A general asymptotic description of turbulent boundary layers

B. Scheichl *, A. Kluwick*

A comprehensive rational theory of incompressible nominally steady and two-dimensional turbulent boundary layers (TBLs) is developed by adopting a minimum of assumptions regarding the flow physics and properly investigating the Reynolds-averaged equations of motion in the limit of high Reynolds number, denoted by Re. The logarithmic law of the wall is established and shown to be associated with an asymptotically small rotational streamwise velocity defect with respect to the external potential bulk flow on top of the viscous wall layer. Consequently, the classical scaling of two-tiered TBLs provides the simplest feasible flow structure. It is, however, possible to extend this concept and to formulate a three-tiered splitting of TBLs having a slightly larger, i.e. a 'moderately' large, velocity defect. This allows for, amongst others, the prediction of the in previous studies intensely discussed phenomenon of non-unique equilibrium flows for a prescribed pressure gradient[1].

Most important, it is observed that all commonly employed closures contain small numbers which are seen to be independent of Re and may serve as a perturbation parameter measuring the slenderness of the shear layer. Exploiting this characteristic finally leads to a fully self-consistent asymptotic description of TBLs which exhibit a velocity defect of $O(1)$ and a slip velocity of $O(1)$ at their bases by considering the formal limit $Re^{-1} = 0$. In turn, they closely resemble turbulent free shear layers and can even undergo marginal separation. This situation is accompanied by the occurrence of a weak singularity in the solutions of the boundary layer approximation. The analysis then reveals the necessity to adopt a viscous/inviscid interaction technique in order to capture the feedback of the pressure induced by the local boundary layer displacement in the external flow[2], also see figure 1. Additionally, we will outline, how the underlying asymptotic concept allows for further analytical and numerical progress and, amongst others, provides the adequate basis to tackle the extremely challenging problem of turbulent gross separation. First investigations indicate that, in contrast to the laminar case, here the Brillouin–Villat condition is not met.

*Institute of Fluid Mechanics and Heat Transfer, Vienna University of Technology, Resselgasse 3/E322, A-1040 Vienna, Austria.

[1]Scheichl and Kluwick, *Theor. Comput. Fluid Dyn.*, submitted in revised form.

[2]Scheichl and Kluwick, *AIAA paper 2005-4936* (2005).

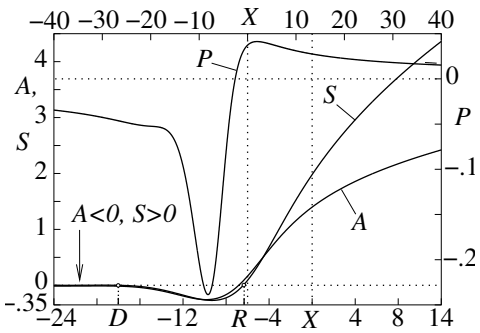

Figure 1: Typical solution of the interaction problem using suitably rescaled local variables: The streamwise coordinate and the locations of flow detach- and reattachment are denoted by X, D, and R, respectively. The top abscissa refers to the induced pressure P, the one at the bottom to the surface slip velocity S and the local boundary layer displacement $-A$.