process must take into account the local interaction of the boundary layer and the external potential ¤ow. This is a topic of the subsequent research.

[1] SCHEICHL, B. & KLUWICK, A. 2004 Non-unique turbulent boundary layer ¤ows having a moderately large velocity defect. *J. Fluid Mech.*, submitted.

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[3] SCHEICHL, B. & KLUWICK, A. 2004 Non-Unique Self-Similar Turbulent Boundary Layers in the Limit of Large Reynolds Number. Proceedings of the *iTi Conference on Turbulence, 21–24 September 2003, Bad Zwischenahn, Germany*, in press.

[4] SCHEICHL, B. & KLUWICK, A. 2004 On a novel wake-type limit of marginally separating turbulent boundary layers. *J. Fluid Mech.*, in preparation.