Survey on an Asymptotic Description of (Marginally) Separating Turbulent Boundary Layers *

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Basic assumptions and analysis

- nominally steady incompressible 2D TBL along solid surface $y = 0$
- no free-stream turbulence
- globally defined Reynolds number $Re = \tilde{U}\tilde{L}/\tilde{\nu} \to \infty$
- $\boxed{\text{Hypothesis}}$

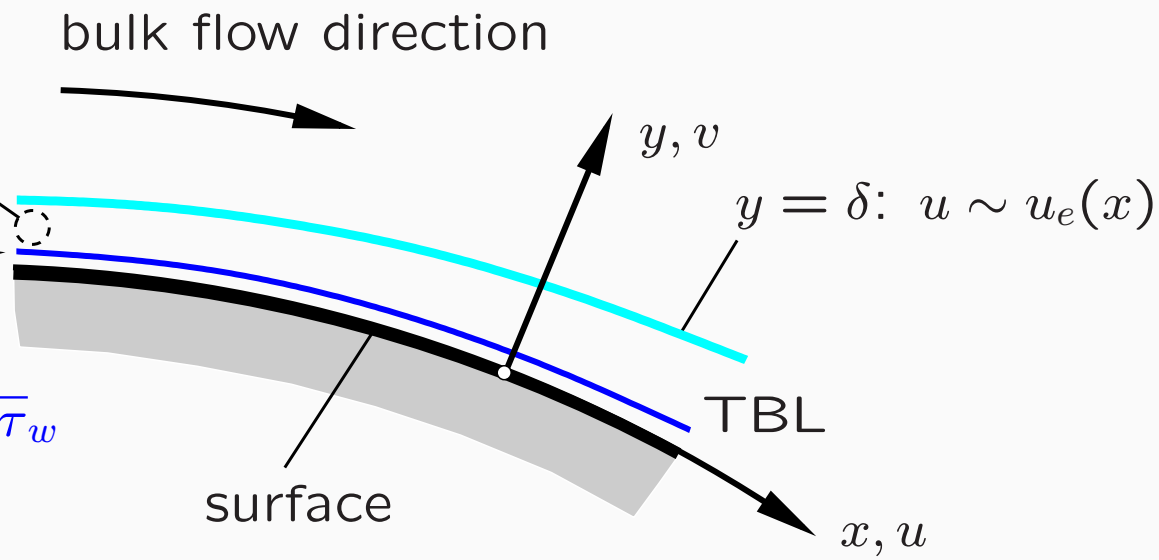
  layer of thickness Δ, assume local turbulent velocity scale $U_t \ll 1$:

  $\partial u/\partial y = O(U_t/\Delta) \;\Rightarrow\; -\langle u'^2\rangle, -\langle v'^2\rangle, -\langle u'v'\rangle \;=\; O(U_t^2)$

$$\Rightarrow \boxed{1 - \frac{u}{u_e} \ll 1}$$

overlap: log-law

$$\frac{\partial u}{\partial y} \sim \frac{U_t}{\kappa y}, \quad U_t = u_\tau = \sqrt{\tau_w}$$



near-wall scaling $u = O(u_\tau)$ from match with fully turbulent region!

Axiomatic reformulation of classical asymptotic theory

. . . by adopting a minimum of assumptions:

- outer main regime: velocity defect $1 - u/u_e \ll 1$
- viscous wall layer:

  order-of-magnitude analysis $\Rightarrow$ velocity scale u_τ $\Rightarrow$ log-law

$$u^+ = u/u_\tau \sim \kappa^{-1} \ln y^+ + C^+, \quad y^+ = y u_\tau / \tilde{\nu} \to \infty$$

- common overlap $\Rightarrow$ two-tiered TBL:

$$\gamma = u_\tau / u_e \sim \kappa / \ln Re, \;\; \delta = O(\gamma)$$

$$1 - u/u_e \sim \gamma(x)\, F'(x,\eta), \;\; -\langle u'v' \rangle \sim \gamma(x)^2\, T(x,\eta), \;\; \eta = y/\delta(x)$$

Extensions/shortcomings of small-defect approach

- 3D TBL, flow near plane of symmetry
  Degani, Smith & Walker 1992, 1993
- compressible TBL Walker 1995
- weak viscous/inviscid interaction:
  - ◇ flow past trailing edge of aligned flat plate
    Melnik & Chow 1975, Bogucz & Walker 1988, Neish & Smith 1988
  - ◇ TBL over humps Sykes 1980
    shock wave impingement
    Adamson & Feo 1975, Melnik & Grossman 1976,
    Adamson & Messiter 1977, 1980, Messiter 1980
    compression corner flow Agrawal & Messiter 1984
    small-defect assumption prevents separation
  - ◇ bluff-body separation Neish & Smith 1992
    separation at rear stagnation point
  - ◇ TBL exposed to smooth APG Neish & Smith 1992, Scheichl 2001
    separation not provoked at all
- Wall layer dynamics for attached time-mean flow Walker 1989

Extended small-defect theory – multi-valued solutions

- viscous wall layer: $\delta_v \sim (Re\, u_\tau)^{-1}$, $u_\tau^2 = Re^{-1}\, \partial u/\partial y|_{y=0}$ $U_t = u_\tau$
- defect region: $D = 1 - u/u_e \ll 1$

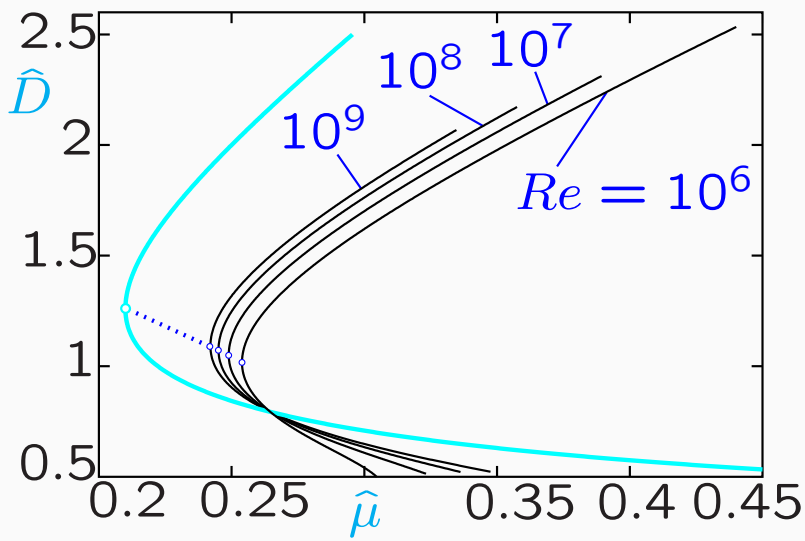
(i) classical theory: two-tiered $U_t = u_\tau$ $D = O(U_t)$

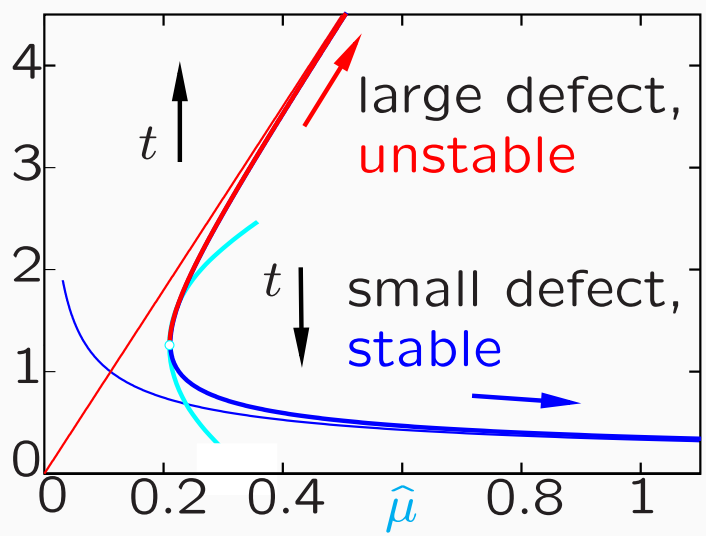
(ii) extended theory: three-tiered (Scheichl & Kluwick 2004)

intermediate layer: $U_t = u_\tau$

outer layer: $U_t \succ u_\tau$ $D = O(U_t)$ $Re^{-1} = 0$?

quasi-equilibrium flow: $u_e \propto x^m$, $m = -1/3 + \mu$, $(U_t, \mu) = O(u_\tau^{2/3})$





- $D = O(1)$ $\Rightarrow$ slenderness parameter $0 < \alpha \ll 1$ as $Re^{-1} = 0$

Serious shortcoming of small-defect approach

. . . with respect to (marginal/internal) separation:

- $\dfrac{\mathrm{d}p_e}{\mathrm{d}x} = -u_e \dfrac{\mathrm{d}u_e}{\mathrm{d}x} = O(1)$, continuous
- separation $\Rightarrow$ need for region where $D = 1 - \dfrac{u}{u_e} = O(1)$
- assumption of small defect $D = O(\epsilon) \ll 1$ in outermost region . . .

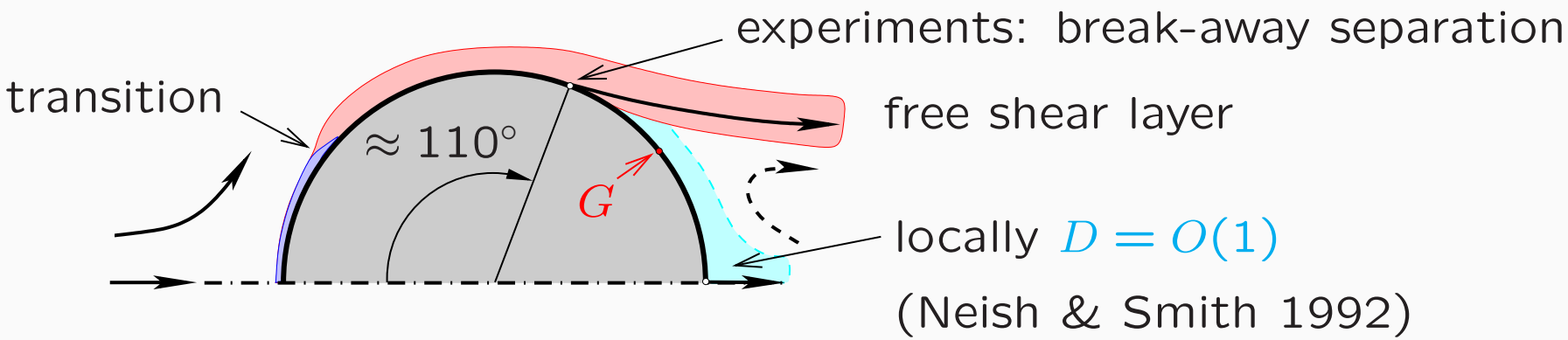
  $$1 - \frac{u}{u_e} \sim \epsilon\, F'(x,\eta) + \cdots\,, \quad \langle u'v' \rangle \sim \epsilon^2\, T(x,\eta) + \cdots\,, \quad \delta \sim \epsilon\, \Delta(x) + \cdots\,, \quad \eta = \frac{y}{\delta},$$

  - ◇ . . . yields $\dfrac{u_\tau}{u_e(x)} \sim \dfrac{\kappa}{\ln Re}$ due to log-law,
  - ◇ . . . does not allow for necessary growth of its order of magnitude!
- physical interpretation:

  insufficient vorticity transfer from wall layer via log-law $\Rightarrow$
  outer region governed by Bernoulli's law $\Rightarrow$ no flow reversal!
- consequence: $\boxed{\text{small-defect TBL does not separate at all}}$

  numerical corroboration: Gersten & Vieth (1995), Scheichl (2001)

case: bluff-body separation



attached bulk flow $\Rightarrow$ Goldstein-type singularity at G

- fundamental question: velocity defect $1 - \dfrac{u}{u_e} = O(1)$: $\alpha(Re) = ?$

Shear layer approximation

- expansions for $\alpha \to 0$:

$$\psi \sim \alpha\, \Psi(x, Y), \quad -\langle u'v' \rangle \sim \alpha\, T(x, Y), \quad \delta \sim \alpha\, \Delta(x), \quad y = \alpha\, Y$$

- BL equations:

$$\frac{\partial \Psi}{\partial Y} \frac{\partial^2 \Psi}{\partial Y \partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial Y^2} = u_e \frac{\mathrm{d}u_e}{\mathrm{d}x} + \frac{\partial T}{\partial Y}, \quad T = \ell^2 \frac{\partial^2 \Psi}{\partial Y^2} \left| \frac{\partial^2 \Psi}{\partial Y^2} \right|$$

- wake-type boundary conditions:

$$(\partial \Psi / \partial Y)\big[x, \Delta(x)\big] = u_e(x), \quad \Psi(x, 0) = T\big[x, \Delta(x)\big] = 0$$

Large velocity defect: BL slenderness α depends on Re ?

- yes: Sychev & Sychev (1980ies): mismatch encountered!
- no:
  - ◇ Melnik (1987, 1989): idea of two-parameter expansion
  - ◇ Schneider (1991): free shear layers
  - ◇ Scheichl & Kluwick (2006):

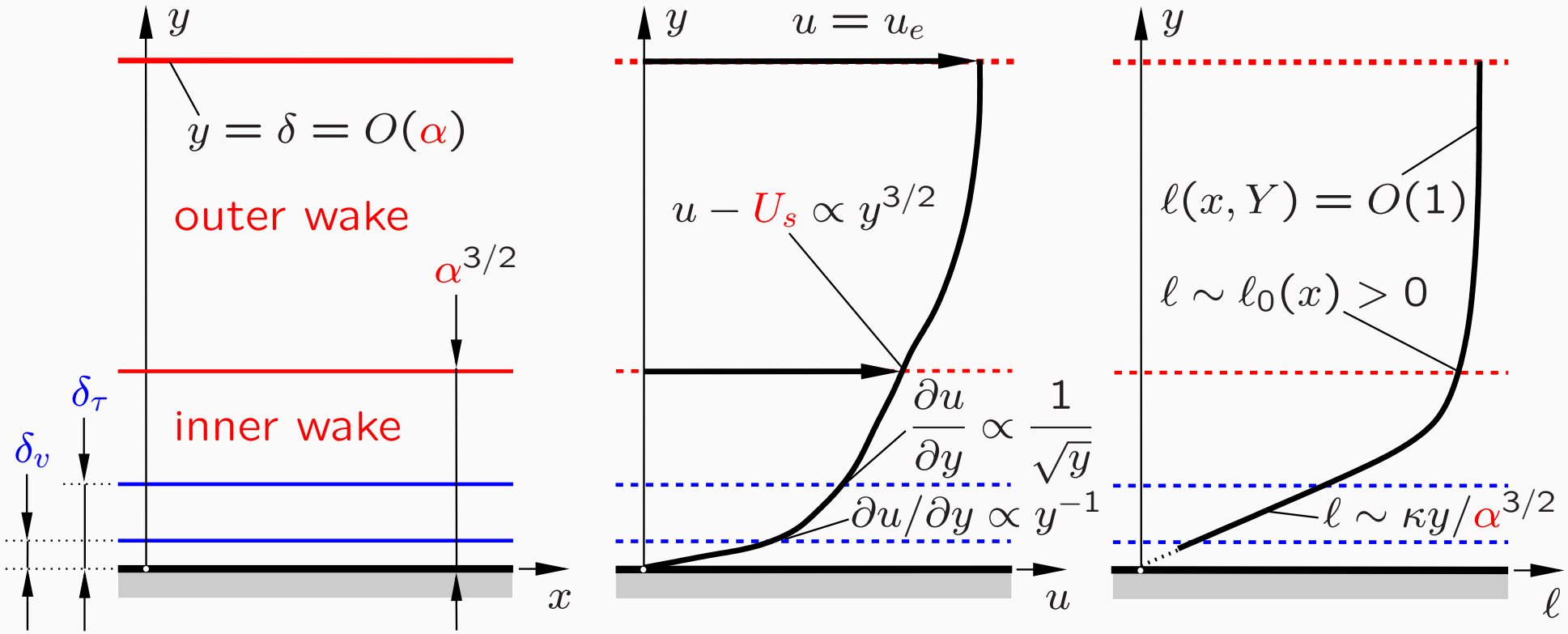
    $U_t = O(1)$ $\Rightarrow$ no small turbulent velocity scale

    $\Rightarrow$ most general structure of TBL, based on dynamical considerations (coherent motions)
  - ◇ note:

    turbulent dissipation $\varepsilon \sim Re^{-1} \left\langle \frac{\partial u_i'}{\partial x_i} \frac{\partial u_j'}{\partial x_j} \right\rangle = O(1)$

    k-budget: $\varepsilon \sim \underbrace{-\langle u'v' \rangle \frac{\partial u}{\partial y}}_{O(1)} - \frac{\partial \langle (k+p')v' \rangle}{\partial y}, \quad k = \frac{u'^2 + v'^2 + w'^2}{2}$

General asymptotic structure of APG-TBL flows



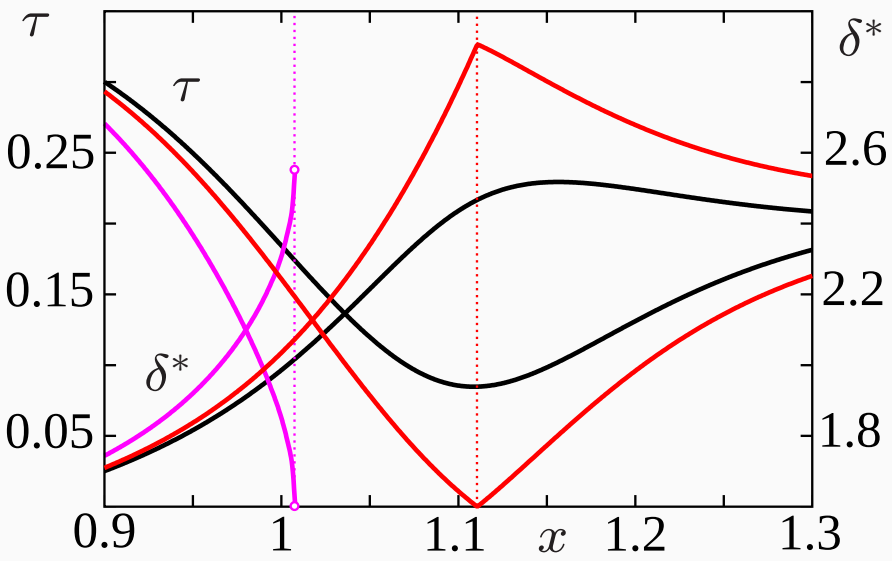
- order-of-magnitude analysis: rotational part of u of $O(U_t)$, $U_t = u_\tau$
- viscous wall layer: $\delta_v \sim (Re\, u_\tau)^{-1}$
- intermediate layer: $\delta_\tau \sim \tau$, transition from log- to half-power-law:

$$\eta \propto \frac{y}{u_\tau^2}, \quad -\frac{\langle u'v' \rangle}{u_\tau^2} \sim 1 + \eta, \quad u - U_s(x) \sim u_\tau\, U(x, \eta) \sim \frac{1}{\kappa} \begin{cases} \ln \eta, & \eta \to 0 \\ 2\sqrt{\eta}, & \eta \to \infty \end{cases}$$

BL solutions: marginally separated flows

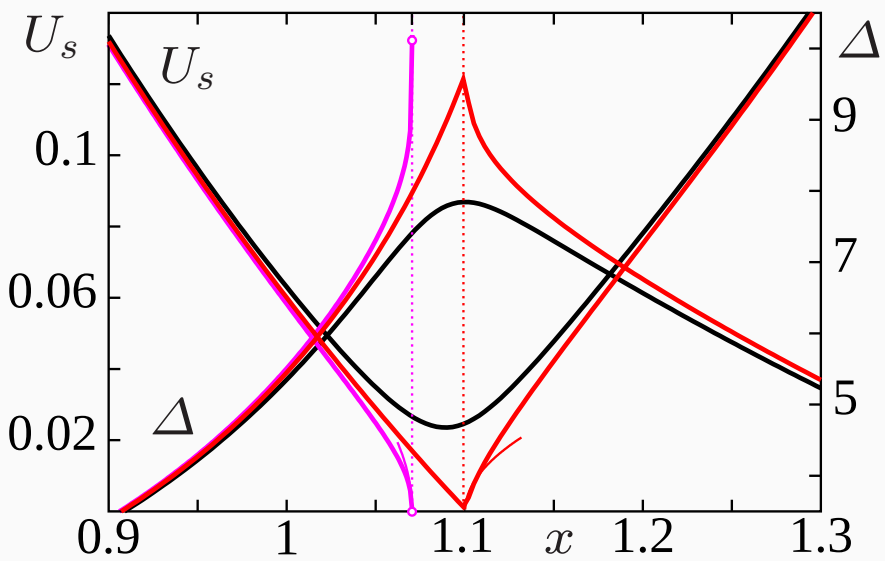
APG controlled by parameter: $u_e(x;\beta)$

laminar (Ruban 1981)



symmetric

turbulent (Scheichl & Kluwick)



asymmetric!

marginal-separation singularity for $\beta = \beta_M$,

solution linearized around separating profile upstream and downstream

nonlinear downstream wake

weak Goldstein singularity for $\beta > \beta_M$

Canonical local representation: bifurcating flows

local variable $s = x - x_M \to 0$

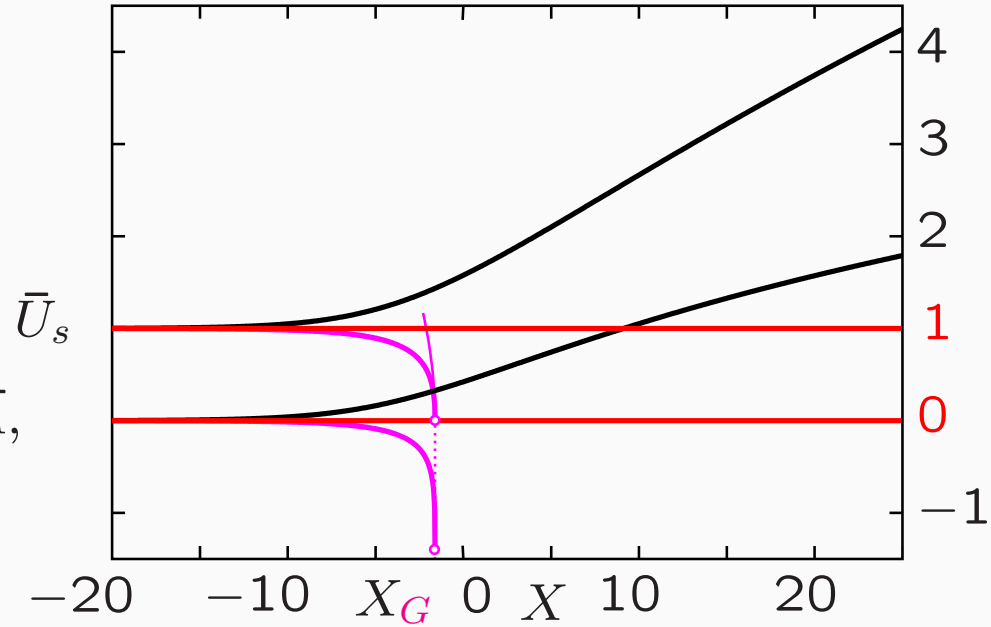
upstream shift $\epsilon \propto \hat{s} - s = O(-1/\ln|\beta - \beta_M|), \quad \beta - \beta_M \to 0$

inner scaling $X = \hat{s}/\epsilon^2, \quad \bar{Y} = Y/\epsilon^{2/3}, \quad \bar{U}_s \propto U_s/\epsilon, \quad \bar{T} \propto T/\epsilon^{2/3}$

exponential branching

$\bar{T} - \bar{Y} \to \bar{A},$

$\bar{Y} \to \infty$



- downstream: locally infinite acceleration (cf. Messiter 1970)

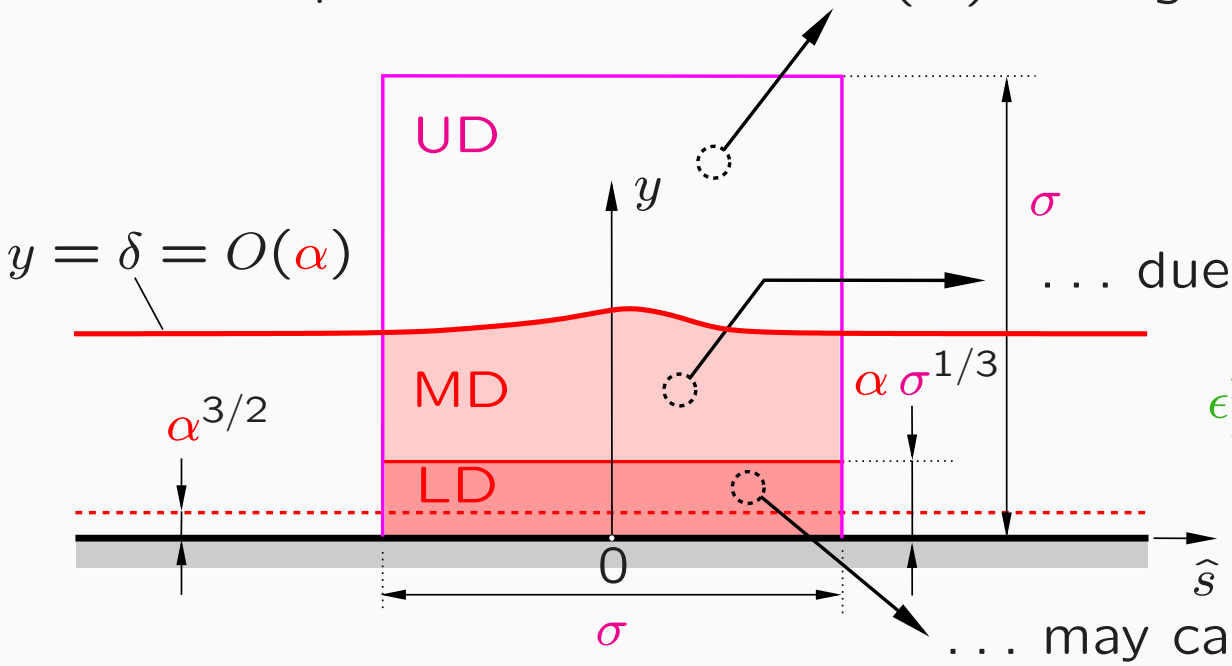
  $\bar{A} \sim 1.0386\ldots X^{1/3}, \quad \bar{U}_s \sim 1.1835\ldots X^{1/2}, \quad X \to \infty$

- upstream: induced *favorable* pressure $\Rightarrow$ *decreasing* $\hat{U}_s$

Local interaction theory: marginal separation

- ... singularity of BL solutions: $u_e = u_{e,M}(x) \;\Rightarrow\; U_s(x_M) = 0$
- $\hat{s} = x - x_M + O(\epsilon)$, $\epsilon \propto -1/\ln|u_e(x_M) - u_{e,M}(x_M)| \to 0_+$

Induced pressure feedback $\sigma\hat{P}(\hat{X})$ acting along $\hat{s} = \sigma\hat{X}$ ...



... due to displacement $-\hat{A}$ ...

$\epsilon^2 \preceq \sigma = (\epsilon/\Gamma)^2 \preceq \alpha^{3/5}$

$1 \geq \Gamma \geq 0$

... may cause separation

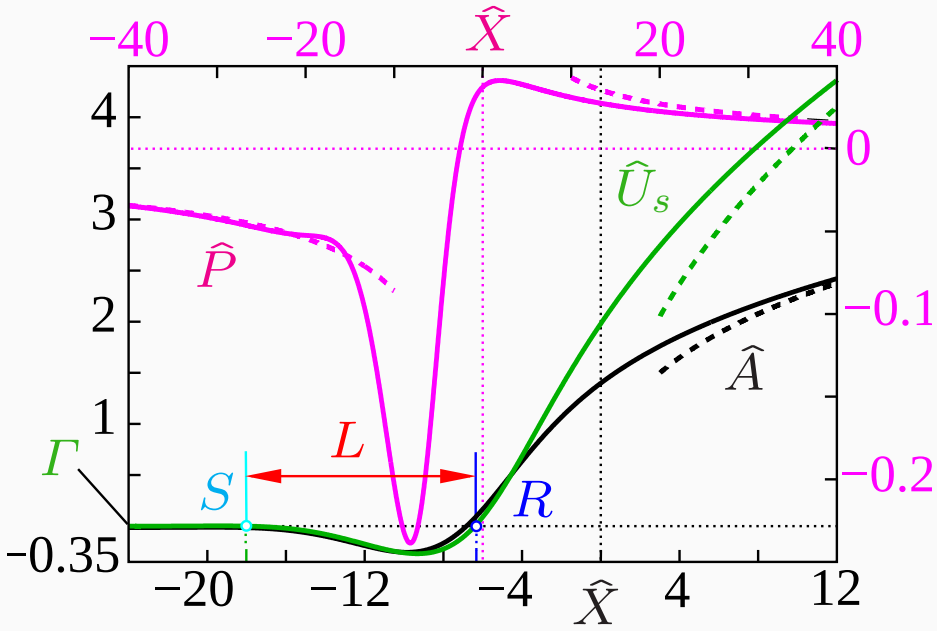
- upper deck UD: $p = P_M + \sigma\left[P'_M\,\hat{X} + \Lambda(\Gamma)\,\hat{P}(\hat{X})\right] + \cdots$
- main deck MD: $\psi/\alpha = \Psi_M(Y) + \sigma^{1/3}\,\ell_{0,M}^{2/3}\,\hat{A}(\hat{X})\,\Psi'_M(Y) + \cdots$, $Y = y/\alpha$
- lower deck LD: $\hat{P} = -\mathcal{H}\{\hat{A}'\}$ enters resulting *interaction problem*

Outer wake: triple-deck solutions

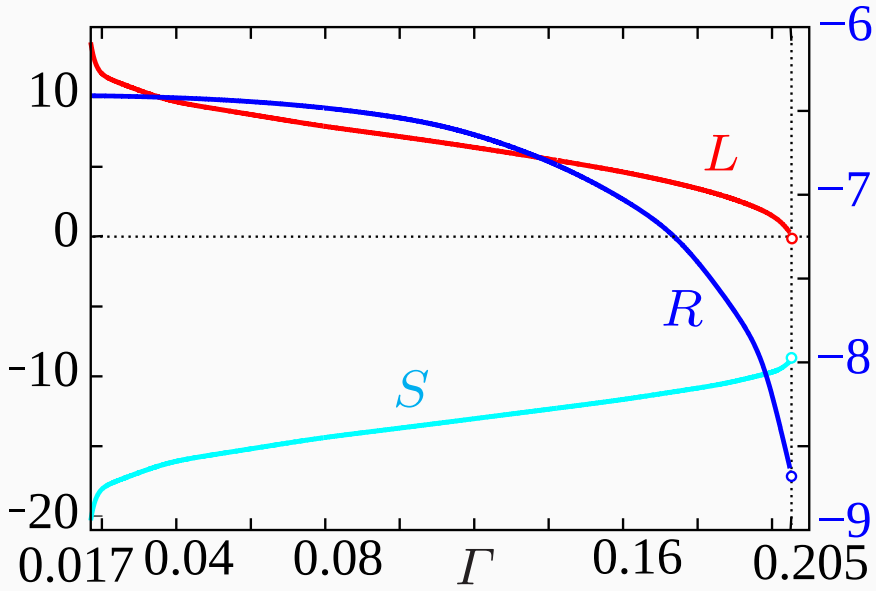
- central FDs, algebraic eqs solved directly (Powell's hybrid method)

  boundary condition for $\hat{X} \to -\infty$: $\hat{U}_s \to \Gamma$

$\Lambda = 3$, $\Gamma = 0.019$



$\Lambda = 3$



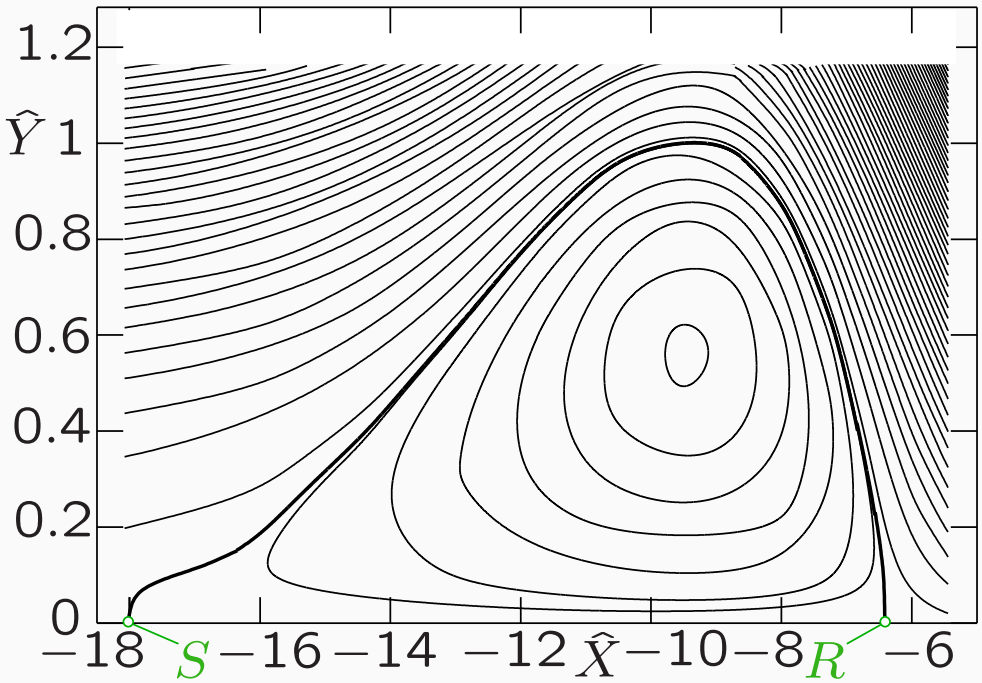
- small second bubble upstream for $0.016\ldots \le \Gamma \le 0.019\ldots$
- $\Gamma \to 0$: exponentially bifurcating eigensolutions vanish, $L \to \infty$
- $\Lambda \to 0$: BL solution – and massively separated solution for $\beta > \beta_M$?

Triple-deck solutions, cont'd

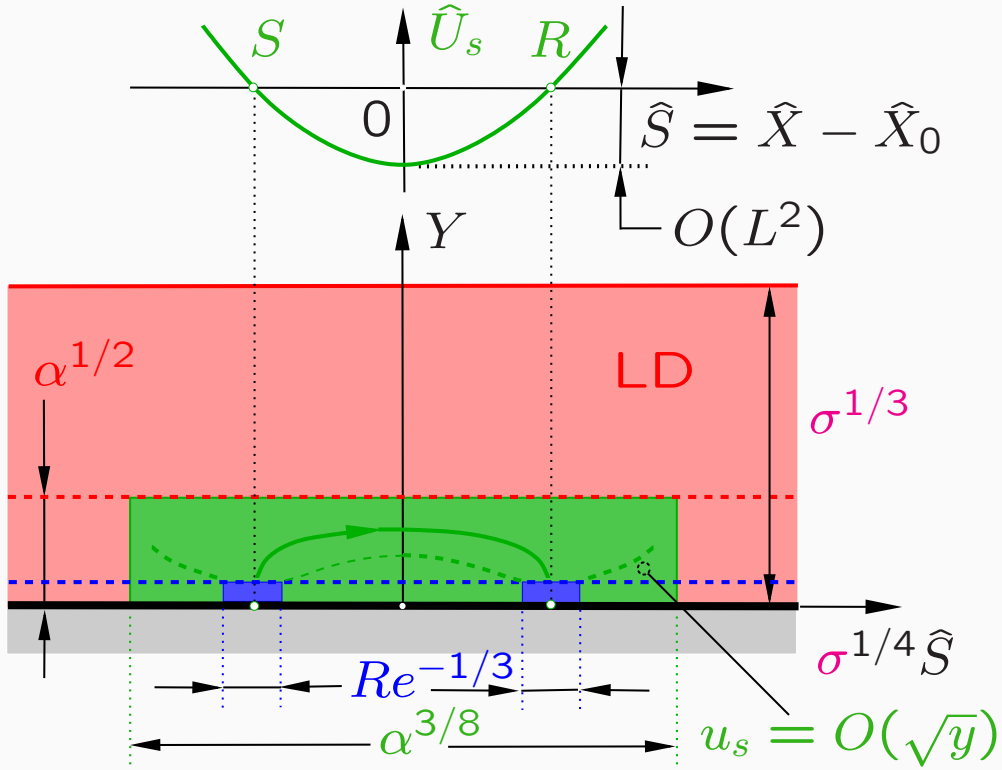
- effect of sublayers near surface on separating solutions?

  overall slip velocity $u_s(x,\alpha) = \sigma^{1/2}\,\hat{U}_s(\hat{X}) + O(\alpha^{3/4})$ $\Rightarrow$ breakdown

LD, $\hat{\Psi} = const$: $\Lambda = 3$, $\Gamma = 0.019$



small bubble, $L \to 0$:



- inner wake, $\hat{X} - \hat{X}_0 = O(L)$, $L = O(\alpha^{9/40})$, $\Gamma \approx 0.205$:

  $-\partial\langle u'v'\rangle/\partial y \sim P_0$, $\partial p/\partial x \sim P_0 = P'_M + \Lambda(\Gamma)\,\hat{P}'(\hat{X}_0)$ governs reverse-flow,

- smoothly separating solution, log-law $\mapsto$ half-power law!

Gradual transition from log-law to half-power law

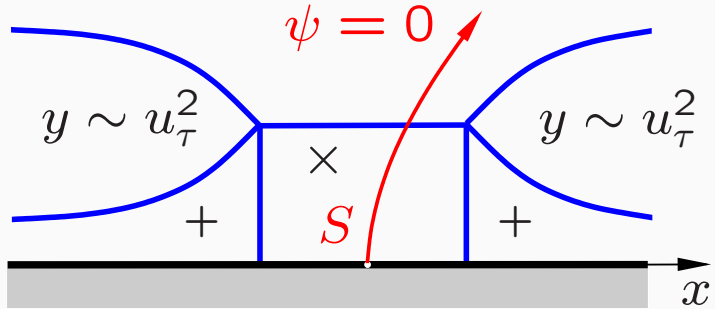
- viscous wall layer $u_\tau \gg Re^{-1/3}$

$$u/u_\tau \sim u(x,y^+) + \cdots, \quad -\langle u'v'\rangle/u_\tau^2 \sim \tau^+(x,y^+) + \cdots, \quad y^+ = y\,u_\tau Re$$

log-law: $\boxed{u \sim A^+(x)\,\ln y^+ + B^+, \quad y^+ \to \infty}$

$$\mathrm{sgn}(A^+) = \mathrm{sgn}(\tau_w), \quad \tau_w > 0: \quad A^+ = \kappa$$

skin friction law: $u_\tau/u_s \sim A^+/\ln(u_\tau^3\,Re)$



- viscous wall layer $u_s,\ u_\tau = O(Re^{-1/3})$

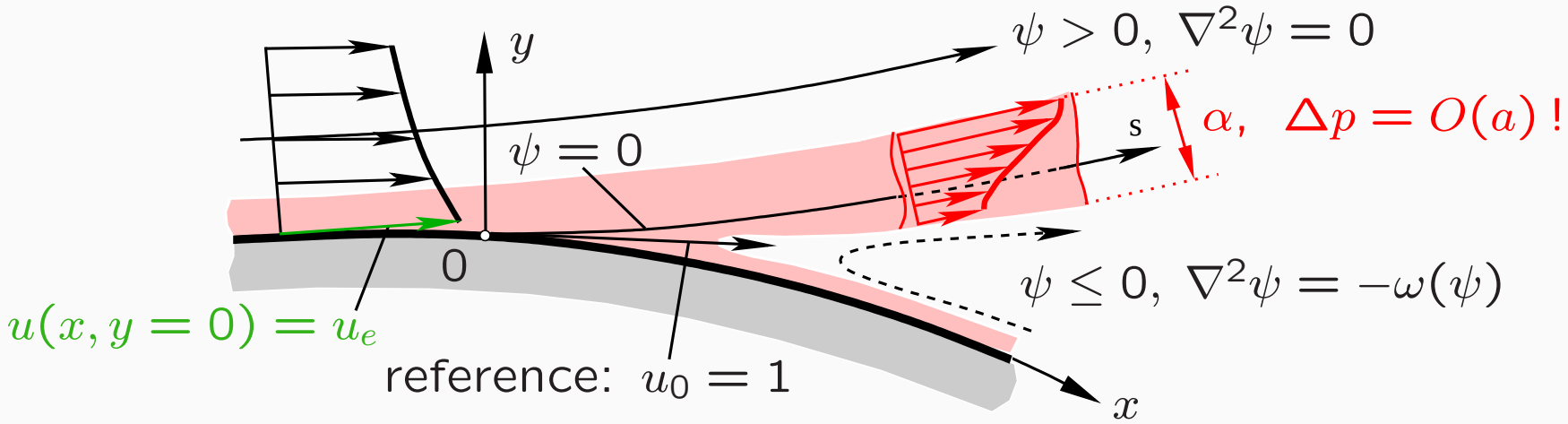
$$u/u_p \sim u(p^\times, y^\times) + \cdots, \quad -\langle u'v'\rangle/u_p^2 \sim \tau^\times(p^\times, y^\times) + \cdots, \quad y^\times = y\,u_p Re,$$

$$p^\times = \mathrm{sgn}(\tau_w)\,(u_p/u_\tau)^3, \quad u_p = (P_0/Re)^{1/3}$$

half-power law: $\boxed{u \sim A^\times(p^\times)\sqrt{y^\times} + B^\times(p^\times), \quad y^\times \to \infty}$

skin friction law: $u_s/u_p \sim B^\times(p^\times)$ . . . separation!

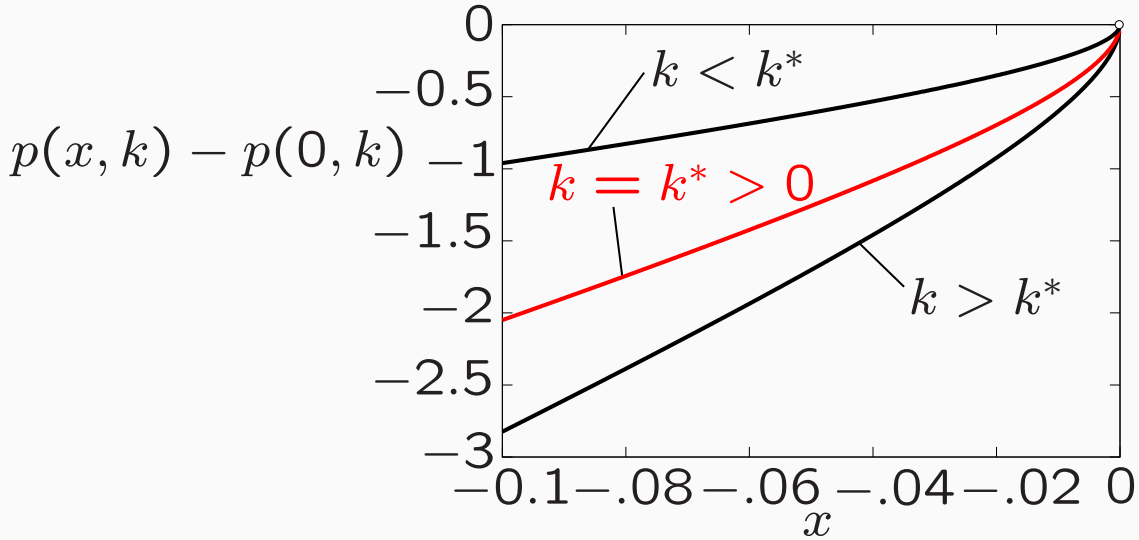
Turbulent gross separation – preliminary results

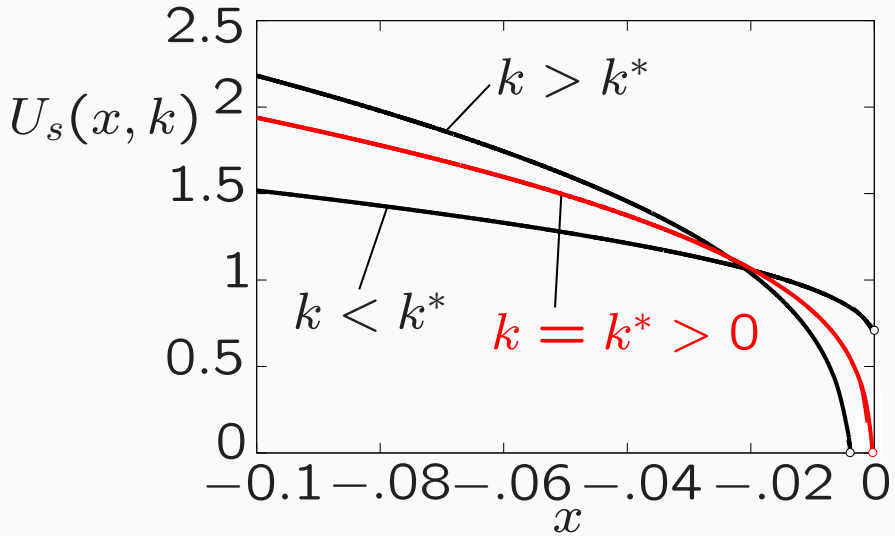


- inviscid-flow theory (including rotational backflow):

$$\frac{\partial p}{\partial x}(x, y = 0) = k\,(-x)^{-1/2} + \frac{16}{3}k^2 + O\big((-x)^{1/2}\big), \quad k \ge 0, \quad x \to 0_-$$

- formal limit $Re^{-1} = 0$, $\alpha \to 0$: Brillouin–Villat condition $k = 0$ not met!





Further outlook

- basic hypothesis for TBL scaling derivedfrom first principles: multiple-scales analysis of Navier-Stokes eqs as $\alpha \to 0$
- closed reverse-flow region: $A^+(x)$, $B^+(x)$
- extended skin-friction formula: $B^\times(p^\times)$
- global separation