EXPERIMENTAL CONTROL OF A FLEXIBLE BEAM USING A STACK-BENDING ACTUATOR PRINCIPLE

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Abstract: Active vibration damping is a well known application to efficiently damp the vibrations of mechanic machines, vehicles and other flexible structures. For the design of an appropriate control system often piezo electric ceramics are used. Actuator patches are commonly used for the active damping of thin structures such as beams and plates. Stack actuators are mostly implemented in truss structures as active members. When the structure to be controlled has considerable stiffness or the actuator cannot be integrated in the structure itself a different kind of actuator type becomes necessary. In this paper an actuator/console concept is presented that combines the advantages of patch and stack actuators, thus leading to a concept that generates a pair of bending moments with considerable amplitudes. Finally, the proposed concept is shown to be capable of effectively reducing flexible vibrations of a simple laboratory beam experiment.

Keywords: actuator/console mounting, active vibration damping, piezo stack actuator, piezo patch, flexible structure identification, laboratory experiment;

1. INTRODUCTION

Active vibration damping has been a region of extensive research during the past decades. Therefore, the design of control systems capable of effectively damping oscillations of mechanical systems nowadays is a standard control application. The goal of any control design method always is to increase the damping of the flexible modes of some structure.

Earlier research interests were focused on the design of active vibration damping systems for large flexible space structures (e.g. (Balas, 1982)). In this case the structure for which the damping has to be increased often is some kind of truss structure for which the accurate positioning of some point of the structure is to be achieved. For such vibration damping problems different approaches to increase the damping of the flexible modes were thoroughly investigated. Often collocated control is used, which requires that the sensor and the actuator are placed at the same location (see (Song *et al.*, 1998; Wang, 2003)). The reason for the high popularity of collocated control schemes comes from the fact, that the resulting transfer functions from sensor to actuator have minimum phase. Special collocated control concepts like Positive Position Feedback (PPF) and Direct Velocity Feedback (DVF) have guaranteed stability properties under neglected actuator dynamics (see (Preumont, 1999)) and benefit from the fact that no model of the structure is needed for controller design. On the other hand one can achieve a stronger vibration reduction with the help of a non-collocated control scheme. This is because in the case when the sensor and the actuator are separated from each other the response at the location of the sensor has to be controlled through the flexible structure.

More recent control experiments utilize piezo electric elements as sensors and actuators, respectively. As sensors normally a piezo patch is bonded to the structure producing a voltage signal that is proportional to the average curvature. In comparison to position, velocity and acceleration measurement this may be advantageous when only the vibrations of the elastic modes have to be measured and the signal part of the rigid body modes has to eliminated. As actuator piezo electric materials are used as patches as well as stacks. Again, the patches are bonded to the structure and driven by an electrical voltage. This kind of actuator concept produces a pair of moments as control input and is limited in the size of the moment. Stack actuators are normally used as active members implemented in a truss structure attached to the structure by some kind of joint. They produce forces normally acting in line with the middle axis of the local structure . Both concepts are shown schematically in figure 1.

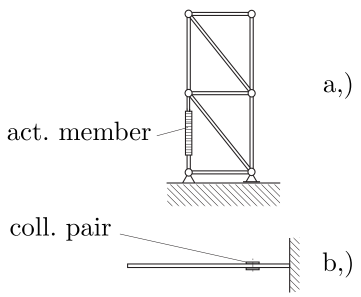

Fig. 1. Examples of active control problems

For the active control of large flexible structures the forces or moments necessary are usually much larger than those generated by patch actuators. On the other hand the implementation of stacks via joints is not always possible, especially when the structure has to transmit large bending moments. Then the weakening of some kind of side member may not be possible for safety reasons. For that situation a concept using an actuator mounted in a console which itself is bonded or welded to the flexible structure is of advantage. With the help of such an actuator/console concept large control forces can be generated without weakening the mainly supporting cross section and without introducing joints in the side member that transmits the main part of the load.

The remainder of the paper is organized as follows. First the laboratory experiment and the actuator/console concept are presented to be followed by a description of the identification procedure and identification results. Then the controller design is discussed and some experimental results are given to show the capability of the proposed actuator/console concept. Finally, the results are summarized and some outlook on future aspects are given.

2. LABORATORY EXPERIMENT

In figure 2 the laboratory experiment is shown schematically. The flexible structure considered is a beam clamped at one side with the geometrical data listed in table 1.

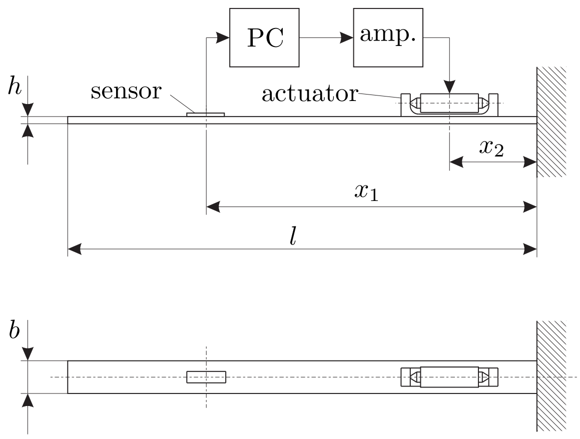

Fig. 2. Experimental set-up

Table 1. Beam Geometry.

Dimension	Value [mm]
Beam length l	450
Beam width b	40
Beam height h	3
Distance: sensor - clamped end x_1	281
Distance: actuator - clamped end x_2	62

As a measurement device a piezoelectric patch is bonded to the beam at the distance x from it's clamped beam and the actuator is a piezoelectric stack. The sensor produces a voltage that is proportional to the average curvature of the beam at the place of the sensor. This signal is fed to a laboratory PC, where, according to an appropriate control algorithm, a control signal is generated. Then the control signal is fed to a high voltage amplifier which drives the actuator. The mentioned control algorithm is thereby implemented in MATLAB/Simulink using the Realtime Workshop Tool.

Figure 3 shows the working principle of the piezoelectric actuator. The stack is mounted in a console and the console is bonded to the beam.

At the actuator ends tips are mounted, that fit into conical sinkings in the console. This special kind of mounting ensures, that only forces pointing in the direction of the actuator middle axes

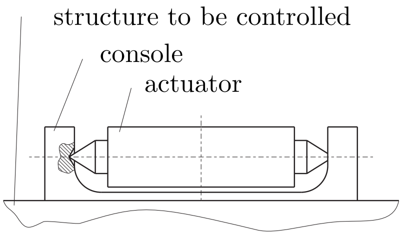

Fig. 3. Actuator-console principle

are acting on the actuator. During the mounting the clearance between actuator and console can be eliminated by an adjusting screw that is not displayed in figure 3.

Finally, the actuator has to work for positive as well as negative curvatures and therefore voltages generated by the sensor, since the piezoelectric actuator cannot directly produce tensile forces. This is achieved by preloading the actuator by an offset voltage. Any voltage smaller than the offset voltage causes in the tensile range stress and any voltage larger than the medium operating voltage works in the pressure region.

The concept presented is advantageous for all kind of structures that need a larger amount of strength than generated by thin piezo patches. It also has advantages when the preloading of the actuator is not done by weight of the structure itself or when introducing joints in the structure is not feasible.

3. TRANSFER FUNCTION IDENTIFICATION

In order to design a controller for the flexible structure experiment a model of the system is needed. Such a model can be generated by physically modeling the system which was done in (Halim and Moheimani, 2001). Another possibility is to identify a model of the system from input/output data. In this paper a structural system identification is used to obtain a mathematical model of the beam experiment.

The identification algorithm used is based on the estimation of Markov parameters, from which a state space model of the system can be estimated. A description of the Markov parameter approach can be found in (Gawronski, 2004). The matrices estimated are the system matrix $\boldsymbol{A}$, the input matrix $\boldsymbol{B}$, the output matrix $\boldsymbol{C}$ and the feedthrough matrix $\boldsymbol{D}$, where in the case of a SISO-system $\boldsymbol{D}$ is a scalar d. The state space system with state vector $\boldsymbol{x}$ given by these matrices consists of the equations

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \tag{1}$$

$$y = \boldsymbol{C}\boldsymbol{x} + du \tag{2}$$

where (1) is the state equation and (2) is the output equation. A transfer function representation $G(s)$ of (1) and (2) is given by

$$G(s) = d + \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}. \tag{3}$$

In order to obtain input/output data for the identification procedure a PRBS-signal was applied to the input of the actuator and the response at the sensor was measured. Part of these data were used in the identification procedure that yielded a transfer function G of seventeenth order from actuator voltage u to sensor voltage y which is displayed in figure 4.

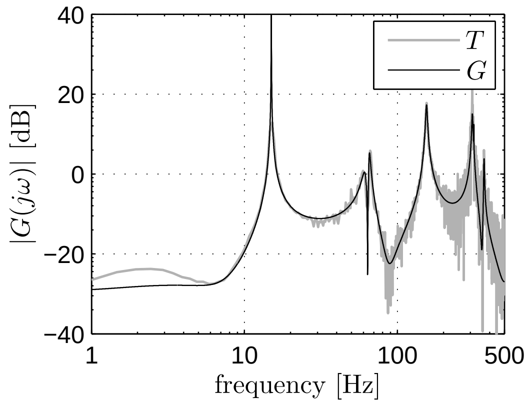

Fig. 4. Identified transfer function G from actuator to sensor

Additionally the non-parametric transfer function estimate T is shown, where T is

$$T = \frac{P_{uy}}{P_{uu}} \tag{4}$$

the quotient of the cross power spectral density of u and y, P_{uy} and the power spectral density of u, P_{uu} (see (Hayes, 1996)). The comparison of G and T indicates that G is a good model of the behavior of the flexible beam. This can also be seen from figure 5 where the model is cross-validated with the second part of the experimental data in time domain.

The responses of the actual system and the obtained model indicate that, besides slight deviations, the model is indeed a good representation of the flexible beam suited for controller design. Especially the response of the first mode can be observed from y which is the most dominant eigenfrequency of the beam response. This also can be seen from the large peak at approximately 15Hz in figure 4. With the help of the mathematical model G of the structure a controller can be designed.

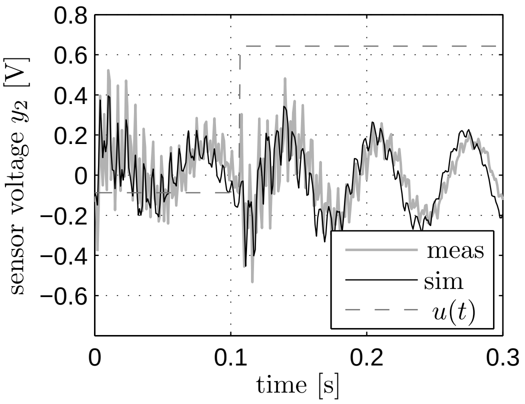

Fig. 5. Validation: measured and simulated sensor signal y

4. CONTROLLER DESIGN

For the controller design the structure of the control system is assumed to be of the form displayed in figure 6.

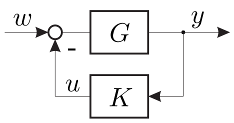

Fig. 6. System with input disturbance $w(t)$, control signal $u(t)$ and measurement $y(t)$

The flexible beam is assumed to be excited by some disturbance $w(t)$ entering the system over the same channel as the control signal $u(t)$. The control signal is generated by filtering the measurement signal $y(t)$ with the controller $K(s)$, where $K(s)$ is designed using the LQR-approach (Linear Quadratic Regulator, see e.g. (Franklin *et al.*, 1990)).
The LQR-controller is an optimal controller that is optimal in the sense that it minimizes the following cost function

$$J = \int_0^\infty (\boldsymbol{x}(t)^T \boldsymbol{Q} \boldsymbol{x}(t) + R u^2(t)) dt. \tag{5}$$

In (5) the weighting matrices $\boldsymbol{Q}$ and R are the design parameters to be chosen and $\boldsymbol{x}$ is the state vector of the system defined by (1) and (2). The goal of the minimization of (5) yields the following state vector feedback control law

$$u = -\boldsymbol{H}\boldsymbol{x}, \tag{6}$$

where the optimal feedback vector $\boldsymbol{H}$

$$\boldsymbol{H} = R^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{7}$$

includes the solution $\boldsymbol{P}$ of the Matrix-Ricatti equation

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}R^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0}. \tag{8}$$

(8) is often referred to as Controller Algebraic Ricatti Equation (CARE, see (Gawronski, 2004)). Since the actual state vector $\boldsymbol{x}$ is not directly measured, the control law (9) has to be replaced by

$$u = -\boldsymbol{H}\hat{\boldsymbol{x}}, \tag{9}$$

where $\hat{\boldsymbol{x}}$ is an estimate of the current state vector. This estimate is generated by a state observer with the (optimal) observer feedback matrix

$$\boldsymbol{L} = \boldsymbol{S}\boldsymbol{C}^T R^{-1}, \tag{10}$$

that contains the solution of the FARE (Filter Algebraic Ricatti Equation)

$$\boldsymbol{A}\boldsymbol{S} + \boldsymbol{S}\boldsymbol{A}^T - \boldsymbol{S}\boldsymbol{C}^T R^{-1}\boldsymbol{C}\boldsymbol{S} + \boldsymbol{Q}. \tag{11}$$

In (8) and (11) $\boldsymbol{Q}$ and R are not necessarily the same.

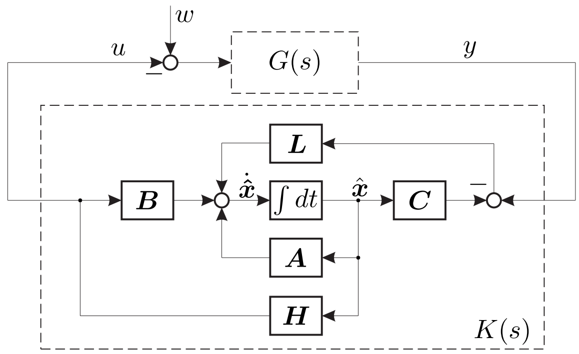

Fig. 7. Structure of the controller $K(s)$

The controller $K(s)$, which is comprised by state vector feedback and observer, is displayed in figure 7. To generate the state estimate $\hat{\boldsymbol{x}}$ the observer includes a model of the system to be controlled.

For the design of state vector feedback and observer the model first must be transformed into modal coordinates. Such a transformation yields a system matrix $\boldsymbol{A}$ that is of block diagonal form

$$\boldsymbol{A} = \mathrm{diag}\{\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_\mathrm{n}\}. \tag{12}$$

If $\boldsymbol{A}_j$ corresponds to a real pole $\boldsymbol{A}_j$ is a scalar, otherwise it corresponds to a complex pair of poles and is of the form

$$\boldsymbol{A}_j = \begin{bmatrix} 0 & 1 \\ -\omega_j^2 & -2\zeta_j\omega_j \end{bmatrix}, \tag{13}$$

where ζ_j and ω_j are the (modal) damping coefficient and the eigenfrequency of the j-th mode. Finally the system matrix $\boldsymbol{A}$ was sorted, such that column 1 to 14 contained the first 7 elastic modes of the flexible beam and the last three columns contained three real poles coming from the model of the sensor. The weighting matrices $\boldsymbol{Q}$ and R where chosen to be

$$\boldsymbol{Q} = \operatorname{diag}\{0.4,\ 0.01,\ 0.01,\ \ldots,\ 0.01\} \tag{14}$$

and

$$R = 1. \tag{15}$$

Closing the loop from the measurement $y(t)$ to the control signal $u(t)$ via the controller $K(s)$ the closed loop transfer function from the disturbance $w(t)$ to the measurement $y(t)$ is obtained by

$$G_c = \frac{y}{w} = \frac{G}{1+GK}. \tag{16}$$

The bode diagram of the closed loop transfer function $G_c(s)$ in comparison to the open loop transfer function $G(s)$ is shown in figure 8.

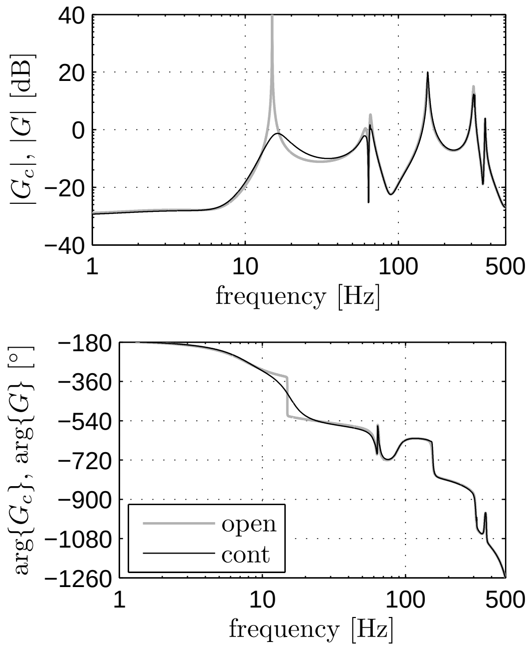

Fig. 8. Bode diagram: closed vs. open loop

Since only the first modal coordinate is assigned a larger weight only the damping of the first elastic mode of the flexible beam is considerably increased. All other elastic modes are almost unchanged by the control action.

5. EXPERIMENTAL RESULTS

The designed controller is implemented in Matlab/SIMULINK and operated in the Realtime Workshop environment. Measurements are obtained with a sampling frequency of 1kHz and then fed to the controller.

With the help of the experiment the damping properties of the closed loop system are investigated. For that reason different disturbance signals $w(t)$ are fed to the actuator. In figure 9 the closed loop disturbance step response is displayed. Since the disturbance enters through the same channel as the control signal first only the disturbance is fed to the actuator (0.5s) and 0.1s later the control is switched on (compare figure 10).

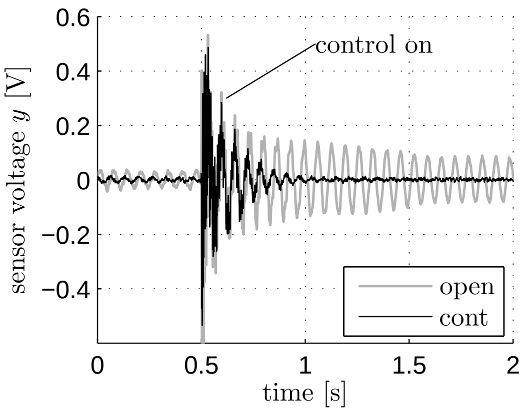

Fig. 9. Step response: sensor signal y

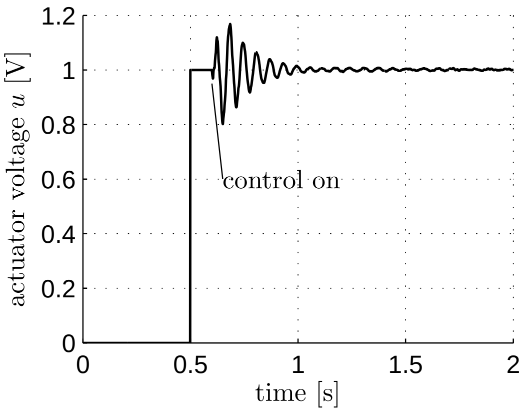

Fig. 10. Step response: actuator input signal (u and w)

It is observed, that in the closed loop case the sensor signal dies out much faster than in the open loop case. This proves, in accordance to figure 8, that the damping of the first mode has been increased significantly.

The second disturbance signal fed to the actuator is a white noise signal passed through a low-pass filter

$$G_f(s) = \frac{100}{s+100}. \tag{17}$$

In contrast to the step disturbance continuous excitation occurs. Figure 11 again shows the closed loop and the open loop response in form of the sensor signal $y(t)$. It is observed, some time is needed to store vibration energy in the first mode.

After approximately 0.7s the response is mainly comprised by the first mode and in the closed loop case that mode is significantly damped due to the control action.

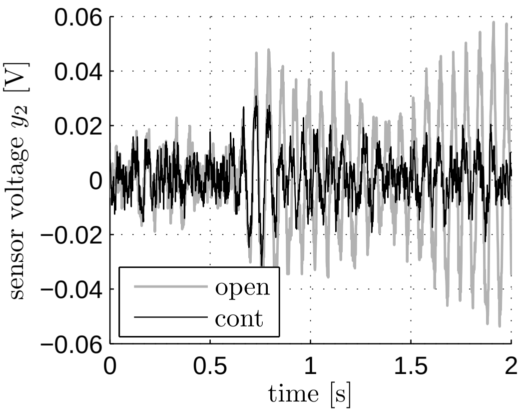

Fig. 11. Noise response: sensor signal y

In the case of the noise disturbance control the control and the disturbance signal are both switched on at $t = 0s$. Therefore, figure 12 shows the combined input signal consisting of $u(t)$ and $w(t)$.

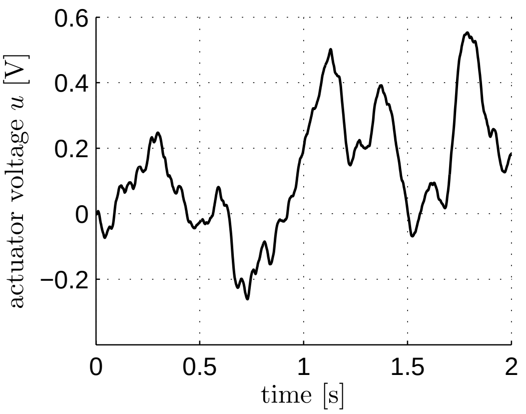

Fig. 12. Noise response: actuator input signal (u and w)

6. CONCLUSION

A new actuator/console concept for the implementation of an active control concept has been tested with the help of a simple laboratory experiment. For this purpose a state space model of the system under consideration has been identified using standard Markov parameter identification techniques. With the help of this model a state vector feedback controller has been designed. Both the state vector feedback as well as the observer have been obtained using LQG-techniques. The obtained controller was then implemented in the Matlab/SIMULINK Realtime Toolbox environment to close the loop from measurement $y(t)$ to control signal $u(t)$. With the help of the so defined closed loop the controller was tested for step and noise disturbances. In both cases strongly reduced vibration amplitudes were observed.

The approach considered in this paper is of advantage if the forces generated by a simple piezo electric patch are to small to actuate a structure with high stiffness. In this case large forces generated by stack actuators are necessary. Additionally, the proposed console concept enables the use of active members that need not be directly integrated into the structure itself.
The results obtained with the flexible beam experiment show that strong vibration reduction can be achieved with the proposed concept.

In the future the presented concept will be implemented on a larger flexible structure to test its capability on a structure with considerably larger stiffness than the beam. Further topics of research include the optimal placement of actuators and sensors for the proposed concept and the applicability of different collocated and non-collocated control concepts.

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