A comparison of measurement techniques to determine the acoustic impedance at oblique sound incidence angle

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Introduction

For obtaining the acoustic field, the acoustic wave equation has to be solved, which reads in the frequency domain (Helmholtz equation) as

$$k^2 p' + \Delta p' = 0 \; . \tag{1}$$

In (1) $k = \omega/c$ denotes the wave number, ω the angular frequency, c the speed of sound of the fluid and p' the acoustic pressure. The acoustic impedance in frequency domain is defined by

$$Z_\mathrm{a} = \frac{p'}{\vec{v'} \cdot \vec{n}} \; . \tag{2}$$

In (2) $\vec{v'}$ is the acoustic particle velocity and $\vec{n}$ is the vector normal to a surface Γ surrounding a domain Ω.

The weak formulation of (1) by using the conservation of momentum and (2) results in

$$\int_\Omega \varphi k^2 p' \mathrm{d}\Omega - \int_\Omega \nabla\varphi \cdot \nabla p' \mathrm{d}\Omega + \int_\Gamma \frac{\varphi \rho j \omega p'}{Z_\mathrm{a}} \mathrm{d}\Gamma = 0 \; . \tag{3}$$

In (3) φ is an arbitrary test function and ρ the density of the fluid. In consequence, to calculate the acoustic field and thus, to solve the acoustic wave equation, the impedance Z_a on the surface Γ has to be known. Hence, a measurement technique to determine the acoustic impedance is of great importance, because a realistic sound field can only be calculated, if the real acoustic impedance at oblique angle of sound incidence is known.

The acoustic quantity determined in the methods, which will be presented in this contribution, are mostly the complex sound pressure reflection coefficient

$$r = \frac{p'_\mathrm{re}}{p'_\mathrm{in}} \; , \tag{4}$$

the ratio of the amplitude of the reflected wave p'_re and the amplitude of the incoming wave p'_in.

The acoustic impedance of a plane wave at the surface computes to

$$Z_\mathrm{a} = \rho c \frac{1+r}{1-r} \; . \tag{5}$$

Methods to determine the acoustic impedance

For the measurement of acoustic material properties as normal incidence absorption coefficient or normal specific acoustic impedance, well established procedures are available. There are standardized laboratory techniques for measuring these quantities. The traditional method to determine the sound absorption of a material is the standing wave tube (Kundt's tube)[1][2], where the acoustic quantity can be determined with a plane wave assumption in a rigid tube. Another laboratory method [3] can be used to calculate the acoustic absorption coefficient of a sample at random incidence. The measurement takes place in a reverberation room and a diffuse sound field is used. Via the measurement of the reverberation time, the absorption coefficient can be obtained by Sabine's reverberation equation.

The measurement of the acoustic quantities of the sample in such procedures can only be performed in the laboratory and not under real conditions. No real sound excitation will be a perfect plane wave or a diffuse sound field. Furthermore, the Kundt's tube method needs a trimmed sample. However, the cutting process changes the acoustic behavior. And last, the measurements cannot be done in situ. This advantage is the most significant, because every material changes its acoustic behavior during installation.

Several in situ approaches can be found in literature and there are a lot of papers, which give an overview of current techniques for a determination of the acoustic properties. Besides the methods discussed in this paper, a variety of other approaches exist, e.g. a reflection method. In this method the reflected wave is isolated by a subtraction technique [4].

This paper gives an overview of techniques, which allow for a determination of the acoustic quantities at oblique sound incidence angles.

Tamura method

In [5] and [6] a method is presented, which uses a spatial Fourier transformation, to determine the pressure reflection coefficient at oblique sound incidence. The outline

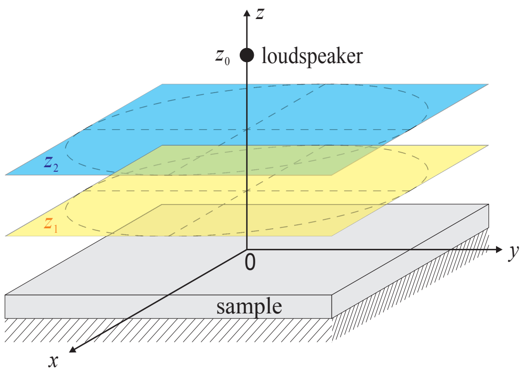

Figure 1: Outline of the measurement principle in the Tamura method. The Loudspeaker placed at z_0 and the two measurement planes at z_1 and z_2 are shown.

of the measurement principle can be seen in Fig. 1. A sound source, radiating spherical waves into a three dimensional space, is placed at z_0 above a sample. The surface of the sample coincides with the xy plane. In this method the pressure distribution on two planes (z_1, z_2) has to be measured. This can either be realized by one microphone on a scanner or with a microphone array [7]. In [6] a number of 200 measurement points is reported.

By two-dimensional Fourier transform

$$P(k_x, k_y, z) = \int\int p' e^{-j(k_x x + k_y y)} \mathrm{d}x\mathrm{d}y \ , \qquad (6)$$

the complex pressure distribution is decomposed into plane-wave components. In case of a pressure amplitude of a wave, which travels from the plane z_2 to z_1, the pressure at z_1 can be calculated by [5]

$$P(k_x, k_y, z_1) = P(k_x, k_y, z_2) e^{jk_z(z_1 - z_2)} \ , \qquad (7)$$

where $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$ is the wave number in z-direction. The pressure reflection coefficient can be computed by separating the incident and reflected plane wave component

$$r = \frac{P(k_x, k_y, z_2) e^{-jk_z z_1} - P(k_x, k_y, z_1) e^{-jk_z z_2}}{P(k_x, k_y, z_1) e^{jk_z z_2} - P(k_x, k_y, z_2) e^{jk_z z_1}} \qquad (8)$$

at the angle of sound incidence

$$\Theta = \sin^{-1}\left[\sqrt{k_x^2 + k_y^2}/k_0\right] \ . \qquad (9)$$

This measurement method is a very time-consuming (in case of a scanner with one microphone) or a costly (in case of a microphone array) technique. Moreover, there will be errors in calculating the pressure reflection coefficient, because the spatial resolution in measuring the pressure distribution is not infinite and the measurement planes cannot be infinitely wide.

Two microphone method

In [8] a measurement technique is presented, which determines the normal acoustic impedance at oblique angles of sound incidence with two microphones in a free field with no reflections. An outline of the measurement setup is shown in Fig. 2. Two microphones M1 and M2 with the distances d_1 and d_2 are placed above a sample. The sample is located on a rigid surface and the sound incidence at angle Θ is realized by means of a loudspeaker with distance r_0 to the surface of the sample.

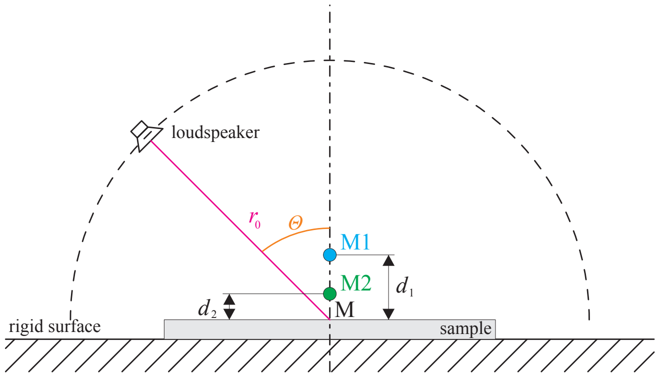

Figure 2: Outline of the measurement principle in the two microphone method. The loudspeaker is placed at angle Θ at distance r_0. M1 and M2 with a distances d_1 and d_2 are placed above a sample.

The pressure reflection coefficient at M (surface of the sample) for a plane wave computes to

$$r_p = \frac{e^{-jkd_2\cos\Theta} - \frac{p_2}{p_1} e^{-jkd_1\cos\Theta}}{\frac{p_2}{p_1} e^{jkd_1\cos\Theta} - e^{jkd_2\cos\Theta}} \ . \qquad (10)$$

As shown in [8], a small distance between loudspeaker and sample provides insufficient results, because the wave at the sample is not plane. Therefore, a spherical-wave approximation is supposed in [8] and the spherical pressure reflection coefficient calculates as

$$r_s = \frac{\frac{1}{r_2} e^{jkr_2} - \frac{p_2}{p_1}\frac{1}{r_1} e^{jkr_1}}{\frac{p_2}{p_1}\frac{1}{r_1'} e^{jkr_1'} - \frac{1}{r_2'} e^{jkd_2}} \ . \qquad (11)$$

In (11) r_1 and r_2 are the distances between loudspeaker and M1 and M2 and r_1' and r_2' are the distances between the mirror source (reflected loudspeaker at the rigid surface) and M1 and M2. The acoustic impedance at a sound incidence of angle Θ computes to

$$Z_{\mathrm{a,s}} = \frac{1}{\cos\Theta}\frac{r_{\mathrm{s}} + 1}{r_{\mathrm{s}} - 1}\frac{jkr_0}{jkr_0 - 1} \ . \qquad (12)$$

Measured impedance of a glass fiber at 2 kHz and 3 kHz in comparison to the impedance obtained by a model are shown in [9]. The distance between loudspeaker and sample r_0 was about 4 m, whereas $d_1 = 1.5$ cm and $d_2 = 0.5$ cm. The surface of the panel was about $1\,\mathrm{m}^2$. The results show good agreement at those two frequencies for angles of sound incidence up to 60°.

In situ method

An easy and quick method to determine the pressure reflection coefficient is proposed in [10]. This method is based on a simultaneous measurement of sound pressure and particle velocity in contrast to the previous described methods, where only the sound pressure is measured. In Fig. 3 the measurement principle with a pu-probe placed at distance $h_\mathrm{s} - h$ in front of a loudspeaker is shown. At first, the method with sound incidence of angle 0° is discussed and first measurement results are shown. The impedance at free radiation a) Z_ff and the impedance near the sample b) Z_meas has to be measured. With the assumption, that the sound source radiates spherical waves, the pressure reflection coefficient can be calculated by

$$r = \frac{\dfrac{Z_\mathrm{meas}}{Z_\mathrm{ff}} - 1}{\dfrac{Z_\mathrm{meas}}{Z_\mathrm{ff}} \left(\dfrac{h_\mathrm{s} - h}{h_\mathrm{s} + h}\right) \left(\dfrac{jk(h_\mathrm{s} + h) + 1}{jk(h_\mathrm{s} - h) + 1}\right) + 1} \frac{h_\mathrm{s} + h}{h_\mathrm{s} - h} e^{jk2h} \tag{13}$$

with h, the distance between pu-probe and sample.

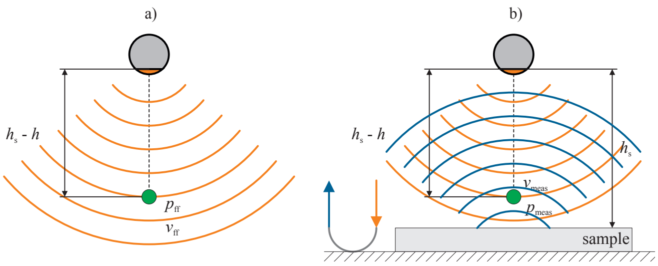

Figure 3: Outline of the measurement principle in the in situ method. a) Freefield measurement b) Measurement of the impedance at distance h above a sample.

The pu-probe we used, is the Microflown Ultimate Sound Probe (USP), capable of measuring the acoustic pressure and the particle velocity in 3D. We calibrated the USP by means of our simultaneous calibration technique [11] and identified the orientation of the USP by using the ACCMF-system [12]. The ACCMF-system is a combination of a 3D acceleration sensor and the USP in order to know the direction of the particle velocity sensor by means of the local gravity field.

Figure 4 displays the obtained sound reflection coefficient of a lacquered wood sample. It can be seen, that the sound reflection coefficients obtained with (13) match in

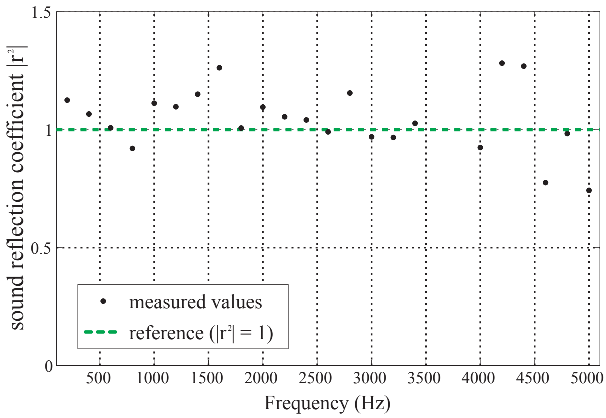

Figure 4: Measurement of the sound reflection coefficient at nearly rigid sample ($|r^2| \approx 1$) obtained by (13) with $h = 5$ mm, $h_\mathrm{s} = 40$ cm and a loudspeaker in a rigid spherical package.

some degree good to the reference. The sound reflection coefficient was assumed to be $|r^2| \approx 1$. The main deviations are at frequencies below 2 kHz and above 4 kHz. In the mid frequency range the obtained reflection coefficients make more sense.

Furthermore, the sound reflection coefficient of a foam sample was obtained by (13). The results can be seen in Fig. 5 in comparison to the sound reflection coefficient obtained by the Brüel & Kjaer 4206-T Transmission Loss Tube referred to the transfer matrix method [13]. There are deviations from the reference at frequencies below 2 kHz, but above 2 kHz, the measured values match with the reference.

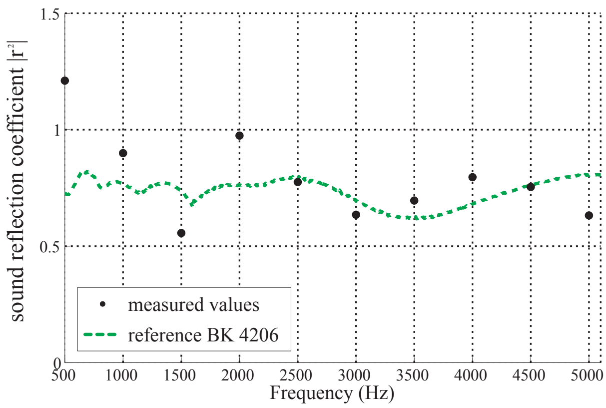

Figure 5: Measurement of the sound reflection coefficient at a foam sample obtained by (13) in comparison to the reference measured with a Brüel & Kjaer 4206-T.

The in situ method for determining the reflection coefficient at 0° sound incidence can be extended to measure the pressure reflection coefficient at oblique angle of sound incidence Θ

$$r(\Theta) = e^{jk(r_1 - r_1')} \frac{r_1'}{r_1} \frac{Z_\mathrm{a}\left(\dfrac{jkr_1 - 1}{jkr_1}\right)\cos\Theta - \rho c}{Z_\mathrm{a}\left(\dfrac{jkr_1' - 1}{jkr_1'}\right)\cos\Theta' + \rho c}, \tag{14}$$

where Z_{a} is the measured impedance perpendicular to the surface of the sample. Further parameters in (14) can be found in Fig. 6, where the outline of the in situ method at sound incidence at angle Θ is shown. Microflown recommends, that the 1D pu-probe should be rotated to the desired angle of sound incidence and the reflection coefficient can be calculated with (14) by determining the acoustic impedance at height h. In that case, the pu-probe has to be adjusted perpendicular to the surface of the sample.

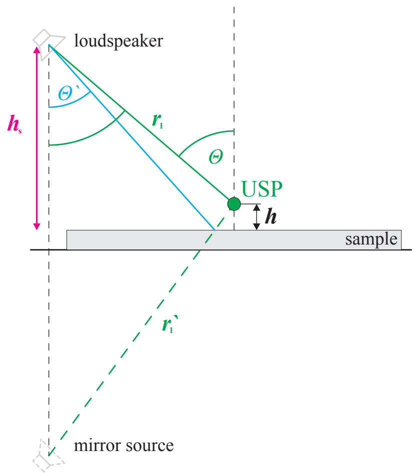

Figure 6: Outline of the measurement principle of the in situ method with an oblique angle of sound incidence.

There are advantages using the ACCMF instead of a 1D pu-probe. First, there is no need to rotate the pu-probe to a desired angle, because the USP is able to measure the particle velocity in 3D. Second, the direction of the USP can easily be found by means of the 3D acceleration sensor in the ACCMF. This arrangement allows us to perform a scan of any angle by using a rotating table and measure the reflection coefficient at any desired angle without a modification of the loudspeaker-ACCMF arrangement.

Conclusion

In this paper we gave an overview of existing methods to obtain the acoustic parameters such as reflection coefficient in situ and at oblique angles of sound incidence. We showed first measurement results using the in situ method with our USP to obtain the sound reflection coefficient at sound incidence of 0° and proposed a new method to make the determination of the reflection coefficient at oblique sound incidence more convenient. In this new method, the USP is used instead of a 1D pu-probe, which allows us to scan a huge number of angles in an automated process. Furthermore, no modification of the measurement set-up is needed, which avoids measurement errors.

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