

LQG/LTR Controller Design for a Gas Engine

W. Hofbauer*, P. Dolovai*, H. P. Joergl*
Department of Mechanics and Mechatronics
Technical University of Vienna
1040 Vienna, Austria

J. Hirzinger**
Jenbacher Gas Engines
GE Energy
6200 Jenbach, Austria

Abstract—The dynamics of a gas engine vary strongly over its power range with controlled engine speed. The major nonlinearities result from the turbo charger driven by exhaust gas. The recirculation valve adjusts the power of the turbo charger, by leading part of the compressed mixture back to the inlet. The position of the throttle valve affects the mass flow into the engine. Both actuators are used to control the engine speed. In order to optimize the switching between the actuators, LQG control is applied.

For linear state feedback control suitable states have to be determined and unmeasured states must be estimated in an acceptable way. This paper shows that LQG control is able to enhance the performance of the engine speed control by using both actuators simultaneously.

I. INTRODUCTION

Modern gas engines are charged by exhaust gas turbo chargers. Their performance is influenced either by a wastegate on the exhaust side or by a recirculation valve on the fresh air side. Both methods cause better but nonlinear performance of the gas engine. Additionally, a throttle valve is commonly used to control the pressure load. Black/grey box models have been developed by identification methods [4] and physical formulations [5]. Anyway, turbo charger models persist in compressor and turbine maps and are based on efficiency and power output.

The gas engines used are internal combustion engines with 6 to 20 cylinders of 2 to 6 liters displacement each. The mixture of fresh air and natural gas is charged by an exhaust gas driven turbo charger with up to 3 atmospheres. The engines are working either as part of a main supply or as independent power supply stations. In the first working mode the generator forces the engine at a fixed speed, hence, frequency of the main supply and the power output of the engine has to be controlled. In the second mode the engine is isolated and must provide a certain frequency of the delivered AC voltage. Therefore, the engine speed is controlled and power output changes act as the disturbance. The quality of this control is important for competitive ability and reliability and is therefore object of the main research.

The engines are controlled by a throttle valve and a recirculation valve. In addition, the gas concentration is adjusted to obtain the exhaust gas quality regulated by legislation. The engine speed is disturbed by the generator consumption, which must be compensated. The most important sensors, which are installed in the engines measure the power consumption of the generator, the engine speed and the boost

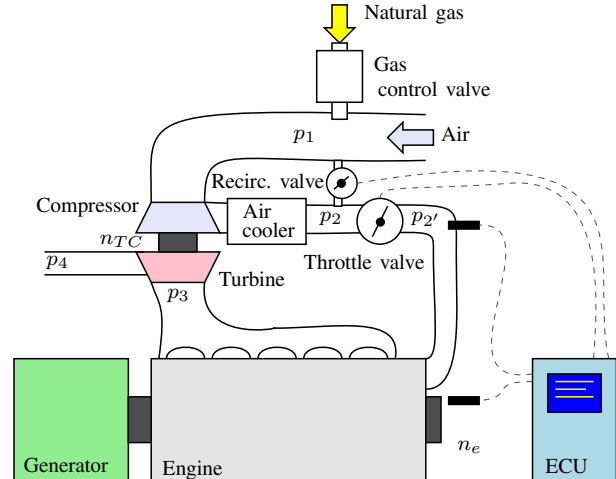


Fig. 1. Airsystem of a Gas engine

pressure.

The current control design consists of independent PID compensators for throttle valve and recirculation valve. The parameters of the compensators are adjusted over the power range to handle the varying dynamics of the engine. To avoid interaction instability the two actuators are switched by a stateflow algorithm. The main focus is to replace this switching by an optimized multi input control. Based on a linear feedback rule and a quadratic criterium the LQ regulator minimizes the energy consumption of more than one actuators. This requires a linearized state space model of the plant and an estimation of the states, which cannot be measured.

The gas engine is modeled by five states, two of them are measured. A linear Kalmanfilter is able to estimate the remaining states shown in [3]. The engine speed must be hold at a desired value, therefore the integrated tracking error is an additional state for the LQ compensator design.

II. PHYSICAL MODEL

The engine model can be divided into the thermodynamical model on one hand and the mechanical model on the other hand. The main dynamics are represented by the moments of inertia of the crank shaft together with the generator J_e and the turbo charger J_{TC} .

The airsystem of a turbo charged engine with recirculation

valve is depicted in Fig. 1. Fresh air enters the inlet and is mixed with natural gas either by Venturi effect or by injection. After compression the mixture is cooled to a temperature of $320K$ and passes into the engine via the throttle valve. The recirculation valve leads a part of the mixture back to the inlet.

The aircsystem of a gas engine can be divided into the ducts (commonly enumerated as)

- inlet (1)
- compressor - throttle valve (2)
- throttle valve - engine (2')
- engine - turbine (3)
- turbine - outlet (4)

The inlet and the outlet can be seen as infinite mass reservoirs under constant pressure. The pressure rates p_i in the other ducts can be derived from the ideal-gas equation ([1] and [2]) with the corresponding gas constant R , temperature T_i and volume V_i . The influence of the temperature in comparison with the mass flows is neglected and therefore the temperatures are assumed constant. The mass flows over recirculation valve \dot{m}_r and throttle valve \dot{m}_v can be defined using the isentropic throttle equation. The theoretical mass consumption of the engine computed by multiplying the engine speed with displacement is corrected by the volumetric efficiency η_{vol} which depends on the pressure load.

The pressure rise of the compressor and its efficiency are provided by the manufacturer as a stationary map depending on volume flow and turbo charger speed. In order to obtain the mass flow through the compressor and its power consumption the maps must be converted.

The turbine is also described by the isentropic throttle equation with the gas constant R_E and the isentropic coefficient κ_E of the exhaust gas, whereas the effective cross-section varies over the pressure drop of the turbine. The temperature of the exhaust gas is derived from the pressure drop of the turbine. The effective cross section area A_t and the efficiency η_t are provided by the manufacturer in form of numerical maps.

The common form of a nonlinear state space system linear in input and disturbance is denoted by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} + \mathbf{e}(\mathbf{x})z \quad (1)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}). \quad (2)$$

As mentioned above the state and input vector are:

$$\mathbf{x} = [p'_2 \quad p_2 \quad p_3 \quad n \quad n_{TC}]^T \quad (3)$$

$$\mathbf{u} = [A_v \quad A_r]^T \quad (4)$$

$$z = P. \quad (5)$$

In all equations mass flow and power relate the time derivation of the pressure of the three ducts and the speed of engine and turbo charger. Therefore these quantities form the 5 dimensional state vector \mathbf{x} .

The effective cross section areas of throttle and recirculation valve are treated as inputs into the system. The power consumption is treated as disturbance input. Defining the engine speed as output the state space model is a MISO system of order 5 with 3 inputs. Since the boost pressure is measured directly it can also be used for observing purposes, so the system is a MIMO system with 2 outputs, 2 actuator inputs and 1 disturbance input.

The nonlinear equations are modeled in MATLAB/Simulink. Validation of the simulation model is done by measurement data. With the *Control Design Toolbox*, which is provided by MathWorks for calculating operating points and the Jacobian, one obtains the common linear state space form of

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}z \quad (6)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}. \quad (7)$$

A computational result of a gas engine with 20 cylinders with 3 liters displacement each at an operating point of $350kW$ is

$$\mathbf{A} = \begin{bmatrix} -33.43 & 25.94 & 0 & -914.12 & 0 \\ 11.48 & -22.73 & 0 & 0 & 77.14 \\ 28.95 & 0 & -54.64 & 1985.97 & 0 \\ 0.0043 & 0 & 0 & 0.30 & 0 \\ 0.0002 & -0.033 & 0.14 & 0 & -0.26 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 26868.68 & 0 \\ -15347.59 & -1030.33 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00128 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For observing purposes the measurement matrix \mathbf{C}^* takes the boost pressure as additional output into account,

$$\mathbf{C}^* = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \quad (8)$$

The state space model of the gas engine has only real poles. In open loop operation the engine accelerates the turbo charger, which increases the boost pressure and hence the engine speed. This self exiting action causes an unstable pole also close to the imaginary axis. The three dominant stable poles due to the gas dynamics are fast and stable. When the operating point is shifted to higher power output, the two poles near the imaginary axis move towards it, but the three other poles get even faster, because the mass flows grow but the volumes of the ducts remain the same.

The state space model of the gas engine meets all conditions for a LQG design, as it is fully observable and controllable. There is no direct effect of the input on the output signals, i. e. $\mathbf{D} = 0$.

III. LQG/LTR THEORY

A. Kaman-filter and LQG

A linear state feedback law

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \quad (9)$$

together with a quadratic cost function

$$\int_0^\infty \frac{1}{2}(\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (10)$$

forms the Linear Quadratic Regulator. Real systems are influenced by system noise \mathbf{w} which is described by the additional input matrix \mathbf{B}_w and imperfect sensors cause measurement noise \mathbf{v} .

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{B}_w \mathbf{w} \quad (11)$$

$$\mathbf{y} = \mathbf{C}\hat{\mathbf{x}} + \mathbf{v}. \quad (12)$$

If states are estimated by a Kalman-filter with noise based weighting matrices,

$$\mathbf{Q}_K = E(\mathbf{w}'\mathbf{w}) \quad \text{and} \quad \mathbf{R}_K = E(\mathbf{v}'\mathbf{v}) \quad (13)$$

the design is a so called Linear Quadratic Gaussian Regulator. The relation between system noise and measurement noise assigns the dynamics of the Kalman-filter. It is limited by the noise covariances, because too fast observation amplifies noise and the estimated states are useless. The feedback gain matrix of the Kalman-filter is $\mathbf{H} = \mathbf{P}_K \mathbf{C}^T \mathbf{R}_K^{-1}$. It is calculated with \mathbf{P}_K as the unique stabilizing solution of the Riccati Equation (14).

$$\mathbf{A}\mathbf{P}_K + \mathbf{P}_K \mathbf{A}^T - \mathbf{P}_K \mathbf{C}^T \mathbf{R}_K^{-1} \mathbf{C} \mathbf{P}_K + \mathbf{Q}_K = 0 \quad (14)$$

Hence, the state feedback gain is calculated as special LQ regulator design with $\mathbf{K} = \mathbf{R}_C^{-1} \mathbf{B}^T \mathbf{P}_C$ and \mathbf{P}_C the unique stabilizing solution of the Riccati Equation (15)

$$\mathbf{A}^T \mathbf{P}_C + \mathbf{P}_C \mathbf{A} - \mathbf{P}_C \mathbf{B} \mathbf{R}_C^{-1} \mathbf{B}^T \mathbf{P}_C + \mathbf{Q}_C = 0 \quad (15)$$

The state space representation of the closed Kalman-filter is

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{H}\mathbf{C}^*)\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{H}\mathbf{y}$$

The state space representation of the closed compensator together with the closed Kalman-filter yields

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= (\mathbf{A} - \mathbf{H}\mathbf{C}^* - \mathbf{B}\mathbf{K})\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{H}\mathbf{y} \\ \mathbf{u} &= -\mathbf{K}\hat{\mathbf{x}} \end{aligned}$$

In Fig.2 the state vector is fed back twice, one time via the Kalman gain matrix and the second time via the state feedback matrix, which can also be seen as the output matrix of the compensator, whereas the Kalman gain matrix acts as the input matrix. The aggregation of the observer and the

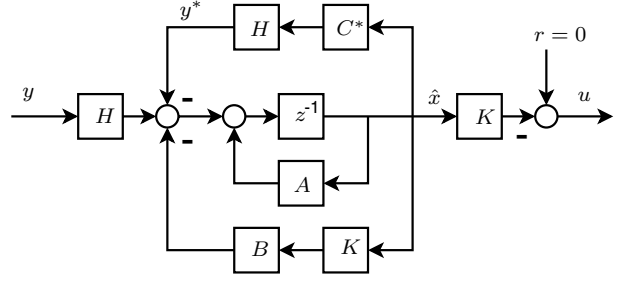


Fig. 2. State space representation of state feedback compensator together with Kalman-filter

state feedback makes the implementation of the compensator into the hardware easier.

In order to avoid a tracking error of the engine speed the integral of it is introduced as an additional state e

$$\dot{e} = \hat{y} - w = \mathbf{C}\hat{\mathbf{x}} \quad (16)$$

Note that the only output for the compensator is the engine speed, therefore \mathbf{C} is used instead of \mathbf{C}^* . With the new state vector the dimension of the Kalman-filter remains the same whereas the dimension of the compensator feedback increases by 1. The compensator dynamics together with the tracking error dynamics in state space form yields

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{H}\mathbf{C}^* - \mathbf{B}\mathbf{K} & 0 \\ \mathbf{C} - \mathbf{B}\mathbf{K} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ e \end{bmatrix} + \begin{bmatrix} \mathbf{H} \\ \mathbf{0} \end{bmatrix} \mathbf{y}$$

If the measured signal is to be used to compute the tracking error the state space representation of the compensator changes to

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{H}\mathbf{C}^* - \mathbf{B}\mathbf{K} & 0 \\ -\mathbf{B}\mathbf{K} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ e \end{bmatrix} + \begin{bmatrix} \mathbf{H} \\ [01] \end{bmatrix} \mathbf{y}$$

In both cases the control signal yields $\mathbf{u} = -\mathbf{K}\hat{\mathbf{x}}$. The Kalman-filter design for the gas engine shows good performance of linear observers over the nonlinear power range of the gas engine

B. Loop transfer recovery

Due to the separation principle of observer and compensator, the dynamics of the compensator only depends on the demands of the closed loop. It is commonly recommended to design the observer much faster than the compensator, otherwise the used estimates are wrong and control performance is lost. State space feedback provides a phase margin of at least 60deg and a gain margin of 2, see [6], if all states can be measured. If states must be estimated this robustness can be recovered by approximating the open loop dynamics to the dynamics of the Kalman-filter. This method is called *loop transfer recovery*. Loop transfer recovery is often shown by the frequency response of the open loop. The Kalman gains are computed the same way as the compensator feedback matrix.

TABLE I
THE MOST IMPORTANT PARAMETERS OF THE SIMULATED
GAS ENGINE

Quantity	Value	Unit
volume 2	0.1	m^3
volume 2'	0.041	m^3
volume 3	0.052	m^3
inertia of engine	47.51	kgm^2
inertia of charger	0.069	kgm^2

If a state feedback regulator is designed separately by pole placement or LQ design, the singular values of the closed loop will be smaller than the values of the open loop shaping filter. Therefore, the control system doesn't reach the quality and the robustness of the Kalman-filter. By reducing the weight R_C on the control variables successively the open loop can be made faster and the singular values of the open loop will reach the values of the estimator in the most important frequency range.

However, this design procedure does not result in good control performance for practical purposes since the open and the closed loop behavior respectively become too fast and therefore highly noise sensitive. Nor are constraints on the control variables regarded. Thus for achieving good performance the compensator and the filter are still designed separately.

For practical purposes in the following the system is discretized with the engine control units sampling time of $80ms$. The Kalman gain and the state feedback gain are calculated with the MATLAB function `dare.m`, which solves the unique stabilizing solution of the discrete time equivalents of equations (14) and (15).

IV. SIMULATION RESULTS

Measurement data from the test bench are used to compute the noise covariances for designing a Kalman-filter. The gas engine used for the simulations is an engine with 20 cylinders with 3 liter displacement each. The gas is added by a gas mixture valve. The quantities pressure after compressor, pressure before turbine and turbo charger speed are measured to compare them with the estimated values of the observer.

The nonlinear simulation model [3] is implemented in MATLAB/Simulink. All relevant parameters of the engine are stored in a structure variable. Most important are the volumina of the three ducts, the moment of inertia of the crank shaft and the the turbo charger as well as the characteristic maps of mass flow and efficiency of the turbo charger. By changing the parameter set, all other engine types can be simulated without changing the model itself.

The values of the most important parameters are listed in Table I

LQG with integration of control error

At the operating point of $350kW$ the weighted system noise covariance matrix Q_K and the measurement noise

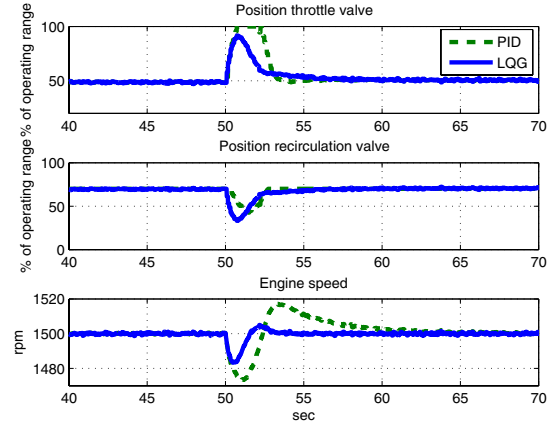


Fig. 3. Engine speed (bottom) and throttle valve (top) and recirculation valve (middle) positions at a load step step of $+50kW$ taking place at $t = 50s$, operating point $350kW$, compared with the PID controllers. Simulation data of nonlinear engine

covariance matrix R_K for the filter design were chosen to $Q_{Kii} = [10e2 \ 10e2 \ 10e2 \ 10e3 \ 1]$ and $R_{Kii} = [0.1 \ 0.1]$, leading to a Kalman gain

$$H = \begin{bmatrix} 0.99951 & -1.9306 \cdot 10^{-6} \\ 0.42133 & 0.00044225 \\ -0.21568 & 0.002055 \\ -1.8137 \cdot 10^{-6} & 0.99987 \\ 0.054369 & 0.00024816 \end{bmatrix}$$

The choice of the weighting matrices for controller design to diagonal matrices $Q_{Cii} = [1 \ 1 \ 1 \ 1 \ 1 \ 1e10]$ and $R_{Cii} = [1e9 \ 1e8]$ results in a feedback matrix

$$K = \begin{bmatrix} 0.00020049 & -0.00019839 \\ 0.00020805 & -0.00020774 \\ 1.6604 \cdot 10^{-5} & -1.348 \cdot 10^{-5} \\ 0.94277 & -0.94652 \\ 0.0066191 & -0.0054273 \\ 0.8655 & -0.82395 \end{bmatrix}^T$$

The performance of this LQG-designed compensator compared with the PID compensators is depicted in Figure 3. The engines speed response to a load step of $+50kW$ at the operating point of $350kW$ is shown in the lower part of the figure, above the absolute positions of the the throttle valve and the recirculation valve are plotted.

In comparison to the conventional PID control, where the two actuators are switched to avoid destabilizing interaction, the desired engine speed is achieved much faster after a step in the disturbance, the power output of the generator. Both actuators react together in a more efficient way on the power output changes, the recirculation valve is more active resulting in an improved step response.

Using the weighting matrix R_c the interaction of the actuators can be adjusted. Putting more weight on the recirculation valve forces the the throttle valve to be more active. Within the states the weighting of the tracking error

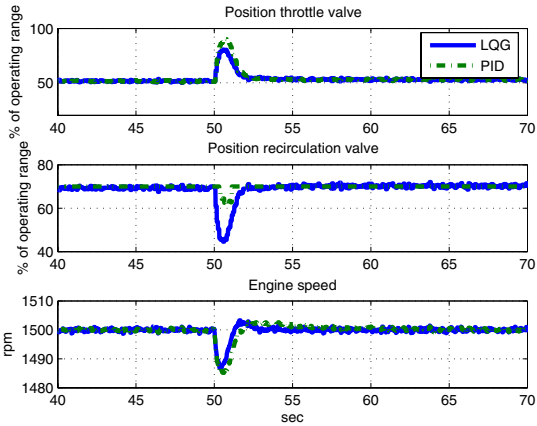


Fig. 4. Engine speed (bottom) and throttle valve (top) and recirculation valve (middle) positions at a load step of $+50kW$ taking place at $t = 50s$, operating point $500kW$, compared with the PID controllers. Simulation data of nonlinear engine

has the most important role. A weighting factor between $1e4$ and $1e6$ adjusts the settling time between $20s$ and $5s$ without seriously affecting the overshoot. But this causes the actuators to move faster. Simulations help to limit the positioning velocity of the actuators.

Due to the good filter performance over the whole operating range the compensator can be tested at other operating points as well. Figure 4 shows the performance of the same LQG - Regulator/Filter at an engine load of $500kW$ with a load step of $+50kW$ taking place again. It can be clearly seen that there is no lack of performance, the engine speed response is even better.

V. EXPERIMENTAL RESULTS

Experiments were performed at a test bench with an engine with similar characteristics as the engine investigated in simulations. The test bench engine has 12 cylinders with 4 liters displacement each, the maximum power output is also at $1000kW$. The most important values of parameters of the test bench engine are listed in Table II. All control loops of the engine are calculated by a programmable logic controller (PLC). Besides of the engine speed control which can be switched to the power control if the engine produces electrical energy for the general power supply, the gas concentration is controlled by the gas mixer. There are even more control loops implemented for charged air temperature, knocking or oil temperature.

The PLC can be programmed by ANSI C or structured language, There is no direct Hardware-in-the-loop access for MATLAB/Simulink. Therefore, the simulation results have to be converted manually to PLC-able program code. This procedure is accelerated by a self made GUI of MATLAB/Simulink which exports the state space matrices of the obtained compensator via OPC. All values can be changed manually while operating the engine, but the matrices must be changed as a whole, so this way is more convenient. Since

TABLE II
THE MOST IMPORTANT PARAMETERS OF THE TEST BENCH
GAS ENGINE

Quantity	Value	Unit
volume 2	0.3	m^3
volume 2'	0.0884	m^3
volume 3	0.071	m^3
inertia of engine	98.3	kgm^2
inertia of charger	0.069	kgm^2

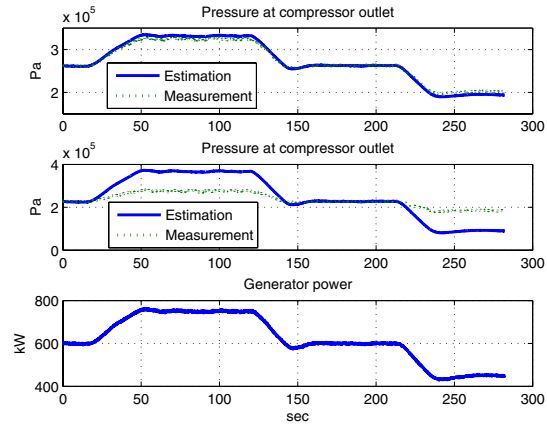


Fig. 5. Estimated values of pressure at compressor outlet (top) and turbine inlet (middle) in comparison with measured values and the recorded power (bottom) at an operating point of $600kW$

there does not exist a library for matrix manipulation the most important calculations are programmed as new library

The Kalman-filter provides a good estimation of the five states if the engine remains near to the operating point, as it is shown in Fig. 5

The engine is controlled at a fixed setpoint of $1500rpm$. The power consumption of the generator defines the disturbance input.

At this first test run it was only possible to operate the LQG controlled engine stable at lower power consumption of the generator. At this operating points the engine generally behaves as it is controlled by PID compensators. The recirculation valve remains constant at a position of 70%, whereas the engine speed is controlled by the throttle valve. This is shown in Fig. 6. A direct comparison with the PID controller was not possible at that test bench.

The LQG compensator can be adjusted by the weighting matrices Q_C and R_C of the states and the inputs. The loop transfer recovery suggests to put less weight on the inputs to make the closed loop faster until it reaches the speed of the estimation. The experiment shows that this is limited by the actuator speed. The design of the Kalman-filter and the LQG compensator is based on the same weighting matrices as they are used for simulation. But tests show that the weight of the throttle valve can be reduced to $1e9$. This also results in better control performance. Decreasing the weight of on the throttle valve also affects the damping of the closed loop.

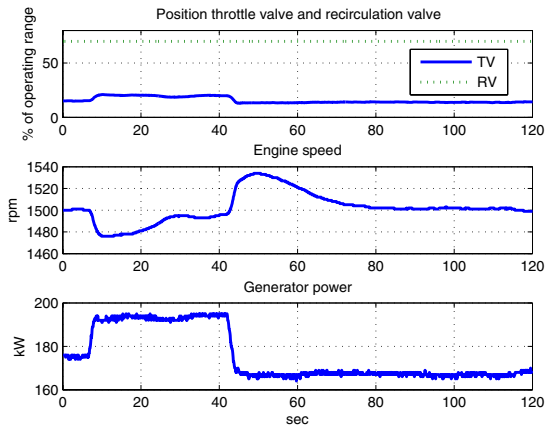


Fig. 6. Engine speed (middle) and throttle valve and recirculation valve positions (top) at generator power step (bottom), operating point 200kW. Experimental data of nonlinear engine.

At higher weights the throttle is too slow and the closed loop is oscillating.

However, as it can be seen the control performance would not be satisfactory for practical operation and does not reach the quality of performance of the PID controllers yet. But it must be mentioned that this is a first application of an LQG controller for a gas engine ever and therefore in practice still not fully developed.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

The engine speed is controlled by two actuators, the throttle valve and the recirculation valve. Simulations show high potential for compensators which use both actuators simultaneously. The transient behavior of the closed loop is faster and shows smaller overshoot than independent PID compensators switched by state flow design.

The stationary positions of the actuators are important to assure the minimal gas concentration and less interactions with other control loops like NO_x control. At higher power output the throttle valve remains open to maximize the volumetric efficiency of the engine.

Simulations show that the stationary positions are similar when using PID control. The experimental results show the potential of the LQG compensator.

B. Future Works

The weighting factors of the LQG design are constant over the frequency range. If frequency dependent weighting factors are used together with the quadratic cost function, H_2 compensators with higher robustness can be obtained. Furthermore, the use of other cost functions, like the maximum peak of the singular values, leads to H_∞ control. It is also considered to use the gas concentration as third control variable for all those engine speed controller design procedures.

VII. ACKNOWLEDGEMENTS

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