Quantum Scholasticism: On Quantum Contexts, Counterfactuals, and the Absurdities of Quantum Omniscience

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Contents

Main issues

- Counterfactuals and context–dependence
- Alternatives to contextuality

Quasi-experimental status

- Einstein–Podolsky–Rosen (EPR): "Explosion view" of two contexts in a singlet state
- Boole's "conditions of possible experience" & the associated Bell-type inequalities, such as Clauser–Horne–Shimony–Holt (CHSH): bounds on correlation functions from convexity conditions of classical probabilities.
- Kochen–Specker (KS) theorem: finite proof by contradiction that on quantum logics ($\dim > 3$) there does not exist any two-valued state associated truth assignments on propositions about a quantized system.
- Greenberger–Horne–Zeilinger (GHZ) theorem: multipartite argument yielding a complete contradiction between classical and quantum predictions.
- Now what?

Quantum context

- A context is a maximal collection of co-measurable observables associated with commuting operators.
- Every context can also be characterized by a single (but nonunique) maximal operator. All operators within a context are functions thereof.
- In quantum logic, contexts are represented by Boolean subalgebras or blocks pasted together to form the Hilbert lattice. (For the sake of nontriviality, Hilbert spaces of dimension higher than two are considered.)
- Heuristically, a context represents a "classical mini–universe," which is distributive and allows for as many two–valued states — interpretable as classical truth assignments — as there are atoms.

Features of quasi–experimental setup

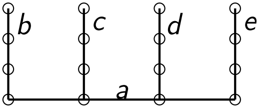
- the proofs require the assumption of counterfactuals; i.e., of "potential" observables which, due to quantum complementarity, are incompatible with the "actual" measurement context; yet could have been measured if the measurement apparatus were different. These counterfactuals are organized into groups of interconnected contexts which, due to quantum complementarity, are incompatible and therefore cannot be measured simultaneously; not even in Einstein-Podolsky-Rosen (EPR) type setups.
- The proofs by contradiction have no direct experimental realizations. As has already been pointedly stated by Robert Clifton, "how can you measure a contradiction?"
- So–called "experimental tests" inspired by Bell-type inequalities, KS as well as GHZ measure the incompatible contexts which are considered in the proofs one after another; i.e., temporally sequentially, and not simultaneously. Hence, different contexts can only be measured on different particles.

Principle of explosion

- The *principle of explosion*: *“ex falso quodlibet,”* or *“contradictione sequitur quodlibet”* amounts to “anything follows from a contradiction.”
- Due to the pasting construction of Hilbert lattices, the principle of explosion holds also in quantum logic.

Star configurations

For the n-dimensional Hilbert space, an n-star configuration represents n different contexts joined in n different atoms of the center context.



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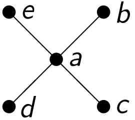
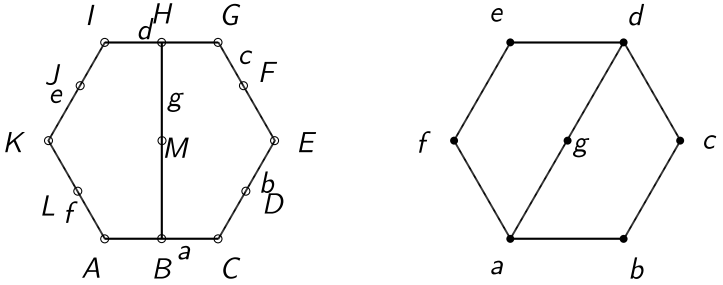


b)

Four-star configuration in four-dimensional Hilbert space a) Greechie diagram representing atoms by points, and contexts by maximal smooth, unbroken curves. b) Dual Tkadlec diagram representing contexts by filled points, and interconnected contexts by lines.

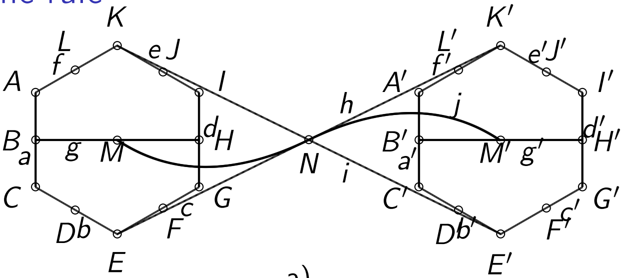
The one–zero rule



a) b)

Configuration of observables in three-dimensional Hilbert space implying that whenever E is true, E must be false. The seven interconnected contexts $a=\{A,B,C\}$, $b=\{C,D,E\}$, $c=\{E,F,G\}$, $d=\{G,H,I\}$, $e=\{I,J,K\}$, $f=\{K,L,A\}$, $g=\{B,H,M\}$, consist of the 13 projectors associated with the one dimensional subspaces spanned by $A=(1,\sqrt{2},-1)$, $B=(1,0,1)$, $C=(-1,\sqrt{2},1)$, $D=(-1,\sqrt{2},-3)$, $E=(\sqrt{2},1,0)$, $F=(1,-\sqrt{2},-3)$, $G=(-1,\sqrt{2},-1)$, $H=(1,0,-1)$, $I=(1,\sqrt{2},1)$, $J=(1,\sqrt{2},-3)$, $K=(\sqrt{2},-1,0)$, $L=(1,\sqrt{2},3)$, $M=(0,1,0)$.

The one–one rule



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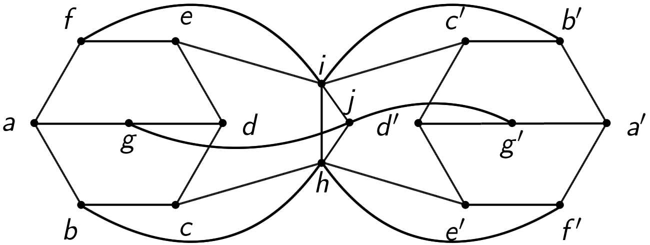
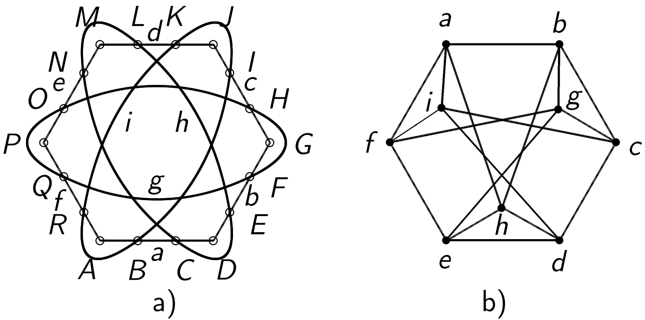


b)

Kochen-Specker Constructions



Proof of the Kochen-Specker theorem by *Cabello et al.* in four-dimensional real vector space. The nine tightly interconnected contexts *a*–*i* consist of the 18 projectors associated with the one dimensional subspaces spanned by $A = (0, 0, 1, -1)$, …, $R = (0, 0, 1, 1)$. b) Tkadlec diagram of the nine contexts interlinked in a four-star form; hence every observable proposition occurs in exactly two contexts. Enumeration of the four observable propositions of each of the nine contexts: there appears to be an *even* number of true propositions; yet the number of contexts — with a single true proposition each — is *odd*.

Contextuality and its alternatives

(i) abandonment of classical omniscience: it is wrong to assume that all observables which could in principle ("potentially") have been measured also co–exist, irrespective of whether or not they have or even could have been actually measured. Realism might still be assumed for a *single* context, in particular the one in which the system was prepared;

(ii) abandonment of realism: it is wrong to assume that physical entities exist even without being experienced by any finite mind. Quite literary, with this assumption, the proofs of KS and similar decay into thin air because there are no counterfactuals or unobserved physical observables or inferred (rather than measured) elements of physical reality.

(iii) contextuality; i.e., the abandonment of context independence of measurement outcomes: it is wrong to assume that the result of an observation is independent not only of the state of the system but also of the complete disposition of the apparatus — possible test in the $\{A, B, C\} - \{C, D, E\}$ system of observables with an explosion type EPR setup & a singlet state of two spin–one particles.

Thank you for your attention!