ACTIVE DAMPING OF A QUADRATIC PLATE FOR A NON-COLLOCATED DISTURBANCE

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Abstract
In this work the active damping of a quadratic plate using two piezo patch actuators is described. A numerical evaluation of an optimality criterion for placement of sensors and actuators defines the location for the piezo patches. A state-space controller in combination with an observer and an alternative 2nd-order controller with acceleration feedback are designed to add additional damping to the existing modes. Specific attention was paid to suppress spillover (amplification of modes due to neglected dynamics), and a frequency response mode was added to account for the effect of higher-order modes. With two actuators in optimal positions only a small band of maximal two eigenmodes could be efficiently damped. Nevertheless, a mode at 167Hz showed an attenuation of over 17dB. Additional problems were posed by a bifurcation of the modal behaviour and the relation between plate and sensor thickness.

INTRODUCTION

Active structural damping of spatial structures is an important field of mechatronics, since the application of an embedded control system with low-cost components can be much easier and less costly than a different design with higher structural complexity or expensive materials. The corresponding control theory is highly evolved and recently excellent text books in this field have been published [5, 11, 12].

In practical applications, however, some of the conditions for these methods may not be fulfilled, and it is of interest for the user, what problems arise and what results can be achieved with alternate methods. In this paper the active damping of a quadratic plate is the subject, and the non-collocated disturbance poses one of the main problems. Due to this fact, only a small bandwidth can be attenuated with a minimal set of collocated sensor/actuator pairs. Additional problems are posed by the symmetric eigenmodes of the quadratic plate, and the relatively high stiffness of collocated actuators and sensors compared to the sheet metal. In spite of these problems a controller consisting of two parallel loops and acceleration feedback could achieve a strong damping of the dominant eigenmode.

The remainder of this paper is structured as follows: First, the system is described and a mathematical model for controller and observer design is designed. Special care is taken to incorporate Frequency Response Modes into the model and to guarantee optimal Sensor/actuator placement. Simulation results and experimental findings show that a simple state space approach is not satisfactory in this case, but local control loops are superior. A short summary with some proposals for improvements concludes the work.

SYSTEM AND MODEL

Quadratic Plate

The vertical elastic deflection $w(t)$ of a quadratic elastic plate with length a and thickness h is given by the partial differential equation

$$\rho h \frac{\partial^4 w}{\partial t^2} + D\nabla^4 w(x,y,t) = \frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} \tag{1}$$

where ρ is the density of the material, and M_{px} and M_{py} constitute the external moments per length applied by actuators [6, 10].

Eigenmodes of the Quadratic Plate

In fig.1 the first 3 eigenmodes of the quadratic plate are depicted. The maximum deflection is normalized to 1. The associated eigenfrequencies are 36.3Hz for the first mode and 74.1Hz for both second and third mode (symmetric modes). In the case of a quadratic plate diagonal

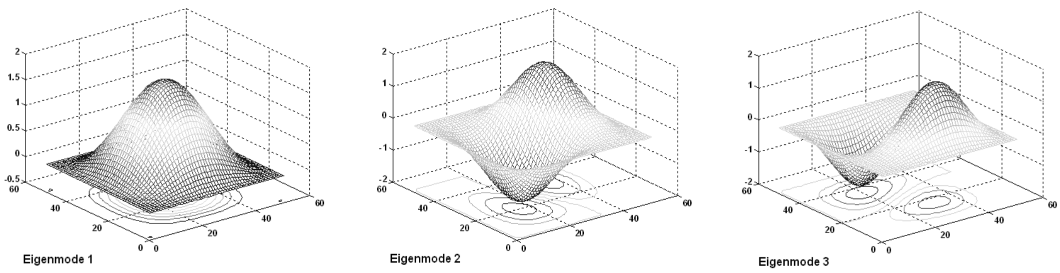

Figure 1: Eigenmodes 1 to 3 of the Quadratic Plate

node lines exist while for the rectangular plate only edge-parallel node lines occur (fig.2). In fig.2 the eigenmodes of a rectangular plate with a difference in side lengths of 0.5 % are depicted. For the rectangular plate eigenfrequencies 2 and 3 are slightly different, while for the quadratic plate they are identical.

State Space Model

Each of the above introduced eigenmodes ϕ_i contributes to the overall deflection $w(t)$ by superposition according to

$$w(t) = \Phi\mathbf{q}(t) = \sum_{i=1}^{n} \phi_i q_i(t), \tag{2}$$

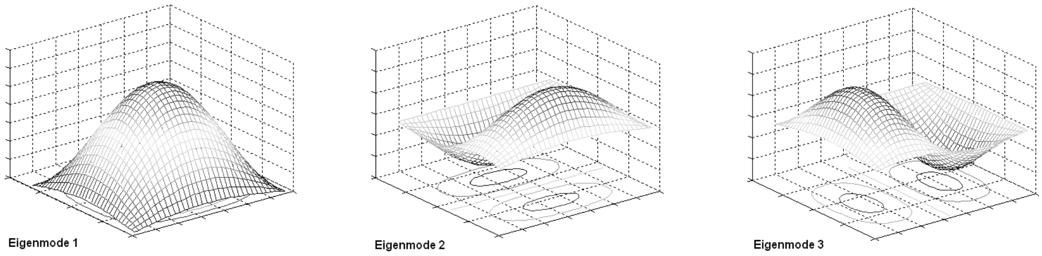

Figure 2: Eigenmodes 1 to 3 of the Rectangular Plate with nearly equal side lengths

where the $q_i(t)$ are the modal coordinates. Choosing the modal coordinates and their time-derivatives as state-vector **x** the state and output equation can be written as

$$\dot{\mathbf{x}} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -diag(\omega_i^2) & -diag(2\xi_i\omega_i) \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ \underline{B} \end{pmatrix} \mathbf{u} + \begin{pmatrix} 0 \\ \underline{E} \end{pmatrix} \mathbf{z} \tag{3}$$

$$\mathbf{y} = \begin{pmatrix} \underline{C} & \mathbf{0} \end{pmatrix} \mathbf{x}, \tag{4}$$

where the ξ_i are Raleigh damping-coefficients and the ω_i are the eigenfrequencies for the i-th eigenmode. The $\underline{B}$-matrix has as many rows as eigenmodes and as many columns as actuators. The matrix $\underline{C}$ has as many rows as sensors and as many columns as eigenmodes. A voltage applied to an actuator corresponds to a certain curvature and vice versa for the sensor. Therefore, the moments on the right-hand side of eq.1 can be produced by piezo patches, and the modal coordinates $q_i(t)$ can be reconstructed from the measured local curvature. Mathematical piezo patch models can be found in [6, 10], and the specific coefficients for materials are provided by the manufacturer (see e.g. [2]).

Controller and Observer Design

The control law for a state space controller is given by $\mathbf{u} = \mathbf{K}_w\,\mathbf{w} - \mathbf{K}\,\mathbf{x}$, where the set point **w** may be considered zero. A possible design procedure is to rotate the position of the open-loop poles in the complex plane towards the real axis therefore increasing the damping.

A state space controller requires all states to be known. Since only physical quantities can be measured the modal coordinates $q_i(t)$ and their derivatives $\dot{q}_i(t)$ must be reconstructed by an observer. For the above defined state space system (3,4) a 4th-order Luenberger-Observer is designed by

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{HC})\hat{\mathbf{x}} + \mathbf{Bu}, \tag{5}$$

where the observer gain matrix **H** is computed by minimizing a quadratic criterion by solving the resulting Riccati-equation.

FREQUENCY RESPONSE MODE (FRM)

Every continuous linear system has an infinite number of eigenmodes with associated eigenvalues. In order to achieve acceptable accuracy a small number of modes n_{red} associated with the lowest eigenvalues is typically chosen for controller design. However, due to external forces or moments a large number of modes may be necessary to guarantee a desired accuracy. This problem may be overcome by the use of a particular solution, which incorporates the effect of higher order modes.

The particular solution for a modal model with external excitation $\mathbf{f}(t) = \mathbf{F}e^{j\Omega_0 t}$ is given by

$$\mathbf{W}_p = \sum_{i=1}^{n} \frac{\Phi_i \Phi_i^T}{\omega_i^2 - \Omega_0^2 + 2j\zeta_i\Omega_0\omega_i}\mathbf{F}, \qquad \mathbf{W}_s = \mathbf{W}_p|_{\Omega_0=0} = \sum_{i=1}^{n} \frac{\Phi_i \Phi_i^T}{\omega_i^2}\mathbf{F} = \mathbf{K}_s\mathbf{F}, \tag{6}$$

where $\Omega_0 = 0$ yields a particular static mode $\mathbf{W}_s$ which may be treated as an additional eigenmode of the system. However, the definition of proper boundary conditions for the computation of (6) is difficult, especially in the presence of rigid body modes.

One way to overcome this problem is the FRM. Using an Ω_0 equal to half the first eigenfrequency of the system the FRM $\mathbf{W}_{FRM}$ is also computed by (6). The approximate overall solution $\mathbf{w}(t)$ of the modal system is then given by

$$\mathbf{w}(t) \simeq \sum_{i=1}^{n} \phi_i \mathbf{q}_i(t) + \mathbf{W}_{FRM}\alpha_{FRM}(t), \tag{7}$$

where $\alpha_{FRM}(t)$ is a scaling factor for the FRM. If the bandwidth of the external excitation is equal or smaller than the bandwidth of the model eq.(7) will yield a good result. For each position of an external excitation a dedicated FRM has to be added. The FRM adds a constant to the transfer functions of the system, thus altering only gain and zeros but not the poles. In fig.3 the frequency response of a high-order model, a reduced version, and a reduced version plus FRM are plotted. More details can be found in [1], [12], [4], and [13]. It should

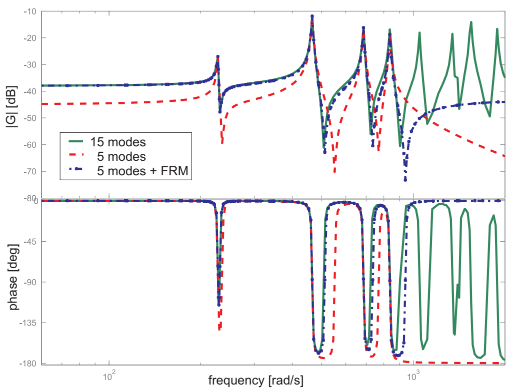

Figure 3: Effect of the Frequency Response Mode (FRM)

be noted that incorporation of the FRM is an important means to avoid or at least minimize spillover phenomena where unmodelled higher modes affect the state vector estimate (observation spillover).

PLACEMENT OF SENSORS AND ACTUATORS

The above mentioned spillover problem is closely associated with optimality in observability and controllability. In order to optimize these properties a quantitative criterion must be defined. Using either the observability gramian W_o or the controllability gramian W_c the following criterion is utilized [7]:

$$C = trace(W)\frac{\sqrt[2N]{det(W)}}{\sigma(\lambda_i)} \tag{8}$$

In this criterion $\sigma(\lambda_i)$ is the standard deviation of the eigenvalues λ_i of the gramian. This formulation can be shown to be equal to criterions using the H_2-norm [9], [8].

Symmetric Modes of the Quadratic Plate

In the case of the quadratic plate symmetric modes with equal eigenfrequencies arise (fig.1). Using only one sensor or actuator, respectively, the gramians will become singular. In the case of closely lying eigenvalues the gramians will be ill-conditioned. This problem can only be overcome by applying additional sensors or actuators.

Numerical Issues

The implementation of (8) requires that for each point along a grid the criterion has to be evaluated and stored. Each evaluation of (8) comprises the calculation of the current gramians, their eigenvalues and determinant. To speed up the computation the position grid is refined iteratively, symmetries are exploited, and the boundaries of the plate are disregarded for physical reasons. A plot of the criterion (8) is given in fig.4. The first sensor position is fixed at the ”x” and the second sensor position is optimized.

SIMULATION AND EXPERIMENT

The experiment consists of a quadratic plate made of 1mm sheet metal (steel) and a side length of 0.5m clamped on all sides in a massive frame. Piezo patches [2] with a length of 35mm and a thickness of 1mm (maximum voltage 250V) were applied to the optimal positions. In fig.5 the plate with applied sensors and actuators is shown. Actuators 1 and 2 are used for control input, the third patch is used for excitation. Additional accelerometers are used for validation or control input, respectively.

Simulation Results

In fig.6 the transfer function from disturbance to actuator 2 is plotted. Only the first 3 eigenmodes are included, the observer is designed for only 2 eigenmodes. This simulation result indicates a fundamental limitation of the plate: Additional damping can only be added over a small frequency range, since the disturbance and the control input are not collocated. In order to achieve a larger bandwidth more actuators have to be applied. Symmetric modes require additional pairs of actuators.

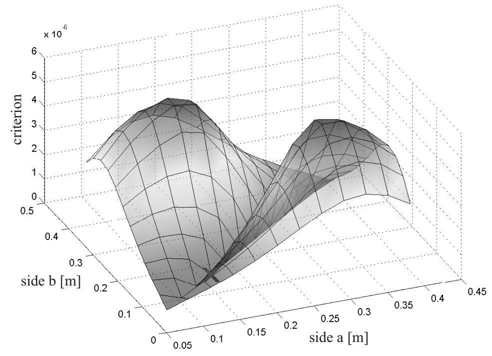

Figure 4: Criterion for optimal actuator position

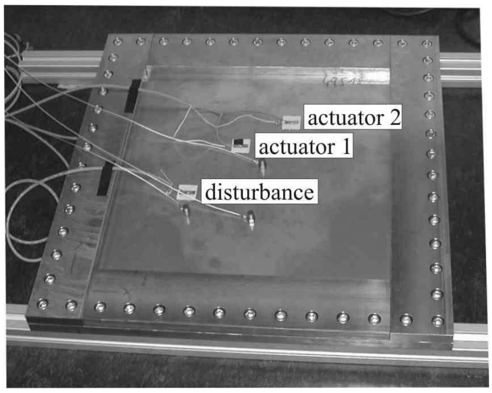

Figure 5: Quadratic plate with sensors and actuators

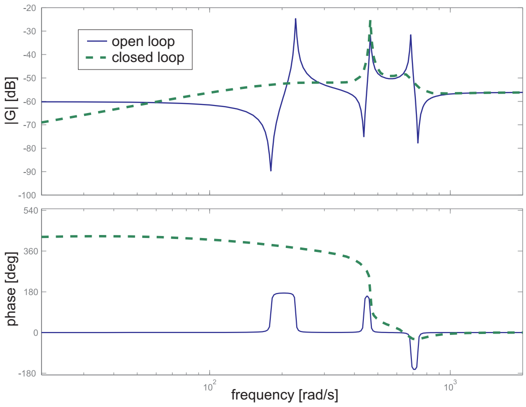

Figure 6: Simulated transfer functions from disturbance to actuator 2 (open and closed loop)

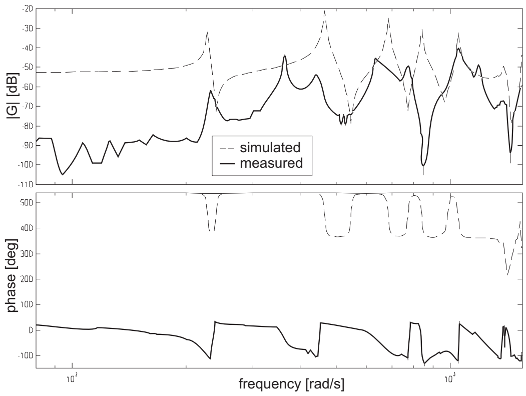

Figure 7: Transfer functions from disturbance and sensor 1 (theoretical and real)

Experimental Results

Model Validation

In fig.7 the predicted and measured transfer functions between disturbance and sensor 1 are plotted. Only the first eigenfrequency shows a good agreement, all other modes exhibit considerable deviations due to increased mass and bending stiffness at sensor and actuator locations, geometrical imperfections, non-uniform clamping conditions, and different mode shapes (see section ”Eigenmodes of the Quadratic Plate”).

During closed loop operation the collocated sensor patch delivered good results, while an additional collocated accelerometer detected poor performance. This can be referred to the fact that using collocation the system response is only controlled locally and a good performance of different measurements cannot be guaranteed [3].

PT2-Controller with Accelerometer

Using the accelerometer as sensor two 2nd-order local controllers were implemented at a resonance frequency of 167Hz, where a significant peak in the open-loop transfer function was measured. The controller transfer function is given by

$$G_c = \frac{P}{\frac{1}{(2\pi 167)^2}s^2 + \frac{2\cdot 0.02}{2\pi 167}s + 1} = \frac{P}{9.083^{-7}s^2 + 3.812^{-5}s + 1}. \tag{9}$$

This controller acts at a resonance frequency of 167Hz with a damping of $\zeta = 0.02$ (small bandwidth). Due to the non-symmetric mode shapes different gains P_i were used for each collocated sensor/actuator pair ($P_1 = 45, P_2 = 12$). Using these gains peak control inputs of 200V were observed. The dominant eigenmode at 167Hz could be attenuated by 17dB. Results are plotted in figures 8 and 9.

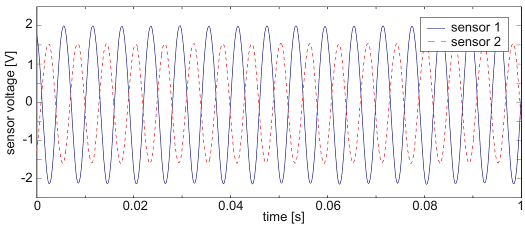

Figure 8: Open-loop accelerations

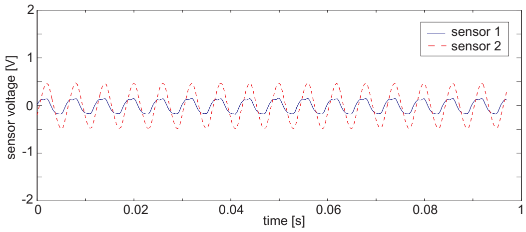

Figure 9: Closed-loop accelerations

CONCLUSIONS

The active damping of a quadratic plate clamped on all sides using piezo patches as actuators is the subject of this work. A state-space controller with modal feedback as well as local 2nd-order controllers with acceleration feedback have been implemented and compared. In order to account for actuator effects and to avoid spillover the Frequency Response Modes (FRM) were incorporated into the model.

Actuators and sensors have been optimally placed by numerically evaluating a criterion. Due to non-collocated disturbance and actuators only a small bandwidth for vibration damping is feasible if a small number of actuators is applied. This fact could be shown both in simulations and experimentally. Additional problems were posed by the symmetric eigenmodes and by the local reinforcement of the plate at collocated sensor/actuator pairs. A 2nd-order controller with acceleration feedback was designed as alternative, gaining a reduction of 17dB for mode at 167Hz.

Improvements in this application can be expected from non-collocated sensor/actuator pairs and from thinner yet stronger patches. A detailed FE-analysis would produce a more realistic model. Finally, the most suitable control structure for non-collocated disturbances is an $\mathcal{H}_\infty$-design, which allows for maximum damping bandwidth for a given set of sensors and actuators.

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