On the Brillouin–Villat Condition in Connection with Turbulent Massive Separation

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Abstract

The prediction of self-induced flow separation from an impermeable solid smooth surface in the limit $Re \to \infty$, here Re denotes a suitably defined Reynolds number representative for the global flow configuration, has provided a long-standing challenging problem not only in theoretical fluid dynamics but also with respect to practical engineering applications. The latter require a sound understanding of the underlying physical mechanisms, specifically, in order to allow for controlling and thus inhibiting the unwanted associated effects like pressure losses. As an important example, internal flows in diffuser ducts designed to achieve a maximum pressure rise are very sensitive to gross separation.

However, as far as incompressible two-dimensional laminar steady flows are considered, the local asymptotic structure of the boundary layer and the induced interaction mechanism with the external inviscid and irrotational bulk flow near the position of separation is already well-understood, see the pioneering papers [3] and [2]. There a rational local theory of separation is derived where the potential flow limit is sought in the class of flows exhibiting a free streamline departing smoothly from the surface. To this end, it is anticipated that the local pressure variations must be less singular inside the reverse-flow regime than upstream of the point of flow detachment. A basic local analysis then yields well-known result that on the surface

$$\frac{\mathrm{d}p}{\mathrm{d}s} \sim \begin{cases} k(-s)^{-1/2} + 16\,k^2/3\,, & s \to 0_-\,, \\ 0\,, & s \to 0_+\,. \end{cases} \tag{1}$$

Herein s and p denote the distance from the detachment point in flow direction, nondimensional with a global length scale, and the pressure, nondimensionalised on the (uniform) fluid density times $\tilde{U}^2$ where $\tilde{U}$ is the value of the velocity along the detaching streamline at $s = 0$. The constant k depends upon the location of flow detachment. In turn, the (non-dimensional) curvature κ of the free streamline assumes the form $\kappa \sim -ks^{-1/2} + \kappa_0$, $s \to 0_+$, where κ_0 denotes the surface curvature at $s = 0$ (such that $k \geq 0$ for geometrical reasons). A more precise estimate for the location of separation is seen to be only possible by taking into account viscous effects. Most important, it is demonstrated in [3] that for $Re \to \infty$ the singular adverse pressure gradient given by Equ. 1 causes separation of the boundary layer which allows for a self-consistent asymptotic formulation on basis of the Navier-Stokes equations only if it initiates the pressure gradient induced more downstream by the locally strong boundary layer displacement. The analysis then shows that the distinguished limit

$$kRe^{1/16} = O(1), \quad Re \to \infty, \tag{2}$$

has to be considered. In the inviscid limit this relationship reduces to the so-called Brillouin-Villat condition, the case $k = 0$, which predicts a regular potential free-stream flow, cf. Equ. 1. In addition, we note that the numerical solution obtained in [2] of the underlying

canonical problem which governs the resulting local viscous/inviscid interaction mechanism finally indeed fixes the value of k and, thus, the location of separation for a given flow configuration (upstream of separation) up to the accuracy included by Equ. 2.

On the other hand, it is a well-known fact that for sufficiently high Reynolds numbers all separated flows are virtually turbulent. Nevertheless, for the time being the much more important turbulent counterpart to the theory outlined above is still lacking. In our contribution we present, amongst others, (i) the basic properties of a novel asymptotic theory of the nominally steady and two-dimensional incompressible turbulent boundary layer under the action of an adverse pressure gradient on basis of the Reynolds-averaged Navier–Stokes equations in order to provide the proper framework for tackling the separation problem; (ii) local solutions of the leading-order boundary layer approximation when the pressure gradient approaches the singular behaviour given by Equ. 1 for $s \to 0_-$. We stress that, in striking difference to laminar boundary layers, in the turbulent case the slenderness of the boundary layer is found to be measured by a small parameter which remains finite even in the formal limit $Re^{-1} = 0$; for a more detailed discussion the reader is referred to [1]. Moreover, in that limit the boundary layer exhibits a surface slip velocity $u_s = O(1)$, here also non-dimensional with $\tilde{U}$ introduced before. By considering item (ii) above, the boundary layer solution inevitably terminates in a singularity at $s = s^* \leq 0$ where u_s vanishes if $s^* < 0$. It is a highly remarkable first result that k must be controlled such that both $s^* = 0$ and $u_s = 0$ in order to allow for a self-consistent description of the separated flow region further downstream, see Fig. 1. As an interesting consequence at this stage of the analysis, the in the past intensely debated question if the Brillouin-Villat condition holds also for turbulent flows then is negated, agreeing well with the experimental findings shown in [4].

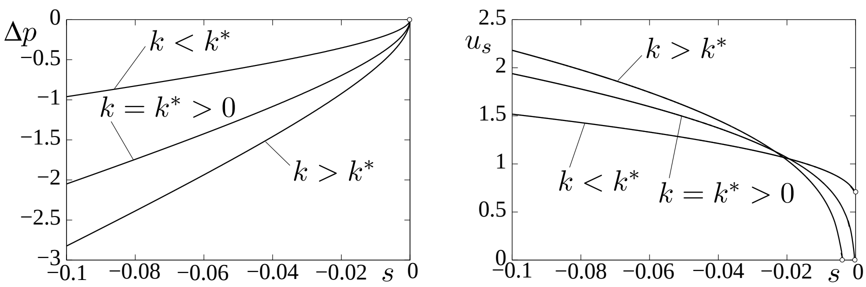

Fig. 1. Local distributions $\Delta p = p(s,k) - p(0,k)$, $u_s(s,k)$ with k^* chosen such that $u_s(0,k^*) = 0$.

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